Handout 8: Introduction to DPL+GQ

Semantics C (Spring 2010)

1. Dynamic Predicate Logic (DPL)

1. A^{x} house-elf fell in love with a$^{x'}$ witch.
2. He$_{x}$ bought her$_{x'}$ an$^{x''}$ alligator purse.

3. Every farmer who owns a^{x} donkey beats it$_{x}$.
4. Every house-elf who falls in love with a^{x} witch buys her$_{x}$ an$^{x'}$ alligator purse.
5. If a^{x} farmer owns a$^{x'}$ donkey, he$_{x}$ beats it$_{x'}$.
6. If a^{x} house-elf falls in love with a$^{x'}$ witch, he$_{x}$ buys her$_{x'}$ an$^{x''}$ alligator purse.

The particular version of dynamic semantics we look at is (based on) DPL (Groenendijk & Stokhof 1991) – and for three reasons:

- the syntax of the system is a fairly close variant of the familiar syntax of classical first-order logic; this enables us to focus on what is really new, namely the semantics;
- the semantics of DPL is minimally different from the standard Tarskian semantics for first-order logic: instead of interpreting a formula as a set of variable assignments (i.e., the set of variable assignments that satisfy the formula in the given model), we interpret it as a binary relation between assignments[1]; moreover, this minimal semantic modification encodes in a transparent way the core dynamic idea that meaning is not merely *truth-conditional content*, but *context change potential*;
- third, just as classical predicate logic can be straightforwardly generalized to static type logic, DPL can be easily generalized to a dynamic version of type logic, which is what Muskens' Compositional DRT is; and CDRT enables us to introduce compositionality at the sub-sentential/sub-clausal level in the tradition of Montague semantics.

[1] Alternatively, and in certain respects equivalently, we can think of the interpretation of a formula as a function taking as argument a set of assignments and returning another set of assignments – this is the view underlying FCS, for example. However, in both cases the update is defined pointwise – and a relational view of update reflects this more directly. There are other differences between FCS and DPL (e.g., using partial and total assignments respectively and disallowing vs. allowing reassignment) – see the dynamic cube in Krahmer (1998): 59 for an overview. In particular, the fact that DPL (and CDRT) allows reassignment will be an essential ingredient in accounting for the interaction between anaphora and generalized conjunction (see section **5** of Chapter 1 below). The "destructive reassignment" or "downdate problem" associated with reassignment can be solved using stacks / 'referent systems': see Nouwen (2003) for a recent discussion and Bittner (2006) for a set of 'stack' axioms for dynamic type logic.

Also, DPL is able to translate the donkey sentences in (3) through (6) above compositionally, with sentences / clauses as the building blocks (i.e., basically, as compositional as one can get in first-order logic).

Sentences (3) and (5) above are translated as shown in (7) and (8) below and, when interpreted dynamically, the translations capture the intuitively correct truth-conditions.

7. $\forall x(farmer(x) \wedge \exists y(donkey(y) \wedge own(x, y)) \rightarrow beat(x, y))$
8. $\exists x(farmer(x) \wedge \exists y(donkey(y) \wedge own(x, y))) \rightarrow beat(x, y)$

Consider (7) first:

- *every* is translated as universal quantification plus implication and the indefinite as existential quantification plus conjunction
- the *syntactic* scope of the existential quantification is 'local' (restricted to the antecedent of the implication), but it does *semantically* bind the occurrence of the variable y in the consequent.

Similarly, in (8):

- the conditional is translated as implication and the indefinites are translated as existentials plus conjunction, again with syntactically 'local' but semantically 'non-local' scope.

DPL has two crucial properties that enable it to provide compositional translations for donkey sentences – the equivalences in (9) and (10) below valid.

9. $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$[2]
10. $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$

So:

- indefinites can semantically bind outside their syntactic scope and indefinitely to the right
- in combination with the definition of dynamic implication, this allows them to scope out of the antecedent and universally bind in the consequent of the implication.

1.1. Definitions and Abbreviations

The 'official' definition of a well-formed formula (wff) of DPL is easily recoverable on the basis of the definition of the interpretation function $\|\cdot\|$ in (11) below – the syntax is therefore not provided.

11. **Dynamic Predicate Logic (DPL).** The definition of the DPL interpretation function $\|\phi\|_{DPL}^{M}$ relative to a standard first-order model $\boldsymbol{M}$=<$\boldsymbol{D^M}$, $\boldsymbol{I^M}$>, where $\boldsymbol{D}$ is the domain of entities and $\boldsymbol{I}$ is the interpretation function which assigns to each n-place relation 'R' a

[2] The symbol '$\Leftrightarrow$' should be interpreted as requiring the identity of the semantic value of two formulas.

subset of $\boldsymbol{D}^n$. For readability, I drop the subscript and superscript on $\|\cdot\|_{DPL}{}^{\boldsymbol{M}}$, $\boldsymbol{D}^{\boldsymbol{M}}$ and $\boldsymbol{I}^{\boldsymbol{M}}$. 'T' and 'F' stand for the two truth values.

For any pair of $\boldsymbol{M}$-variable assignments $\langle g, h\rangle$:

a. Atomic formulas ('lexical' relations and identity):

$\|R(x_1, \ldots, x_n)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and $\langle g(x_1), \ldots, g(x_n)\rangle \in \boldsymbol{I}(R)$

$\|x_1{=}x_2\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and $g(x_1){=}g(x_2)$

b. Connectives (dynamic conjunction and dynamic negation):

$\|\phi;\ \psi\|\langle g, h\rangle = \mathrm{T}$ iff there is a k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$ and $\|\psi\|\langle k, h\rangle = \mathrm{T}$

$\|{\sim}\phi\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$,

i.e., $\|{\sim}\phi\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and $g \notin \mathbf{Dom}(\|\phi\|)$,

where $\mathbf{Dom}(\|\phi\|) := \{g$: there is an h s.t. $\|\phi\|\langle g, h\rangle = \mathrm{T}\}$

c. Quantifiers (random assignment of value to variables):

$\|[x]\|\langle g, h\rangle = \mathrm{T}$ iff for any variable υ, if $\upsilon \neq x$ then $g(\upsilon){=}h(\upsilon)$

d. Truth: A formula ϕ is true with respect to an input assignment g iff there is an output assignment h s.t. $\|\phi\|\langle g, h\rangle = \mathrm{T}$, i.e., $g \in \mathbf{Dom}(\|\phi\|)$.

Given that variable assignments are functions from variables to entities, if two variable assignments assign identical values to all the variables, they are identical. Hence, based on definition (11c), the formula [ ] defines the 'diagonal' of the product $\boldsymbol{G} \times \boldsymbol{G}$, where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments, as shown in (12).

12. $\|[\ ]\| = \{\langle g, g\rangle: g \in \boldsymbol{G}\}$,
where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments.

We define the other sentential connectives and the quantifiers as in (13) below.

13. **a. Abbreviations – Connectives (anaphoric closure, disjunction and implication):**

$!\phi := {\sim}{\sim}\phi$,[3]

i.e., $\|!\phi\| = \{\langle g, h\rangle: g{=}h$ and $g \in \mathbf{Dom}(\|\phi\|)\}$[4]

$\phi \vee \psi := {\sim}({\sim}\phi;\ {\sim}\psi)$,

i.e., $\|\phi \vee \psi\| = \{\langle g, h\rangle: g{=}h$ and $g \in \mathbf{Dom}(\|\phi\|) \cup \mathbf{Dom}(\|\psi\|)\}$

$\phi \to \psi := {\sim}(\phi;\ {\sim}\psi)$,

i.e., $\|\phi \to \psi\| = \{\langle g, h\rangle: g{=}h$ and for any k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$,
there is an l s.t. $\|\psi\|\langle k, l\rangle = \mathrm{T}\}$[5],

i.e., $\|\phi \to \psi\| = \{\langle g, h\rangle: g{=}h$ and $(\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)\}$,

where $(\phi)^g := \{h: \|\phi\|\langle g, h\rangle = \mathrm{T}\}$

b. Abbreviations – quantifiers (existential, universal, multiple random assignment):

$\exists x(\phi) := [x];\ \phi$

$\forall x(\phi) := {\sim}([x];\ {\sim}\phi)$,

i.e., $[x] \to \phi$ or, equivalently, ${\sim}\exists x({\sim}\phi)$,

i.e., $\|\forall x(\phi)\| = \{\langle g, h\rangle: g{=}h$ and
for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|\langle k, l\rangle = \mathrm{T}\}$[6],

i.e., $\|\forall x(\phi)\| = \{\langle g, h\rangle: g{=}h$ and $([x])^g \subseteq \mathbf{Dom}(\|\phi\|)\}$

$[x_1, \ldots, x_n] := [x_1];\ \ldots;\ [x_n]$

Given the definitions of dynamic negation '~' and closure '!', the equivalence in (14) below holds; (14) is very useful in proving that many equivalences of interest hold in DPL (e.g., the one in (15) below). Two formulas are equivalent, symbolized as '⇔', iff they denote the same set of pairs of variable assignments.

14. ${\sim}(\phi;\ \psi) \Leftrightarrow {\sim}(\phi;\ !\psi)$[7]

The equivalence in (15) below exhibits the limited extent to which the existential and universal quantifiers are duals;[8] this will prove useful, for example, when we try to determine the DPL translation of the English determiner *no*.

15. ${\sim}\exists x(\phi) \Leftrightarrow \forall x({\sim}\phi)$[9]

The practice of setting up abbreviations as opposed to directly defining various connectives and quantifiers might seem cumbersome, but it is useful in three ways:

- by setting up explicit abbreviations, we see exactly which component of the basic dynamic system does the work, e.g., we see that the universal 'effect' of universal quantification

[3] I use the symbol '!' for closure, as in van den Berg (1996b) and unlike Groenendijk & Stokhof (1991), who use '◊'.

[4] The connective '!' is labeled 'anaphoric closure' because, when applied to a formula ϕ, it closes off the possibility of subsequent reference to any dref introduced in ϕ. This is because the input and the output assignments in the denotation of $!\phi$ are identical. The operator '!' is important because ϕ and $!\phi$ have the same truth-conditions – see the definition of truth in (11a), i.e., '!' can be said to factor out the truth-conditions of a dynamic formula.

[5] This is shown by the following equivalences:: $\|\phi \to \psi\|\langle g, h\rangle = \mathrm{T}$ iff $\|{\sim}(\phi;\ {\sim}\psi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k s.t. $\|\phi;\ {\sim}\psi\|\langle g, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k and no l s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $\|{\sim}\psi\|\langle l, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k and no l s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $l{=}k$ and $l \notin \mathbf{Dom}(\|\psi\|)$ iff $g{=}h$ and there is no k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$ and $k \notin \mathbf{Dom}(\|\psi\|)$ iff $g{=}h$ and for any k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$, we have that $k \in \mathbf{Dom}(\|\psi\|)$ iff $g{=}h$ and for any k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$, there is an l s.t. $\|\psi\|\langle k, l\rangle = \mathrm{T}$. Summarizing: $\|\phi \to \psi\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and for any k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$, there is an l s.t. $\|\psi\|\langle k, l\rangle = \mathrm{T}$.

[6] This is shown by the following equivalences: $\|\forall x(\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $\|{\sim}([x];\ {\sim}\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k s.t. $\|[x];\ {\sim}\phi\|\langle g, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k and no l s.t. $\|[x]\|\langle g, l\rangle = \mathrm{T}$ and $\|{\sim}\phi\|\langle l, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no k and no l s.t. $g[x]l$ and $l{=}k$ and $l \notin \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and there is no k s.t. $g[x]k$ and $k \notin \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and for any k s.t. $g[x]k$, we have that $k \in \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|\langle k, l\rangle = \mathrm{T}$. Summarizing: $\|\forall x(\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|\langle k, l\rangle = \mathrm{T}$.

[7] The equivalence holds because the following equalities hold (I use two abbreviations: $(\phi)^g := \{h: \|\phi\|\langle g, h\rangle = \mathrm{T}\}$ and $\mathbf{Dom}(\|\phi\|) := \{g$: there is an h s.t. $\|\phi\|\langle g, h\rangle = \mathrm{T}\}$):

$\|{\sim}(\phi;\ \psi)\| = \{\langle g, h\rangle: g{=}h$ and $g \notin \mathbf{Dom}(\|\phi;\ \psi\|)\} = \{\langle g, h\rangle: g{=}h$ and it is not the case that there is a k s.t. $\|\phi;\ \psi\|\langle g, k\rangle = \mathrm{T}\} = \{\langle g, h\rangle: g{=}h$ and it is not the case that there is an l and a k s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $\|\psi\|\langle l, k\rangle = \mathrm{T}\} = \{\langle g, h\rangle: g{=}h$ and there is no l s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $l \in \mathbf{Dom}(\|\psi\|)\} = \{\langle g, h\rangle: g{=}h$ and $(\phi)^g \cap \mathbf{Dom}(\|\psi\|) = \emptyset\} = \{\langle g, h\rangle: g{=}h$ and $(\phi)^g \cap \mathbf{Dom}(\|!\psi\|) = \emptyset\} = \{\langle g, h\rangle: g{=}h$ and $g \notin \mathbf{Dom}(\|\phi;\ !\psi\|)\} = \|{\sim}(\phi;\ !\psi)\|$.

[8] The other 'half' of the duality, i.e., $\exists x({\sim}\phi) \Leftrightarrow {\sim}\forall x(\phi)$, clearly doesn't hold: using the terminology defined in (16), ${\sim}\forall x(\phi)$ is a test, while $\exists x({\sim}\phi)$ isn't.

[9] ${\sim}\exists x(\phi) \Leftrightarrow {\sim}([x];\ \phi) \Leftrightarrow$ (given (14)) ${\sim}([x];\ !\phi) \Leftrightarrow {\sim}([x];\ {\sim}{\sim}\phi) \Leftrightarrow \forall x({\sim}\phi)$.

$\forall x(\phi)$, just as the universal unselective binding 'effect' of implication $\phi \rightarrow \psi$, is in fact due to dynamic negation[10]

- distinguishing basic definitions and derived abbreviations will prove useful when we start generalizing the system in various ways. The official definition is the logical 'core' that undergoes modifications when we define extensions of DPL; the system of abbreviations, however, remains more or less constant across extensions. In this way, we are able to exhibit in a transparent way the commonalities between the various systems we consider and also between the analyses of natural language discourses and within these different systems.
- the abbreviations indicate explicitly the relation between the 'core' dynamic system and related systems (e.g., DRT). From this perspective, it is useful to add to the core layer of definitions in (11) above and the layer of abbreviations in (13) (which 'recovers' first-order logic) yet another and final layer of abbreviations that 'recovers' DRT (Kamp 1981, Kamp & Reyle 1993).

1.2. Discourse Representation Structures (DRSs) in DPL

The semantic notion of *test* and the corresponding syntactic notion of *condition* are defined in (16) and (17) below (see Groenendijk & Stokhof (1991): 57-58, Definitions 11 and 12). The relation between them is stated in (18) (see Groenendijk & Stokhof (1991): 58, Fact 6).

16. A wff ϕ is a *test* iff $\|\phi\| \subseteq \{<g, g>: g \in \boldsymbol{G}\}$, where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments,
    i.e., in our terms, a wff ϕ is a *test* iff $\|\phi\| \subseteq \|[\,]\|$ [11].
17. The set of *conditions* is the smallest set of wffs containing atomic formulas, [ ], negative formulas (i.e., formulas whose main connective is dynamic negation '~'[12]) and closed under dynamic conjunction.
18. ϕ is a test iff ϕ is a condition or a contradiction (ϕ is a *contradiction* iff $\|\phi\| = \emptyset$)

We indicate that a formula is a condition by placing square brackets around it.

19. **Conditions:**
    $[\phi]$ is defined iff ϕ is a *condition*; when defined, $[\phi] := \phi$
    $[\phi_1, \ldots, \phi_m] := [\phi_1]; \ldots; [\phi_m]$

We can now define a Discourse Representation Structure (DRS) or linearized 'box' as follows:

20. **Discourse Representation Structures (DRSs), a.k.a. linearized 'boxes':**
    $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := [x_1, \ldots, x_n]; [\phi_1, \ldots, \phi_m]$,
    equivalently: $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := \exists x_1 \ldots \exists x_n([\phi_1, \ldots, \phi_m])$.
    That is, $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m]$ is defined iff $\phi_1, \ldots, \phi_m$ are conditions and, if defined:
    $\|[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m]\| := \{<g, h>: g[x_1, \ldots, x_n]h$ and $\|\phi_1\|<h, h> = T$ and $\ldots$ $\|\phi_m\|<h, h> = T\}$

2. Anaphora in DPL

The benefit of setting up this system of abbreviations becomes clear as soon as we begin translating natural language discourses into DPL.

2.1. Cross-sentential Anaphora

Consider again discourse (1-2) above, repeated in (21-22) below.

21. A^x house-elf fell in love with a^y witch.
22. He$_x$ bought her$_y$ an^z alligator purse.

The representation of (21-22) in the unabbreviated system is provided in (23) below.

The 'first-order'-style abbreviation is provided in (24) and the DRT-style abbreviation in (25).

23. $[x]$; $house\_elf(x)$; $[y]$; $witch(y)$; $fall\_in\_love(x, y)$;
    $[z]$; $alligator\_purse(z)$; $buy(x, y, z)$
24. $\exists x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)))$;
    $\exists z(alligator\_purse(z); buy(x, y, z))$
25. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$;
    $[z \mid alligator\_purse(z), buy(x, y, z)]$

2.2. Relative-clause Donkey Sentences

Consider now the relative-clause donkey sentence in (26) below (repeated from (4) above). The 'first-order'-style translation in terms of universal quantification and implication is provided in (27) and the DRT-style translation in (28).

One way to see that the two translations are equivalent is to notice that both of them are equivalent to the formula in (29).

26. Every^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.
27. $\forall x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y))$
    $\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$
28. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$
    $\rightarrow [z \mid alligator\_purse(z), buy(x, y, z)]$
29. $[x]$; $house\_elf(x)$; $[y]$; $witch(y)$; $fall\_in\_love(x, y)$
    $\rightarrow [z]$; $alligator\_purse(z)$; $buy(x, y, z)$

[10] See the observations in van den Berg (1996b): 6, Section **2.3**.

[11] Note that $\phi \Leftrightarrow !\phi$ iff ϕ is a test; see Groenendijk & Stokhof (1991): 62.

[12] Note that, given our abbreviations in (13) above, the set of negative formulas includes closed formulas (i.e., formulas of the form '$!\phi$'), disjunctions, implications and universally quantified formulas.

Moreover, the three translations in (27), (28) and (29) are all equivalent (in DPL) to the formula in (30) below, which is the formula that assigns sentence (26) the intuitively correct truth-conditions when interpreted as in classical first-order logic.

30. $\forall x \forall y(\mathit{house\_elf}(x); \mathit{witch}(y); \mathit{fall\_in\_love}(x, y)$
$\quad \rightarrow \exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z)))$

As already noted, the formulas in (27) through (30) are equivalent because DPL validates the equivalence in (10) above, i.e., $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$.[13]

2.3. Conditional Donkey Sentences

Finally, the conditional donkey sentence in (31) below (repeated from (6)) is truth-conditionally equivalent to the relative clause donkey sentence in (26), as shown by the fact that they receive the same DRT-style translation – provided in (32).

The 'first-order'-style compositional translation – equivalent to the DRT-style translation and all the other formulas listed above – is given in (33).

31. If a^x house-elf falls in love with a^y witch, he$_x$ buys her$_y$ an^z alligator purse.
32. $[x, y \mid \mathit{house\_elf}(x), \mathit{witch}(y), \mathit{fall\_in\_love}(x, y)]$
$\quad \rightarrow [z \mid \mathit{alligator\_purse}(z), \mathit{buy}(x, y, z)]$
33. $\exists x(\mathit{house\_elf}(x); \exists y(\mathit{witch}(y); \mathit{fall\_in\_love}(x, y)))$
$\quad \rightarrow \exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z))$

I conclude this section with the DPL analysis of two negative donkey sentences.

34. No^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.
35. If a^x house-elf falls in love with a^y witch, he$_x$ never buys her$_y$ an^z alligator purse.

If we follow the canons of classical first-order logic in translating sentence (34), we have a choice between a combination of negation and existential quantification and a combination of negation and universal quantification. But the limited duality exhibited by existential and universal quantification in DPL (see (15) above) is of help here. To see this, note first that the duality can be generalized to the equivalence in (36) below.

36. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$ [14,15]

[13] $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$ iff $([x]; \phi) \rightarrow \psi \Leftrightarrow \sim([x]; \sim(\phi \rightarrow \psi))$ iff $\sim(([x]; \phi); \sim\psi) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; !(\phi; \sim\psi))$. The last equivalence holds because it is an instance of the more general equivalence $\sim(\phi; \psi) \Leftrightarrow \sim(\phi; !\psi)$ (see (14) above).

[14] The equivalence holds because: $\sim\exists x(\phi; \psi) \Leftrightarrow$ (by (15)) $\forall x(\sim(\phi; \psi)) \Leftrightarrow$ (by (14)) $\forall x(\sim(\phi; !\psi)) \Leftrightarrow \forall x(\sim(\phi; \sim\sim\psi)) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$.

[15] The equivalence $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$ in (36) is a generalization of the equivalence $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$ in (15) expressing the partial duality of the two quantifiers because we can obtain (15) from (36) by inserting [ ] in the place of ϕ in (36). In particular, the two equivalences in (i) and (ii) below hold:

Now, given that the equivalence in (36) holds, we can translate sentence (34) either way, as shown in (37) and (38). Moreover, both translations are equivalent to the formula in (39), which explicitly shows that we quantify universally over all pairs of house-elves and witches standing in the 'fall in love' relation.

37. $\sim\exists x(\mathit{house\_elf}(x); \exists y(\mathit{witch}(y); \mathit{fall\_in\_love}(x, y));$
$\quad \exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z)))$
38. $\forall x(\mathit{house\_elf}(x); \exists y(\mathit{witch}(y); \mathit{fall\_in\_love}(x, y))$
$\quad \rightarrow \sim\exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z)))$
39. $\forall x \forall y(\mathit{house\_elf}(x); \mathit{witch}(y); \mathit{fall\_in\_love}(x, y)$
$\quad \rightarrow \sim\exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z)))$

Consider now sentence (35).

There is a compositional DPL translation for it, which becomes apparent as soon as we consider the intuitively equivalent English sentence in (40) below.

Both sentence (35) and sentence (40) are compositionally translated as in (41).

40. If a^x house-elf falls in love with a^y witch, he$_x$ doesn't buy her$_y$ an^z alligator purse.
41. $\exists x(\mathit{house\_elf}(x); \exists y(\mathit{witch}(y); \mathit{fall\_in\_love}(x, y)))$
$\quad \rightarrow \sim\exists z(\mathit{alligator\_purse}(z); \mathit{buy}(x, y, z))$

It is easily seen that the DPL translations capture the fact that the English sentences in (34), (35) and (40) are intuitively equivalent.

3. Extending DPL with Unselective Generalized Quantification

As the translations of the *every*- and *if*-examples in (26) and (31) above indicate, there is a systematic correspondence in DPL between the generalized quantifier *every* and the unselective implication connective.[16]

The same point is established by the equivalence of the DPL translations of the *no*- and *never*-examples in (34) and (35).

The correspondence between *every* and implication is concisely captured by the equivalence in (42) (which is none other than the equivalence we mentioned at the beginning of the previous section – see (10) above).

42. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi$ [17]

(i) $[\,]; \phi \Leftrightarrow \phi$, hence $\sim\exists x([\,]; \phi) \Leftrightarrow \sim\exists x(\phi)$
(ii) $[\,] \rightarrow \sim\phi \Leftrightarrow \sim([\,]; \sim\sim\phi) \Leftrightarrow \sim\sim\sim\phi \Leftrightarrow \sim\phi$, hence $\forall x([\,] \rightarrow \sim\phi) \Leftrightarrow \forall x(\sim\phi)$

Moreover, we have (by (36)) that $\sim\exists x([\,]; \phi) \Leftrightarrow \forall x([\,] \rightarrow \sim\phi)$; it follows that $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$, i.e., (15), holds.

[16] Implication is unselective basically because it is a sentential connective.

When interpreted relative to an input assignment g, the implication connective $\phi \rightarrow \psi$ boils down to an inclusion relation between two sets of assignments:

- $(\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)$
- $(\phi)^g := \{h: \|\phi\|\langle g, h\rangle = \mathrm{T}\}$,
  i.e., $(\phi)^g$ is the image of the singleton set $\{g\}$ under the relation $\|\phi\|$
- $\mathbf{Dom}(\|\psi\|) := \{h$: there is a k s.t. $\|\phi\|\langle h, k\rangle = \mathrm{T}\}$

The inclusion relation between the two sets is precisely the relation expressed by the static generalized quantifier **EVERY** when applied to the two sets in question, i.e., $\mathbf{EVERY}((\phi)^g, \mathbf{Dom}(\|\psi\|))$.

We can therefore give an alternative definition of implication using the static quantifier **EVERY**:

43. $\|\phi \rightarrow \psi\| = \{\langle g, h\rangle: g{=}h \text{ and } \mathbf{EVERY}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,
    where **EVERY** is the usual static generalized quantifier.

Putting together (42) and (43), we obtain a definition of the natural language quantifier *every* as a binary operator over two DPL formulas:

44. $\|\mathbf{every}_x(\phi, \psi)\| = \{\langle g, h\rangle: g{=}h \text{ and } \mathbf{EVERY}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$

It is easily checked that the equivalence in (42) can be extended as follows:

45. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi \Leftrightarrow \mathbf{every}_x(\phi, \psi)$

This equivalence shows that the operator $\mathbf{every}_x(\phi, \psi)$ can be successfully used to translate donkey sentences with *every* and assign them the intuitively correct truth-conditions.

The 'in love house-elf' example and its DPL translation are repeated in (46) and (47) below. The equivalent translation based on the binary **every** operator is provided in (48).

46. Every^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.
47. $\forall x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y))$
    $\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$
48. $\mathbf{every}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z); buy(x, y, z)))$

We can define in a similar way a binary operator over DPL formulas $\mathbf{no}_x(\phi, \psi)$.

49. $\|\mathbf{no}_x(\phi, \psi)\| = \{\langle g, h\rangle: g{=}h \text{ and } \mathbf{NO}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$,
    i.e., $\|\mathbf{no}_x(\phi, \psi)\| = \{\langle g, h\rangle: g{=}h \text{ and } ([x]; \phi)^g \cap \mathbf{Dom}(\|\psi\|) = \emptyset\}$

[17] $\|\forall x(\phi \rightarrow \psi)\|\langle g, h\rangle = \mathrm{T}$ iff $\|[x] \rightarrow (\phi \rightarrow \psi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and for any k s.t. $\|[x]\|\langle g, k\rangle = \mathrm{T}$, there is an l s.t. $\|\phi \rightarrow \psi\|\langle k, l\rangle = \mathrm{T}$ iff $g{=}h$ and for any k s.t. $g[x]k$, there is an l s.t. $k{=}l$ and for any k' s.t. $\|\phi\|\langle k, k'\rangle = \mathrm{T}$, there is an l' s.t. $\|\psi\|\langle k', l'\rangle = \mathrm{T}$ iff $g{=}h$ and for any k and k' s.t. $g[x]k$ and $\|\phi\|\langle k, k'\rangle = \mathrm{T}$, there is an l s.t. $\|\psi\|\langle k', l\rangle = \mathrm{T}$ iff $g{=}h$ and for any k s.t. $\|[x]; \phi\|\langle g, k\rangle = \mathrm{T}$, there is an l s.t. $\|\psi\|\langle k, l\rangle = \mathrm{T}$ iff $\|([x]; \phi) \rightarrow \psi\|\langle g, h\rangle = \mathrm{T}$.

It is easily checked that the equivalence in (36) above extends as shown in (50).

50. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi) \Leftrightarrow \mathbf{no}_x(\phi, \psi)$

So, we can translate sentence (34)/(51) as in (52):

51. No^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.
52. $\mathbf{no}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z); buy(x, y, z)))$

3.1. Dynamic Unselective Generalized Quantification

The definitions of **every** and **no** in (44) and (49) and the way in which these operators are used to translate the English sentences in (48) and (51) suggest a way to add generalized quantification to DPL so that we can analyze the following donkey sentences:

53. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.
54. Few^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.

Let's first define the family of unselective binary operators **det**. Again, note that they are unselective because they are essentially *sentential* operators.

55. $\|\mathbf{det}(\phi, \psi)\| = \{\langle g, h\rangle: g{=}h \text{ and } \mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,
    where **DET** is the corresponding static determiner.

Given that $\mathbf{Dom}(\|\psi\|) = \mathbf{Dom}(\|!\psi\|)$, it follows that $\mathbf{det}(\phi, \psi) \Leftrightarrow \mathbf{det}(\phi, !\psi)$.

The fact that the **det** sentential operators are *unselective* is semantically reflected in the fact that they express generalized quantification between two sets of *info states* (a.k.a. variable assignments), namely $(\phi)^g$ and $\mathbf{Dom}(\|\psi\|)$.

Their unselectivity is the source of two problems:

- the proportion problem
- no account of weak vs. strong donkey readings

Note that a formula of the form $\mathbf{det}(\phi, \psi)$ is a test. So, we should also extend our syntactic notion of *condition* defined for DPL in (17) above.

56. The set of *conditions* is the smallest set of wffs containing atomic formulas, formulas whose main connective is dynamic negation '~' or a **det** operator and closed under dynamic conjunction.

The definition in (56) enables us to construct DRSs of the form $[\ldots \mid \ldots, \mathbf{det}(\phi, \psi), \ldots]$.

Natural language generalized determiners are defined in terms of the unselective **det** operators, as shown in (57) below.

57. $\mathbf{det}_x(\phi, \psi) := \mathbf{det}([x]; \phi, \psi)$

The determiners $\mathbf{every}_x(\phi, \psi)$ and $\mathbf{no}_x(\phi, \psi)$, i.e., the **every** and **no** instances of the general definition in (57), are just the determiners directly defined in (44) and (49) above.

The generalized determiners defined in this way are still unselective, despite the presence of the variable *x*: the variable *x* in $\mathbf{det}_x$ is only meant to indicate the presence of the additional update [*x*], but the basic operator is still the unselective **det**.

That is, we still determine the denotation of $\mathbf{det}_x(\phi, \psi)$ by checking whether the static determiner **DET** applies to two sets of info states – and not to two sets of individuals.

The definition of det(ϕ, ψ) in (55) above is just the definition of quantificational adverbs in Groenendijk & Stokhof (1991): 81-82, which follows Lewis (1975) in taking adverbs to quantify over *cases*. E.g., *never* is translated in Groenendijk & Stokhof (1991): 82 as the binary implication connective $\rightarrow_{no}$. The definition of $\phi \rightarrow_{no} \psi$ is identical to the definition of **no**(ϕ, ψ).

The analysis can be extended in the obvious way to other adverbs of quantification, e.g., *always* can be interpreted as **every**(ϕ, ψ) (just like bare conditionals), *often* and *usually* as **most**(ϕ, ψ) and *rarely* as **few**(ϕ, ψ) – where the corresponding static determiners **MOST** and **FEW** are interpreted as more than half and less than half respectively.

The definition of $\mathbf{det}_x(\phi, \psi)$ is actually equivalent to the (implicit) definition of generalized quantification in Kamp (1981) and Heim (1982/1988).

A nice consequence of defining $\mathbf{det}_x$ in terms of **det** (as in (57) above) is that the systematic natural language correspondence between adverbs of quantification and generalized quantifiers, e.g., the correspondence between *no* and *never* in examples (34) and (35) above, is explicitly captured.

3.2. Limitations of Unselectivity: Proportions

Consider the translations in (59) and (62) ('predicate logic'-style) and (60) and (63) (DRT-style).

58. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.
59. $\mathbf{most}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z); buy(x, y, z)))$
60. $\mathbf{most}_x([y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)],$
    $[z \mid alligator\_purse(z), buy(x, y, z)])$

61. Few^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.
62. $\mathbf{few}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z); buy(x, y, z)))$
63. $\mathbf{few}_x([y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)],$
    $[z \mid alligator\_purse(z), buy(x, y, z)])$

We do capture the anaphoric connections, but we do not derive the intuitively correct truth-conditions. As shown in Partee (1984),[18] Rooth (1987), Kadmon (1987) and Heim (1990), the analysis has a proportion problem.[19]

This is easy to see if we examine the formula in (64) below (equivalent to (59) and (60)).

64. $\mathbf{most}([x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)],$
    $[z \mid alligator\_purse(z), buy(x, y, z)])$

The representation in (64) makes clear that we are quantifying over most pairs $\langle x, y\rangle$ where *x* is a house-elf that fell in love with a witch *y*. For most such pairs $\langle x, y\rangle$, the requirement in the nuclear scope, i.e., *x* bought *y* some alligator purse *z*, should be satisfied.

But: we can produce a scenario in which the English sentence in (58) is intuitively false while the formula in (64) is true.

- there are ten house-elves that fell in love with some witch or other
- one of them, call him Dobby, is a Don Juan of sorts and he fell in love with more than one thousand witches[20] and bought them all alligator purses
- the other nine house-elves are less exceptional: they each fell in love with only one witch and they bought them new brooms, not alligator purses

Sentence (58) is intuitively false in this scenario, while formula (64) is true: all the Dobby-based pairs that satisfy the restrictor also satisfy the nuclear scope – and these pairs are more than half, i.e., *most*, of the pairs under consideration.

3.3. Limitations of Unselectivity: Weak / Strong Ambiguities

In addition, the unselective analysis of generalized quantifiers fails to account for the fact that the same donkey sentence can exhibit two different readings, a *strong* one and a *weak* one. Consider again the classical sentence in (65) below.

65. Every^x farmer who owns a^y donkey beats it$_y$.

The most salient reading of this sentence: every farmer behaves violently towards *each and every* one of his donkeys, i.e., the so-called *strong* reading.

[18] "[…] when we have to deal with quantification with a complicated and possibly uncertain underlying ontology, we need to specify a 'sort' (for the quantifier to 'live on' in the sense of Barwise & Cooper 1981) separately from whatever further restrictions we want to add (perhaps in terms of 'cases') about which instances of the sort we are quantifying over. In terms of Kamp's framework this means that we have to worry not only about what belongs in the antecedent box but also how to distinguish a substructure within it that plays the role of sortal (the head noun in the NP case)." (Partee 1984: 278).

[19] The 'proportion problem' terminology is due to Kadmon (1987): 312.

[20] To be more precise, one thousand and three witches only in Spain.

The $\textbf{every}_x$ operator correctly captures this reading, as shown in (66) below; the equivalent formulas in (67) and (68) are provided because they display the 'strength' of the reading in a clearer way.

66. $\textbf{every}_x([y \mid farmer(x), donkey(y), own(x, y)],\ [beat(x, y)])$
67. $\textbf{every}([x, y \mid farmer(x), donkey(y), own(x, y)],\ [beat(x, y)])$
68. $\forall x \forall y(farmer(x); donkey(y); own(x, y) \rightarrow beat(x, y))$

However, sentence (65) can receive another, *weak* reading: every farmer beats *some* donkey that he owns, but not necessarily each and every one of them.[21]

Chierchia (1995): 64 provides a context in which the most salient reading is the weak one: imagine that the farmers under discussion are all part of an anger management program and they are encouraged by the psychotherapist in charge to channel their aggressiveness towards their donkeys (should they own any) rather than towards each other. The farmers scrupulously follow the psychotherapist's advice – in which case we can assert (65) even if the donkey-owning farmers beat only some of their donkeys.

Furthermore, there are donkey sentences for which the *weak* reading is the most salient one:

69. Every person who has a dime will put it in the meter.
(Pelletier & Schubert 1989)
70. Yesterday, every person who had a credit card paid his bill with it.
(R. Cooper, apud Chierchia 1995: 63, (3a))

Thus, both readings seem to be *semantically* available[22] and the unselective analysis of dynamic generalized quantifiers does not allow for both of them.

The weak/strong ambiguity also provides an argument against the unselective analysis of conditionals and adverbs of quantification, as shown, for example, by (71) below.

71. If a^x farmer owns a^y donkey, he_x (always/usually/often/rarely/never) beats it_y.

For a detailed discussion of such conditionals, see (among others) Chierchia (1995): 66-69. I will only mention the generalization reached in Kadmon (1987) and summarized in Heim (1990):

"Kadmon's generalization is that a multi-case conditional with two indefinites in the antecedent generally allows three interpretations: one where the QAdverb quantifies over pairs, one where it quantifies over instances of the first indefinite and one where it quantifies over instances of the second." (Heim 1990: 153)

[21] Partee (1984) seems to be (one of) the first to notice weak donkey readings: the example in (i) below is from Partee (1984): 280, fn. 12.

(i) If you have a credit card, you should use it here instead of cash.

[22] See for example the discussion in Chierchia (1995): 62-65, in particular the argument that the strong reading is not a conversational implicature triggered in certain contexts.

A partial solution to the problem posed by weak donkey readings is available in classical DRT / FCS / DPL. As pointed out in Groenendijk & Stokhof (1991): 89, we can define an alternative implication connective, as shown in (72) below.

72. $\phi \mapsto \psi := \sim\phi \vee (\phi; \psi)$,
i.e., $\|\phi \mapsto \psi\| = \{\langle g, h\rangle: g=h \text{ and } g\notin \textbf{Dom}(\|\phi\|) \text{ or } (\phi)^g \cap \textbf{Dom}(\|\psi\|) \neq \emptyset\}$,
i.e., $\|\phi \mapsto \psi\| = \{\langle g, h\rangle: g=h \text{ and } g\notin \textbf{Dom}(\|\phi\|) \text{ or } (\phi; !\psi)^g \neq \emptyset\}$,
i.e., $\|\phi \mapsto \psi\| = \{\langle g, h\rangle: g=h \text{ and } g\notin \textbf{Dom}(\|\phi\|) \text{ or } g\in \textbf{Dom}(\|\phi; !\psi\|)$.
73. $\|\phi \rightarrow \psi\| = \{\langle g, h\rangle: g=h \text{ and } g\notin \textbf{Dom}(\|\phi\|) \text{ or } (\phi)^g \subseteq \textbf{Dom}(\|\psi\|)\}$

Note the 'some' flavor of $\mapsto$: $(\phi)^g \cap \textbf{Dom}(\|\psi\|) \neq \emptyset$.

Compare with the 'every' flavor of $\rightarrow$: $(\phi)^g \subseteq \textbf{Dom}(\|\psi\|)$.

The weak reading of sentence (74) (repeated from above) is presumably analyzed as shown in (75), which is 'unpacked' in (76). The strong reading is given in (77) and (78) for ease of comparison.

74. $Every^x$ farmer who owns a^y donkey beats it_y.
75. **weak reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \mapsto beat(x, y))$
76. **weak reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \mapsto [beat(x, y)])$
77. **strong reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \rightarrow beat(x, y))$
78. **strong reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \rightarrow [beat(x, y)])$

However, this analysis of weak implication faces three problems:

- as we can see from the 'unpacked' formula in (76), we still need the 'strong' implication connective $\rightarrow$ in addition to the 'weak' one $\mapsto$ to capture the correct truth-conditions for the weak reading of sentence (74), i.e., the weak reading is obtained via a combination of 'strong' and 'weak' implication. So, this solution fails to extend to weak readings of conditionals: as argued by Kadmon, the conditional in (79) below can receive a weak reading that is equivalent to the weak reading of the *every* donkey sentence in (74) above. However, this reading is not captured by the formula in (80), precisely because the equivalence $\exists x(\phi) \mapsto \psi \Leftrightarrow \forall x(\phi \mapsto \psi)$ fails for 'weak' implication – and we *do* want it to fail with respect to the indefinite a^y *donkey*, but *not* with respect to the indefinite a^x *farmer*.

79. If a^x farmer owns a^y donkey, he_x beats it_y.
80. $\exists x(farmer(x); \exists y(donkey(y); own(x, y))) \mapsto beat(x, y)$

- the 'weak' implication solution does not generalize to other determiners (e.g., *most*)
- it does not account for the proportion problem.

Summarizing:

- a donkey sentence turns out to be ambiguous between a weak and a strong reading

- the strong reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with an *every* DP
- the weak reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with a *some* DP
- extending DPL with an *unselective* form of generalized quantification fails to account for the weak / strong donkey ambiguity and for the proportion problem – so, we need to further extend DPL with a *selective* form of dynamic generalized quantification.

3.4. Conservativity and Unselective Quantification

Defining dynamic **det**s in terms of static **DET**s (as we did in (55) and (57) above) provides us with a version of *unselective dynamic conservativity* that underlies the definition of selective generalized quantification to be introduced in the next section.

Consider again the definition in (55) above:

$\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$.

Assuming that the static determiner **DET** is conservative, we have that:

$\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|)) = T \text{ iff } \mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|)) = T$.

The r.h.s. formula encodes an intuitively appealing meaning for unselective dynamic generalized quantification:[23] a dynamic generalized determiner relates two sets of info states, the first of which is the set of output states compatible with the restrictor, i.e., $(\phi)^g$, while the second one is the set of output states compatible with the restrictor that can be further updated by the nuclear scope, i.e., $(\phi)^g \cap \mathbf{Dom}(\psi)$.

To reformulate this intuition in a more formal way, note that:

$\mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|)) = T \text{ iff } \mathbf{DET}((\phi)^g, (\phi; !\psi)^g)) = T$.

Thus, assuming that all static generalized determiners **DET** are conservative, we can restate the definition in (55) above as follows:

81. **Built-in unselective dynamic conservativity:**
$\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}((\phi)^g, (\phi; !\psi)^g))\}$

Now, putting together the definition of $\mathbf{det}_x(\phi, \psi)$ in (57), i.e., $\mathbf{det}_x(\phi, \psi) := \mathbf{det}([x]; \phi, \psi)$, and the 'conservative' definition in (81), we obtain the following definition of generalized quantification:

82. **Generalized quantification w/ built-in dynamic conservativity (unselective version):**
$\|\mathbf{det}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x]; \phi)^g, ([x]; \phi; !\psi)^g))\}$

[23] This has been previously noted with respect to the dynamic definition of *selective* generalized quantification – see for example Chiechia (1992, 1995) and Kamp & Reyle (1993) among others.

The definition of *conservative* unselective quantification in (82) can in fact be thought of as the basis for the definition of selective generalized quantification introduced in Chierchia (1995) among others (see section **4** below):

- we have access to the variable x in the restrictor of the static determiner **DET**, i.e., $[x]; \phi$
- we also have access to the variable x in its nuclear scope, i.e., $[x]; \phi; !\psi$
- so, we can be *selective* and (somehow) λ-abstract over the variable x in both formulas
- we thus obtain two sets of *individuals* and can require the static determiner **DET** to apply to these two sets individuals and not to the corresponding sets of info states.

4. Extending DPL with Selective Generalized Quantification (DPL+GQ)

The notion of *selective* generalized quantification introduced in this section has been proposed in various guises by many authors: Bäuerle & Egli (1985), Root (1986) and Rooth (1987) put forth the basic proposal and van Eijck & de Vries (1992) and Chierchia (1992, 1995) were the first to formulate it in DPL terms. The proposal is also adopted in Heim (1990) and Kamp & Reyle (1993).

We use the same notation as above:

- selective dynamic generalized quantification has the form $\mathbf{det}_x(\phi, \psi)$
- x is the bound variable
- ϕ is the restrictor
- ψ is the nuclear scope.

But, since $\mathbf{det}_x(\phi, \psi)$ is selective (it relates two sets of individuals), it will be directly defined – i.e., it isn't an abbreviation of a formula containing the unselective $\mathbf{det}(\phi, \psi)$.

4.1. Dynamic Selective Generalized Quantification

- the fact that $\mathbf{det}_x(\phi, \psi)$ is defined in terms of sets of *individuals* (and not of info states) enables us to account for the proportion problem
- the weak/strong donkey ambiguity is attributed to an ambiguity in the interpretation of the selective generalized quantifier, following the proposals in Bäuerle & Egli (1985), Rooth (1987), Reinhart (1987), Heim (1990) and Kanazawa (1994a, b) – for each dynamic generalized determiner, we will have a *weak* lexical entry $\mathbf{det}^{wk}_x(\phi, \psi)$ and a *strong* lexical entry $\mathbf{det}^{str}_x(\phi, \psi)$
- an English sentence containing a determiner *det* is ambiguous between the two readings

83. $\|\mathbf{det}^{wk}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(\lambda x.\ (\phi)^g,\ \lambda x.\ (\phi; \psi)^g)\}$
$\|\mathbf{det}^{str}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(\lambda x.\ (\phi)^g,\ \lambda x.\ (\phi \to \psi)^g)\}$,
where $(\phi)^g := \{h: \|\phi\|<g, h> = \mathrm{T}\}$
and $\lambda x.\ (\phi)^g := \{h(x): h\in([x]; \phi)^g\}$
and **DET** is the corresponding static determiner.

The abbreviation $\lambda x.\ (\phi)^g := \{h(x): h\in([x]; \phi)^g\}$ is really just λ-abstraction in static terms: $\lambda x.\ (\phi)^g$ is the set of entities a s.t. $\|\phi\|_{static}{}^{g[x/a]} = \mathrm{T}$, where $\|\cdot\|_{static}$ is the usual static interpretation function (I don't know why this connection hasn't been explicitly made in the dynamic literature …).

Both lexical entries are selective: the static determiner **DET** relates two sets of individuals, represented by means of abbreviations of the form $\lambda x.\ (\ldots)^g$.

The only difference between the weak and the strong entries has to do with how the nuclear scope of the static quantification is obtained:

- by means of *dynamic conjunction* $\lambda x.\ (\phi; \psi)^g$ in the *weak* case
- by means of *dynamic implication* $\lambda x.\ (\phi \to \psi)^g$ in the *strong* case
- dynamic conjunction yields the weak reading because an existential quantifier in the restrictor $\lambda x.\ (\phi)^g$ will still be an existential in the nuclear scope $\lambda x.\ (\phi; \psi)^g$: every farmer that owns *some* donkey beats *some* donkey he owns
- dynamic implication yields the strong reading because it has universal quantification built into it (due to dynamic negation '~', since $\phi \to \psi := {\sim}(\phi; {\sim}\psi)$): DPL validates the equivalence $\exists x(\phi) \to \psi \Leftrightarrow \forall x(\phi \to \psi)$, so an indefinite in the restrictor ends up being universally quantified in the nuclear scope: every farmer that owns *some* donkey beats *every* donkey he owns.

The unselective conservative entry defined in (82) above provides the basic format for the selective entries.

Assuming that, in (83) above, $[x]$ is not reintroduced in ψ (and it cannot be if we want the definitions to work properly), it is always the case that:

84. $\lambda x.\ (\phi; \psi)^g = \lambda x.\ (\phi; !\psi)^g$
85. $\lambda x.\ (\phi \to \psi)^g = \lambda x.\ (\phi \to !\psi)^g$
(for dynamic implication $\to$, we have the more general result that $\phi \to \psi \Leftrightarrow \phi \to !\psi$, which follows directly from the equivalence in (14) above)

More generally, the weak and strong selective generalized determiners in (83) above can be defined in terms of generalized quantification over info states if we make use of the closure operator '!' as shown in (86) below.

86. $\|\mathbf{det}^{wk}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid !(\phi; \psi)])^g)\}$
$\|\mathbf{det}^{str}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid !(\phi \to \psi)])^g)\}$[24],
where $(\phi)^g := \{h: \|\phi\|<g, h> = \mathrm{T}\}$
and **DET** is the corresponding static determiner.

It is easily checked that the two pairs of definitions are equivalent given the fact that there is a bijection between the sets of individuals quantified over in (83) and the set of info states (i.e., variable assignments) quantified over in (86):

87. $\lambda x.\ (\phi)^g := \{h(x): h\in([x]; \phi)^g\}$
$= \{a: \text{there is an } h \text{ s.t. } \|[x]; \phi\|<g, h> = \mathrm{T} \text{ and } a{=}h(x)\}$
$= \{a: \text{there is a } k \text{ and an } h \text{ s.t. } g[x]k \text{ and } \|\phi\|<k, h> = \mathrm{T} \text{ and } a{=}h(x)\}$
(since x is not reintroduced in ϕ, $k(x){=}h(x)$)
$= \{a: \text{there is a } k \text{ and an } h \text{ s.t. } g[x]k \text{ and } \|\phi\|<k, h> = \mathrm{T} \text{ and } a{=}k(x)\}$
$= \{a: \text{there is a } k \text{ s.t. } a{=}k(x) \text{ and } g[x]k \text{ and there is an } h \text{ s.t. } \|\phi\|<k, h> = \mathrm{T}\}$
$= \{a: \text{there is a } k \text{ s.t. } a{=}k(x) \text{ and } g[x]k \text{ and } k\in \mathbf{Dom}(\|\phi\|)\}$
$= \{a: \text{there is a } k \text{ s.t. } k\in([x]; !\phi)^g \text{ and } a{=}k(x)\}$.

Let f be a function from the set of assignments $([x]; !\phi)^g$ to the set of individuals $\lambda x.\ (\phi)^g$ s.t., for any assignment h, $f(h){=}h(x)$. By the above equality, f is surjective. Since for any assignment g and individual a there is a unique assignment h s.t. $g[x]h$ and $h(x){=}a$, f is injective.

Note that f is just a 'type-lifted' of the variable x: it is the x-based projection function over variable assignments $\lambda g.\ g(x)$.

Finally, according to definition (83), a formula of the form $\mathbf{det}^{wk}{}_x(\phi, \psi)$ or $\mathbf{det}^{str}{}_x(\phi, \psi)$ is a test. So, we should further extend the syntactic notion of condition with selective generalized determiners. The new definition is:

88. The set of *conditions* is the smallest set of wffs containing atomic formulas, formulas whose main connective is dynamic negation '~', a **det** operator or a $\mathbf{det}^{wk/str}{}_\upsilon$ operator (for any variable υ) and closed under dynamic conjunction.

The definition in (88) enables us to construct DRSs of the form $[\ldots \mid \ldots, \mathbf{det}^{wk/str}{}_x(\phi, \psi), \ldots]$.

4.2. Accounting for Weak / Strong Ambiguities

Let us see how the above definitions derive the weak and strong readings of the classical example in (89) below (repeated from (65)).

89. Every^x farmer who owns a^y donkey beats it$_y$.

The two lexical entries for *every* are given in (90) below and simplified in (91).

[24] Since $!(\phi \to \psi) \Leftrightarrow \phi \to \psi$, the strong determiner can be more simply defined as $\|\mathbf{det}^{str}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid \phi \to \psi])^g)\}$.

90. $\|\mathbf{every}^{wk}{}_x(\phi, \psi)\| = \{\langle g, h\rangle: g=h \text{ and } \mathbf{EVERY}(\lambda x.\ (\phi)^g, \lambda x.\ (\phi; \psi)^g)\}$
$\|\mathbf{every}^{str}{}_x(\phi, \psi)\| = \{\langle g, h\rangle: g=h \text{ and } \mathbf{EVERY}(\lambda x.\ (\phi)^g, \lambda x.\ (\phi \to \psi)^g)\}$

91. $\|\mathbf{every}^{wk}{}_x(\phi, \psi)\| = \{\langle g, h\rangle: g=h \text{ and } \lambda x.\ (\phi)^g \subseteq \lambda x.\ (\phi; \psi)^g\}$
$\|\mathbf{every}^{str}{}_x(\phi, \psi)\| = \{\langle g, h\rangle: g=h \text{ and } \lambda x.\ (\phi)^g \subseteq \lambda x.\ (\phi \to \psi)^g\}$

The weak reading of (89) is represented in (92) and simplified in (93).

92. $\mathbf{every}^{wk}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))$

93. $\|\mathbf{every}^{wk}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))\| =$
$\{\langle g, g\rangle: \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g \subseteq$
$\lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); \mathit{beat}(x, y))^g\} =$
$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in (\boldsymbol{I}(\mathit{own}) \cap \boldsymbol{I}(\mathit{beat})\}\} =$
$\{\langle g, g\rangle$: any farmer a who owns a donkey b is s.t. he owns and beats a donkey $b'\}$[25]

The formula in (92) delivers the weak reading because the donkey-owning farmers do not have to beat *all* the donkeys they own – they only have to beat *some* of their donkeys.

The strong reading of (89) is represented in (94) and simplified in (95).

94. $\mathbf{every}^{str}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))$

95. $\|\mathbf{every}^{str}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))\| =$
$\{\langle g, g\rangle: \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g \subseteq$
$\lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y) \to \mathit{beat}(x, y))^g\} =$
$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{a: \text{any } b \text{ s.t. } a\in \boldsymbol{I}(\mathit{farmer}), b\in \boldsymbol{I}(\mathit{donkey}), \langle a, b\rangle \in \boldsymbol{I}(\mathit{own}) \text{ is s.t. } \langle a, b\rangle \in \boldsymbol{I}(\mathit{beat})\}\} =$
$\{\langle g, g\rangle$: any farmer a who owns a donkey b beats any donkey b' that he owns$\}$[26]

The formula in (94) delivers the strong reading because the donkey-owning farmers have to beat *all* the donkeys they own.

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[25] In more detail, the simplification proceeds as follows:

$\|\mathbf{every}^{wk}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))\| =$

$\{\langle g,g\rangle: \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g \subseteq \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); \mathit{beat}(x, y))^g\}=$

$\{\langle g, g\rangle: \{h(x): h\in([x]; \mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g\} \subseteq$
$\{h(x): h\in([x]; \mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); \mathit{beat}(x, y))^g\}\} =$

$\{\langle g, g\rangle: \{h(x): g[x, y]h, h(x)\in \boldsymbol{I}(\mathit{farmer}), h(y)\in \boldsymbol{I}(\mathit{donkey}), \langle h(x), h(y)\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): g[x, y]h, h(x)\in \boldsymbol{I}(\mathit{farmer}), h(y)\in \boldsymbol{I}(\mathit{donkey}), \langle h(x), h(y)\rangle \in (\boldsymbol{I}(\mathit{own}) \cap \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle: \{a: \text{there is a } b \text{ s.t. } a\in \boldsymbol{I}(\mathit{farmer}), b\in \boldsymbol{I}(\mathit{donkey}), \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{a: \text{there is a } b \text{ s.t. } a\in \boldsymbol{I}(\mathit{farmer}), b\in \boldsymbol{I}(\mathit{donkey}), \langle a, b\rangle \in (\boldsymbol{I}(\mathit{own}) \cap \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in (\boldsymbol{I}(\mathit{own}) \cap \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle$: any farmer a who owns a donkey b is such that he owns and beats a donkey $b'\}$.

[26] In more detail, the simplification proceeds as follows:

$\|\mathbf{every}^{str}{}_x(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y),\ \mathit{beat}(x, y))\| =$

$\{\langle g, g\rangle: \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g \subseteq \lambda x.\ (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y) \to \mathit{beat}(x, y))^g\} =$

$\{\langle g, g\rangle: \{h(x): h\in([x]; \mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y))^g\} \subseteq$
$\{h(x): h\in([x]; (\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y) \to \mathit{beat}(x, y)))^g\}\} =$

$\{\langle g, g\rangle: \{h(x): g[x, y]h, h(x)\in \boldsymbol{I}(\mathit{farmer}), h(y)\in \boldsymbol{I}(\mathit{donkey}), \langle h(x), h(y)\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): h\in([x]; {\sim}(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); {\sim}\mathit{beat}(x, y)))^g\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): \text{there is a } k \text{ s.t. } g[x]k \text{ and } \|{\sim}(\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); {\sim}\mathit{beat}(x, y))\|\langle k, h\rangle = \mathrm{T}\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): g[x]h \text{ and there is no } l \text{ s.t. } \|\mathit{farmer}(x); [y]; \mathit{donkey}(y); \mathit{own}(x, y); {\sim}\mathit{beat}(x, y)\|\langle h, l\rangle = \mathrm{T}\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): g[x]h \text{ and there is no } l \text{ s.t. } h[y]l, l(x)\in \boldsymbol{I}(\mathit{farmer}), l(y)\in \boldsymbol{I}(\mathit{donkey}), \langle l(x), l(y)\rangle \in \boldsymbol{I}(\mathit{own}), \langle l(x), l(y)\rangle \notin \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): g[x]h \text{ and for any } l, \text{ if } h[y]l, l(x)\in \boldsymbol{I}(\mathit{farmer}), l(y)\in \boldsymbol{I}(\mathit{donkey}), \langle l(x), l(y)\rangle \in \boldsymbol{I}(\mathit{own}), \text{ then } \langle l(x), l(y)\rangle \in \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{h(x): g[x]h \text{ and for any } b, \text{ if } h(x)\in \boldsymbol{I}(\mathit{farmer}), b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle h(x), b\rangle \in \boldsymbol{I}(\mathit{own}), \text{ then } \langle h(x), b\rangle \in \boldsymbol{I}(\mathit{beat})\}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(\mathit{farmer}) \text{ and there is a } b \text{ s.t. } b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own})\} \subseteq$
$\{a: \text{any } b \text{ s.t. } a\in \boldsymbol{I}(\mathit{farmer}), b\in \boldsymbol{I}(\mathit{donkey}) \text{ and } \langle a, b\rangle \in \boldsymbol{I}(\mathit{own}) \text{ is s.t. } \langle a, b\rangle \in \boldsymbol{I}(\mathit{beat})\}\}\} =$

$\{\langle g, g\rangle$: any farmer a who owns a donkey b beats any donkey b' that he owns$\}$.

4.3. Solving Proportions

Selective generalized quantification also solves the proportion problem. Consider again sentence (58), repeated in (96) below (alternatively, consider (100)).

The most salient reading of this sentence seems to be the strong one, represented in (97), just as the most salient reading of the structurally similar sentence in (98) is the weak one, represented in (99) below.

96. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.
97. **most**$^{str}{}_x$(*house_elf*(x); [y]; *witch*(y); *fall_in_love*(x, y),
 [z]; *alligator_purse*(z); *buy*(x, y, z))
98. Most^x drivers who have a^y dime will put it$_y$ in the meter.
99. **most**$^{wk}{}_x$(*driver*(x); [y]; *dime*(y); *have*(x, y), *put_in_meter*(x, y))
100. Most^x people that owned a^y slave also owned his$_y$ offspring. (Heim 1990: 162, (49))

The formula in (97) is true iff more than half of the house-elves who fall in love with a witch are such that they buy *any* witch that they fall in love with (*strong* reading) some alligator purse or other. This formula is false in the 'Dobby as Don Juan' scenario above, in agreement with our intuitions about the corresponding English sentence in (96).

The formula in (99) makes similarly correct predictions about the truth-conditions of the English sentence in (98): both of them are true in a scenario in which there are ten drivers, each of them has ten dimes in his/her pocket and nine of them put exactly one dime in their respective meters. Out of the one hundred possible pairs of drivers and dimes, only nine pairs (far less than half) satisfy the nuclear scope of the quantification, but this is irrelevant as long as a majority of drivers (and not of pairs) satisfies it.

5. Limitations of DPL+GQ: Mixed Weak & Strong Donkey Sentences

The dynamic notion of selective generalized quantification introduced in the previous section does not offer a completely general account of weak/strong donkey ambiguities: it fails for more complex weak & strong donkey sentences much as the unselective notion failed for the simplest ones.

Consider again the *dime* example from Pelletier & Schubert (1989), repeated in (101).

101. Every^x person who has a^y dime will put it$_y$ in the meter.

Unselective generalized quantification fails to assign the correct weak interpretation to this example because it cannot distinguish between the various discourse referents (drefs) introduced in the restrictor of the generalized quantifier:

- x (the persons) should be quantified over universally
- y (their dimes) should be quantified over existentially

Selective generalized quantification provides a solution to this problem because it can distinguish between x, which is the dref contributed by the generalized determiner, and y, which is the dref contributed by the indefinite in the restrictor of the determiner.

Thus, selective generalized quantification:

- can distinguish between the 'main' quantified-over dref and the other drefs introduced in the restrictor
- cannot further distinguish between the latter ones, which are collectively interpreted as either weak or strong.

Since the decision about the 'strength' of the drefs introduced in the restrictor is not made on an individual basis, selective generalized quantification as defined in (83) above fails to account for any examples in which two indefinites in the restrictor of a generalized quantifier are not interpreted as both weak or both strong.

102. Every^x person who buys a^y book on amazon.com and has a^z credit card uses it$_z$ to pay for it$_y$.
103. Every^x man who wants to impress a^y woman and who has an^z Arabian horse teaches her$_y$ how to ride it$_z$.

The most salient interpretation of (102) is strong with respect to *a^y book* and weak with respect to *a^z credit card*, i.e., for *every* book bought on amazon.com by any person that is a credit-card owner, the person uses *some* credit card or other to pay for the book.

In particular, note that the credit card might vary from book to book, i.e., the strong indefinite *a^y book* seems to be able to 'take scope' over the weak indefinite *a^z credit card*: I can use my Mastercard to buy set theory books and my Visa to buy sci-fi novels. This means that, despite the fact that it receives a weak reading, the indefinite *a$^{u'}$ credit card* can introduce a possibly non-singleton set of credit cards.

Similarly, in the case of (103), the indefinite *a^y woman* is interpreted as strong and the indefinite *an^z Arabian horse* as weak. Yet again, the strong indefinite seems to 'take scope' over the weak one: the horse used in the pedagogic activity might vary from female student to female student.

We can easily construct examples of this kind if we are willing to countenance other anaphoric expressions besides pronouns. For example, we can replace one of the non-animate pronouns in sentence (102) with a definite description – as shown in (104) below[27].

104. Every^x person who buys a^y book on amazon.com and has a^z credit card uses the$_z$ card to pay for it$_y$.

[27] I substitute a definite description for the pronoun that enters the anaphoric dependency receiving a weak reading; substituting a definite description for the strong pronoun might bring in the additional complexity that the strong reading is in fact due to the use of the (maximal) definite description (see for example the D-/E-type analyses in Neale 1990, Lappin & Francez 1994 and Krifka 1996b).

How can we extend the DPL-style definition of dynamic selective quantification in a way that can discriminate between the drefs introduced by indefinites in the restrictor?

The basic idea: introduce additional lexical entries for generalized determiners that bind universally or existentially the indefinites in their restrictor, e.g., *most* would have:

- a 'single quantifier' entry of the form $\mathbf{most}_x$
- two 'double quantifier' entries of the form $\mathbf{most}_x\forall_y$ and $\mathbf{most}_x\exists_y$
- four 'triple quantifier' entries of the form $\mathbf{most}_x\forall_y\forall_z$, $\mathbf{most}_x\forall_y\exists_z$, $\mathbf{most}_x\exists_y\forall_z$, $\mathbf{most}_x\exists_y\exists_z$ etc.

Note that interpreting English sentences in terms of such determiners is not compositional, e.g., to interpret (103), we need a 'triple quantifier' of the form $\mathbf{every}_x\forall_y\exists_z$, which requires us to look inside the second relative clause, identify the indefinite *an*^{z} *Arabian horse* and assign it a weak interpretation.

The situation is in fact even more complicated and non-compositional:

- the indefinites in the restrictor can enter pseudo-scopal relations since the value of the weak indefinite can vary with the value of the strong indefinite, e.g., the same 'triple quantifier' $\mathbf{every}_x\forall_y\exists_z$ has a choice of scoping $\forall_y$ over $\exists_z$ or the other way around, i.e., $\mathbf{every}_x\exists_z\forall_y$.
- these relations are *pseudo*-scopal because the two donkey indefinites in both (102) and (103) are 'trapped' in a coordination island and none of them can scope out of their VP- or CP-conjunct to take scope over the other.

The impossibility of scoping out of a coordination structure is not dependent on any particular scoping mechanism – the two sentences in (105) and (106) below show that a quantifier like *every* cannot scope out of VP- or CP-coordination structures.

105. #Every person who buys every^x *Harry Potter* book on *amazon.com* and gives it$_x$ to a friend must be a *Harry Potter* addict.
106. #Every boy who wanted to impress every^x girl in his class and who planned to buy her$_x$ a fancy Christmas gift asked his best friend for advice.

Many accounts of weak and strong readings fail to analyze such conjunction-based, mixed weak & strong donkey sentences. The main difficulty:

- they cannot allow for the weak indefinite to be a (possibly) *non-singleton* set and to covary with the value of the strong indefinite.