Mathematical Foundations of Linguistics – Linguistics Dpt., UCSC, Spring 2007
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The course introduces some of the mathematical notions and tools underlying contemporary work in theoretical linguistics. The recurring theme of the course – over and above basic notions of set theory (e.g. ordinals, cardinals and their arithmetic) and basic notions of universal algebra (e.g. lattices, Boolean algebras etc.) – will be the connection between the language of various logics (i.e. their syntax) and the models relative to which they are interpreted (i.e. their semantics): we will begin with the Tarskian definition of satisfaction and truth for classical first-order logic, we will then work our way through the syntax and semantics of other logics (e.g. three-valued, modal, tense and type logics) and we will end with the (sketch of the) proof of the completeness theorem for first-order logic (originally proved by Gödel; following standard practice, we will study Henkin's proof).

The linguistic applications will hopefully cover the three main sub-disciplines, i.e.

(1) phonology (and learnability), e.g. multi-valued logics and the logic of ranking arguments in Optimality Theory (OT), generating the typological space for a simple OT grammar with a small number of constraints and studying how certain learning algorithms behave with respect to this typological space,
(2) syntax, e.g. context-free grammars, categorial grammars, basic notions of model-theoretic syntax,
(3) semantics, e.g. generalized quantifiers, modal & tense logics, classical type logic and compositionality, Skolem functions and wide-scope indefinites, dynamic logics,

but, given the time constraints, a fairly drastic selection will most probably have to be made.

The friendship theorem. (Have fun proving this theorem over the Spring break ☻)
Suppose a party has six people. Consider any two of them; they might be meeting for the first time, in which case we will call them *mutual strangers*, or they might have met before, in which case we will call them *mutual acquaintances*. Prove that, in any party of six people, either at least three of them are pairwise mutual strangers or at least three of them are pairwise mutual acquaintances.

SELECTED BIBLIOGRAPHY – We will only read sections from (some of) the books & papers listed below. This course does **not** have a textbook.

Brasoveanu, A. and Prince, A.: 2005. Ranking and Necessity. Part I, ROA 794. (http://roa.rutgers.edu/view.php3?id=1094)
Burris, S. and Sankappanavar, H.P.: 1981. *A Course in Universal Algebra*, Springer. (http://www.math.uwaterloo.ca/~snburris/htdocs/UALG/univ-algebra.pdf)
Doets, K.: 1996. *Basic Model Theory*, CSLI. (http://citeseer.ist.psu.edu/rd/0%2C380140%2C1%2C0.25%2CDownload/http://citeseer.ist.psu.edu/cache/papers/cs/17548/ftp:zSzzSzftp.cwi.nlzSzpubzSzmdrzSzOLDzSzSiLLIzSzSiLLI-fivezSzbmt.pdf/doets95basic.pdf)
Halmos, P.R.: 1998. *Naive Set Theory*, Springer.
Lewis, H.R. and Papadimitriou, C.: 1981. *Elements of the Theory of Computation*, Prentice Hall.
Manzano, M.: 1999. *Model Theory*, Oxford University Press.
Manzano, M.: 2005. *Extensions of First Order Logic*, Cambridge University Press.
Pullum, G.K. and Scholz, B.C.: 2001. On the Distinction between Model-theoretic and Generative-enumerative Syntactic Frameworks. In P. de Groote, G. Morrill and C. Retoré, *Logical Aspects of Computational Linguistics: 4th International Conference*, Springer, 17–43. (http://people.ucsc.edu/~pullum/LeCroisic.pdf)
Rogers, J.: 1998. *A Descriptive Approach to Language-Theoretic Complexity*, CSLI/FoLLI. (http://citeseer.ist.psu.edu/rd/97678309%2C178324%2C1%2C0.25%2CDownload/http://citeseer.ist.psu.edu/cache/papers/cs/3039/http:zSzzSzlongwood.cs.ucf.eduzSz~jrogerszSzmono.pdf/james96descriptive.pdf)