Muskens 1995 Chap. 7 & 8

presented by Kevin Reschke – May 4, 2010

Chapter 7: Situations, Persistence and Weak Consequence

1 Situations and the part-of relation

- Indices as possible worlds? Indices correspond to models.

1.1 Consider total (non-partial) theory

- *Extension* of term A of type $\langle \alpha_1 \ldots \alpha_n s\rangle$ in index i in model $M = \langle \{D_\alpha\}_\alpha, I\rangle$ under assignment α is the n+1[th] slice function of A.

  E.g., *extension* of $love_{\langle ees\rangle}$ at i is $F^3_{\|love\|}(i)$.

- For each i in D_s we associate interpretation function I_i

  For c of type *e* or *s*,
  $I_i(c) = I(c)$

  For c of type $\langle \alpha_1 \ldots \alpha_n\rangle$,
  $I_i(c)$ = the extension of the corresponding c of type $\langle \alpha_1 \ldots \alpha_n s\rangle$ at index i.

  E.g., $I_i(\text{LOVE}_{\langle ee\rangle}) = F^3_{\|love\|}(i)$.

- So, let $M_i = \langle \{D_\alpha\}_\alpha, I_i\rangle$.

  Then M is the union of M_i for all i.

1.2 What happens when we partialize?

- M is partial, thus the components M_i are partial too.

  M_i is the partial possible world at i – aka **possible situation.**

1.3 Part-of relation

- M_i is *part of* M_j if $I_i(c) = I_j(c)$ for all c of type *e* or *s* and $I_i(c) \sqsubseteq I_j(c)$ for all c of relational type.

  $R_1 \sqsubseteq R_2$ iff R_1^+ is a subset of R_2^+ and R_1^- is a subset of R_2^-

- Let $\leq$ be a non-logical constant of type $\langle ss \rangle$ which represents the *part-of* relation.

  AX9 $\forall ij(i \leq j = \Psi)$
  where Ψ is the conjunction of all formulae of the form
  $\forall x_1 \ldots x_n(cx_1 \ldots x_n i \sqsubseteq cx_1 \ldots x_n j)$
  where c is a constant in the language.

  In other words, *i* is *part-of j* iff for all c, the extension of c at *i* approximates the extension of c at *j*.

2 Persistence

- As information grows, will true expressions remain true, will false expressions remain false, or will there be sentences whose truth value is instable?

2.1 ≤-*persistent*

- relational term *A* is ≤-*persistent* if
  $\forall ij(i \leq j \rightarrow \forall x_1 \ldots x_n(Ax_1 \ldots x_n i \sqsubseteq Ax_1 \ldots x_n j))$

  By AX9, all terms in the language are ≤-*persistent*.

  It can be shown, therefore, that all translations of sentences are ≤-*persistent*.

- Is this desirable?

2.2 Quantification

- Suppose that the domain of quantification is the union of a predicate's denotation and antidenotation.

  Then truth values might vary as situations are enlarged

  (20) Every man loves Mary

  (21) Some woman talks

  (22) The woman does not talk

- Solution: redefine *every*, *some*, and *the*

- Existence predicate

  Let E be a non-logical constant of type $\langle es \rangle$

  Exi means 'x exists in situation i'

  AX10 Exi is either true or false. Not both; not neither.

  Therefore, $\forall ij(i \leq j \rightarrow \forall(Exi = Exj)$

- Inclusion between situations

  Consider for each relational constant in L the formula:

  $\forall x_1 \ldots x_n((Ex_{\sigma 1}i \wedge \ldots \wedge Ex_{\sigma m}i) \rightarrow cx_1 \ldots x_n i = cx_1 \ldots x_n j)$
  where x_σ are the type e variables in $\{x_1, \ldots, x_n\}$

  E.g., $\forall x(Exi \rightarrow walk\ xi = walk\ xj)$

  Let Ξ be the conjunction of all such formula for L

  Let $\subseteq$ be a constant of type $\langle ss \rangle$ meaning *included-in*.

  AX11 $\forall ij(i \subseteq j = \Xi)$

  Since E is in L, by AX11 we know $\forall x(Exi \rightarrow Exi = Exj)$
  …equivalently: $\forall x(Exi \rightarrow Exj)$

  Therefore, $i \subseteq j$ implies that domain of i is contained in domain of j.

- Redefine quantifiers:

$$\begin{aligned}
\text{every}^{\circ} &= \lambda P_1 \lambda P_2 \lambda i \forall x\, (Exi \rightarrow (P_1 xi \rightarrow P_2 xi)) \\
\text{a}^{\circ} &= \lambda P_1 \lambda P_2 \lambda i \exists x\, (Exi \wedge P_1 xi \wedge P_2 xi) \\
\text{the}^{\circ} &= \lambda P_1 \lambda P_2 \lambda i \exists x\, (\forall y\, ((Eyi \wedge P_1 yi) \leftrightarrow x = y) \wedge P_2 xi)
\end{aligned}$$

- Intuitions in (20 – 22) reflect lack of ⊆ -*persistence*, not lack of ≤-*persistence*

2.3 Information states

- Natural language *does* exhibit some cases that defy ≤-*persistence*.

Information state 1. You are presented with two little boxes, box 1 and box 2. The boxes are closed but you know that together they contain three marbles, a blue one, a yellow one, and a red one, and that each box contains at least one of them.

Information state 2. As 1, except that in addition you know that the blue marble is in box 1. Where the other two marbles are remains a secret.

(A) The blue marble may be in box 2.
(B) If the yellow marble is in box 1, the red one is in box 2.

$$\begin{aligned}
\text{if}^{\circ} &= \lambda p \lambda q \lambda i\, (\exists j\, (j \geq i \wedge pj \wedge \neg qj) \neq \top) \\
\text{maybe}^{\circ} &= \lambda p \lambda i\, (\exists j\, (j \geq i \wedge pj) = \top)
\end{aligned}$$

3 Strong consequence and weak consequence

(1) John walks.
(2) John walks and Bill talks or Bill does not talk.
(3) Mary believes that John walks.
(4) Mary believes that John walks and Bill talks or Bill does not talk.

- Good News: we can now account for non-synonymy of (1) and (2).

- Problem, we still want to maintain an entailment relation.

- Solution: weak entailment

Strong entailment: follows from set of trees and AX

Weak entailment: follows from set of trees and *meaning postulates*

E.g., MP1 λi . i is a total and coherent situation (i.e. a *world*)

Chapter 8: Propositional Attitudes

1 Belief, doubt, knowledge and assertion

(25) Mary believes that John walks and Bill talks

(26) Mary believes that Bill talks

(27) Mary believes that John walks

(28) Mary believes that John walks or Bill talks

(29) Mary believes that Bill is a man

(30) Mary believes that a man talks

- Problem: we want to capture entailment relations

- Let *B* be a constant in our language L of type ⟨*ess*⟩.

  Bxji is true if in situation *i*, situation *j* is compatible with *x*'s beliefs.

- Redefine believe:

$$\text{believe that}^{\circ} = \lambda p \lambda x \lambda i \forall j\, (Bxji \rightarrow Tpj)$$

2 Neutral Perception

$$\text{see}^{\circ} = \lambda p \lambda x \lambda i \exists j\, (see\, xji \wedge Tpj).$$

$$\text{MP2}\ \ \lambda i \forall x \forall jk\, ((see\, xji \wedge see\, xki) \rightarrow j = k)$$

$$\text{MP3}\ \ \lambda i \forall x \forall j\, (see\, xji \rightarrow \exists k\, (k \subseteq i \wedge j \leq k))$$

(5) Mary sees John walk

(6) Mary sees John walk and Bill talk or Bill not talk

(44) Mary sees Bill talk

(45) Mary sees John not walk

(46) Mary does not see John walk

(47) Mary sees John walk and Bill talk

(48) Mary sees John walk and Mary sees Bill talk

(49) Mary sees John walk or Bill talk

(50) Mary sees John walk or Mary sees Bill talk

(51) Mary sees a man walk

a [[a man]^3[Mary [see [he$_0$ walk]^4]^{18}]^4]^{14,0}

$\lambda i \exists x\, (Exi \wedge man\, xi \wedge \exists j\, (see\ \ mary\, ji \wedge T walk\, xj))$

b *[Mary [see [[a man]^3 walk]^4]^{18}]^4

$\lambda i \exists j\, (see\ \ mary\, ji \wedge T \exists x\, (Exj \wedge man\, xj \wedge walk\, xj))$

(52) Mary sees the man walk

a [[the man]^3[Mary [see [he$_0$ walk]^4]^{18}]^4]^{14,0}

$\lambda i \exists x\, (\forall y\, ((Eyi \wedge man\, yi) \leftrightarrow x = y) \wedge \exists j\, (see\ \ mary\, ji \wedge T walk\, xj)$

b *[Mary [see [[the man]^3 walk]^4]^{18}]^4

$\lambda i \exists j\, (see\ \ mary\, ji \wedge T \exists x\, (\forall y\, ((Eyj \wedge man\, yj) \leftrightarrow x = y) \wedge walk\, xj))$

(53) Mary sees every man walk

a [[every man]^3 [Mary [see [he$_0$ walk]^4]^{18}]^4]^{14,0}

$\lambda i \forall x\, (Exi \wedge man\, xi) \rightarrow \exists j\, (see\ \ mary\, ji \wedge T walk\, xj))$

b *[Mary [see [[every man]^3 walk]^4]^{18}]^4

$\lambda i \exists j\, (see\ \ mary\, ji \wedge T \forall x\, ((Exj \wedge man\, xj) \rightarrow walk\, xj))$

(54) Mary sees no man walk

a [[no man]^3 [Mary [see [he$_0$ walk]^4]^{18}]^4]^{14,0}

$\lambda i \neg \exists x\, (Exi \wedge man\, xi \wedge \exists j\, (see\ \ mary\, ji \wedge T walk\, xj)$

b *[Mary [see [[no man]^3 walk]^4]^{18}]^4

$\lambda i \exists j\, (see\ \ mary\, ji \wedge T \neg \exists x\, (Exj \wedge man\, xj \wedge walk\, xj))$

Definition 48 An analysis tree is called *admissible* if for all its subtrees of the form $[\xi\vartheta]^{18}$ the term ϑ° is $\subseteq$-persistent.

Muskens 1995 Chapter 7 & 8
Presented by Kevin Reschke: part II

Chapter 7 cont.
Strong consequence and weak consequence

-In classical Montagovian logic, (1) and (2) are synonymous because they denote equivalent sets of possible worlds.

(1) John walks
(2) John walks and Bill talks or Bill does not talk

-Since the semantics is compositional, and (1) and (2) are synonymous, they are also interchangeable, thus (3) and (4) are equivalent.

(3) Mary believes that John walks
(4) Mary believes that John walks and Bill talks or Bill does not talk

-This runs counter to fact: (3) may be true while (4) is false.

-Partialization solves the problem. Consider the situation *i* where *walk*(*john*) is true and *talk*(*bill*) is undefined.

(1') *walks john i* = T
(2') *walks john i* ^ (*talks bill i* v ¬ *talks bill i*) = T ^ (N v ¬ N)
= T ^ (N v N)
= T ^ N
= N

-Since there is a situation *i* such that (1) and (2) are different, it follows that (1) and (2) denote different sets of possible situations, thus they are not synonymous, and thus not interchangeable. Therefore (3) does not entail (4), just as we want.

Drawbacks

-By going partial we eliminate the unwanted entailment between (3) and (4), but we also lose the desired entailment between (1) and (2). We have the intuition that in a normal world, (2) does follow from (1), yet the entailment does not hold in our partial semantics (see (1') and (2')).

Solution: distinguish strong and weak entailment

-synonymy (mutual entailment) surfaces in judgments of interchangeability in natural language. Pairs of sentences fall into one of three categories.

i) always interchangeable

(5) John walked and Bill talked
(6) Bill talked and John walked
(7) Mary believes that John walked and Bill talked
(8) Mary believes that Bill talked and John walked

ii) only interchangeable in non-intensional contexts

cf. (1 – 4)

iii) never interchangeable

(9) John walked
(10) Bill talked

-type (i) sentence pairs stand in *strong entailment* relations. Type (ii) sentence pairs stand in *weak entailment* relations. Type (iii) sentence pairs have no entailment relation.

Definitions

Strong entailment

X strongly entails Y iff for every model that satisfies the basic axioms (e.g. p.11 AX1-8), and for all possible situations *i*, the extension of X is included in the extension of Y.

Weak entailment

-the axioms restrict the type of models that we consider. We introduce meaning postulates (MP) to restrict the type of situations we consider.

For example,

MP1: *i* is total and complete

(i.e., for all c in L, for all $x_1 \ldots x_n(cx_1 \ldots x_n i$ = T or F)

X weakly entails Y if for any valid model, and for every *i* that satisfies MP, the extension of X is included in the extension of Y.

(1) John walks
(2) John walks and Bill talks or Bill doesn't talk

-Recall our counterevidence for (1) entailing (2) was the situation *i* such that *talks*(*bill*) = N. But *i* does not satisfy MP1, thus we ignore it for weak entailment. (1) does weakly entail (2).

Summary

-The notion of strong entailment in a partial semantics adequately accounts for the non-interchangeability of classically synonymous expressions in intensional contexts. However, this notion misses the intuition that these expressions are interchangeable in normal contexts. The notion of weak entailment account for this normal interchangeability.

Chapter 8
Propositional Attitudes

Purpose: to show that verb translations, axioms, and meaning postulates can be defined within our partial worlds semantics to account for the predicates believe, assert, know, doubt, and see. Here I'll cover believe and see.

Believe

(25) Mary believes that John walks and Bill talks
(26) Mary believes that Bill talks

-Intuition: (25) entails (26). But current system does not capture this.

-Solution: we define believe in Hintikka's way:

Let *Bxji* mean that in situation *i* situation *j* is compatible with *x*'s beliefs.

$believe\ that^{o} = \lambda p \lambda x \lambda i \forall j(Bxji \rightarrow Tpj)$

Example:

(26) $\lambda i \forall j(Bmary\ ji \rightarrow T\ walk\ bill\ j)$

First, note that there are four possibilities for *j*.

I) *B mary ji* = T
II) *B mary ji* = F
III) *B mary ji* = N
IV) *B mary ji* = B

(I and IV: doxastic options. I and III: doxastic alternatives)

-Truth conditions for (26)

$X \rightarrow Y = \neg X \vee Y$

			(*Tpj*)	
			T	**F**
	I	**T**	T	F
(*Bmary ji*)	**II**	**F**	T	T
j-type	**III**	**N**	T	N
	IV	**B**	T	B

-(26) is true at *i* just if Bill walks at all Mary's doxastic alternatives; it will be false just if Bill does not walk at some of Mary's doxastic options

-Now we show that (25) entails (26)

i) Suppose (25) is true.

ii) Then *T*(*walks john ^ talks bill*)*j* for all *j* of type I and III (Mary's doxastic alternatives).

iii) *T*(*walks john ^ talks bill*) => *T*(*talks bill*)

iv) So, *T*(*talks bill*) for all *j* of type I and III (Mary's doxastic alternatives).

v) Thus "Mary believes that Bill talks" is true.

See

$see^{o} = \lambda p \lambda x \lambda i \exists j(see\ xji \wedge Tpj)$

(5) Mary sees John walk

True in *i* just if there is a situation *j* that Mary sees and John walks is true in *j*.

-Note that this definition only requires that the proposition be true in the situation *j* which represents Mary's visual field. However, we have an intuition that *p* also holds in our own world *i*. Solution: add a meaning postulate to relate the filed of view to the real world.

MP3: $\lambda i \forall x \forall j(see\ xji \rightarrow \exists k\ (k \subseteq i \wedge j \leq k))$

I.e., the field of vision *j* must be part of a situation *k* that is included in the real world *i*.

(Recall: *j* is part of *k* if *j* approximates *k* for all terms. In other words, *k* is a more defined version of *j*.
Recall: *k* is included in *i* if *k*'s domain is a subset of *i*'s domain.)

-We now show that (5) weakly entails "John walks"

i) Suppose that *walk john j* = T

ii) by MP3, *j* is part of *k*, thus *walk john k* = T

iii) by MP3, *k* is included in *i*, thus *walk john i* = T

-Suppose we lacked MP3.
This would allow two unattested scenarios.
i) *walk john j* = T, but *walk john i* = N
ii) *walk john j* = T, but *john* is not in domain of *i*.