Handout 3: PTQ Revisited (Muskens 1995, Ch. 4)

Semantics C (Spring 2010)

Montague's PTQ article (Montague 1973), the paper in which he gave his 'Proper Treatment of Quantification', functions as the paradigm of Montague Grammar:

- a fragment of ordinary English was provided with a semantics, via a translation into the logic IL

Plan – we take a second look at the PTQ fragment and:

- reformulate its syntax
  - we define certain structures called *analysis trees*: languages are ambiguous and expressions may have more than one reading – analysis trees represent those readings
  - we give an inductive assignment of *phrases* to analysis trees: if a phrase is assigned to an analysis tree, the tree will be a reading of that phrase
- give it a simplified semantics on the basis of our relational type logic TT_2
  - analysis trees will be translated into TT_2 by a separate induction
  - this will associate truth conditions and other semantic values with trees and will induce a relation of entailment on them
  - if a tree with certain truth conditions functions as a possible reading of a sentence, we can say that the sentence has those truth conditions given that particular reading
  - it will also be possible to characterize an argument as valid or invalid, given readings of its premises and conclusion
  - the relation of entailment that is induced on the fragment by this procedure is provably orthodox: it equals the entailment relation given in the textbook Dowty et al. (1981)[1]

1 Syntactic Categories and Analysis Trees

The syntactic system described here – a categorial grammar:

- all syntactic objects are required to have some category, just as all objects in type logic are subsumed under some type

The set of categories is defined in the following way.

(1) (Categories)

i. E is a category; S is a category;

[1]The main difference between Montague's own semantics for the PTQ fragment and that given in DWP concerns the use of 'individual concepts' (type se functions). Montague employed these in order to circumvent certain difficulties with sentences like Partee's *The temperature is ninety but it is rising*, but DWP skip the use of individual concepts altogether, following Bennett (1974), who saw that these did not only complicate the theory considerably, but moreover created as many problems as they were supposed to solve. For a careful discussion see DWP; for a contrary opinion Janssen (1984). We accept Bennett's Simplification here. However, it is possible to reintroduce individual concepts in the present relational setting by treating them as $\langle se\rangle$ type relations that happen to be functional. See also Chapter 9 where individual concepts are reintroduced in a somewhat different manner.

ii. If A and B are categories and $A \neq E$, then A/B and $A//B$ are categories.

The idea: an object of category A/B or category $A//B$ combines with an object of category B to an object of category A.

For example:

- if Mary is assigned to category $S/(S/E)$ and run is given category S/E, then the two expressions can combine into Mary runs.

Table 1 lists the categories we actually use, the way in which we abbreviate these categories and their traditional name.

Category A	Abbreviation	Traditional name
S		Sentence
S/E	IV	Verb Phrase/Intransitive Verb
$S//E$	CN	Common Noun
S/S		Sentence Adverb
S/IV	T or NP	Noun Phrase/Proper Name
IV/S		Sentence-complement Verb
IV/IV	IAV	Verb Phrase Adverb
$IV//IV$		Infinitive-complement Verb
IV/T	TV	Transitive Verb
T/CN	DET	Determiner
IAV/T		Preposition

Table 1: Syntactic categories

Next step: define the lexicon of our fragment.

Each category A comes with a set of basic expressions B_A.

(2) (Basic Expressions)

$$
\begin{array}{lcl}
B_{IV} & = & \{\text{run}, \text{walk}, \text{talk}\} \\
B_{CN} & = & \{\text{man}, \text{woman}, \text{park}, \text{fish}, \text{pen}, \text{unicorn}\} \\
B_{S/S} & = & \{\text{necessarily}\} \\
B_{T} & = & \{\text{John}, \text{Mary}, \text{Bill}, \text{he}_0, \text{he}_1, \text{he}_2, \ldots\} \\
B_{IV/S} & = & \{\text{believe that}, \text{assert that}\} \\
B_{IAV} & = & \{\text{rapidly}, \text{slowly}, \text{voluntarily}, \text{allegedly}\} \\
B_{IV//IV} & = & \{\text{try to}, \text{wish to}\} \\
B_{TV} & = & \{\text{find}, \text{lose}, \text{eat}, \text{love}, \text{date}, \text{be}, \text{seek}, \text{conceive}\} \\
B_{DET} & = & \{\text{every}, \text{the}, \text{a}\} \\
B_{IAV/T} & = & \{\text{in}, \text{about}\} \\
B_{A} & = & \emptyset \quad \text{if } A \text{ is any category other than those mentioned above}
\end{array}
$$

The words in this lexicon can be combined into larger units by means of certain modes of combination:

- each clause in the definition below corresponds to one such mode of combination
- analysis trees simply summarize the basic combinatorics of an expression (they do not capture all syntactic information and there is no semantic information)

(3) (Analysis Trees) For each category A the set AT_A of *analysis trees* of category A is defined as follows.

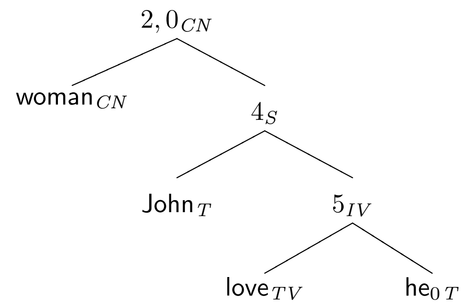
Basic rule

G1. $B_A \subseteq AT_A$ for every category A.

Relative clause rule For each natural number n:

G2. If $\xi \in AT_{CN}$ and $\vartheta \in AT_S$ then $[\xi\vartheta]^{2,n} \in AT_{CN}$.

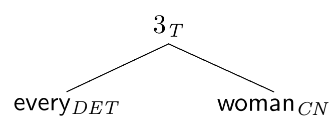
[woman [John [love he_0]^5]^4]^{2,0}



Rules of functional application

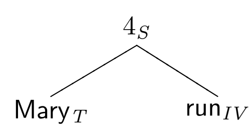
G3. If $\xi \in AT_{DET}$ and $\vartheta \in AT_{CN}$ then $[\xi\vartheta]^3 \in AT_T$.

[every woman]^3



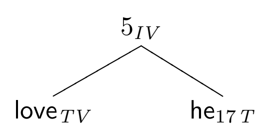
G4. If $\xi \in AT_T$ and $\vartheta \in AT_{IV}$ then $[\xi\vartheta]^4 \in AT_S$.

[Mary run]^4



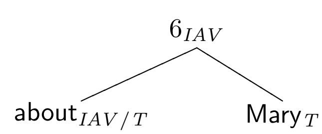
G5. If $\xi \in AT_{TV}$ and $\vartheta \in AT_T$ then $[\xi\vartheta]^5 \in AT_{IV}$.

[love he_{17}]^5



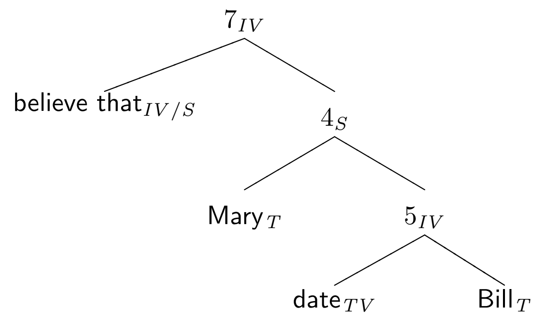
G6. If $\xi \in AT_{IAV/T}$ and $\vartheta \in AT_T$ then $[\xi\vartheta]^6 \in AT_{IAV}$.

[about Mary]^6



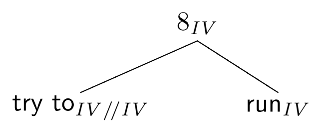
G7. If $\xi \in AT_{IV/S}$ and $\vartheta \in AT_S$ then $[\xi\vartheta]^7 \in AT_{IV}$.

[believe that [Mary [date Bill]^5]^4]^7



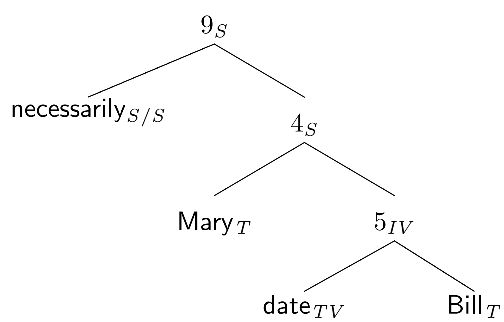
G8. If $\xi \in AT_{IV//IV}$ and $\vartheta \in AT_{IV}$ then $[\xi\vartheta]^8 \in AT_{IV}$.

[try to run]^8



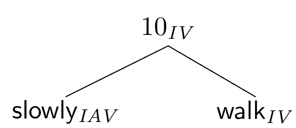
G9. If $\xi \in AT_{S/S}$ and $\vartheta \in AT_S$ then $[\xi\vartheta]^9 \in AT_S$.

[necessarily [Mary [date Bill]^5]^4]^9



G10. If $\xi \in AT_{IAV}$ and $\vartheta \in AT_{IV}$ then $[\xi\vartheta]^{10} \in AT_{IV}$.

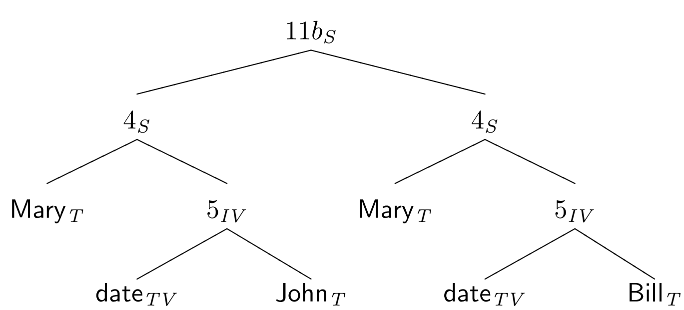
[slowly walk]^{10}



Rules of conjunction and disjunction

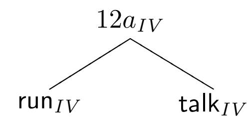
G11. If $\xi, \vartheta \in AT_S$ then $[\xi\vartheta]^{11a}$, $[\xi\vartheta]^{11b} \in AT_S$.

[[Mary [date John]^5]^4 [Mary [date Bill]^5]^4]^{11b}



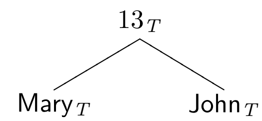
G12. If $\xi, \vartheta \in AT_{IV}$ then $[\xi\vartheta]^{12a}$, $[\xi\vartheta]^{12b} \in AT_{IV}$.

[run talk]^{12a}



G13. If $\xi, \vartheta \in AT_T$ then $[\xi\vartheta]^{13} \in AT_T$.

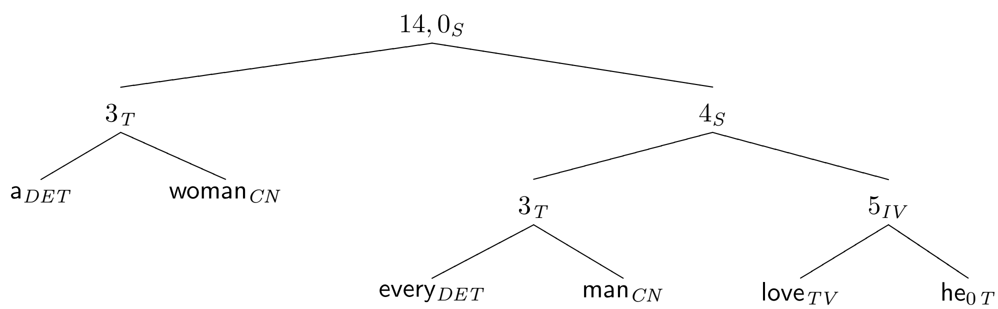
[Mary John]^{13}



Quantification rules For each natural number n:

G14. If $\xi \in AT_T$ and $\vartheta \in AT_S$ then $[\xi\vartheta]^{14,n} \in AT_S$.

[[a woman]^3 [[every man]^3 [love he_0]^5]^4]^{14,0}



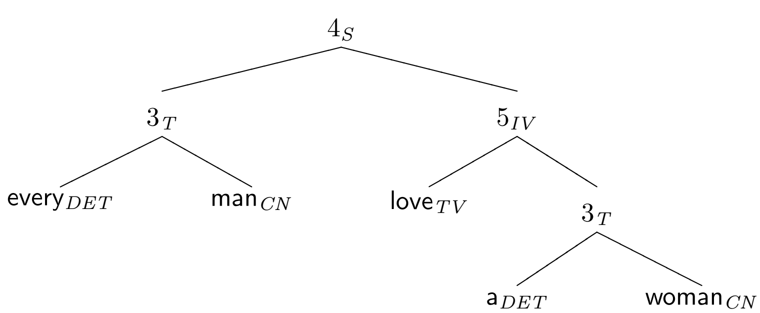
G15. If $\xi \in AT_T$ and $\vartheta \in AT_{CN}$ then $[\xi\vartheta]^{15,n} \in AT_{CN}$.
G16. If $\xi \in AT_T$ and $\vartheta \in AT_{IV}$ then $[\xi\vartheta]^{16,n} \in AT_{IV}$.

Negation and tense rules

G17. If $\xi \in AT_T$ and $\vartheta \in AT_{IV}$ then $[\xi\vartheta]^{17a}$, $[\xi\vartheta]^{17b}$, $[\xi\vartheta]^{17c}$, $[\xi\vartheta]^{17d}$, $[\xi\vartheta]^{17e} \in AT_S$.

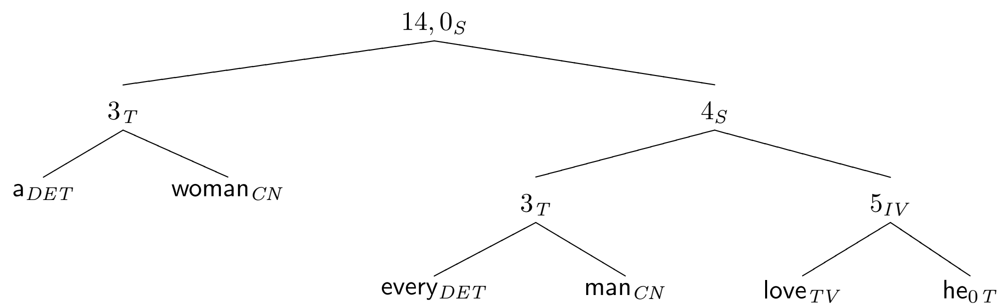
For example, typical examples of quantifier scope ambiguities are captured in terms of different analysis trees (both elements of AT_S):

(4) a. [[every man]^3 [love [a woman]^3]^5]^4
b.



(5) a. [[a woman]^3 [[every man]^3 [love he_0]^5]^4]^{14,0}

b.



2 Syntax of the Fragment

Our main interest lies in the semantic interpretation of analysis trees – but we must assign them English for our syntax to be complete.

- we do this by letting the expression that is to be assigned to a complex tree be a function of the expressions that are assigned to its parts
- the functions we use are Montague's operations F_3–F_{15}
- using these functions we can give a compositional assignment of phrases to trees – we only need to stipulate which operation is used in which case
- Montague's syntax is awkward and is not the reason for studying Montague Grammar; however, it does show the basics of how a categorial syntax can be coupled with Montagovian semantics; we'll give a slightly modified version of Montague's original syntax
- all modern syntactic theories (CG, HPSG, MP etc.) are usually coupled with a version Montague semantics

(6) (Syntactic operations) Let γ and δ be strings. Define:

$F_{3,n}(\gamma, \delta) = \gamma$ such that δ'; and δ' comes from δ by replacing each occurrence of he_n or him_n by he/she/it or him/her/it respectively, according as the first B_{CN} in γ is of masc./fem./neuter gender.

$F_4(\gamma, \delta) = \gamma\delta'$, and δ' is the result of replacing the main verbs in δ by their third person singular present.

$F_5(\gamma, \delta) = \gamma\delta$ if δ does not have the form he_n, otherwise $F_5(\gamma, \text{he}_n) = \gamma\text{him}_n$.

$F_6(\gamma, \delta) = F_7(\delta, \gamma) = \gamma\delta$.

$F_8(\gamma, \delta) = \gamma$ and δ.

$F_9(\gamma, \delta) = \gamma$ or δ.

$F_{10,n}(\gamma, \delta)$ comes from δ by replacing the first occurrence of he_n or him_n by γ and all other occurrences of he_n or him_n by he/she/it or him/her/it respectively, according as the first B_{CN} or B_T in γ is masc./fem./neuter, if γ does not have the form he_k, otherwise $F_{10,n}(\text{he}_k, \delta)$ comes from δ by replacing all occurrences of he_n or him_n by he_k or him_k respectively.

$F_{11}(\gamma, \delta) = \gamma\delta'$ and δ' is the result of replacing the first verb in δ by its negative third person singular present.

$F_{12}(\gamma, \delta) = \gamma\delta''$ and δ'' is the result of replacing the first verb in δ by its third person singular future.

$F_{13}(\gamma, \delta) = \gamma\delta'''$ and δ''' is the result of replacing the first verb in δ by its negative third person singular future.

$F_{14}(\gamma, \delta) = \gamma\delta''''$ and δ'''' is the result of replacing the first verb in δ by its third person singular present perfect.

$F_{15}(\gamma, \delta) = \gamma\delta'''''$ and δ''''' is the result of replacing the first verb in δ by its negative third person singular present perfect.

(7) (Phrases) For each analysis tree ξ, define a *phrase* $\sigma(\xi)$ by induction on the complexity of analysis trees:

S1. $\sigma(\xi) = \xi$ if $\xi \in B_A$

S2–S17. If g is a rule number and $S(g)$ is as in Table 2, then $\sigma([\xi\vartheta]^g) = F_{S(g)}(\sigma(\xi), \sigma(\vartheta))$.

g	$S(g)$	g	$S(g)$	g	$S(g)$	g	$S(g)$
G2,n	$F_{3,n}$	G8	F_6	G12b	F_9	G17b	F_{12}
G3	F_6	G9	F_6	G13	F_9	G17c	F_{13}
G4	F_4	G10	F_7	G14,n	F_{10}, n	G17d	F_{14}
G5	F_5	G11a	F_8	G15,n	$F_{10,n}$	G17e	F_{15}
G6	F_5	G11b	F_9	G16,n	$F_{10,n}$		
G7	F_6	G12a	F_8	G17a	F_{11}		

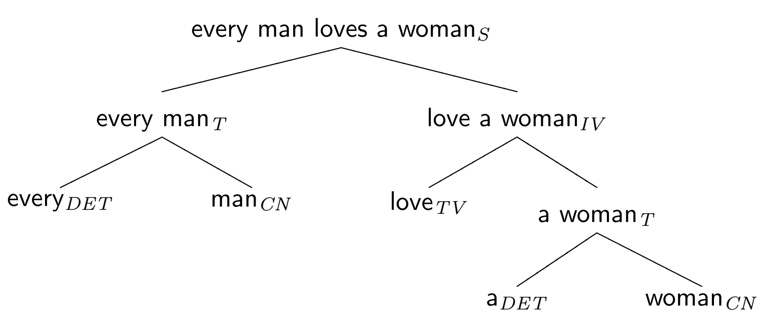
Table 2: Rule-to-operation correspondence

For example, the phrases associated with the analysis trees in (4) and (5) above are derived as follows.

(8) a.

$$\begin{aligned} \sigma([[\text{every man}]^3\ [\text{love } [\text{a woman}]^3\ ]^5\ ]^4) &= \\ F_4(F_6(\text{every}, \text{man}), F_5(\text{love}, F_6(\text{a}, \text{woman}))) &= \\ F_4(\text{every man}, \text{love a woman}) &= \\ \text{every man loves a woman} & \end{aligned}$$

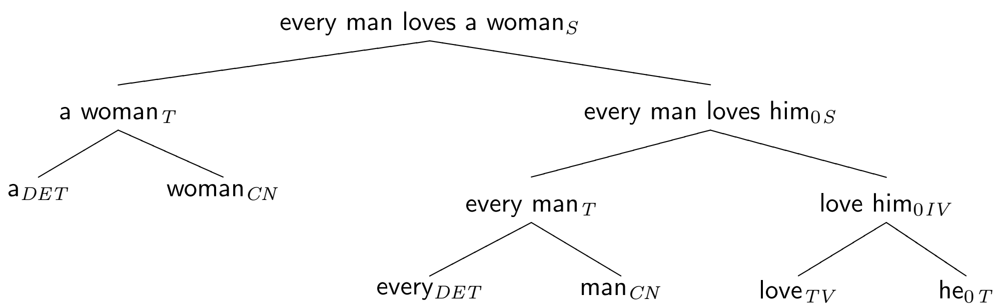
b.



(9) a.

$$\begin{aligned} \sigma([[\text{a woman}]^3\ [[\text{every man}]^3\ [\text{love he}_0]^5\ ]^4\ ]^{14,0}) &= \\ F_{10,0}(F_6(\text{a}, \text{woman}), F_4(F_6(\text{every}, \text{man}), F_5(\text{love}, \text{he}_0))) &= \\ F_{10,0}(\text{a woman}, F_4(\text{every man}, \text{love him}_0)) &= \\ F_{10,0}(\text{a woman}, \text{every man loves him}_0) &= \\ \text{every man loves a woman} & \end{aligned}$$

b.



That is, the function σ is not one-one: the same phrase may be associated with different trees.

- we say that an analysis tree ξ is a *reading for* the phrase $\sigma(\xi)$
- thus, a phrase may have different readings – and each reading consists of exactly one analysis tree

The above way of defining the syntax of the PTQ fragment differs slightly from Montague's original setup:

- we defined analysis trees first and assigned them English expressions inductively
- Montague defines English expressions directly and uses analysis trees as a way to track their construction process
- the gain of our approach: we now have a distinct language of analysis trees and we assign meanings directly to these trees
- this approach conforms to the general program Montague set out in 'Universal Grammar' (Montague 1970), where ambiguous languages are interpreted through the mediation of 'disambiguated' languages
- note also the parallel between this grammar architecture and the Y-model of syntax in GB/MP: analyses trees are spelled out on the PF branch (i.e., English expressions are associated with them) and are interpreted on the LF branch (i.e., model-theoretic objects are associated with them, possibly via translation into an intermediate logical language)
- this grammar architecture has another advantage (again, compare with GB/MP): the separation of the syntactical operations of the language (F_3–F_{15} in the present case) from the grammatical rules (G1–G17 here) makes a distinction between those parts of the grammar that are language-dependent and those that are not
- Dowty (1982) has an interesting discussion of this point and traces the idea of separating grammatical rules and syntactic operations back to Curry (1963), who calls the universal part of language 'tectogrammatics' and the language-particular part 'phenogrammatics'
- for example, grammatical relations like 'subject', 'object' and 'indirect object' are universal / tectogrammatical in nature – and such relations can be easily defined on the basis of our analysis trees (just as they are in GB/MP)

3 Semantics of the Fragment

To formalize the way in which meanings are attached to expressions of English, we give a translation function $^\circ$ that associates analysis trees with terms of our relational logic TT_2.

Given that

- the terms of our logic are interpreted model-theoretically and
- the translation is be well behaved

we can think of these translations as standing proxy for *meanings*.

Each phrase Φ is associated with a set of trees ξ by our function σ: $\{\xi \mid \sigma(\xi) = \Phi\}$. We associate Φ with a set of meanings by translating each tree ξ into TT_2 – the resulting TT_2 term is ξ°.

Thus, the set of meanings associated with a phrase Φ is:

$$\{\xi^\circ \mid \sigma(\xi) = \Phi\}$$

3.1 Category-to-Type Correspondence

Analysis trees of syntactic category A are translated into terms of a fixed type that can be obtained from A as follows:

(10) (Category-to-type Rule)

i. $\tau(E) = e$; $\tau(S) = \langle s \rangle$;

ii. $\tau(A/B) = \tau(A//B) = \tau(B) * \tau(A)$,
where $\beta * \langle \alpha_1 \dots \alpha_n \rangle = \langle \beta\alpha_1 \dots \alpha_n \rangle$ for all β, α_1, ..., α_n.

The idea:

- the meaning of a sentence is a proposition – a type $\langle s \rangle$ object, i.e., a set of indices
- the meaning of an expression of category E (these do not actually occur in the fragment) is a possible individual
- the meaning of an expression of any category expecting a B in order to form an A has a type that expects a $\tau(B)$ to form a $\tau(A)$
- the translation of an expression will be its meaning, i.e., its intension, not its extension
- the extension of an expression at any index can always be obtained from its intension

Lewis (1974) gives the following category-to-type rule (using functional types):

i. $\tau_L(E) = se$; $\tau_L(S) = st$;

ii. $\tau_L(A/B) = \tau_L(A//B) = (\tau_L(B)\tau_L(A))$

Adopting Bennett's Simplification – from *se* to *e*, we obtain the following rule:

i. $\tau'(E) = e$; $\tau'(S) = st$;

ii. $\tau'(A/B) = \tau'(A//B) = (\tau'(B)\tau'(A))$

Our category-to-type rule for TT_2 is equivalent to this last one in the sense that $\tau'(A) = \Sigma(\tau(A))$, where the function Σ is as in Chapter 2.

The extensional version of the τ' function is defined below:

i. $\tau'_{ext}(E) = e$; $\tau'_{ext}(S) = t$;

ii. $\tau'_{ext}(A/B) = \tau'_{ext}(A//B) = (\tau'_{ext}(B)\tau'_{ext}(A))$

Table 3 provides the values of the function τ for those categories that are actually used in the PTQ fragment. The third column gives the functional types that are assigned to these categories in Lewis (1974) (with Bennett's simplification) and the fourth column gives the corresponding extensional types.

To see that objects of type $\tau(A)$ are the kind of objects one would like to assign to expressions of category A, we can use the slice functions discussed previously. For example:

Category A	$\tau(A)$	$\tau'(A)$	$\tau'_{ext}(A)$
S	$\langle s\rangle$	st	t
S/E	$\langle es\rangle$	$e(st)$	et
$S//E$	$\langle es\rangle$	$e(st)$	et
S/S	$\langle\langle s\rangle s\rangle$	$(st)(st)$	tt
S/IV	$\langle\langle es\rangle s\rangle$	$(e(st))(st)$	$(et)t$
IV/S	$\langle\langle s\rangle es\rangle$	$(st)(e(st))$	$t(et)$
IV/IV	$\langle\langle es\rangle es\rangle$	$(e(st))(e(st))$	$(et)(et)$
$IV//IV$	$\langle\langle es\rangle es\rangle$	$(e(st))(e(st))$	$(et)(et)$
IV/T	$\langle\langle\langle es\rangle s\rangle es\rangle$	$((e(st))(st))(e(st))$	$((et)t)(et)$
DET	$\langle\langle es\rangle\langle es\rangle s\rangle$	$(e(st))((e(st))(st))$	$(et)((et)t)$
IAV/T	$\langle\langle\langle es\rangle s\rangle\langle es\rangle es\rangle$	$((e(st))(st))((e(st))(e(st)))$	$((et)t)((et)(et))$

Table 3: Categories and corresponding TT_2 and functional types (intensional and extensional)

- one would arguably like the intension of a CN or an IV to be a property of individuals, i.e., a function from possible worlds to sets of entities
- the second slice function of any type $\langle es\rangle$ object is just this kind of thing
- the meaning of a term is a property of properties (a quantifier), a function from possible worlds to sets of properties
- the meaning of an IV/IV is a function from properties to properties
- the intension of a determiner can be seen as:
  - a function from properties to quantifiers (use the first slice function)
  - a so-called relation-in-intension between properties, a function from possible worlds to relations between properties (use the third slice function)

Examples:

Constants	Type
john, *bill*, *mary*	e
run, *walk*, *talk*	$\langle es\rangle$
man, *woman*, *park*, *fish*, *pen*, *unicorn*	$\langle es\rangle$
believe, *assert*	$\langle\langle s\rangle es\rangle$
find, *lose*, *eat*, *love*, *date*	$\langle ees\rangle$
seek, *conceive*	$\langle\langle\langle es\rangle s\rangle es\rangle$
rapidly, *slowly*, *voluntarily*, *allegedly*	$\langle\langle es\rangle es\rangle$
try, *wish*	$\langle\langle es\rangle es\rangle$
in	$\langle\langle\langle es\rangle s\rangle\langle es\rangle es\rangle$
about	$\langle e\langle es\rangle es\rangle$
$<, \approx$	$\langle ss\rangle$

Table 4: Constants and their types

3.2 Translation Rules

We now can translate each expression of the fragment into type theory inductively by:

- giving translations to all lexical items

- specifying how the translation of a complex expression is to depend on the translations of its parts

Notational conventions:

- x, y and z range over individuals (type e)
- i and j range over indices (type s)
- p and q range over propositions (type $\langle s \rangle$)
- P ranges over properties (type $\langle es \rangle$)
- Q ranges over quantifiers (type $\langle\langle es\rangle s\rangle$)

(11) (Translation) Let the constants in the first column of Table 4 have types as indicated in the second column. For each analysis tree ξ define its translation ξ° by induction on the complexity of analysis trees:

Basic rule

T1. $\mathsf{run}^\circ = run$, $\mathsf{walk}^\circ = walk$, $\mathsf{talk}^\circ = talk$;
$\mathsf{John}^\circ = \lambda P\,(P john)$, $\mathsf{Mary}^\circ = \lambda P\,(P mary)$,
$\mathsf{Bill}^\circ = \lambda P\,(P bill)$, $\mathsf{he}_n^\circ = \lambda P\,(P x_n)$;
$\mathsf{believe\ that}^\circ = believe$, $\mathsf{assert\ that}^\circ = assert$
$\mathsf{find}^\circ = \lambda Q \lambda y\, Q(\lambda x\,(find\, xy))$, $\mathsf{lose}^\circ = \lambda Q \lambda y\, Q(\lambda x\,(lose\, xy))$,
$\mathsf{eat}^\circ = \lambda Q \lambda y\, Q(\lambda x\,(eat\, xy))$, $\mathsf{love}^\circ = \lambda Q \lambda y\, Q(\lambda x\,(love\, xy))$,
$\mathsf{date}^\circ = \lambda Q \lambda y\, Q(\lambda x\,(date\, xy))$, $\mathsf{be}^\circ = \lambda Q \lambda y Q(\lambda x \lambda i\,(x = y))$,
$\mathsf{seek}^\circ = seek$, $\mathsf{conceive}^\circ = conceive$
$\mathsf{rapidly}^\circ = rapidly$, $\mathsf{slowly}^\circ = slowly$
$\mathsf{voluntarily}^\circ = voluntarily$, $\mathsf{allegedly}^\circ = allegedly$;
$\mathsf{try\ to}^\circ = try$, $\mathsf{wish\ to}^\circ = wish$;
$\mathsf{man}^\circ = man$, $\mathsf{woman}^\circ = woman$ $\mathsf{park}^\circ = park$,
$\mathsf{fish}^\circ = fish$, $\mathsf{pen}^\circ = pen$, $\mathsf{unicorn}^\circ = unicorn$;
$\mathsf{every}^\circ = \lambda P_1 \lambda P_2 \lambda i\, \forall x(P_1 xi \to P_2 xi)$,
$\mathsf{a}^\circ = \lambda P_1 \lambda P_2 \lambda i\, \exists x(P_1 xi \wedge P_2 xi)$,
$\mathsf{the}^\circ = \lambda P_1 \lambda P_2 \lambda i\, \exists x(\forall y(P_1 yi \leftrightarrow x = y) \wedge P_2 xi)$;
$\mathsf{necessarily}^\circ = \lambda p \lambda i\, \forall j (pj)$;
$\mathsf{in}^\circ = \lambda Q \lambda P \lambda y\, Q(\lambda x\,(in\, xPy))$, $\mathsf{about}^\circ = about$

Relative clause rule. For each natural number n:

T2. $([\xi\vartheta]^{2,n})^\circ = \lambda x_n \lambda i\,(\xi^\circ x_n i \wedge \vartheta^\circ i)$;

Rules of functional application.

T3–T10. $([\xi\vartheta]^{k})^\circ = \xi^\circ \vartheta^\circ$ if $3 \leq k \leq 10$;

Rules of conjunction and disjunction.

T11. $([\xi\vartheta]^{11a})^\circ = \lambda i\,(\xi^\circ i \wedge \vartheta^\circ i)$;
$([\xi\vartheta]^{11b})^\circ = \lambda i\,(\xi^\circ i \vee \vartheta^\circ i)$;

T12. $([\xi\vartheta]^{12a})^\circ = \lambda x \lambda i\,(\xi^\circ xi \wedge \vartheta^\circ xi)$;
$([\xi\vartheta]^{12b})^\circ = \lambda x \lambda i\,(\xi^\circ xi \vee \vartheta^\circ xi)$;

T13. $([\xi\vartheta]^{13})^\circ = \lambda P \lambda i(\xi^\circ Pi \vee \vartheta^\circ Pi)$;

Quantification rules. For each natural number n:

T14. $([\xi\vartheta]^{14,n})^\circ = \xi^\circ \lambda x_n\,(\vartheta^\circ)$;

T15. $([\xi\vartheta]^{15,n})^\circ = \lambda y(\xi^\circ \lambda x_n\,(\vartheta^\circ y))$;

T16. $([\xi\vartheta]^{16,n})^\circ = \lambda y\,(\xi^\circ \lambda x_n\,(\vartheta^\circ y))$;

Negation and tense rules.

T17. $([\xi\vartheta]^{17a})^\circ = \lambda i \neg \xi^\circ \vartheta^\circ i;$
$([\xi\vartheta]^{17b})^\circ = \lambda i \exists j\, (i < j \wedge i \approx j \wedge \xi^\circ \vartheta^\circ j);$
$([\xi\vartheta]^{17c})^\circ = \lambda i \neg\exists j\, (i < j \wedge i \approx j \wedge \xi^\circ \vartheta^\circ j);$
$([\xi\vartheta]^{17d})^\circ = \lambda i \exists j\, (j < i \wedge i \approx j \wedge \xi^\circ \vartheta^\circ j);$
$([\xi\vartheta]^{17e})^\circ = \lambda i \neg\exists j\, (j < i \wedge i \approx j \wedge \xi^\circ \vartheta^\circ j).$

The translation of (8) is provided below.

(12) a. $\mathsf{a}^\circ = \lambda P_1 \lambda P_2 \lambda i\, \exists x (P_1 x i \wedge P_2 x i)$

b. $\mathsf{woman}^\circ = woman$

c. $([\mathsf{a\ woman}]^3)^\circ = \lambda P_1 \lambda P_2 \lambda i\, \exists x (P_1 x i \wedge P_2 x i)\ [woman] \rightsquigarrow$
$\lambda P_2 \lambda i\, \exists x (woman\, xi \wedge P_2 x i)$

d. $\mathsf{love}^\circ = \lambda Q \lambda y\, Q(\lambda x\, (love\, xy))$

e. $([\mathsf{love}\ [\mathsf{a\ woman}]^3\ ]^5)^\circ \rightsquigarrow$
$\lambda Q \lambda y\, Q(\lambda x\, (lovexy))\ [\lambda P_2 \lambda i \exists x\, (woman\, xi \wedge P_2 x i)] \rightsquigarrow$
$\lambda y\, (\lambda P_2 \lambda i\, \exists x (woman\, xi \wedge P_2 x i)\ [\lambda x (love\, xy)]) \rightsquigarrow$
$\lambda y \lambda i\, \exists x (woman\, xi \wedge (\lambda x\, (love\, xy)\ [xi])) \rightsquigarrow$
$\lambda y \lambda i\, \exists x (woman\, xi \wedge love\, xyi))$

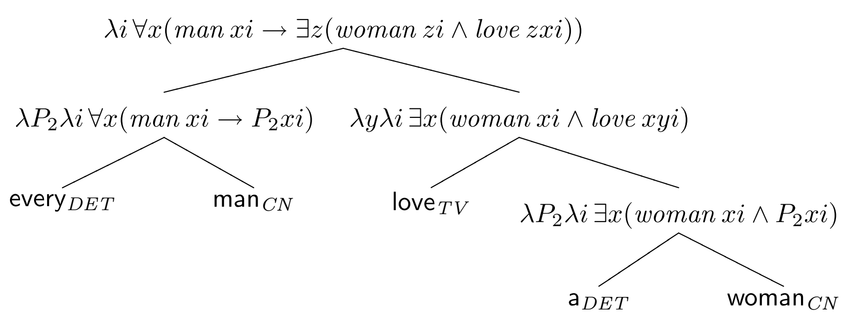
f. $\mathsf{every}^\circ = \lambda P_1 \lambda P_2 \lambda i\, \forall x (P_1 x i \rightarrow P_2 x i)$

g. $\mathsf{man}^\circ = man$

h. $([\mathsf{every\ man}]^3)^\circ = \lambda P_1 \lambda P_2 \lambda i\, \forall x (P_1 x i \rightarrow P_2 x i)\ [man] \rightsquigarrow$
$\lambda P_2 \lambda i\, \forall x (man\, xi \rightarrow P_2 x i)$

i. $([[\mathsf{every\ man}]^3\ [\mathsf{love}[\mathsf{a\ woman}]^3\ ]^5\ ]^4)^\circ \rightsquigarrow$
$\lambda P_2 \lambda i\, \forall x (man\, xi \rightarrow P_2 x i)\ [\lambda y \lambda i\, \exists x (woman\, xi \wedge love\, xyi)] \rightsquigarrow$
$\lambda i\, \forall x (man\, xi \rightarrow (\lambda y \lambda i\, \exists x (woman\, xi \wedge love\, xyi)\ [xi])) \rightsquigarrow$
$\lambda i\, \forall x (manxi \rightarrow (\lambda y \lambda i\, \exists z (woman\, zi \wedge love\, zyi)\ [xi])) \rightsquigarrow$
$\lambda i\, \forall x (man\, xi \rightarrow \exists z (woman\, zi \wedge love\, zxi))$

(13)



The translation of (9) is provided below.

(14) a. $\mathsf{love}^\circ = \lambda Q \lambda y\, Q(\lambda x\, (love\, xy))$

b. $\mathsf{he}_0^\circ = \lambda P\, (P x_0)$

c. $([\mathsf{love\ he}_0]^5)^\circ = \lambda Q \lambda y\, Q(\lambda x\, (love\, xy))\ [\lambda P (P x_0)] \rightsquigarrow$
$\lambda y\, (\lambda P\, (P x_0)\ [\lambda x\, (love\, xy)]) \rightsquigarrow$
$\lambda y\, (\lambda x\, (love\, xy)\ [x_0]) \rightsquigarrow$
$\lambda y\, (love\, x_0 y) \rightsquigarrow$
$love\, x_0$

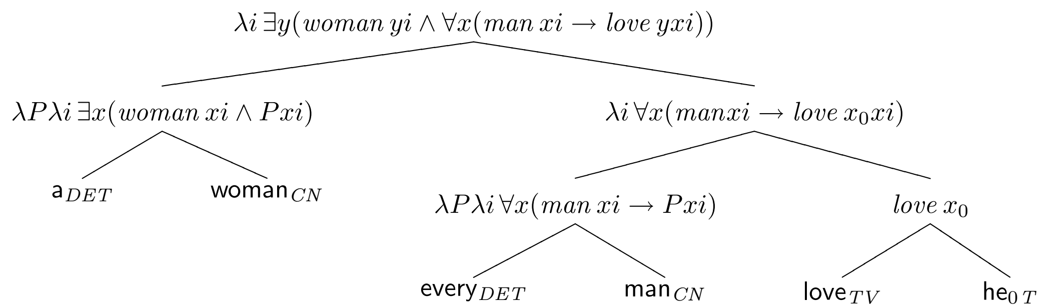
d. $([\mathsf{every\ man}]^3)^\circ \rightsquigarrow$
$\lambda P \lambda i\, \forall x (man\, xi \rightarrow P x i)$

e. $([[\mathsf{every\ man}]^3\ [\mathsf{love\ he}_0]^5\ ]^4)^\circ \rightsquigarrow$
$\lambda P \lambda i\, \forall x (man\, xi \rightarrow P x i)\ [love\, x_0] \rightsquigarrow$
$\lambda i\, \forall x (manxi \rightarrow love\, x_0 x i)$

f. $([\mathsf{a\ woman}]^3)^\circ \leadsto$
$\lambda P \lambda i\, \exists x(woman\, xi \wedge Pxi)$

g. $([[\mathsf{a\ woman}]^3\ [[\mathsf{every\ man}]^3\ [\mathsf{love\ he_0}]^5\ ]^4\ ]^{14,0})^\circ \leadsto$
$\lambda P \lambda i\, \exists x(woman\, xi \wedge Pxi)\ [\lambda x_0 \lambda i\, \forall x(man\, xi \rightarrow love\, x_0 xi)] \leadsto$
$\lambda P \lambda i\, \exists y(woman\, yi \wedge Pyi)\ [\lambda x_0 \lambda i\, \forall x(man\, xi \rightarrow love\, x_0 xi)] \leadsto$
$\lambda i\, \exists y(woman\, yi \wedge (\lambda x_0 \lambda i\, \forall x(man\, xi \rightarrow love\, x_0 xi)\ [yi])) \leadsto$
$\lambda i\, \exists y(woman\, yi \wedge \forall x(man\, xi \rightarrow love\, yxi))$

(15)



4 Entailment

A precise definition of the notion of logical consequence on our natural language fragment (in all categories – we need not restrict it to sentences) is provided below. Notational convention:

- if Γ and Δ are sets of terms of type $\langle \alpha_1 \ldots \alpha_n \rangle$, we write $\Gamma \models_{\mathrm{AX}} \Delta$ for:

$$\Gamma, \{\lambda x_{\alpha_1} \ldots \lambda x_{\alpha_n} \varphi \mid \varphi \in \mathrm{AX}\} \models_s \Delta$$

where AX is the set $\{\mathrm{AX1}, \ldots, \mathrm{AX8}\}$.

We say that an analysis tree ϑ of any category *follows from* a set of trees Ξ of the same category if and only if it holds that $\Xi^\circ \models_{\mathrm{AX}} \vartheta^\circ$.

For analysis trees of sentence category S, i.e., that have translations of type $\langle s \rangle$, this amounts to stipulating that (in each model in which indices behave like world-time pairs):

- at each index at which all propositions expressed by the premises are true the proposition expressed by the conclusion is true

This entailment relation is equivalent to the one given in DWP. The following theorem states this; its proof and more precise information about the translation function $'$ and set of IL sentences Δ that it mentions are given in the Appendix.

(16) For each analysis tree ξ let ξ' be the translation it is given in DWP. Let Δ be a set of DWP meaning postulates, to be specified in the Appendix, and let $\Xi \cup \{\vartheta\}$ be a set of analysis trees, then:

$$\Xi^\circ \models_{\mathrm{AX}} \vartheta^\circ \text{ in } \mathrm{TT}_2 \ \text{ iff } \ \Xi', \Delta \models \vartheta' \text{ in IL.}$$

References

Bennett, M.: 1974, *Some Extensions of a Montague Fragment of English*, Doctoral Dissertation, UCLA.

Curry, H.B.: 1963, 'Some Logical Aspects of Grammatical Structure', in *Structure of Language and its Mathematical Aspects: Proceedings of the Twelfth Symposium in Applied Mathematics*, 56–68. AMS.

Dowty, D.R.: 1982, 'Grammatical Relations and Montague Grammar', in P. Jacobson and G. Pullum (eds.), *The Nature of Syntactic Representation*, 79–130. Reidel, Dordrecht.

Dowty, D.R., R.E. Wall, and S. Peters: 1981, *Introduction to Montague Semantics.* Reidel, Dordrecht.

Janssen, T.: 1984, 'Individual Concepts are Useful', in F. Landman and F. Veltman (eds.), *Varieties of of Formal Semantics*, 171–192. Foris, Dordrecht.

Lewis, D.: 1974, ''Tensions', in M. Munitz and P. Unger (eds.), *Semantics and Philosophy.* New York University Press, New York.

Montague, R.: 1970, 'Universal Grammar', in *Montague (1974)*, 222–246.

Montague, R.: 1973, 'The Proper Treatment of Quantification in Ordinary English', in *Montague (1974)*, 247–270.

Montague, R.: 1974, *Formal Philosophy.* Yale University Press, New Haven.