Chapter 7. Structured Modal Reference: Modal Anaphora and Subordination

1. Introduction

This chapter shows that PCDRT can be extended to analyze structured discourse reference in the modal domain. In particular, adding a new type **w** for possible worlds is the only extension to our underlying logic Dynamic Ty2 that is needed to account for the discourse in (1) below, i.e. to derive its intuitively correct truth-conditions and explicitly capture the individual-level and modal anaphoric connections established in it.

1. **a.** [A] man cannot live without joy.
   b. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures
   (Thomas Aquinas[1]).

We will focus on only one of the meaning dimensions of this discourse, namely the *entailment* relation established by *therefore* between the modal premise (1a) and the modal conclusion in (1b)[2]. We are interested in the following features of this discourse. First, we want to capture the meaning of the entailment particle *therefore*, which relates the *content* of the premise (1a) and the *content* of the conclusion (1b) and requires the latter to be entailed by the former. I take the content of a sentence to be *truth-conditional* in nature, i.e. to be the set of possible worlds in which the sentence is true, and entailment to be *content inclusion*, i.e. (1a) entails (1b) iff for any world *w*, if (1a) is true in *w*, so is (1b)[3].

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[1] Attributed to Thomas Aquinas, http://en.wikiquote.org/wiki/Thomas_Aquinas#Attributed.

[2] For the multi-dimensionality of the meaning of *therefore*-discourses, see for example Grice (1975) and Potts (2003).

[3] I am grateful to a *Logic & Language 9* reviewer for pointing out that modeling the entailment relation expressed by *therefore* as a truth-conditional relation, i.e. as requiring inclusion between two sets of possible worlds, cannot account for the fact that the discourse *Pi is an irrational number, therefore Fermat's last theorem is true* is not intuitively acceptable as a valid entailment and it cannot be accepted as a mathematical proof despite the fact that both sentences are necessary truths (i.e. they are true in every possible world). I think that at least some of the available accounts of hyper-intensional phenomena are compatible with my proposal, so I do not see this as an insurmountable problem.

Second, we are interested in the *meanings* of (1a) and (1b). I take meaning to be *context-change potential*, i.e. to encode both content (truth-conditions) and anaphoric potential. Thus, on the one hand, we are interested in the contents of (1a) and (1b). They are both modal quantifications: (1a) involves a circumstantial modal base (to use the terminology introduced in Kratzer 1981) and asserts that, in view of the circumstances, i.e. given that God created man in a particular way, as long as a man is alive, he must find some thing or other pleasurable; (1b) involves the same modal base and elaborates on the preceding modal quantification: in view of the circumstances, if a man is alive and has no spiritual pleasure, he must have a carnal pleasure. Note that we need to make the contents of (1a) and (1b) accessible in discourse so that the entailment particle *therefore* can relate them.

On the other hand, we are interested in the anaphoric potential of (1a) and (1b), i.e. in the anaphoric connections between them. These connections are explicitly represented in discourse (2) below, which is intuitively equivalent to (1) albeit more awkwardly phrased.

2. **a.** If a^{u_1} man is alive, he$_{u_1}$ must find something^{u_2} pleasurable / he$_{u_1}$ must have a^{u_2} pleasure.

   b. Therefore, if he$_{u_1}$ doesn't have any^{u_3} spiritual pleasure, he$_{u_1}$ must have a^{u_4} carnal pleasure.

Note in particular that the indefinite *a*^{u_1} *man* in the antecedent of the conditional in (2a) introduces the dref u_1, which is anaphorically retrieved by the pronoun *he*$_{u_1}$ in the antecedent of the conditional in (2b). This is an instance of *modal subordination* (Roberts 1989), i.e. an instance of simultaneous modal and invididual-level anaphora (see Geurts 1995/1999, Frank 1996 and Stone 1999): the interpretation of the conditional in (2b) is such that it seems to covertly duplicate the antecedent of the conditional in (2a), i.e. the conditional in (2b) asserts that, if *a man is alive* and doesn't have any spiritual pleasure, he must have a carnal one.

I will henceforth analyze the simpler and more transparent discourse in (2) instead of the naturally occurring discourse in (1). The challenge posed by (2) is that, when we *compositionally* assign meanings to (**i**) the modalized conditional in (2a), i.e. the premise, (**ii**) the modalized conditional in (2b), i.e. the conclusion, and (**iii**) the entailment particle *therefore*, which relates the premise and the conclusion, we have to capture both the intuitively correct *truth-conditions* of the whole discourse and the modal and individual-level *anaphoric connections* between the two sentences of the discourse and within each one of them.

The structure of the chapter is the following. Section **2** outlines the proposed account of the Aquinas discourse in (1/2) above. The discourse is basically analyzed as a network of structured anaphoric connections and the meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora.

Section **3** defines the formal system, dubbed Intensional PCDRT (IP-CDRT), i.e. the extension of PCDRT with (dref's for) possible worlds. Section **4** shows how modalized conditionals and the entailment particle *therefore* are analyzed in IP-CDRT, while section **5** introduces the IP-CDRT analysis of modal subordination: modal subordination is basically analyzed as an instance of restricting the domain of modal quantifiers via structured modal anaphora; that is, the antecedent of (2b) is simultaneously anaphoric to the set of worlds and the set of individuals introduced by the the antecedent of (2a) and, also, to the quantificational dependency established between these two sets.

In order to make the presentation simpler and, hopefully, clearer, the development of Intensional PCDRT in sections **3**, **4** and **5** builds on the simpler PCDRT system introduced in chapter **5**, which does not contain all the extensions introduced in chapter **6** for the PCDRT analysis of quantificational subordination (e.g. the dummy individual, distributivity operators over individual dref's etc.).

It is only in section **6** that I revise the analysis of modal quantification, modal anaphora and modal subordination within an intensional system that incorporates and extends the PCDRT system of chapter **6**. The revised analysis introduced in section **6** will explicitly and systematically capture the intuitive parallel between quantificational subordination and modal subordination – in particular, the intuitive parallel between the quantificational subordination discouse *Harvey courts a*^{u} *girl at every convention. She*$_u$ *always comes to the banquet with him* (Karttunen 1976) and the modal subordination discourse *A*^{u} *wolf might come in. It*$_u$ *would attack Harvey first* (based on Roberts 1989).

The final section (section **7**) compares IP-CDRT with alternative analyses of modalized conditionals and modal subordination.

2. Structured Reference across Domains

This section outlines the account of the Aquinas discourse in (1/2) above. I first show how to extend Plural Compositional DRT (PCDRT) with (dref's for) possible worlds (**2.1**). The extension enables us to analyze the discourse in (1/2) as a network of structured anaphoric connections. The meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora (**2.2**).

2.1. Extending PCDRT with Possible Worlds

To analyze discourse (1/2), I will extend Dynamic Ty2 (and PCDRT) with a new basic type **w** for possible worlds. Thus, we will work with a Dynamic Ty3 logic with four basic types: t (truth-values), e (individuals; variables: x, x' etc.) and **w** (possible worlds; variables: w, w' etc.) and s ('variable assignments'; variables: i, j, i', j' etc.). The only modifications we have to make to the Dynamic Ty2 logic introduced in chapter **3** are: (**i**) resetting the set of basic static types **BasSTyp** to $\{t, e, \mathbf{w}\}$ and (**ii**) redefining the notion of standard frame for Dynamic Ty3 so that $\boldsymbol{D}_t$, $\boldsymbol{D}_e$, $\boldsymbol{D}_{\mathbf{w}}$ and $\boldsymbol{D}_s$ are non-empty and pairwise disjoint sets. In particular, the set of four axioms that ensures that the objects in the domain $\boldsymbol{D}_s$ actually behave like variable assignments in the relevant respects remain the same.

In the spirit of Stone (1999), I will analyze modal anaphora by means of dref's for *static* modal objects; in this way, we will explicitly capture the intuitive parallel between anaphora and quantification in the individual and modal domains argued for in Geurts

(1995/1999), Frank (1996), Stone (1999), Bittner (2001) and Schlenker (2005) among others. I will call the resulting system Intensional Plural CDRT (IP-CDRT). IP-CDRT takes the research program in Muskens (1996), i.e. the unification of Montague semantics and DRT, one step further: IP-CDRT unifies – in classical type logic – the static Lewis (1973) / Kratzer (1981) analysis of modal quantification and van den Berg's Dynamic Plural Logic.

Throughout this chapter, I will continue to subscript terms with their types, e.g. x_e, $w_{\mathbf{w}}$, i_s. I will also subscript lexical relations with their world variable, e.g. $see_w(x, y)$ is meant to be interpreted as x sees y in world w.

Just as in CDRT+GQ and PCDRT, a dref for individuals u will be a function of type se from 'assignments' i_s to individuals x_e; intuitively, the individual $u_{se}i_s$ is the individual that i assigns to the dref u. In addition, IP-CDRT has dref's for possible worlds p, p', …, p_1, p_2, which are functions of type $s\mathbf{w}$ from 'assignments' i_s to possible worlds $w_{\mathbf{w}}$; intuitively, the world $p_{s\mathbf{w}}i_s$ is the world that i assigns to the dref p.

As in PCDRT, dynamic info states are sets of 'variable assignments', i.e. terms I, J etc. of type st. A sentence is still interpreted as a DRS, i.e. a relation of type $(st)((st)t)$ between an input and an output info state. An individual dref u stores a set of individuals with respect to an info state I, abbreviated $uI := \{u_{se}i_s\text{: } i_s \in I_{st}\}$ (that is, uI is the image of the set of 'assignments' I under the function u). A dref p stores a set of worlds, i.e. a *proposition*, with respect to an info state I, abbreviated $pI := \{p_{s\mathbf{w}}i_s\text{: } i_s \in I_{st}\}$ (that is, pI is the image of the set of 'assignments' I under the function p).

Propositional dref's have two uses: (**i**) they store contents, e.g. the content of the entire conditional in (2a) (i.e. the content of the premise of the Aquinas argument); (**ii**) they store possible scenarios (in the sense of Stone 1999), e.g. the set of worlds introduced by the conditional antecedent in (2a), i.e. a possible scenario containing a man that is alive and on which the consequent of the conditional in (2a) further elaborates

As before, we use plural info states to store sets of individuals and propositions instead of simply using dref's for sets of individuals or possible worlds (their types would be $s(et)$ and $s(\mathbf{w}t)$) because we need to store in our discourse context (i.e. in our information states) both the *values* assigned to various dref's and the *structure* associated with those values, as shown in (3) below.

3.

Info State I	…	u	u'	p	p'	…
i_1	…	x_1 (i.e. ui_1)	y_1 (i.e. $u'i_1$)	w_1 (i.e. pi_1)	v_1 (i.e. $p'i_1$)	…
i_2	…	x_2 (i.e. ui_2)	y_2 (i.e. $u'i_2$)	w_2 (i.e. pi_2)	v_2 (i.e. $p'i_2$)	…
i_3	…	x_3 (i.e. ui_3)	y_3 (i.e. $u'i_3$)	w_3 (i.e. pi_3)	v_3 (i.e. $p'i_3$)	…
…	…	…	…	…	…	…

Values (sets of individuals or worlds): $\{x_1, x_2, x_3, \ldots\}$, $\{w_1, w_2, w_3, \ldots\}$ etc.

Structure (relations between individuals and / or worlds): $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \ldots\}$, $\{<x_1, y_1, w_1>, <x_2, y_2, w_2>, <x_3, y_3, w_3>, \ldots\}$, $\{<w_1, v_1>, <w_2, v_2>, <w_3, v_3>, \ldots\}$ etc.

Mixed reading donkey sentences, donkey anaphora to structure (both analyzed in chapter **5**) and quantificational subordination (analyzed in chapter **6**) provide empirical motivation for plural info states. The example of modal subordination in (5) below, which is intuitively parallel to the example of quantificational subordination in (4), provides independent empirical support.

4. **a.** Every^{u_1} man saw a^{u_2} woman. **b.** They$_{u_1}$ greeted them$_{u_2}$.
5. **a.** A^{u_1} wolf might^{p_1} enter the cabin. **b.** It$_{u_1}$ would$_{p_1}$ attack John.

In both discourses, we do not simply have anaphora to sets of values (individuals and / or possible worlds), but anaphora to *structured* sets.

In particular, if man m_1 saw woman n_1 and m_2 saw n_2, (4b) is interpreted as asserting that m_1 greeted n_1, not n_2, and that m_2 greeted n_2, not n_1; the structure of the greeting is the same as the structure of the seeing[4]. Similarly, (5b) is interpreted as asserting that, if a wolf entered the cabin, it would attack John, i.e. if a black wolf x_1 enters the cabin in world w_1 and a white wolf x_2 enters the cabin in world w_2, then x_1 attacks John in w_1, not in w_2, and x_2 attacks John in w_2, not in w_1.

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[4] The fact that *correspondence* interpretation of discourse (4) – in which the structure of the greeting is the same as the structure of the seeing – is a distinct reading for this discourse and not simply a particular understanding of a vague / underspecified *cumulative*-like reading is argued for in Krifka (1996b) and Nouwen (2003).

A plural info state I stores the *quantificational structure* associated with sets of individuals and possible worlds: (4a) requires each variable assignment $i \in I$ to be such that the man $u_1 i$ saw the woman $u_2 i$; (4b) elaborates on this structured dependency by requiring that, for each $i \in I$, the man $u_1 i$ greeted the woman $u_2 i$. The structured dependency can be represented in the (by now) familiar way, i.e. by means of a matrix like the one in (6) below.

6. Info state I	…	u_1 (**men**)	u_2 (**women**)	…
i_1	…	m_1 $(=u_1 i_1)$	n_1 $(=u_2 i_1)$	…
		man m_1 saw woman n_1		
i_2	…	m_2 $(=u_1 i_2)$	n_2 $(=u_2 i_2)$	…
i_3	…	m_3 $(=u_1 i_3)$	n_3 $(=u_2 i_3)$	…
…	…	…	…	…

Similarly, (5a) outputs an info state I such that, for each $i \in I$, the wolf $u_1 i$ enters the cabin in the world $p_1 i$; (5b) elaborates on this structured dependency: for each assignment $i \in I$, it requires the wolf $u_1 i$ to attack John in world $p_1 i$.

7. Info state I	…	u_1 (**wolves**)	p_1 (**worlds**)	…
i_1	…	x_1 $(=u_1 i_1)$	w_1 $(=p_1 i_1)$	…
		wolf x_1 enters the cabin in world w_1		
i_2	…	x_2 $(=u_1 i_2)$	w_2 $(=p_1 i_2)$	…
i_3	…	x_3 $(=u_1 i_3)$	w_3 $(=p_1 i_3)$	…
…	…	…	…	…

Moreover, we need plural info states to capture structured anaphora between the premise(s) and the conclusion of entailment discourses like (1/2) above or (8) and (9) below.

8. **a.** Every^{u_1} man saw a^{u_2} woman. **b.** Therefore, they$_{u_1}$ noticed them$_{u_2}$.

9. **a.** A^{u_1} wolf might^{p_1} enter the cabin. **b.** It$_{u_1}$ would$_{p_1}$ see John^{u_2}.

c. Therefore, it$_{u_1}$ would$_{p_1}$ notice him$_{u_2}$.

2.2. Structured Reference in Modal Discourse

Let us return now to discourse (2), which is analyzed as shown in (10) below.

10. **CONTENT**^{p_1} **: if**^{p_2} (a^{u_1} man$_{p_2}$ is alive$_{p_2}$);

must^{p_3} $_{p_1, \mu, \omega}$ (p_2, p_3); he$_{u_1}$ has$_{p_3}$ a^{u_2} pleasure$_{p_3}$.

THEREFORE^{p_4} $_{p^*, \mu^*, \omega^*}$ (p_1, p_4)**:**

if^{p_5} ($p_5 \Subset p_2$; **not**(he$_{u_1}$ has$_{p_5}$ a^{u_3} spiritual pleasure$_{p_5}$));

must^{p_6} $_{p_4, \mu, \omega}$ (p_5, p_6); he$_{u_1}$ has$_{p_6}$ a^{u_4} carnal pleasure$_{p_6}$.

The representation in (10) is basically a network of structured anaphoric connections. Consider the conditional in (2a) first. The morpheme *if* introduces a dref p_2 that stores the content of the antecedent – we need this distinct dref because the antecedent in (2b) is anaphoric to it (due to modal subordination). The indefinite *a man* introduces an individual dref u_1, which is later retrieved: (**i**) by the pronoun *he* in the consequent of (2a), i.e. by donkey anaphora, and (**ii**) by the pronoun *he* in the antecedent of (2b), i.e. by modal subordination.

The modal verb *must* in the consequent of (2a) contributes a tripartite quantificational structure and it relates three propositional dref's. The dref p_1 stores the content of the whole modalized conditional. The dref p_2, which was introduced by the antecedent and which is anaphorically retrieved by *must*, provides the restrictor of the modal quantification. Finally, p_3 is the nuclear scope of the modal quantification; it is introduced by the modal *must*, which constrains it to contain the set of *ideal* worlds among the p_2-worlds – ideal relative to the p_1-worlds, a *circumstantial* modal base μ and an *empty* ordering source ω. Finally, we test that the set of ideal worlds stored in p_3 satisfies the remainder of the consequent.

Consider now the entailment particle *therefore*. I take it to relate *contents* and not meanings. We can see this by examining the discourses in (8) and (9) above: in both cases, the contents (i.e. truth-conditions) of the premise(s) and the conclusion stand in an inclusion relation, but not their meanings (i.e. context change potentials). Further support is provided by the fact that the felicity of *therefore*-discourses is *context-dependent* – which is expected if *therefore* relates contents because contents are determined in a context-sensitive way. Consider, for example, the discourse in (11) below: entailment obtains if (11) is uttered on a Thursday in a discussion about John, but not otherwise.

11. **a.** He_{John} came back three days $\text{ago}_{Thursday}$.
    b. Therefore, John came back on a Monday.

Moreover, I propose that *therefore* in (2b) should be analyzed as a modal relation, in particular, as expressing *logical consequence*; thus, I analyze discourse (1/2) as a modal quantification that relates two embedded modal quantifications, the second of which is modally subordinated to the first. Just as the modal *must*, *therefore* contributes a necessity modal relation and introduces a tripartite quantificational structure: the restrictor is p_1 (the content of the premise) and the nuclear scope is the newly introduced dref p_4, which stores the set of ideal p_1-worlds – ideal relative to the dref p^* (the designated dref for the actual world w^*), an *empty* modal base μ^* and an *empty* ordering source ω^* (the modal base μ^* and the ordering source ω^* are empty because *therefore* is interpreted as logical consequence). Since μ^* and ω^* are empty, the dref p_4 is identical to p_1.

Analyzing *therefore* as an instance of modal quantification makes at least two welcome predictions. First, it predicts that we can interpret it relative to different modal bases and ordering sources – and this prediction is borne out. *Therefore* expresses *causal* consequence in (12) below and it seems to express a form of *practical inference* in (13).

12. Reviewers are usually people who would have been poets, historians, biographers, etc., if they could; they have tried their talents at one or the other, and have failed; therefore they turn critics.
    (Samuel Taylor Coleridge, *Lectures on Shakespeare and Milton*)

13. We cannot put the face of a person on a stamp unless said person is deceased. My suggestion, therefore, is that you drop dead.
(attributed to J. Edward Day; letter, never mailed, to a petitioner who wanted himself portrayed on a postage stamp)

Second, it captures the intuitive equivalence between the *therefore* discourse *A man saw a woman, therefore he noticed her* and the modalized conditional *If a man saw a woman, he (obviously / necessarily) noticed her* (they are equivalent provided we add the premise *A man saw a woman* to the conditional).

The conditional in (2b) is interpreted like the conditional in (2a), with the additional complication that its antecedent is anaphoric to the antecedent of the conditional in (2a), i.e. to the dref p_2. The dref p_5 is a *structured* subset of p_2, symbolized as $p_5 \Subset p_2$. We need *structured inclusion* because we want p_5 to preserve the structure associated with the p_2-worlds, i.e. to preserve the quantificational correspondence between the p_2-worlds and the u_1-men that are alive in them. The modal verb *must* in (2b) is anaphoric to p_5, it introduces the set of worlds p_6 containing all the p_5-worlds that are ideal relative to the p_4-worlds, μ and ω (the same as the modal base and ordering source in the premise (2a)) and it checks that, in each ideal p_6-world, all its associated u_1-men have a carnal pleasure.

3. Intensional Plural CDRT (IP-CDRT)

In an intensional Fregean / Montagovian framework, the compositional aspect of interpretation is largely determined by the types for the extensions of the 'saturated' expressions, i.e. names and sentences, plus the type that enables us to build intensions out of these extensions. Let us abbreviate them as **e**, **t** and **s**, respectively. In IP-CDRT, we assign the following dynamic types to the 'meta-types' **e**, **t** and **s**: a sentence is interpreted as a DRS, i.e. as a relation between info states, hence $\mathbf{t} := (st)((st)t)$ (the same as in PCDRT); a name is interpreted as an individual dref, hence $\mathbf{e} := se$ (again, the same as in PCDRT). Finally, $\mathbf{s} := s\mathbf{w}$, i.e. we use the type of propositional dref's to build intensions.

To interpret a noun like 'man', we define an atomic $man_p\{u\}$ based on the static one $man_w(x)$, as shown in (14) below. The IP-CDRT atomic conditions are the obvious intensionalized versions of the corresponding PCDRT conditions.

14. **Atomic conditions – first attempt.**

$$man_p\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in} I(man_{pi}(ui)).$$

In general, the IP-CDRT basic meanings for lexical items are the usual intensionalized versions of the corresponding extensional PCDRT meanings, as shown in table (45) below. I use the following notational conventions:

u, u' etc. for dref's of type $\mathbf{e}{:=}se$ (recall that they are constants in our Dynamic Ty3 logic) and v, v' etc. for variables of type $\mathbf{e}{:=}se$;

p, p' etc. for dref's of type $\mathbf{s}{:=}s\mathbf{w}$ (which are also constants in our Dynamic Ty3 logic) and q, q' etc. for variables of type $\mathbf{s}{:=}s\mathbf{w}$;

$\mathbb{P}$, $\mathbb{P}'$ etc. for variables over dynamic propositions of type $\mathbf{st}$, where $\mathbf{s}{:=}s\mathbf{w}$ and $\mathbf{t}{:=}(st)((st)t)$;

P, P' etc. for variables over dynamic intensional properties of type $\mathbf{e}(\mathbf{st})$, where $\mathbf{e}{:=}se$;

Q, Q' etc. for variables over dynamic intensional quantifiers of type $(\mathbf{e}(\mathbf{st}))(\mathbf{st})$.

15. **TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	**Translation**	**Type** $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
$[sleep]_{V_{in}}$	$\rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [sleep_q\{v\}]$, where $sleep$ is of type $e(\mathbf{w}t)$	$\mathbf{e}(\mathbf{st})$
$[own]_{V_{tr}}$	$\rightsquigarrow \lambda Q_{(\mathbf{e}(\mathbf{st}))(\mathbf{st})}.\ \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ Q(\lambda v'_{\mathbf{e}}.\ \lambda q'_{\mathbf{s}}.\ [own_{q'}\{v, v'\}])(q)$, where own is of type $e(e(\mathbf{w}t))$	$((\mathbf{e}(\mathbf{st}))(\mathbf{st}))(\mathbf{e}(\mathbf{st}))$
	equivalently: $\rightsquigarrow$ $\lambda Q_{(\mathbf{e}(\mathbf{st}))(\mathbf{st})}.\ \lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [own_q\{v, v'\}])$	
$[buy]_{V_{di}}$	$\rightsquigarrow \lambda Q'_{(\mathbf{e}(\mathbf{st}))(\mathbf{st})}.\ \lambda Q_{(\mathbf{e}(\mathbf{st}))(\mathbf{st})}.\ \lambda v_{\mathbf{e}}.$ $Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v''_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [buy_q\{v, v', v''\}]))$, where buy is of type $e(e(e(\mathbf{w}t)))$	$(\mathbf{e}(\mathbf{st})(\mathbf{st}))((\mathbf{e}(\mathbf{st})(\mathbf{st}))$ $(\mathbf{e}(\mathbf{st})))$
$[house\text{-}elf]_{N}$	$\rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [house\_elf_q\{v\}]$, where $house\_elf$ is of type $e(\mathbf{w}t)$	$\mathbf{e}(\mathbf{st})$

15. TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
$[he_u]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e}(\mathbf{st})}.\ P(u_{\mathbf{e}})$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
$[the_u]_{\mathrm{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ [\mathbf{unique}_q\{u\}];\ P'(u)(q);\ P(u)(q)$, where $\mathbf{unique}_q\{u\} :=$ $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(qi{=}qi' \rightarrow ui{=}ui')$, i.e. anaphoric and 'weakly' unique. $\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ P'(u)(q);\ P(u)(q)$, i.e. anaphoric.	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
$[t_v]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e}(\mathbf{st})}.\ P(v_{\mathbf{e}})$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
$[he_{Dobby}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e}(\mathbf{st})}.\ P(Dobby_{\mathbf{e}})$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
$[Dobby^u]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ [u \mid u{=}Dobby];\ P(u)(q)$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
$[who]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e}(\mathbf{st})}.\ P$	$(\mathbf{e}(\mathbf{st}))(\mathbf{e}(\mathbf{st}))$
$[\emptyset]_{\mathrm{I}}/[\text{-}ed]_{\mathrm{I}}/[\text{-}s]_{\mathrm{I}}$	$\rightsquigarrow \lambda \mathbb{P}_{(\mathbf{st})}.\ \mathbb{P}$	$(\mathbf{st})(\mathbf{st})$
$[doesn't]_{\mathrm{I}}/[didn't]_{\mathrm{I}}$	$\rightsquigarrow \lambda \mathbb{P}_{(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ [\sim\mathbb{P}(q)]$, where: $\sim D := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall H_{st}(H{\neq}\emptyset \wedge H{\subseteq}I \rightarrow \neg\exists K_{st}(DHK))$, where D is a DRS (type $\mathbf{t}$)	$(\mathbf{st})(\mathbf{st})$
$[a^{\mathrm{wk}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ [u];\ P'(u)(q);\ P(u)(q)$, i.e. $\lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \exists u(P'(u)(q);\ P(u)(q))$, where $\exists u(D) := [u];\ D$	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
$[a^{\mathrm{str}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q);\ P(u)(q))$, where: $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$, i.e. $\lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \exists^{\mathbf{m}}u(P'(u)(q);\ P(u)(q))$, where $\exists^{\mathbf{m}}u(D) := \mathbf{max}^u(D)$	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
$[the^u]_{\mathrm{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q));\ [\mathbf{unique}_q\{u\}];\ P(u)(q)$, where $\mathbf{unique}_q\{u\} :=$ $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(qi{=}qi' \rightarrow ui{=}ui')$ and $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$, i.e. $\lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \exists^{\mathbf{m}}u(P'(u)(q));\ [\mathbf{unique}_q\{u\}];\ P(u)(q)$, i.e. existence and uniqueness – the Russellian analysis $\rightsquigarrow \lambda P'_{\mathbf{e}(\mathbf{st})}.\ \lambda P_{\mathbf{e}(\mathbf{st})}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q));\ P(u)(q)$, where:	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$

15. **TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	**Translation**	**Type** $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
	$\mathbf{max}^{u}(D) := \lambda I_{st}J_{st}.\ ([u];\, D)IJ \wedge \forall K_{st}(([u];\, D)IK \rightarrow uK \subseteq uJ)$, i.e. $\lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \exists^{\mathbf{m}}u(P'(u)(q));\, P(u)(q)$, i.e. existence and maximality	
$[det^{u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [{}_{q}\mathbf{det}_{u}(P'(u)(q),\, P(u)(q))]$, where:	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
e.g. *every*[u], *no*[u], *most*[u]… (but not $a^{\mathbf{wk}:u}$, $a^{\mathbf{str}:u}$, the_{u} or the^{u})	${}_{p}\mathbf{det}_{u}(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(u_p[D_1 I],\ u_p[(D_1;\, D_2)I])$, where $u_p[DI] := \cup\{uJ\colon ([u \mid \mathbf{unique}_p\{u\}];\, D)IJ\}$ and $\mathbf{unique}_p\{u\} :=$ $\lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(pi = pi' \rightarrow ui = ui')$ and **DET** is the corresponding static determiner	
$[and]_{\mathrm{Conj}}$	$\rightsquigarrow \lambda v_1.\ \ldots\ \lambda v_n.\ v_1 \sqcap \ldots \sqcap v_n$	$\tau(\ldots(\tau\tau)\ldots)$
$[or]_{\mathrm{Conj}}$	$\rightsquigarrow \lambda v_1.\ \ldots\ \lambda v_n.\ v_1 \sqcup \ldots \sqcup v_n$	$\tau(\ldots(\tau\tau)\ldots)$

The IP-CDRT definitions of generalized conjunction $\sqcap$ and generalized disjunction $\sqcup$ are the same as the PCDRT ones.

3.1. An Example: Indicative Sentences in IP-CDRT

Let us now look at the IP-CDRT analysis of a simple indicative sentence like the one in (16) below. I will assume that the LF of such a sentence contains an indicative mood morpheme in the complementizer head *C*, whose meaning is provided in (1) below: the indicative mood stakes the dynamic proposition $\mathbb{P}_{\mathbf{st}}$ denoted by the remainder of the sentence and applies it to the designated dref for the actual world p^*. We capture the fact that the dref p^* refers to the actual world w^* by requiring that $p^*I=\{w^*\}$, where I is the input information state relative to which the sentence is interpreted.

Furthermore, I assume that *alive* functions as an intransitive verb and that *is* functions as a semantically vacuous inflectional head *I*, much like $[\emptyset]_{\mathrm{I}}$/$[\text{-}ed]_{\mathrm{I}}$/$[\text{-}s]_{\mathrm{I}}$ – and it is assigned the same kind of meaning, i.e. an identity function over dynamic propositions: $\lambda \mathbb{P}_{(\mathbf{st})}.\ \mathbb{P}$.

16. A $^{\mathbf{wk}:u_1}$ man is alive.

17. $[\mathbf{ind}_{p^*}]_C \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ \mathbb{P}(p^*)$

18. $a^{\mathbf{wk}:u_1} \rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u_1];\ P'(u_1)(q);\ P(u_1)(q)$

$man \rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [man_q\{v\}]$

$a^{\mathbf{wk}:u_1}\ man \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u_1 \mid man_q\{u_1\}];\ P(u_1)(q)$

$alive \rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [alive_q\{v\}]$

$a^{\mathbf{wk}:u_1}\ man\ is\ alive \rightsquigarrow \lambda q_{\mathbf{s}}.\ [u_1 \mid man_q\{u_1\},\ alive_q\{u_1\}]$

$\mathbf{ind}_{p^*}\ a^{\mathbf{wk}:u_1}\ man\ is\ alive \rightsquigarrow [u_1 \mid man_{p^*}\{u_1\},\ alive_{p^*}\{u_1\}]$

Note that, before introducing the meaning of the indicative mood morpheme $\mathbf{ind}_{p^*}$, the composition makes available the dynamic proposition (of type **st**) $\lambda q_{\mathbf{s}}.\ [u_1 \mid man_q\{u_1\},\ alive_q\{u_1\}]$ and it is based on this proposition that the meaning of the conditional antecedent in (2a) is obtained – as the following section endeavors to show.

4. Conditionals, Modals and **Therefore** *in IP-CDRT*

In this section, I show how to compositionally analyze in Intensional Plural CDRT (IP-CDRT):

- modalized conditionals, i.e. the meaning of the particle *if* (**4.1**) and the meaning of modals (**4.2**);
- the entailment particle *therefore* (**4.3**).

4.1. *If*

To interpret the conditional in (2a) above, we need to: (**i**) extract the content of the antecedent of the conditional and store it in a propositional dref p_2 and (**ii**) define a dynamic notion of *structured* subset of a set of worlds.

We will first see how to extract the content of the antecedent of the conditional. For this purpose, I define two operators over a propositional dref p and a DRS D: a maximization operator $\mathbf{max}^p(D)$ and a distributivity operator $\mathbf{dist}_p(D)$. The maximization operator over propositional dref's, defined in (19) below, is identical to the maximization operator over individual dref's in PCDRT.

19. $\mathbf{max}^p(D) := \lambda I_{st}\lambda J_{st}.\ ([p];\ D)IJ \wedge \forall K_{st}(([p];\ D)IK \rightarrow pK \subseteq pJ)$

The definition of the distributivity operator in (20) below follows the basic format (but not the exact implementation) of the corresponding operator over individual dref's in van den Berg (1994, 1996a) and incorporates an amendment of van den Berg's definition proposed in Nouwen (2003)[5]. Just like $\mathbf{max}^p$, the $\mathbf{dist}_p$ operator is an operator over DRS's: its argument is a DRS D, i.e. a term of type $\mathbf{t} := (st)((st)t)$ and its value is another DRS (of type $\mathbf{t}$), i.e. $\mathbf{dist}_p(D)$.

20. **Selective distributivity over modal dref's in IP-CDRT.**

$$\mathbf{dist}_p(D) := \lambda I_{st}\lambda J_{st}.\ pI{=}pJ \wedge \forall w{\in}pI(DI_{p=w}J_{p=w}),$$

where $I_{p=w} := \{i_s{\in}I\text{: } pi{=}w\}$

and p is of type $\mathbf{s} := s\mathbf{w}$ and D is of type $\mathbf{t} := (st)((st)t)$.

The basic idea behind *distributively* updating an input info state I with a DRS D is that we first partition the info state I and then *separately* update each partition cell (i.e. subset of I) with D. Moreover, the partition of the info state I is induced by a dref p as follows: consider the set of worlds $pI := \{pi\text{: } i{\in}I\}$; each world w in the set pI generates one cell in the partition of I, namely the subset $\{i{\in}I\text{: } pi{=}w\}$. Clearly, the family of sets $\{\{i{\in}I\text{: } pi{=}w\}\text{: } w{\in}pI\}$ is a partition of the info state I: the union of the family of sets is the info state I and, for any two distinct worlds w and w' in pI, the sets $\{i{\in}I\text{: } pi{=}w\}$ and $\{i{\in}I\text{: } pi{=}w'\}$ are disjoint.

Thus, updating an info state I with a DRS D *distributively* over a dref p means updating each cell in the p-partition of I with the DRS D and then taking the union of the resulting output info states. The first conjunct in definition (20) above, i.e. $pI{=}pJ$, is required to ensure that there is a bijection between the partition cells induced by the dref

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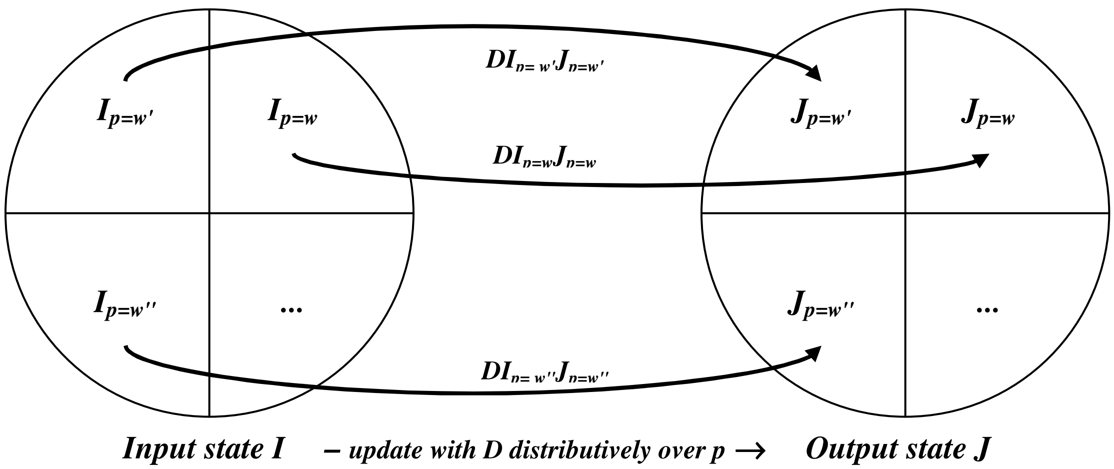
[5] See van den Berg (1994): 14, (43), van den Berg (1996a): 145, (18) and Nouwen (2003): 87, (4.17).

p over the input state I and the partition cells induced by p over the output state J; without this requirement, we could introduce arbitrary new values for p in the output state J, i.e. arbitrary new partition cells[6,7].

The second conjunct, i.e. $\forall w \in pI(DI_{p=w}J_{p=w})$, is the one that actually defines the distributive update: every partition cell in the input info state I is related by the DRS D to the corresponding partition cell in the output state J. The figure in (21) below schematically represents how the input state I is p-distributively updated with the DRS D.

21. **Updating the info state *I* with the DRS *D* distributively over the dref *p*.**



The Appendix to the chapter studies in more detail the formal properties of selective distributity, generalizes it to distributivity over multiple dref's and defines distributivity operators over arbitrary distributable types over and above the basic distributable type **t** := $(st)((st)t)$.

The operators $\mathbf{max}^p(D)$ and $\mathbf{dist}_p(D)$ enable us to 'dynamize' λ-abstraction over possible worlds, i.e. to extract and store contents: the $\mathbf{dist}_p(D)$ update checks one world at a time that the set of worlds stored in p satisfies the DRS D and the $\mathbf{max}^p(D)$ update collects in p all the worlds that satisfy D. I will analyze *if* as a dynamic λ-abstractor over

[6] See Nouwen (2003): 87.

[7] Note that the first conjunct could be replaced with the biconditional $\forall w(I_{p=w}{\neq}\emptyset \leftrightarrow J_{p=w}{\neq}\emptyset)$.

possible worlds, i.e. as a morpheme that extracts the *content* of a dynamic proposition $\mathbb{P}_{\mathbf{st}}$ and stores it in a newly introduced propositional dref p, as shown in (22) below. The representation in (23) shows how the meaning of *if* combines with the dynamic proposition contributed by *A*$^{\mathbf{wk}:u_1}$ *man is alive* and stores its content in the dref p_2.

22. *if*^{p} $\rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ \mathbf{max}^{p}(\mathbf{dist}_{p}(\mathbb{P}(p)))$

23. *a*$^{\mathbf{wk}:u_1}$ *man is alive* $\rightsquigarrow \lambda q_{\mathbf{s}}.\ [u_1 \mid man_q\{u_1\},\ alive_q\{u_1\}]$

if^{p_2} *a*$^{\mathbf{wk}:u_1}$ *man is alive* $\rightsquigarrow \mathbf{max}^{p_2}(\mathbf{dist}_{p_2}([u_1 \mid man_{p_2}\{u_1\},\ alive_{p_2}\{u_1\}]))$

We need one last thing to translate the antecedent in (2a). The donkey indefinite *a man* receives a *strong* reading, i.e. the conditional in (2a) is interpreted as asserting that *every* (and not only *some*) man that is alive must have a pleasure. Thus, the antecedent of (2a) is translated in IP-CDRT as shown in (24) below[8].

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[8] Thus, I assume that the strong reading associated with the indefinite *a man* is contributed by the indefinite article itself and not by the modal verb *must* (and / or the morpheme *if*). I have chosen this analysis because it is parallel to the analysis of weak / strong readings of relative-clause donkey sentences in chapter **5** above. However, it might very well be that modal verbs in modalized conditionals might bind certain indefinites in the antecedent of the conditional, i.e. they might be instances of *multiply selective quantification*. See, for example, Chierchia (1995) for the use of the notion of dynamic multiply selective quantification in the analysis of extensional conditionals with adverbs of quantification like *always*, *usually* etc.

It seems clear to me that the analysis of conditional donkey sentences like *If a man buys a book on amazon.com and has a credit card, he always / usually uses it to pay for it* should allow for more readings than the corresponding relative-clause donkey sentences, i.e. *Every man who buys a book on amazon.com and has a credit card uses it to pay for it / Most men who buy a book on amazon.com and have a credit card use it to pay for it*. The *usually* donkey sentence has a reading in which we consider most cases in which a man buys a book, while the *most* donkey sentence seems to lack this reading – or, in any case, it is a lot less clear that the *most* donkey sentence has such a reading (see also the contrast between *If a farmer owns a donkey, he usually beats it* and *Most farmers who own a donkey beat it*).

Since conditional donkey sentences allow for more readings than the corresponding relative-clause donkey examples, it seems clear that this is due to the conditional structure itself, i.e. to the adverb of quantification together with the morpheme *if* – and I am inclined at this point to allow for a multiply selective analysis of such donkey conditionals in which the adverb binds indefinites in the antecedent (the analysis of *always* proposed in chapter **6** can be fairly easily extended to accomplish this).

It is not as clear to me that modal verbs in modalized conditionals should receive a similar, multiply selective interpretation, i.e. it is not at all clear to me that modal verbs and adverbs of quantification should be analyzed in parallel (I am indebted to Maribel Romero, p.c., for emphasizing the importance of this issue). Heim (1982), for example, proposes such a parallel analysis; note also that such a parallel analysis is an almost immediate consequence of a situation-based D-/E-type approach to donkey anaphora, since the

24. *if*^{p_2} *a*$^{\mathbf{str}:u_1}$ *man is alive* ⇝ $\mathbf{max}^{p_2}(\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}([man_{p_2}\{u_1\}, alive_{p_2}\{u_1\}])))$

The IP-CDRT representation in (24) provides the empirical motivation for the introduction of *selective* distributivity: we need the $\mathbf{dist}_{p_2}$ operator over and above the *unselective* distributivity built into the atomic conditions because, in the standard Kripke-style modal system that I assume, the same individual may exist in multiple worlds (though not necessarily in all of them). Therefore, it can be possible for a man to be alive in two distinct possible worlds – in which case, we want to introduce this man with respect to *each* of the possible worlds in which he is alive – and this is what the selective distributivity operator over the modal dref p_2 achieves: $\mathbf{dist}_{p_2}$ ensures that we separately consider every possible world stored in p_2 and relate it to all the men that are alive in it. Should we omit the selective distributive operator, we could introduce *all* the men that are alive in some world or other, but we might fail to introduce *each* man with respect to *each* possible world in which he is alive.

Thus, at least for the particular example we are considering, the need for selective distributivity is partly due to the assumed underlying ontology[9]. However, the introduction of selective distributivity has a more general motivation, namely the parallel treatment of the dynamics of values and structure in PCDRT and IP-CDRT. More precisely, maximization together with *selective* distributivity enables us to 'dynamize' λ-abstraction over *structure* as well as over values: one the one hand, selective distributive operators, e.g. $\mathbf{dist}_{p_2}$ in (24) above, enable us to λ-abstract one value at a time; on the other hand, selective maximization operators make it possible to extract the desired set and, when we maximize *under* the scope of a selective distributive operator, e.g.

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same domain of situations is used for modal quantification (see Kratzer 1989) and anaphora (see Heim 1990 among others).

However, conditionals with adverbs of quantification are intuitively extensional, while conditionals with modal verbs are intensional, so it is not obvious that we should have a parallel analysis of the two. Unfortunately, I have to leave the investigation of these crucial issues for future research.

[9] Had we used a counterpart-based system of the kind proposed by Lewis (see Lewis 1968 among others), we wouldn't have needed selective distributivity over modal dref's because, in such a system, an individual exists in exactly one possible world.

$\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}(\ldots))$ in (24) above, we are able to extract the full u_1-structure associated with each single p_2-value.

4.2. Modals

We have seen in the previous section how to extract the content of the antecedent of the conditional and store it in a propositional dref p_2. We turn now to the second notion needed for the interpretation of the conditional in (2a), namely the definition of a dynamic notion of *structured* subset of a set of worlds. We need a notion of structured inclusion because:

- the modal *must* and the donkey pronoun *he* in the consequent of (2a) are simultaneously anaphoric to the p_2-worlds and the u_1-men and we need to preserve the structured dependencies between them;
- the modally subordinated antecedent of the conditional in (2b) is also anaphoric to p_2 and u_1 in a structured way.

In the spirit of van den Berg's (extensional) Dynamic Plural Logic, who makes use of a dummy / 'undefined' individual #, I will assume that there is a dummy world # (of type **w**) relative to which all lexical relations are false (the dummy world # can be thought of as the world in which no individual whatsoever exists) and I will use this world to define the *structured inclusion* condition in (25) below [10].

25. $p \Subset p' := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(pi = p'i \vee pi = \#)$.

However, unlike van den Berg, who makes use of the dummy individual # within a partial logic (the dummy individual yields undefinedness), we will continue to work with a classical (bivalent, total) type logic and assume that the dummy world # yields falsity (i.e. any lexical relation of the form $R_w(x_1, \ldots, x_2)$ is false if w is #). We can think of the dummy world $\#_\mathbf{w}$ as the world where no individual whatsoever exists, hence all the

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[10] The corresponding notion of structured inclusion in the individual domain is defined and justified in section **3.2** of chapter **6** above.

lexical relations are false because a relation between individuals obtains at a particular world only if the individuals exist in that possible world.

The dummy world # is used to signal that an 'assignment' i such that pi=# is irrelevant for the evaluation of conditions, so we need to slightly modify the definition of atomic conditions as shown in (26) below. The matrix in (27) represents an info state I that satisfies the structured inclusion requirement $p \Subset p'$ and the atomic condition $man_p\{u\}$. The shaded rows i_2 and i_4 represent the 'assignments' that are discarded in the evaluation of the atomic condition $man_p\{u\}$ – and they are discarded because they both assign the dummy world # to the propositional dref p, i.e. $pi_2=pi_4=\#$.

26. **Atomic conditions.**

$$man_p\{u\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s \in I_{p\neq\#}(man_{pi}(ui)),$$
$$\text{where } I_{p\neq\#} := \{i_s \in I\text{: } pi\neq\#\}.$$

27. Info state I: $p \Subset p'$ and $man_p\{u\}$	p' (**superset worlds**)	p (**subset worlds**)	u (**men**)
i_1	w_1 (=$p'i_1$)	w_1 (=pi_1)	x_1 (=ui_1)
i_2	w_1 (=$p'i_2$)	# (=pi_2)	x_2 (=ui_2)
i_3	w_1 (=$p'i_3$)	w_1 (=pi_3)	x_3 (=ui_3)
i_4	w_2 (=$p'i_4$)	# (=pi_4)	x_1 (=ui_4)
i_5	w_2 (=$p'i_5$)	w_2 (=pi_5)	x_4 (=ui_5)

In a similar vein, we need to slightly modify the way we make use of selective distributivity: we will discard the 'dummy' partition cell $I_{p=\#}$ when we distributively update with the DRS D, which is formally captured by the first conjunct in definition (28) below, which requires the equality of the input and output 'dummy' partition cells. The second conjunct $I_{p\neq\#}\neq\emptyset$ is needed to rule out the degenerate case in which the distributive update $\mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$ is vacuously satisfied.

28. **Selective distributivity modulo the dummy world $\#_\mathbf{w}$.**

$${}_p(D) := \lambda I_{st}\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$$

where $I_{p=\#} := \{i_s{\in}I\text{: } pi{=}\#\}$, $I_{p\neq\#} := \{i_s{\in}I\text{: } pi{\neq}\#\}$,
p is of type $\mathbf{s} := s\mathbf{w}$ and D is of type $\mathbf{t} := (st)((st)t)$ [11].

Finally, we also need to slightly modify the definition of the maximization operator, as shown in (29) below.

29. **Selective maximization modulo the dummy world $\#_\mathbf{w}$.**

$$\mathbf{max}^p(D) := \lambda I_{st}.\lambda J_{st}.\ ([p];\ D)IJ \wedge \forall K_{st}(([p];\ D)IK \rightarrow pK_{p\neq\#}{\subseteq}pJ_{p\neq\#})$$

We are now ready to give the lexical entries for modal verbs. The modal verb *must* is interpreted in terms of a modal condition $\mathbf{nec}_{p,\mu,\omega}(p', p'')$, defined in (30) below. The condition is relativized to: (**i**) a propositional dref p storing the content of the entire modal quantification, (**ii**) an modal base dref μ and (**iii**) an ordering source dref ω.

30. $\mathbf{nec}_{p,\mu,\omega}(p', p'') := \lambda I_{st}.\ I_{p\neq\#}{\neq}\emptyset \wedge$

$$\forall w{\in}pI_{p\neq\#}(\mathbf{NEC}_{\mu I_{p=w,\mu\neq\{\#\}},\ \omega I_{p=w,\omega\neq\{\#\}}}(p'I_{p=w,p'\neq\#},\ p''I_{p=w,p''\neq\#}) \wedge$$

$$(p''{\sqsubseteq}p')I_{p=w})$$

The definition crucially relies on the notion of structured inclusion defined in (25) above. However, we need to strengthen this notion of structured inclusion as shown in (31) below. The reason is that the notion of structured inclusion in (25) merely requires the subset dref to store *only* the superset structure, but modal quantifications in general additionally require the subset dref to store *all* the superset structure – which is what the second conjunct in (31) ensures. To see that we need to store all the superset structured, consider example (32) below, which is interpreted as asserting that, in every deontically

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[11] Strictly speaking, we should also modify the translation of the indicative morpheme from the one in (17) above to the one in (i) below, which makes use of the $_{p^*}(\ldots)$ operator. However, I will ignore this complication throughout most of the chapter (more precisely, until section **6**, where the parallel between singular pronouns and the indicative morpheme is explicitly captured). This simplification will not affect any of the analyses in this chapter. Indeed, the translation in (17) and the one in (i) below are equivalent with respect to any input info state I such that p^*I is a singleton set, namely the singleton set containing only the actual world, i.e. $\{w^*\}$. We can in fact achieve this (and therefore preserve the simpler translation of the indicative morpheme in (17)) by assuming that any discourse starts with a default update of the form $[p^* \mid p^*{=}w^*]$, where $p^*{=}w^* := \lambda I_{st}.\ p^*I_{p^*\neq\#}{=}\{w^*\}$.

(i) $[\mathbf{ind}_{p^*}]_C \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ _{p^*}(\mathbb{P}(p^*))$.

ideal world among the worlds in which there is a murder, for *each and every* murder (and not merely some of the murders) in said ideal world, the murder is investigated in that world[12].

31. **Structured inclusion for dynamic modal quantification.**

$$p''\sqsubseteq p' := \lambda I_{st}.\ (p''\Subset p')I \wedge \forall i\in I(p'i\in p''I_{p''\neq\#} \rightarrow p'i=p''i)$$

32. If$^{p'}$ there is a^{u} murder, it$_{u}$ must$_{p}^{p''\sqsubseteq p'}$ be investigated.

Both μ and ω are dref's for sets of worlds, i.e. they are of type $s(\mathbf{w}t)$[13], a significant simplification compared to the type of static modal bases and ordering sources in Kratzer (1981), i.e. $\mathbf{w}((\mathbf{w}t)t)$. We can simplify these types in IP-CDRT because we have plural info states: every world $w\in pI$ is associated with a sub-state $I_{p=w}$ and we can use this sub-state to associate a set of propositions with the world w, e.g. the set of propositions $\{\mu i$: $i_s\in I_{p=w}\}$, where each μi is of type $\mathbf{w}t$. A similar procedure enables us to associate an ordering source ω with each p-world.

NEC is the static modal relation, basically defined as in Lewis (1973) and Kratzer (1981). In particular, the dref's μ and ω in (30) above associate with each p-world two sets of propositions M and O of type $(\mathbf{w}t)t$: for each world $w\in pI_{p\neq\#}$, the set of propositions M is the modal base $\{\mu i$: $i\in I_{p=w}\}$ and the set of propositions O is the ordering source $\{\omega i$: $i\in I_{p=w}\}$. The set of propositions O induces a strict partial order $<_O$ on the set of all possible worlds as shown in (33) below.

33. $w<_O w'$ iff $\forall W_{\mathbf{w}t}\in O(w'\in W \rightarrow w\in W) \wedge \exists W_{\mathbf{w}t}\in O(w\in W \wedge w'\notin W)$

I assume that all the strict partial orders of the form $<_O$ satisfy the Generalized Limit Assumption in (34) – therefore, the **Ideal** function in (35) is well-defined. This function extracts the subset of O-ideal worlds from the set of worlds W.

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[12] See the corresponding strengthened notion of structured inclusion in the individual domain defined in section **3.2** of chapter **6** above and its parallel justification.

[13] I take the dummy value for modal base and OS dref's to be the singleton set whose member is the dummy world, i.e. {#}.

34. **Generalized Limit Assumption.**

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$,

$$\forall w \in W \exists w' \in W((w' <_O w \vee w' = w) \wedge \neg \exists w'' \in W(w'' <_O w'))$$

35. **The Ideal function.**

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\mathbf{Ideal}_O(W) := \{w \in W: \neg \exists w' \in W(w' <_O w)\}$$

Possibility modals are interpreted in the same way, we only need to replace the static modal relation **NEC** with **POS**; both static modal relations are defined in (36) below. The definition of the dynamic modal relation **pos**, parallel to the definition of the dynamic relation **nec** in (30) above, is given in (37).

36. $\mathbf{NEC}_{M,O}(W_1, W_2) := W_2 = \mathbf{Ideal}_O((\cap M) \cap W_1)$

$\mathbf{POS}_{M,O}(W_1, W_2) := W_2 \neq \emptyset \wedge W_2 \subseteq \mathbf{Ideal}_O((\cap M) \cap W_1)$

37. $\mathbf{pos}_{p,\mu,\omega}(p', p'') := \lambda I_{st}.\ I_{p\neq\#} \neq \emptyset \wedge$

$$\forall w \in pI_{p\neq\#}(\mathbf{POS}_{\mu I_{p=w,\mu\neq\{\#\}}, \omega I_{p=w,\omega\neq\{\#\}}}(p'I_{p=w,p'\neq\#}, p''I_{p=w,p''\neq\#}) \wedge$$

$$(p'' \sqsubseteq p')I_{p=w})$$

The dref p' is the restrictor of the dynamic modal quantification and the dref p'' is the nuclear scope, containing the ideal worlds among the p'-worlds – this is ensured by the second conjunct in (30) and (37) above, which takes care of the *values* (i.e. sets of worlds) associated with the dref's p' and p''. The third and fourth conjuncts make sure that we associate the correct *structure* with these dref's: the third conjunct (i.e. structured inclusion) requires that p'' (the set of ideal worlds) stores *only* the p'-structure, while the fourth conjunct ensures that p'' stores *all* the p'-structure associated with the ideal worlds, i.e. for any assignment i such that $p'i$ is an ideal world, we require p'' to store the same ideal world, thereby ruling out the possibility that p'' stores the dummy world #.

The structural requirements are necessary if we want to capture donkey anaphora between the nuclear scope, i.e. the consequent, and the restrictor, i.e. the antecedent of the modalized conditional in (2a): storing in p'' all and only the structure in p' boils down in this case to the requirement that each ideal world should be associated with all the men that are alive in it.

The matrix in (38) below shows an info state I satisfying the modal relation $\mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)$: w_1 is an ideal world among the p_2-worlds, so p_3 inherits all the p_2-rows (i.e. 'assignments') that store w_1, i.e. p_3 inherits all the structure associated with w_1 by the dref p_2. In contrast, w_2 is not an ideal world among the p_2-worlds, so p_3 stores the 'dummy' world in all the p_2-rows that store w_2; all these rows are shaded because we discard all of them when we compute atomic conditions that contain the dref p_3.

38. Info state I: $\mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)$	p_2 (**antecedent worlds**)	u_1 (**men**)	p_3 (**consequent worlds, i.e. ideal worlds**)
i_1	w_1 $(=p_2i_1)$	x_1 $(=u_1i_1)$	w_1 $(=p_3i_1)$
	man x_1 is alive in world w_1		
i_2	w_1 $(=p_2i_2)$	x_2 $(=u_1i_2)$	w_1 $(=p_3i_2)$
i_3	w_1 $(=p_2i_3)$	x_3 $(=u_1i_3)$	w_1 $(=p_3i_3)$
i_4	w_2 $(=p_2i_4)$	x_2 $(=u_1i_4)$	$\#$ $(=p_3i_4)$
i_5	w_2 $(=p_2i_5)$	x_4 $(=u_1i_5)$	$\#$ $(=p_3i_5)$
…	…	…	…

Thus, the modal verb *must* in (2a) above is translated as shown in (39) below. Note that the type of its denotation is (**st**)**t**, which is parallel to the type of modal quantifiers in static Montague semantics.

39. $must^{p_3 \hat{o}\, p_2}{}_{p_1,\mu,\omega} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ [\mu, \omega \mid \mathbf{circumstantial}_{p*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}];$

$$[p_3 \mid \mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)];\ {}_{p_3}(\mathbb{P}(p_3))$$ [14]

[14] I assume, for simplicity, that the modal base and ordering source dref's μ and ω are introduced by the modal verb *must*. As Kratzer (1981) argues, they are in fact contextually supplied, i.e. the modal *must* is, in this respect, very much like the deictic pronouns discussed in section **3.7** of chapter **6** above. The update $[\mu, \omega \mid \mathbf{circumstantial}_{p*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}]$ is, therefore, either contributed by the 'deixis' associated with

Let us examine the translation in (39) in more detail. First, we introduce the modal base μ and the ordering source ω and relate them to the dref p_1 (which stores the content of the entire modalized conditional) by the **circumstantial** and **empty** conditions defined in (40) below. The $\mathbf{circumstantial}_{p*}\{p_1, \mu\}$ condition is *context-dependent*, i.e. it is relativized to the dref for the actual world $p*$; we need this because the argument put forth by Aquinas in discourse (1/2) goes through only if we add another premise to the one explicitly stated, namely that pleasures are either spiritual or carnal.

Thus, the condition $\mathbf{circumstantial}_{p*}\{p_1, \mu\}$ is meant to contrain the modal quantification expressed by the modalized conditional in (2a) so that it is evaluated only with respect to worlds whose circumstances are identical to the actual world $w*$ in the relevant respects – in particular, the proposition in (41) below has to be true in all the p_1-worlds just as it is in $w*$.

40. $\mathbf{circumstantial}_p\{p', \mu\} := \lambda I_{st}.\ I_{p\neq\#,p'\neq\#}\neq\emptyset \wedge$

$$\forall w\in pI_{p\neq\#}(\forall w'\in p'I_{p=w,p'\neq\#}(\mathbf{circumstantial}_w(w',\mu I_{p=w,p'=w'}))$$

$\mathbf{empty}\{p, \omega\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s\in I(\omega i=\{\#\})$

$\mathbf{empty}\{p, \mu\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s\in I(\mu i=\{\#\})$

41. $\{w_{\mathbf{w}}: \forall x_e(pleasure_w(x) \rightarrow spiritual_w(x) \vee carnal_w(x))\}$

The remainder of the lexical entry in (39) ensures that the propositional dref p_3 stores all and only the ideal p_2-worlds and then checks that the dynamic proposition $\mathbb{P}$ contributed by the consequent of the conditional in (2a) is satisfied in each such ideal world.

In sum, the modalized conditional in (2a) above is translated in IP-CDRT as shown in (42) below. Since the contrast between the weak and the strong reading of the

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the use of the modal verb *must* or, alternatively, it is accommodated to satisfy the requirements that this 'deixis' places on its (local) discourse context.

indefinite *a pleasure* is irrelevant for the discourse as a whole[15], I will take the indefinite to have the formally simpler weak reading.

42. *if*^{p_2} *a*$^{\mathbf{str}:u_1}$ *man is alive, he*$_{u_1}$ *must*$^{p_3 \hat{o}\ p_2}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk}:u_2}$ *pleasure*

$$\rightsquigarrow \mathbf{max}^{p_2}({}_{p_2}(\mathbf{max}^{u_1}([man_{p_2}\{u_1\}, alive_{p_2}\{u_1\}])));$$

$$[\mu, \omega \mid \mathbf{circumstantial}_{p*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}];$$

$$[p_3 \mid \mathbf{nec}_{p_1,\mu,\omega}(p_2, p_3)];\ {}_{p_3}([u_2 \mid pleasure_{p_3}\{u_2\}, have_{p_3}\{u_1, u_2\}])$$ [16]

The IP-CDRT representation in (42) encodes the following sequence of updates: consider all the worlds in which at least one man is alive and consider all the men that are alive in these worlds; store them in p_2 and u_1 respectively. Now consider a circumstantial modal base μ and an empty ordering source ω. Then, every p_2-world that is ideal relative to μ and ω (these ideal worlds are stored in p_3) is such that each of its corresponding u_1-men have some pleasure or other.

4.3. Therefore

Like *must*, the particle *therefore* introduces a necessity quantificational structure, as shown in (43) below. Since *therefore* expresses logical consequence, both its modal base μ^* and its ordering source ω^* are empty.

43. *therefore*$^{p_4 \hat{o}\ p_1}$ $_{p^*,\mu^*,\omega^*}$ $\rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ [\mu^*, \omega^* \mid \mathbf{empty}\{p^*, \mu^*\}, \mathbf{empty}\{p^*, \omega^*\}];$

$$[p_4 \mid \mathbf{nec}_{p^*,\mu^*,\omega^*}(p_1, p_4)];\ {}_{p_4}(\mathbb{P}(p_4))$$

The effect of the update is that the dref p_4 is identical to p_1 both in its value and in its structure, i.e., if J is the output state after processing the **nec** condition in (43) above,

[15] The weak vs. strong constrast is irrelevant in this case because there is no subsequent anaphora to the indefinite *a pleasure* and both readings yield the same truth-conditions for the discourse as a whole.

[16] The use of the operator $_p(\ldots)$ in the definition of modal quantification builds an existential commitment into its meaning – see the corresponding discussion for individual-level quantification in section **3.4** of chapter **6**. The revised definition of modal quantification in section **6** below will employ the operator $_{\langle p\rangle}(\ldots)$ and solve this problem.

we have that $p_1j=p_4j$ for any 'assignment' $j\in J$. Consequently, p_1 can be freely substituted for p_4 and we can simplify the translation of *therefore* as shown in (44) below [17].

44. *therefore*$^{p_4 \hat{o}\ p_1}{}_{p^*,\mu^*,\omega^*} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ {}_{p_1}(\mathbb{P}(p_1))$

I assume that the anaphoric nature of the entailment particle *therefore*, which requires a propositional dref p_1 as the restrictor of its quantification, triggers the accommodation of a *covert* 'content-formation' morpheme *if*^{p_1} that takes scope over the entire modalized conditional in (2a), i.e. the premise of the Aquinas argument, and stores its content in p_1.

5. Modal Subordination in IP-CDRT

The conditional in (2b) is different from the one in (2a) in three important respects. First, given that (2b) elaborates on the modal quantification in (2a), the modal verb *must* in (2b) is anaphoric to the previously introduced modal base μ (circumstantial) and ordering source ω (empty), so it is translated as shown in (45) below.

45. *must*$^{p_6 \hat{o}\ p_5}{}_{p_1,\mu,\omega} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ [p_6 \mid \mathbf{nec}_{p_1,\mu,\omega}(p_5, p_6)];\ {}_{p_6}(\mathbb{P}(p_6))$

Second, the negation in the antecedent of (2b) is translated as in table (45) above, i.e. ***not*** $\rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ \lambda q_{\mathbf{s}}.\ [\sim\mathbb{P}(q)]$.

Finally and most importantly, the modally subordinated antecedent in (2b) is translated in terms of an update requiring the newly introduced dref p_5 to be a *maximal structured* subset of p_2, as shown in (46) below. Thus, modal subordination is capture by establishing a *modal* anaphoric connection that is parallel to the individual-level anaphora between the pronoun *he* in the antecedent of (2b) and the strong donkey indefinite *a man* in the antecedent of (2a).

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[17] See the parallel simplification of the meaning of the generalized quantifier *every* in chapter **6**, section **4.1**, definition (65).

46. $\mathit{if}^{p_5 Ð p_2} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ \mathbf{max}^{p_5 Ð p_2}(\ _{p_5}(\mathbb{P}(p_5)))$

The crucial component of the modally subordinated, i.e. modally anaphoric, $\mathit{if}^{p_5 Ð p_2}$ is the maximization operator $\mathbf{max}^{p_5 Ð p_2}$, which is defined in (47) below and which maximizes *both value and structure*. This makes the $\mathbf{max}^{p\Subset p'}$ operator crucially different from the simpler $\mathbf{max}^{p}$ operator defined in (19) above and which maximizes only values.

47. $\mathbf{max}^{p\Subset p'}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H([p \mid p\Subset p']IH \wedge DHJ \wedge$

$\forall K_{st}([p \mid p\Subset p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p\neq\#}\subseteq H_{p\neq\#}))$

We need *structure* maximization over and above value maximization in the analysis of modal subordination because the antecedent of the modalized conditional in (2b) is interpreted as quantifying over *all* the p_2-worlds in which there is at least one u_1-man without spiritual joys and over *all* such u_1-men, i.e. over the maximal structure associated with these p_2-worlds that satisfies the antecedent of (2b).

The effect of the $\mathbf{max}^{p_5 Ð p_2}$ operator is represented by the matrix in (48) below. Note that we can keep in p_5 some of the rows (i.e. 'assignments') associated with a particular possible world and shade (i.e. discard) other rows associated with the same world. This contrasts with the structured inclusion required by dynamic modal relations (see in particular the matrix in (38) above) where, if a row with a given possible world is shaded / discarded, then all the other rows in the matrix with that possible world also have to be shaded / discarded[18].

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[18] Why do we need to use $\mathbf{max}^{p\Subset p'}(D)$ instead of the simpler $\mathbf{max}^{p}([p\Subset p']; D)$? The reason is that the latter has value maximization (due to $\mathbf{max}^{p}$) and structured inclusion (due to $p\Subset p'$), but it does not also have *structure maximization*, which we get in (47) by the *info state* inclusion requirement $K_{p\neq\#}\subseteq H_{p\neq\#}$. And, to derive the correct truth-conditions for (15b), we need *structure* (and not only value) maximization: if a man is alive and he doesn't have any spiritual pleasure, he must have a carnal pleasure, i.e. we look at *every p_2-world* and at *every u_1-man* in it that is deprived of spiritual joys, then we select the ideal subset among these worlds and check that *every u_1-man* in each ideal world has a carnal pleasure. Thus, the antecedent of the conditional in (15b) has to introduce all the p_2-worlds where some u_1-man is alive and without spiritual joy and *all the structure* associated with these worlds, i.e., *all the u_1-men* in question, so that we can check in

48. Info state *I*	p_2 (**premise worlds**)	u_1 (**men**)	p_5 (**conclusion worlds, i.e. modally subordinated worlds**)
i_1	w_1 (=p_2i_1)	x_1 (=u_1i_1)	w_1 (=p_3i_1)
	man x_1 *is alive in world* w_1		
i_2	w_1 (=p_2i_2)	x_2 (=u_1i_2)	w_1 (=p_3i_2)
i_3	w_1 (=p_2i_3)	x_3 (=u_1i_3)	# (=p_3i_3)
i_4	w_2 (=p_2i_4)	x_2 (=u_1i_4)	w_2 (=p_3i_4)
i_5	w_2 (=p_2i_5)	x_4 (=u_1i_5)	# (=p_3i_5)
…	…	…	…

In sum, the antecedent of the modalized conditional in (2b) is translated as shown in (49) below. Just as before, the weak vs. strong contrast is otiose with respect to the indefinite *any spiritual pleasure*, so I interpret it as weak.

49. *if* $^{p_5 \text{Ð} p_2}$ *he*$_{u_1}$ *doesn't have any*$^{\mathbf{wk}:u_3}$ *spiritual pleasure*

$$\rightsquigarrow \mathbf{max}^{p_5 \text{Ð} p_2}(_{p_5}([\sim[u_3 \mid spiritual_{p_5}\{u_3\}, pleasure_{p_5}\{u_3\}, have_{p_5}\{u_1, u_3\}]]))$$

The translation of the consequent of (2b) is parallel to the translation of the consequent of (2a) – hence, the entire modalized conditional in (2b) is translated in IP-CDRT as shown in (50) below. The representation in (50) shows that modal subordination is basically analyzed in IP-CDRT as *quantifier domain restriction via structured modal anaphora*; that is, the antecedent of (2b) is simultaneously anaphoric to the set of worlds and the set of individuals introduced by the the antecedent of (2a) and, also, to the quantificational dependency established between these two sets.

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the consequent that each and every such man has a carnal pleasure. The update $\mathbf{max}^p([p \Subset p']; D)$ would introduce *all* the relevant p_2-worlds, but only *some* of the relevant u_1-men.

Moreover, the update $\mathbf{max}^p([p \sqsubseteq p']; D)$ would also be inadequate because it would store in p only the worlds in which each and every u_1-man that is alive has no spiritual pleasure, while incorrectly discarding all the worlds in which only some of the u_1-men that are alive have no spiritual pleasure, but not all of them.

50. *if* $^{p_5 Ð p_2}$ *he*$_{u_1}$ *doesn't have any*$^{\mathbf{wk}:u_3}$ *spiritual pleasure, he*$_{u_1}$ *must*$^{p_6 ô\, p_5}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk}:u_4}$ *carnal pleasure*

$\rightsquigarrow$ $\mathbf{max}^{p_5 Ð p_2}(_{p_5}([\sim[u_3 \mid spiritual_{p_5}\{u_3\}, pleasure_{p_5}\{u_3\}, have_{p_5}\{u_1, u_3\}]]));$

$[p_6 \mid \mathbf{nec}_{p_1,\mu,\omega}(p_5, p_6)];\ _{p_6}([u_4 \mid carnal_{p_6}\{u_4\}, pleasure_{p_6}\{u_4\}, have_{p_6}\{u_1, u_4\}])$

One final observation before providing the IP-CDRT translation of the entire Aquinas discourse: the IP-CDRT analysis of modal subordination requires us to assign two different translations to the antecedent of the conditional in (2a) and the modally subordinated antecedent in (2b). Note, however, that the discourse-initial antecedent in (2a) can also be assigned a translation of the form $\mathbf{max}^{p \Subset p'}(D)$; since the conditional is discourse initial, the superset dref p' will have to be accommodated and it will be completely unrestricted, i.e. it will store the set of all possible worlds $\boldsymbol{D}_{\mathbf{w}}^{\boldsymbol{M}}$ [19]. Hence, this more complex translation will ultimately be equivalent to the simpler one in (23) above.

The entire translation of the Aquinas discourse is provided in (51) below. The reader can check that, given the PCDRT definition of truth, which is repeated in (52), we assign the intuitively correct truth-conditions to this discourse. And, according to the translation in (51), the argument does indeed go through: the premise (2a) establishes that the set of ideal worlds among the p_2-worlds is such that any man x has at least one pleasure y. The conclusion follows because in all the ideal p_2-worlds pleasures are spiritual or carnal (just as in the actual world w^*) and any man has at least one pleasure: hence, if a man x has no spiritual pleasure, he must have at least one carnal pleasure y.

51. *If*^{p_2} *a*$^{\mathbf{str}:u_1}$ *man is alive, he*$_{u_1}$ *must*$^{p_3 ô\, p_2}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk}:u_2}$ *pleasure*.

Therefore$^{p_4 ô\, p_1}$ $_{p^*,\mu^*,\omega^*}$, *if* $^{p_5 Ð p_2}$ *he*$_{u_1}$ *doesn't have any*$^{\mathbf{wk}:u_3}$ *spiritual pleasure, he*$_{u_1}$ *must*$^{p_6 ô\, p_5}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk}:u_4}$ *carnal pleasure*

$\rightsquigarrow$ $\mathbf{max}^{p_1}(_{p_1}(\mathbf{max}^{p_2}(_{p_2}\mathbf{max}^{u_1}([man_{p_2}\{u_1\}, alive_{p_2}\{u_1\}]))));$

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[19] We can make sure that p' stores the set of all possible worlds $\boldsymbol{D}_{\mathbf{w}}^{\boldsymbol{M}}$ if we introduce it by means of an update $\mathbf{max}^{p'}(_{p'}([p' \Subset p']))$.

$$[\mu,\ \omega \mid \mathbf{circumstantial}_{p*}\{p_1,\ \mu\},\ \mathbf{empty}\{p_1,\ \omega\}];$$

$$[p_3 \mid \mathbf{nec}_{\,p_1,\,\mu,\,\omega}(p_2,\ p_3)];\ \ _{p_3}([u_2 \mid \mathit{pleasure}_{\,p_3}\{u_2\},\ \mathit{have}_{\,p_3}\{u_1,\ u_2\}]);$$

$$_{p_1}(\mathbf{max}^{p_5 \text{Ð} p_2}(\,_{p_5}([\sim[u_3 \mid \mathit{spiritual}_{\,p_5}\{u_3\},\ \mathit{pleasure}_{\,p_5}\{u_3\},\ \mathit{have}_{\,p_5}\{u_1,\ u_3\}]])));$$

$$[p_6 \mid \mathbf{nec}_{\,p_1,\,\mu,\,\omega}(p_5,\ p_6)];\ \ _{p_6}([u_4 \mid \mathit{carnal}_{\,p_6}\{u_4\},\ \mathit{pleasure}_{\,p_6}\{u_4\},\ \mathit{have}_{\,p_6}\{u_1,\ u_4\}])$$

52. **Truth:** A DRS D (type **t**) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

6. A Parallel Account of Modal and Quantificational Subordination

In this section, I will slightly revise the analysis of modal quantification proposed in section **4** above and make it parallel to the analysis of individual-level quantification proposed in chapter **6**. The benefit of the revised analysis is that we can give a compositional account of modal subordination examples like the one in (53) below (based on an example in Roberts 1989) that is completely parallel to the analysis proposed in chapter **6** of the quantificational subordination example in (54) below (from Karttunen 1976).

53. **a.** A^u wolf might come in. **b.** It_u would attack Harvey first.

54. **a.** Harvey courts a^u girl at every convention.

 b. She_u always comes to the banquet with him.

 [**c.** The_u girl is usually also very pretty.]

Under its most salient interpretation, discourse (53) asserts that, for all the speaker knows, it is a possible that a wolf comes in. Moreover, for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first.

The modal subordination discourse in (53) is parallel to the quantificational subordination discourse in (54) because the interaction between the indefinite *a^u wolf* and the modal *might* on the one hand and the singular pronoun *it_u* and the modal *would* on the other hand is parallel to the interaction between *a^u girl-every convention* and *she_u-always*.

6.1. Redefining Modal Quantification

This section introduces the main definitions and abbreviations needed for the revised definition of dynamic modal quantification. They are point-for-point parallel to the definitions given in chapter **6** for individual-level quantification (see appendix **0** of chapter **6**).

As already indicated in section **4.2** above, we need a notion of structured inclusion to define dynamic modal quantification and we need to introduce a dummy / exception world $\#_{\mathbf{w}}$ to be able to define structured inclusion. The dummy world $\#_{\mathbf{w}}$ makes every lexical relation false, much like the dummy / exception individual $\#_e$ introduced in chapter **6** yields falsity.

Just as before, the new definition of intensional atomic conditions – provided in (55) below – relies on static lexical relations $R_w(x_1, \ldots, x_n)$ of the expected intensional type $e^n(\mathbf{w}t)$[20]. The definition in (55), however, is different from the corresponding definition of lexical relations in section **3** because now we also have to take into account the dummy individual $\#_e$ over and above the dummy world $\#_{\mathbf{w}}$ (since the intensional system introduced in this section builds on the extended PCDRT system in chapter 6, which makes use of the dummy / exception individual $\#_e$).

The definitions in (56) through (61) are identical to the corresponding definitions introduced in section **4** above and they are repeated here only to make the comparison with the individual-level definitions in chapter **6** easier.

55. $R_p\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{p \neq \#, u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset \wedge$

$$\forall i_s \in I_{p \neq \#, u_1 \neq \#, \ldots, u_n \neq \#}(R_{pi}(u_1 i, \ldots, u_n i))$$

56. $[p] := \lambda I_{st} J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[p]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[p]j)))$

57. $p' \Subset p := \lambda I_{st}.\ \forall i_s \in I(p'i = pi \vee p'i = \#)$

58. $p' \sqsubseteq p := \lambda I_{st}.\ (p' \Subset p)I \wedge \forall i_s \in I(pi \in p'I_{p' \neq \#} \rightarrow pi = p'i)$

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[20] Where $e^n\tau$ (for any type τ) is defined as in Muskens (1996): 157-158, i.e. $e^0\tau := \tau$ and $e^{m+1}\tau := e(e^m\tau)$.

59. $\mathbf{max}^p(D) := \lambda I_{st}.\lambda J_{st}.\ ([p];\ D)IJ \wedge \forall K_{st}(([p];\ D)IK \rightarrow pK_{p\neq\#}\subseteq pJ_{p\neq\#})$

60. $\mathbf{max}^{p'\sqsubseteq p}(D) := \mathbf{max}^{p'}([p'\sqsubseteq p];\ D)$

61. $\mathbf{dist}_p(D) := \lambda I_{st}.\lambda J_{st}.\ \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \leftrightarrow J_{p=w}\neq\emptyset) \wedge \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \rightarrow DI_{p=w}J_{p=w})$,

i.e. $\mathbf{dist}_p(D) := \lambda I_{st}J_{st}.\ pI{=}pJ \wedge \forall w_{\mathbf{w}}{\in}pI(DI_{p=w}J_{p=w})$.

The most important novelties introduced in this section are the definition of modal quantification and the definition of the indicative mood in (68) and (69) below.

Just as the generalized determiners in chapter **6** above relate dynamic properties *P*, *P'* etc. of type **et**, modal verbs relate dynamic propositions $\mathbb{P}$, $\mathbb{P}'$ etc. of type **st**, as shown in (68).

Moreover, just as a singular pronoun anaphorically retrieves an individual dref, requires it to be unique and makes sure that a dynamic property holds of that dref (see the translation of he_u in chapter **6**), the indicative mood anaphorically retrieves p^*, which is the designated dref for the actual world, requires it to be unique (since there is a unique actual world) and makes sure that a dynamic proposition holds of p^*, as shown in (69).

62. $_p(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}{=}J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$

63. $_{\langle p\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}{=}J_{p=\#} \wedge (I_{p\neq\#}{=}\emptyset \rightarrow I{=}J) \wedge (I_{p\neq\#}\neq\emptyset \rightarrow \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#})$

64. $\mathbf{unique}\{p\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s,i'_s{\in}I_{p\neq\#}(pi{=}pi')$

65. $\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}{=}\emptyset \wedge \mathbf{unique}\{q\}I \wedge$

$\mathbf{MODAL}_{\mu I\neq\{\#\},\omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

where μ and ω (dref's for a modal base and an ordering source respectively) are of type $s(\mathbf{w}t)$[21].

[21] Note that the first two conjuncts in (65), i.e. $I_{q=\#}{=}\emptyset \wedge \mathbf{unique}\{q\}I$, entail that qI is a singleton set $\{w\}$, where w cannot be the dummy world $\#_{\mathbf{w}}$.

The third conjunct in (65) is of the form $\mathbf{MODAL}_{M,O}\{W, W'\}$, where w is a possible world (of type **w**), M and O are sets of sets of worlds (of type $(\mathbf{w}t)t$), i.e. a modal base and ordering source respectively, and W and W' are sets of possible worlds (of type $\mathbf{w}t$), i.e. the restrictor and the nuclear scope of the modal quantification. The formula $\mathbf{MODAL}_{M,O}\{W, W'\}$ is defined following the Lewis (1973) / Kratzer (1981) semantics for modal quantification (see section section **4.2** above for more details).

66. **Example – the necessity condition (type** $(st)t$**):**

$\mathbf{NEC}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{NEC}_{\mu I\neq\{\#\},\omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

where $\mathbf{NEC}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M)\cap W_1)\subseteq W_2$ [22]

67. **Example – the possibility condition (type** $(st)t$**):**

$\mathbf{POS}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{POS}_{\mu I\neq\{\#\},\omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

where $\mathbf{POS}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M)\cap W_1)\cap W_2\neq\emptyset$

68. if^{p} + $modal_{\mu,\omega}{}^{p'\sqsubseteq p}$ ⇝

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$$

69. **indicative**$_{p^*}$ ⇝ $\lambda\mathbb{P}_{\mathbf{st}}.\ [\mathbf{unique}\{p^*\}];\ {}_{p^*}(\mathbb{P}(p^*))$,

where p^* is the dref for the actual world.

Note that the definition in (68) can be easily modified to allow for the kind of modal quantification instantiated by the second conditional in our Aquinas discourse (i.e. by the conditional in (2b) above). As shown in (70) below, we only need to make use of the maximization operator $\mathbf{max}^{p\Subset p'}(D)$ introduced in section **5** above, whose definition is repeated in (71) for convenience.

70. $if^{p\Subset p''}$ + $modal_{\mu,\omega}{}^{p'\sqsubseteq p}$ ⇝

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p\Subset p''}({}_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$$

71. $\mathbf{max}^{p\Subset p'}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H([p \mid p\Subset p']IH \wedge DHJ \wedge$

$$\forall K_{st}([p \mid p\Subset p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p\neq\#}\subseteq H_{p\neq\#}))$$

The most important difference between the definition of modal quantification in (68) and the definition in section **4** above is that we now introduce the *maximal* nuclear scope set of worlds *unrestricted / not parametrized by a modal base or an ordering source*. The modal parametrization comes in only later on, in the modal condition relating

[22] The definitions of $\mathbf{NEC}_{M,O}(W_1, W_2)$ and $\mathbf{POS}_{M,O}(W_1, W_2)$ differ slightly from the corresponding definitions in section **4.2** above, but they still rely on the **Ideal** function defined in that section.

the unparametrized maximal restrictor set and the unparametrized maximal nuclear scope set.

In contrast, the definition in section **4** introduces only the maximal restrictor set of worlds and, if the modal relation is necessity (**NEC**), it also introduces the maximal set of ideal worlds among the restrictor worlds. That is, the old definition introduces a maximal nuclear scope set only in some cases and, even then, it is a *parametrized* nuclear scope set (parametrized by a modal base and an ordering source).

Thus, what distinguishes the definition of dynamic modal quantification in (68) from the previous one – and, to my knowledge, from any other analysis of modal quantification in the previous dynamic literature[23] – is that: (**i**) it introduces maximal restrictor and nuclear scope sets and (**ii**) these maximal sets are unparametrized by modal bases or ordering sources. As we will see in the next section, the new definition has several theoretical and empirical advantages over the definition in section **4** above and the definitions in the previous dynamic semantics literature.

6.2. Advantages of the Novel Definition

The novel definition of modal quantification, which introduces the maximal unparametrized (i.e. not restricted by any modal base or ordering source) nuclear scope set of worlds over and above the maximal restrictor set of worlds, has several advantages over the definition in section **4** above (which introduces only the maximal restrictor set) – and over various other definitions proposed in the previous dynamic semantics literature.

For ease of comparison, I will restate the old definition of dynamic modal quantification using the new format (i.e. the format of the definition in (68) above), as shown in (72) below.

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[23] Most previous analyses of modal quantification differ from the new IP-CDRT analysis because either they did not have any modal dref's at all (Roberts 1987, 1989) or, if they had, the dref's had dynamic objects as values, e.g. <world, variable assignment> pairs (see, for example, Geurts 1995/1999 and Frank 1996 among others). Stone (1999) does propose an analysis of modal quantification that relates dref's for static objects (in particular, dref's for accessibility relations of type $s(\mathbf{w}(\mathbf{w}t))$, but his restrictor and nuclear scope sets, which are introduced by means of an ***if***-update (see Stone 1999: 17, (34)), are parametrized – their maximality is relativized to a Lewis-style similarity ordering source built into the ***if***-update.

72. **The previous definition of modal quantification** (see section **4** above)**.**

if^{p} + *modal*$_{\mu,\omega}$$^{p'\sqsubseteq p}$ $\rightsquigarrow$

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{\langle p\rangle}(\mathbb{P}(p)));\ [p' \mid p'\sqsubseteq p,\ \mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}];\ {}_{\langle p'\rangle}(\mathbb{P}'(p'))$$

The definition in (72) is formally simpler than the one in (68) because it has only one maximization operator. But the additional complexity of (68) is both theoretically and empirically motivated.

The theoretical advantage of the new definition in (68) over the previous definition in (72) is that the new definition systematically and explicitly captures the parallel between modal quantification and individual-level quantification as analyzed in chapter **6**. For convenience, I repeat the definition of individual-level quantification in (73) below. The reader can easily check that it is point-for-point parallel to the definition in (68) above.

73. *det*$^{u,u'\sqsubseteq u}$ $\rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}({}_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}({}_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

Empirically, the new definition is better than the previous one in at least two respects. As we will see, these two empirical advantages are a direct consequence of the parallel between the dynamic definition of modal quantification and its individual-level counterpart in (73) above. Let us examine them in turn.

First, the new definition generalizes to downward monotonic modal quantifiers (i.e. to modal determiners / modal relations that are downward monotonic in their right argument) like *impossible*, *improbable*, *unlikely* etc. To see this, note that, just as the individual-level quantification in (74) below is incompatible with (75) (i.e. *Few men left* entails that *It is not the case that most men left*), the modal quantification in (76) is incompatible with (77) (i.e. *Given the available evidence, it is improbable / unlikely that it will rain* entails that *It is not the case that, given the available evidence, it is probable / likely that it will rain*).

74. Few men left.

75. Most men left.

76. Given the available evidence, it is improbable / unlikely that it will rain.

77. Given the available evidence, it is probable / likely that it will rain.

This shows that, when computing the meaning of updates containing (individual-level or modal) determiners that are downward monotonic in their right argument, we need to have access to the maximal nuclear scope set (of individuals or possible worlds). To see this, consider the definition of dynamic individual-level quantification in (78) below, which does not introduce the maximal nuclear scope set and which is parallel to the old definition of modal quantification in (72) above.

78. **A definition of individual-level quantification that fails for determiners that are downward monotonic in their right argument:**

$$det^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}({}_{\langle u\rangle}(P(u)));\ [u' \mid u'\sqsubseteq u, \mathbf{DET}\{u, u'\}];\ {}_{\langle u'\rangle}(P'(u'))$$

The definition in (78) captures the meaning of upward monotonic quantifiers, e.g. *Most men left* is correctly interpreted as: introduce the maximal set u of individuals that satisfies the restrictor dynamic property, i.e. the maximal set of men; then, nondeterministically introduce some subset u' of the restrictor set u that is a *most*-subset (i.e. it is more than half of the restrictor set). If there is at least one such non-deterministically introduced subset u' that also satisfies the nuclear scope dynamic property, then the *most*-quantification is successful.

However, the definition in (78) fails to capture the meaning of downward monotonic quantifiers, e.g. *Few men left* is incorrectly interpreted as: introduce the maximal set u of individuals that satisfies the restrictor dynamic property, i.e. the maximal set of men; then, nondeterministically introduce some subset u' of the restrictor set u that is a *few*-subset (i.e. it is less than half of the restrictor set, possibly the empty set). If there is at least one such non-deterministically introduced subset u' that also satisfies the nuclear scope dynamic property (let us assume that the empty set vacuously satisfies any property), then the *few*-quantification is successful.

This meaning for *few* fails to capture the fact that *Few men left* is incompatible with *Most men left* because, even if we are successful in introducing a *few*-subset that satisfies the nuclear scope property, it can still be the case that a *most*-subset, for example, also satisfies that property, i.e. a successful update with *Few men left* does not rule out a successful update with *Most men left* (this is a direct consequence of the proposition relating witness sets and quantifier monotonicity in Barwise & Coopeer 1981: 104[24]; for a closely related discussion, see fn. **15** in section **3.3** of chapter **6**).

And, just as the definition of individual-level quantification in (78) above fails to account for the fact that *Few men left* is incompatible with *Most men left*, the parallel definition of modal quantification in (72) above fails (*mutatis mutandis*) to account for the fact that *Given the available evidence, it is improbable that it will rain* is incompatible with *Given the available evidence, it is probable that it will rain*.

The second advantage of the new definition of dynamic modal quantification over the previous one is that we predict without any additional stipulation that anaphora to the nuclear scope set of a modal quantification is always maximal – which is exactly what we need to account for the standard case of modal subordination in (53) above (i.e. *A^u wolf might come in. It_u would attack Harvey first*) and, also, for the more complex example involving interactions between *therefore* and modal subordination in (9) above (i.e. *A^u wolf might enter the cabin. It_u would see $John^{u'}$. Therefore, it_u would notice $him_{u'}$*).

In more detail: recall that, under its most salient interpretation, discourse (53) is interpretating as asserting that: (**i**) for all the speaker knows, it is a possible that a wolf comes in, and, in addition, (**ii**) for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first. That is, the modal *would* is anaphoric to all the epistemically accessible worlds in which a wolf comes in and not only to some of them.

However, according to the old definition, the modal verb *might* introduces only some (and not necessarily all) the epistemically accessible worlds in which a wolf comes in. Consequently, we would need an additional stipulation to the effect that, at least in discourse (53), *might* introduces the maximal set of epistemically accessible worlds satisfying the nuclear scope.

I can think of two ways of justifying the additional maximality stipulation associated with anaphora to *might* in discourse (53), namely: (**i**) modal anaphora is

[24] Page references to Partee & Portner (2002).

parallel to donkey anaphora and, in discourse (53), we have an instance of strong donkey-like modal anaphora and (**ii**) modal anaphora is parallel to plural anaphora and plural anaphora is always maximal.

However, as we will presently see, these two justifications do not hold under scrutiny. In contrast, the fact that the novel definition of modal quantification in (68) introduces the maximal unparametrized nuclear scope set of worlds is independently motivated by the need to capture the meaning of downward monotonic modal quantifiers.

Moreover, this *explanation* for the maximality of modal anaphora – i.e. the fact that the maximality of modal anaphora (analyzed as structured anaphora to quantifier domains) emerges as a consequence of independently justified meanings for dynamic generalized quantifiers – is parallel to the explanation provided in section **3.3** of chapter **6** above for the maximality of E-type anaphora in the invidual domain (recall the Evans examples *Few^{u} congressmen admire Kennedy and they$_{u}$ are very junior* and *Harry bought some^{u} sheep. Bill vaccinated them$_{u}$*).

Let us examine the first suggestion above, namely the idea that modal anaphora is, in general, parallel to donkey anaphora (and not parallel to E-type anaphora to quantifier domains) and that discourse (53) is basically an instance of strong donkey anaphora in the modal domain.

This hypothesis derives the intuitively correct truth-conditions for discourse (53) since the modal *might* in (53a) has a strong donkey reading and, therefore, introduces the *maximal* set of epistemically accessible possible worlds in which a wolf comes in (see the PCDRT analysis of weak / strong donkey ambiguities in chapter **5**). The modal anaphor *would* in (53b) will then retrieve this maximal set of worlds and further elaborate on them, much like the anaphor *it$_{u}$* in *Every farmer who owns a$^{\mathbf{str}:u}$ donkey beats it$_{u}$* retrieves all the donkeys owned by any particular farmer.

The problem with this hypothesis is that we expect to find instances of modal anaphora that have weak donkey-like readings – and I am not aware of any examples of this kind. All the examples of cross-sentential modal anaphora to *might* of the same form as discourse (53) above seem to require maximality – and the same maximality requirement seem to be obligatory in cases in which *might* occurs embedded in conditional structures. Consider, for example, the conditional in (79) below, where the (putatively donkey-like) modal *might* occurs in the antecedent of a conditional and the purpose infinitival clause *to kill it* in the consequent is (presumably) anaphoric to the epistemically accessible possible worlds introduced by *might*.

79. If you think a rat might come in, you should bring some poison to kill it. [25]
80. If you think a rat might come in, then you should bring some poison so that:
    if a rat does come in, you'll have a way to kill it / #you might have a way to kill it.

As the intuitively correct paraphrase in (80) above shows, the modal anaphora does not have a weak reading: the infinitival clause is anaphoric to all the worlds in which a rat comes in and not only to some of the (epistemically accessible) worlds in which a rat comes in.

The second suggestion made above is that modal anaphora is parallel to plural anaphora and, given that plural anaphora is always maximal, this explains why modal anaphora to *might* is always maximal. Much like the previous hypothesis, this one also derives the intuitively correct truth-conditions for discourse (53). But it ultimately faces the same problems as the "modal anaphora as donkey anaphora" idea – and this is because plural anaphora is in fact not always maximal / strong.

Plural donkey anaphora to *some* does indeed seem to always be maximal / strong, both in cross-sentential cases (the Evans example: *Harry bought some*^{u} *sheep. Bill vaccinated them*$_{u}$[26]) and in the case of intra-sentential plural donkey anaphora – see for example (81) below.

81. Every person who has some^{u} dimes will put them$_{u}$ in the meter.

However, the maximality effect in all these cases seems to be due to the determiner *some*, because plural anaphora to cardinal indefinites can very well be non-maximal /

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[25] This example incorporates several modifications suggested to me by Jessica Rett (p.c.).

[26] Based on Evans (1980): 217, (8) (page references are to Evans 1985).

weak, as shown by the cross-sentential example and the donkey sentence in (82) and (83) below.

82. Harry bought two^{u} sheep and Bill vaccinated them$_{u}$.
But Bill didn't vaccinate all the sheep that Harry bought on the same occasion / But Bill didn't vaccinate the three other sheep that Harry bought on the same occasion.
83. Every person who has two^{u} dimes will put them$_{u}$ in the meter.

Thus, the idea that modal anaphora is parallel to individual-level plural anaphora is problematic for the same reason as the "modal anaphora as donkey anaphora" hypothesis, because there seem to be no non-maximal instances of modal anaphora to *might* – which is exactly what we would expect under the "modal anaphora as anaphora to quantifier domains" view pursued throughout this section.

6.3. Conditional Antecedents vs. Modal Bases

As (68) indicates, I take modal generalized determiners to have a composite, conditional-like structure. The observation that antecedents of conditionals contribute to the restrictor, i.e. the modal base, of a modal quantification goes back at least to Kratzer (1981). A typical example (which, incidentally, provides an argument for ordering sources over an above modal bases) is given in (84) below.

84. If^{p} there is a$^{\mathbf{str}:u}$ murder, the$_{u}$ murderer must$_{\mu,\omega}{}^{p'\sqsubseteq p}$ go to jail.

The modalized conditional in (84) is interpreted as a modal quantification relativized to a contextually provided empty modal base μ and a contextually provided deontic ordering source ω (e.g. *in view of the law in the actual world*).

The antecedent of the conditional contributes the set p of all worlds where there is some murder or other. The modalized conditional is true if the consequent of the conditional is satisfied in all the deontically ideal worlds among the p-worlds intersected with the modal base worlds; since, in this case, the modal base is empty (hence it is vacuously satisfied in any possible world), the restrictor of the quantification is just the set of p-worlds.

However, despite the fact that antecedents and modal bases should be lumped together in the evaluation of a modal quantification (as Kratzer 1981 has it), they should in fact be distinguished for anaphoric purposes: as example (84) shows, we can have donkey anaphora between the definite *the*$_u$ *murderer* in the consequent and the indefinite *a*^{u} *murder* in the antecedent. This is the reason for the systematic distinction between the conditional antecedent (i.e. the restrictor *stricto sensu*) and the modal base in the definition of dynamic modal quantification in (68) above.

The necessity to distinguish between conditional antecedents and modal bases is further supported by the discourses in (85) and (86) below (based on examples (7) and (10) in Stone 1999: 4-5), where we have instances of cross-sentential (modally subordinated) anaphora to dref's introduced in antecedents of conditionals[27].

85. **a.** If a^u wolf came in, John could escape (from it$_u$). **b.** It$_u$ might eat Mary though.

86. **a.** If a^u wolf came in, John could not legally kill it$_u$. **b.** But he still would have to.

6.4. Anaphoric Modal Quantifiers

Finally, just as quantifiers like *always* in (54b) above anaphorically retrieves its restrictor (more exactly: it is anaphoric to the nuclear scope dref introduced by the determiner *every* in (54a)), the modal quantifier *would* in (53b) anaphorically retrieves its restrictor – and, in a parallel way, *would* in (53b) is anaphoric to the nuclear scope dref introduced by the modal verb *might* in (53a). The general format for the translation of such anaphoric modal quantifiers is provided in (87) below.

87. $\mathit{modal}_{\mu,\omega,p}{}^{p'\sqsubseteq p} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

[27] Unlike the Aquinas discourse in (1/2) above, where the modally subordinated pronoun is located in the restrictor of the modal quantification, the modally subordinated pronoun in discourses (85) and (86) is located in the nuclear scope.

This concludes our brief survey of the version of Intensional PCDRT (IP-CDRT) that builds on the extended PCDRT system introduced in chapter **6**[28].

6.5. Subordination across Domains

We finally turn to the IP-CDRT analysis of the modal subordination discourse in (53). As desired, this analysis is the exact modal counterpart of the analysis of quantificational subordination in section 4 of chapter **6**.

Under its most salient interpretation, discourse (53) asserts that, for all the speaker knows, it is a possible that a wolf comes in and that, for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first. Thus, we are interested in the "narrow-scope indefinite" reading of discourse (53), wherein the indefinite *a wolf* in (53a) has narrow scope relative to the modal *might* and sentence (53b) preserves and elaborates on this *de dicto* reading.

The meanings for the two modal quantifiers *might* in (53a) and *would* in (53b) are provided in (88) and (89) below. Given that the modal relation **POS** contributed by *might* has a built-in existential commitment, i.e. there must be a non-empty restrictor set of worlds p of a non-empty nuclear scope set of worlds p' (see the definition in (67) above), we can simplify the meaning of *might* by replacing the operators $_{\langle p\rangle}(\ldots)$ and $_{\langle p'\rangle}(\ldots)$ with $_{p}(\ldots)$ and $_{p'}(\ldots)$. The same applies to the meaning of anaphoric *would* because, according to definition (66) above, if the restrictor set of *would* (i.e. p') is non-empty, then so must be its nuclear scope set p'' (given that *would* is parametrized by the same modal base as *might*).

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[28] For completeness, I provide below the revised intensional meanings for dynamic properties, generalized determiners, indefinite articles, pronouns and proper names.

(**i**) *girl* $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [girl_q\{v\}]$

(**ii**) **det**$^{u,u'\sqsubseteq u}$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{u}(_{\langle u\rangle}(P(u)(q)));\ \mathbf{max}^{u'\sqsubseteq u}(_{\langle u'\rangle}(P'(u')(q)));\ [\mathbf{DET}\{u, u'\}]$

(**iii**) *a*$^{\mathbf{wk}:u}$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u];\ _{u}(P(u)(q));\ _{u}(P'(u)(q))$

(**iv**) *a*$^{\mathbf{str}:u}$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{u}(_{u}(P(u)(q));\ _{u}(P'(u)(q)))$

(**v**) *he*$_u$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\mathbf{unique}\{u\}];\ _{u}(P(u)(q))$

(**vi**) *Harvey*^{u} $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u \mid u \in Harvey];\ _{u}(P(u)(q))$, where $Harvey := \lambda i_s.\ harvey_e$.

88. *might*$_{\mu,\omega}{}^{p,p'\sqsubseteq p}$ ⇝ $\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.$ $\mathbf{max}^{p}({}_{p}(\mathbb{P}(p)))$; $\mathbf{max}^{p'\sqsubseteq p}({}_{p'}(\mathbb{P}'(p')))$; $[\mathbf{POS}_{q,\mu,\omega}\{p, p'\}]$

89. *would*$_{\mu,\omega,p'}{}^{p''\sqsubseteq p'}$ ⇝ $\lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.$ $\mathbf{max}^{p''\sqsubseteq p'}({}_{p''}(\mathbb{P}(p'')))$; $[\mathbf{NEC}_{q,\mu,\omega}\{p', p''\}]$

The contextually supplied modal base μ (of type $s(\mathbf{w}t)$) for both *might* and *would* is *epistemic*, e.g. it associates with each q-world the set of propositions that the speaker believes in that q-world. The contextually supplied ordering source ω is empty[29], which means that it does not contribute anything to the meaning of the two modal quantifications – and we will henceforth ignore it.

Given that the *might* quantification is discourse initial, we have to accommodate a restrictor proposition $\mathbb{P}_{\mathbf{st}}$ – and a natural choice is the trivial dynamic proposition $\lambda q_{\mathbf{s}}.$ $[q\Subset q]$. This ensures that the restrictor dref p introduced by *might* stores the set of all possible worlds (since the restrictor DRS is $\mathbf{max}^{p}({}_{p}([p\Subset p])))$, which in turn entails that we quantify over each and every world compatible with the epistemic modal base μ – and this is intuitively correct: when uttered out of the blue, sentence (53a) is interpreted as asserting that, for all the speaker knows, it is possible that a wolf comes in.

Finally, both modal quantifications in (53a) and (53b) are interpreted relative to the actual world, since the epistemic modal base μ for both quantifications is in fact the set propositions believed by the actual speaker in the actual world. I will capture this means of an indicative mood morpheme taking scope over the modal verbs. Thus, I will assume that sentences (53a) and (53b) have the logical forms provided in (90) and (91) below; the logical forms are followed by their compositionally derived IP-CDRT translations[30].

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[29] Emptiness can be required by a condition of the form **empty**$\{\omega\} := \lambda I_{st}.\ \forall i_s\in I(\omega i=\{\#\})$, i.e., throughout the plural info state I, we assign to the dref ω of type $s(\mathbf{w}t)$ the dummy object of type $\mathbf{w}t$, which is the singleton set of the dummy world $\{\#_{\mathbf{w}}\}$.

[30] I employ the notational abbreviations and equivalences introduced in chapter **6** above (see appendix **0** of chapter **6** for the entire list), in particular:

(i) ${}_{p}(C) := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall w\in pI_{p\neq\#}(CI_{p=w})$, where C is a condition (of type $(st)t$)

(ii) ${}_{p}(\alpha_1, \ldots, \alpha_n) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}[\alpha_1, \ldots, \alpha_n]J_{p\neq\#}$,
where $p\notin\{\alpha_1, ..., \alpha_n\}$ and $[\alpha_1, ..., \alpha_n] := [\alpha_1]; ...; [\alpha_n]$

(iii) ${}_{p}([C_1, \ldots, C_m]) = [{}_{p}(C_1), \ldots, {}_{p}(C_m)]$

90. $\mathbf{ind}_{p*}$($might_{\mu}{}^{p,p'\Subset p}$ ($\lambda q_{\mathbf{s}}.[q\Subset q]$) ($a^{\mathrm{wk}:u}$ *wolf come in*))

$$[\mathbf{unique}\{p^*\}];\ {}_{p*}(\mathbf{max}^{p}({}_{p}([p\Subset p]));\ \mathbf{max}^{p'\Subset p}([{}_{p'}(u)\mid {}_{p'}(wolf_{p'}\{u\}),\ {}_{p'}(come\_in_{p'}\{u\})]);$$
$$[\mathbf{POS}_{p*,\mu}\{p,p'\}])$$

91. $\mathbf{ind}_{p*}$($would_{\mu,p'}{}^{p''\Subset p'}$ (it_u *attack Harvey first*))

$$[\mathbf{unique}\{p^*\}];{}_{p*}(\mathbf{max}^{p''\Subset p'}([{}_{p''}(\mathbf{unique}\{u\}),\ {}_{p''}(attack_{p''}\{u,Harvey\})]);$$
$$[\mathbf{NEC}_{p*,\mu}\{p',p''\}])$$

Intuitively, the DRS in (90) instructs us to update the default info state $\{i_{\#}\}$ as follows. First, since the entire modal quantification is relativized to the actual world and the epistemic modal base provided by the speaker's beliefs, we need to introduce the actual world dref p^* and the epistemic modal base dref μ. These updates are default start-up updates for any discourse whatsoever, i.e. they are what Stalnaker (1978) refers to as "commonplace" updates that "will include any information which the speaker assumes his audience can infer from the performance of the [assertion] speech act" (Stalnaker 1978; see also the related discussion about deictic pronouns in section **3.7** of chapter **6** above). Thus, I will assume that the DRS in (90) is in fact preceded by the start-up update in (92) below.

92. $[p^*, \mu \mid p^*{=}w^*, \mathbf{epistemic}_{p*}\{\mu\}]$,

where $p^*{=}w^* := \lambda I_{st}.\ p^*I_{p*\neq\#}{=}\{w^*\}$

We are now able to test that the dref p^* contains a unique non-dummy world (in particular, the actual world w^*), as the first update in (90) instructs us to do.

Then, we introduce the dref p relative to the actual world dref p^* and store in it the set of all possible worlds (given that the condition $p\Subset p$ is vacuously satisfied). The next update instructs us to introduce the dref p' and store in it the maximal subset of p-worlds

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(iv) ${}_{p}([\alpha_1, \ldots, \alpha_n \mid C_1, \ldots, C_m]) = [{}_{p}(\alpha_1, \ldots, \alpha_n) \mid {}_{p}(C_1), \ldots, {}_{p}(C_m)]$.

that contain a wolf u that comes in. Given that p is the set of all possible worlds, p' will in fact store the set of all worlds that contain a wolf u that comes in.

We finally test that the nuclear scope p' is a μ-epistemic possibility relative to the restrictor p, which basically means that there is at least one possible world w which is both a p-world and a μ-world and which, in addition, is also a p'-world. In other words, the DRS in (90) is true iff there is an epistemic possibility of a wolf coming in.

The DRS in (91) instructs us to update the info state that we have obtained after processing (90) as follows. First, we test again that the dref p^* contains a unique non-dummy world, which we know is true (we have performed the same test in (90)). Then, we introduce the nuclear scope dref p'', which stores the maximal subset of the p'-worlds relative to which we have introduced a unique u-wolf and in which said wolf attacks Harvey first.

We finally test that the nuclear scope p'' is a μ-epistemic necessity relative to the restrictor p', which basically means that any possible world w which is both a p'-world and a μ-world is also a p''-world. In other words, the DRS in (91) is true iff *any* epistemically accessible possible world in which a wolf comes in is such that the wolf attacks Harvey first.

Thus, the IP-CDRT representation in (90) + (91) captures the intuitively correct truth-conditions for the modal subordination discourse in (53) above. Moreover, as desired, the representation in (90) + (91) is parallel to the corresponding PCDRT representation that captures the "narrow-scope indefinite" reading of the quantificational subordination discourse in (54) above. For convenience, I repeat this representation in (93) below (see **4.3** section of chapter **6** for more discusssion).

93. $\mathbf{max}^{u_1}([convention\{u_1\}]);\ [_{u_1}(u_2)];\ [_{u_1}(girl\{u_2\}),\ _{u_1}(court\_at\{Harvey, u_2, u_1\})];$
$[_{u_1}(\mathbf{unique}\{u_2\}),\ _{u_1}(come\_to\_banquet\_of\{u_2, u_1\})]$

The differences between the two representations are only an artifact of the fact that, in the analysis provided in (93), we have conflated the restrictor and nuclear scope dref's for both the determiner *every* in (54a) and the anaphoric adverb *always* in (54b). This simplification is, however, not possible for the modal representation in (90) + (91) because, unlike individual-level quantification, modal quantification is parametrized by a non-empty modal base and the restrictor and nuclear scope dref's of *might* and *would* cannot be conflated. The conflation is possible only of both the modal base and the ordering source are empty, as it was the case for the entailment particle *therefore* analyzed in section **4.3** of the present chapter and whose translation was simplified in much the same way as the translations of *every* and *always* in chapter **6**.

Thus, anaphora and quantification in the individual and modal domains are analyzed in a systematically parallel way in IP-CDRT, from the types of the dref's to the general format of the meanings associated with quantificational and anaphoric expressions.The fact that this formal feature is empirically and theoretically desirable has been repeatedly observed in the literature – see Stone (1997, 1999), Frank (1996), Geurts (1999), Bittner (2001), Schlenker (2005) among others, extending the parallel between the individual and temporal domains argued for in Partee (1973, 1984).

IP-CDRT – which builds on and unifies Muskens (1996), van den Berg (1996a) and Stone (1999) – is, to my knowledge, the first dynamic system that systematically captures the anaphoric and quantificational parallels between the individual and modal domains while, at the same time, keeping the underlying logic classical and preserving the Montagovian approach to compositionality.

6.6. *De Re* Readings

Consider the discourses in (94) and (95) below. In both cases, the only intuitively available reading for the indefinite a^{u} *wolf* is a *de re* reading, that is, the anaphoric pronoun it_u in the indicative sentences (94b)/(95b) rules out the "narrow-scope indefinite" reading $might_{\mu}{}^{p,p'\sqsubseteq p}$>>$a^{u}$ *wolf* for sentence (94a/95a).

94. **a.** $A^{\mathbf{wk}:u}$ wolf $might_{\mu}{}^{p,p'\sqsubseteq p}$ come in.

b. It_u escaped yesterday from the zoo.[31]

[31] Or: *A wolf might come in. It's the wolf that escaped yesterday from the zoo.*

95. **a.** $A^{\mathbf{wk}:u}$ wolf $\text{might}_{\mu}^{p,p'\sqsubseteq p}$ come in.

b. John saw it_u yesterday night standing dangerously close to the cabin.

Discourses (94) and (95) are parallel to the first discourse from Karttunen (1976) analyzed in chapter **6**, repeated in (96) below. Much as in (94) and (95) above, the anaphoric pronoun she_u in sentence (96b) rules out the "narrow-scope indefinite" reading $every^{u'}$ *convention*>>$a^{\mathbf{wk}:u}$ *girl* for sentence (96a).

96. **a.** Harvey courts $a^{\mathbf{wk}:u}$ girl at $\text{every}^{u'}$ convention.

b. She_u is very pretty.

According to the analysis in section **4.2** of chapter **6** above, this is a consequence of the fact that the two readings of sentence (96a) are effectively conflated by the condition $\mathbf{unique}\{u\}$ condition contributed by the singular number morphology on the pronoun she_u. The PCDRT representations of the entire discourse in (96), derived on the basis of the two (conflated) quantifier scopings of (96a), are repeated in (97) and (98) below.

97. $a^{\mathbf{wk}:u}$ *girl*>>$every^{u'}$ *convention*:

$[u \mid girl\{u\}];\ _u(\mathbf{max}^{u'}([convention\{u'\}]));\ [_u(court\_at\{Harvey, u, u'\})];$

$[\mathbf{unique}\{u\}, very\_pretty\{u\}]$

98. $every^{u'}$ *convention*>>$a^{\mathbf{wk}:u}$ *girl*:

$\mathbf{max}^{u'}([convention\{u'\}]);\ [_{u'}(u) \mid {}_{u'}(girl\{u\}),\ _{u'}(court\_at\{Harvey, u, u'\})];$

$[\mathbf{unique}\{u\}, very\_pretty\{u\}]$

This analysis, however, does not generalize to the modal case – and for a simple reason. The *de re* reading of the modal discourses in (94) and (95) above requires the common noun *wolf* to be interpreted relative to the dref for the actual world p^* over and above the fact that the indefinite a^u *wolf* in (94a/95a) brings to salience a single individual. The $\mathbf{unique}\{u\}$ condition contributed by the pronoun it_u in (94b)/(95b) can constrain only the cardinality of the set of individuals introduced by the indefinite a^u *wolf* – but it cannot require them to be wolves in the actual world, i.e. to satisfy the condition $wolf_{p^*}\{u\}$.

To see the problem more clearly, consider the two IP-CDRT representations of discourse (94) above provided in (99) and (100) below. For simplicity, I ignore the

unique$\{p^*\}$ condition and the $_{p^*}(\ldots)$ operator contributed by the indicative mood **ind**$_{p^*}$ (as the reader can check, nothing crucial rests on this assumption).

99. *a^u wolf>>might$_\mu^{p,p'\sqsubseteq p}$*, i.e. **ind**$_{p^*}$([*$a^{\mathbf{wk}:u}$ wolf*]^v *might$_\mu^{p,p'\sqsubseteq p}$* ($\lambda q_{\mathbf{s}}.[q \Subset q]$) (*$t_v$ come in*)):

$[u \mid wolf_{p^*}\{u\}]$; $_u(\mathbf{max}^p(_p([p \Subset p]))$; $\mathbf{max}^{p' \sqsubseteq p}([come\_in_{p'}\{u\}])$; $[\mathbf{POS}_{p^*,\mu}\{p, p'\}])$;

$[\mathbf{unique}\{u\}, escape\_from\_zoo_{p^*}\{u\}]$

100. *might$_\mu^{p,p'\sqsubseteq p}$>>a^u wolf*, i.e. **ind**$_{p^*}$(*might$_\mu^{p,p'\sqsubseteq p}$* ($\lambda q_{\mathbf{s}}.[q \Subset q]$) (*$a^{\mathbf{wk}:u}$ wolf come in*)):

$\mathbf{max}^p(_p([p \Subset p]))$; $\mathbf{max}^{p' \sqsubseteq p}([_{p'}(u) \mid {}_{p'}(wolf_{p'}\{u\}), {}_{p'}(come\_in_{p'}\{u\})])$; $[\mathbf{POS}_{p^*,\mu}\{p, p'\}]$;

$[\mathbf{unique}\{u\}, escape\_from\_zoo_{p^*}\{u\}]$

The representation in (99) provides the intuitively available *de re* reading: there is a *u*-individual that is a wolf in the actual *p**-world and there are some *p'*-worlds in which the *u*-individual comes in and that are μ-epistemic possibilites relative to the actual *p**-world. Note that we do not require the *u*-individual to be a wolf in these *p'*-worlds, but we can assume that, in all the relevant μ-accessible *p'*-worlds, the *u*-individual is a wolf because, on the one hand, it is a wolf in the actual *p**-world and, on the other hand, the μ-accessible worlds are also relativized to the actual *p**-world [32].

It is the representation in (100) that is problematic. Intuitively, the *de dicto* reading *might$_\mu^{p,p'\sqsubseteq p}$>>a^u wolf* should be ruled out, but the IP-CDRT representation in (100) incorrectly predicts that discourse (94) could have the following unavailable *de dicto* reading: there are some *p'*-worlds that are μ-epistemic possibilites relative to the actual *p**-world and there is this unique *u*-individual that is a wolf in each of the *p'*-worlds and that comes in in each of the *p'*-worlds. Moreover, the *u*-individual under consideration is such that it escaped from the zoo in the actual *p**-world. Note, in particular, that the *u*-individual can be a mouse or a giraffe in the actual world – and not necessarily a wolf.

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[32] Note that a similar reasoning can be used to account in IP-CDRT for the discourse in (i) below, due to Stone (1999): 8, (18), and which, as Stone (1999): 8-10 shows, poses significant problems for most alternative dynamic approaches to modal quantification (including Geurts 1995/1999, Frank 1996 and Frank & Kamp 1997).

(i) **a.** A^u wolf might walk in. **b.** We would be safe because John has $a^{u'}$ gun. **c.** He would use $it_{u'}$ to shoot it_u.

Thus, we observe that IP-CDRT over-generates with respect to modal subordination discourses like (94) above because it allows for an intuitively unavailable *de dicto* reading. However, the over-generation is not due to a peculiarity of the IP-CDRT system, but to the fact that the scopal interaction between a modal and an individual-level quantifier is more complex than the interaction between two individual-level quantifiers. In turn, this complexity is a consequence of the fact that the lexical relations contributed by a DP are always relativized to a modal dref and can, therefore, interact with a modal quantifier in a way that is independent from the interaction between that modal quantifier and the determiner heading the DP under consideration.

I will now briefly suggest a solution to this problem, following a proposal in Stone (1999). Stone (1999): 21 derives the infelicity of the example in (101) below by associating a presupposition of existence relative to a particular modal dref with every pronoun. This presupposition is of the form given in (102) below (Stone's actual implementation is different, but the basic proposal is the same as the one in (102), which is formulated in IP-CDRT terms).

101. **a.** John might$^{p'}$ be eating a^{u} cheesesteak. **b.** #It$_{u}$ is$_{p*}$ very greasy.
(Stone 1999: 21, (40))

102. u **exists in** $p := \lambda I_{st}.\ I_{u\neq\#,p\neq\#}\neq\emptyset \wedge \forall i_s\in I_{u\neq\#,p\neq\#}(ui$ **exists in** $pi)$,
where ***exists in*** is a constant of type $e(\mathbf{w}t)$[33].
Abbreviation: **in** := **exists in**,
i.e. we omit '**exists**', e.g. u **in** p, x **in** w etc.

The basic proposal in Stone (1999) (various technical details are, again, different) is that the pronoun *it*$_u$ in (101b) contributes such a presupposition of existence relative to the actual world dref $p*$, i.e. u **in** $p*$. This presupposition, however, is not satisfied because the indefinite *a*^{u} *cheesesteak* in (101a) receives a narrow scope, *de dicto* reading and introduces the u-individual only relative to the epistemically accessible p'-worlds

[33] This particular format for 'relativizing' the domain of individuals to possible worlds is due to Muskens (1995b). I use it in IP-CDRT only for its formal simplicity – and without any particular commitment to the possibilist (as opposed to the actualist) approach to quantified modal logic.

contributed by $might^{p'}$, i.e. u **in** p'. Consequently, the u-individual exists in the p'-epistemically accessible worlds, but not necessarily in the actual p^*-world, which makes the discourse in (101) infelicitous.

The discourse in (101) is infelicitous because the most salient reading for sentence (101a) is the *de dicto*, "narrow-scope indefinite" one, while sentence (101b) requires a *de re*, "wide-scope indefinite" reading to satisfy the existence presupposition contributed by the pronoun it_u. In contrast, the discourse in (94) above is felicitous because the *de re*, "wide-scope indefinite" reading for sentence (94a) is salient enough – but the same presuppositional mechanism that accounts the infelicity of (101) enables us to account for the fact that the only available reading for discourse (94) as a whole is the *de re* one.

More precisely, I propose to revise the IP-CDRT translations for indefinite articles and pronouns as shown in (103), (104) and (105) below. The new translations are identical to the ones proposed above (see fn. **28** in section **6.4** above) except for the addition of Stone-style existence conditions of the form u **in** p. The presuppositional status of such conditions when contributed by pronouns is indicated by underlining. A simplified version of the translation for pronouns – which is good enough for our current purposes – is provided in (106).

103. $a^{\mathrm{wk}:u} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u \mid u\ \mathbf{in}\ q];\ _u(P(u)(q));\ _u(P'(u)(q))$

104. $a^{\mathrm{str}:u} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^u([u\ \mathbf{in}\ q];\ _u(P(u)(q));\ _u(P'(u)(q)))$

105. $it_{u,p} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\underline{u\ \mathbf{in}\ p},\ \underline{q \Subset p}];\ [\mathbf{unique}\{u\}];\ _u(P(u)(q))$ [34]

106. $it_u \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\underline{u\ \mathbf{in}\ q}];\ [\mathbf{unique}\{u\}];\ _u(P(u)(q))$

For concreteness, I will assume that the presuppositional conditions $\underline{u\ \mathbf{in}\ q}$ contributed by pronouns have to be satisfied as such in discourse, i.e. a condition of the

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[34] Alternatively (or: in addition), we can associate every lexical relation $R_q\{v_1, ..., v_n\}$ with a family of existence presuppositions of the form given in (i) below. For our current purposes, the simplified form in (ii) is sufficient. Just as before, underlining indicates presuppositional status.

(i) $\lambda v_n \ldots \lambda v_1.\lambda q.\ [\underline{v_1\ \mathbf{in}\ p_1}, \ldots, \underline{v_n\ \mathbf{in}\ p_n}, \underline{q \Subset p_1}, \ldots, \underline{q \Subset p_n}];\ [R_q\{v_1, ..., u_n\}]$

(ii) $\lambda v_n \ldots \lambda v_1.\lambda q.\ [\underline{v_1\ \mathbf{in}\ q}, \ldots, \underline{v_n\ \mathbf{in}\ q}];\ [R_q\{v_1, ..., u_n\}]$.

form u **in** q has to be available (and 'accessible') in the representation of the previous discourse[35]. The revised (compositionally derived) IP-CDRT representations of discourse (94) are provided in (107) and (108) below. The presupposition $\underline{u \textbf{ in } p^*}$ contributed by the pronoun it_u in (94b) is satisfied in the *de re* representation in (107), but not in the *de dicto* representation in (108) – hence, we correctly predict that the only available reading for discourse (94) as a whole is the *de re* one.

107. a^u $wolf{>>}might_\mu{}^{p,p'\sqsubseteq p}$, i.e. $\textbf{ind}_{p^*}(\ [a^{\textbf{wk}:u}\ wolf]^v\ might_\mu{}^{p,p'\sqsubseteq p}\ (\lambda q_{\textbf{s}}.[q \Subset q])\ (t_v\ come\ in)\ )$:

$[u \mid u \textbf{ in } p^*, wolf_{p^*}\{u\}]$;

$_u(\textbf{max}^p(_p([p \Subset p])); \textbf{max}^{p' \sqsubseteq p}([come\_in_{p'}\{u\}]); [\textbf{POS}_{p^*,\mu}\{p, p'\}])$;

$[\underline{u \textbf{ in } p^*}]$; $[\textbf{unique}\{u\}, escape\_from\_zoo_{p^*}\{u\}]$

108. $might_\mu{}^{p,p'\sqsubseteq p}{>>}a^u\ wolf$, i.e. $\textbf{ind}_{p^*}(\ might_\mu{}^{p,p'\sqsubseteq p}\ (\lambda q_{\textbf{s}}.[q \Subset q])\ (a^{\textbf{wk}:u}\ wolf\ come\ in)\ )$:

$\textbf{max}^p(_p([p \Subset p])); \textbf{max}^{p' \sqsubseteq p}([_{p'}(u) \mid u \textbf{ in } p', {}_{p'}(wolf_{p'}\{u\}), {}_{p'}(come\_in_{p'}\{u\})])$;

$[\textbf{POS}_{p^*,\mu}\{p, p'\}]$;

$[\underline{u \textbf{ in } p^*}]$; $[\textbf{unique}\{u\}, escape\_from\_zoo_{p^*}\{u\}]$

7. Comparison with Alternative Approaches

Summarizing various points made throughout the present chapter (chapter **7**) and the previous two (chapters **5** and **6**), Intensional PCDRT differs from most previous dynamic approaches in at least three respects. The first difference is conceptual: PCDRT captures the idea that reference to structure is as important as reference to value and that the two should be treated in parallel (contra van den Berg 1996a, Krifka 1996b and Nouwen 2003 among others).

The second difference is empirical: the motivation for plural information states is provided by singular and intra-sentential donkey anaphora, in contrast to the previous

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[35] This is more in line with the binding / presupposition-as-anaphora theory of presupposition (van der Sandt 1992, Geurts 1995/1999, Kamp 2001 among others) rather than with the satisfaction theory (Karttunen 1974, Heim 1983b, 1992 among others), but I expect the solution to also be compatible with (some form of) the satisfication theory. See Krahmer (1998), Geurts (1995/1999) and Beaver (2001) for comparative evaluations of the two theories.

literature (see van den Berg 1996a, Krifka 1996b and Nouwen 2003) which relies on plural and cross-sentential anaphora.

Finally, from a formal point of view, Intensional PCDRT accomplishes two non-trivial goals for the first time.

On the one hand, it is not obvious how to recast van den Berg's Dynamic Plural Logic in classical type logic, given that, among other things, the former logic is partial and it conflates discourse-level plurality (i.e. the use of plural information states) and domain-level plurality (i.e. non-atomic individuals) (see chapter **8** below for more discussion about this distinction).

On the other hand, previous dynamic reformulations of the analysis of modal quantification in Lewis (1973) / Kratzer (1981), e.g. the ones in Geurts (1995/1999), Frank (1996) and Stone (1999), are not satisfactory insofar as they fail to associate modal quantifications with *contents* (i.e. the propositions such quantifications express in a particular context) and cannot account for the fact that the entailment particle *therefore* can relate such contents as, for example, in the Aquinas discourse analyzed in the present chapter (see section **7.2** below for more details).

In general, the previous dynamic approaches to modal subordination fall into three broad categories based on the way in which they encode the quantificational dependencies between possible scenarios (e.g. the epistemically accessible possibilities of a wolf coming in) and the individuals that feature in these scenarios (e.g. whichever wolf enters in a particular epistemically accessible possibility):

- accommodation accounts, e.g. Roberts (1987, 1989, 1995, 1996), where there are no modal dref's of any kind and the associations between possible scenarios and the individuals that feature in them is captured at the level of logical form, i.e. by accommodating / copying the DRS's that introduce the relevant individual-level dref's into the restrictor or nuclear scope DRS's of another modal operator;
- analyses like the ones proposed in Kibble (1994, 1995), Portner (1994), Geurts (1995/1999), Frank (1996), Frank & Kamp (1997) and van Rooy (2001), which take modal quantifiers to relate dynamically-valued dref's, i.e. (in the simplest case)

dref's for information states, where, following Heim (1983b), an information state is basically represented as a set of <world, variable assignment> pairs; in these approaches, the dependency between possibilities and individuals is encoded in the dref's for information states: every <world, assignment> pair is such that the assignment stores the individual-level dref's that have been introduced with respect to that world; these approaches to modal subordination are parallel to the "parametrized sum individuals" approaches to donkey anaphora and quantificational subordination in Rooth (1987) and Krifka (1996b): instead of summing atomic individuals that are each parametrized with a variable assignment, these approaches 'sum' possible worlds that are each parametrized with a variable assignment;

- encapsulated quantification accounts, e.g. Stone (1997, 1999) and Bittner (2001, 2006), where modal quantifiers relate dref's for *static* objects (unlike Geurts 1995/1999, Frank 1996 and van Rooy 2001), namely dref's for accessibility relations. Thus, modal dref's in such accounts are of type $s(\mathbf{w}(\mathbf{w}t))$ and individual-level dref's are of type $s(\mathbf{w}e)$, i.e. they are dref's for individual concepts. The quantificational dependency between possibilities and individuals is encoded in the complex static objects that these dref's have as values. For example, in a sentence like *A wolf might come in*, the modal *might* introduces a dref of type $s(\mathbf{w}(\mathbf{w}t))$ which, with respect to a given 'assignment' i_s, stores a function of type $\mathbf{w}(\mathbf{w}t)$ that maps (the current candidates for) the actual world to the set of epistemically accessible worlds in which a wolf comes in; at the same time, the indefinite *a wolf* introduces a dref of type $s(\mathbf{w}e)$ which, relative to a given 'assignment' i_s, stores a function mapping every epistemically accessible world w in which a wolf comes in to the wolf that comes in in w.

Intensional PCDRT (IP-CDRT) makes use of a fourth way of capturing the quantificational dependencies between possibilities and individuals, namely plural information states. Just as in encapsulated quantification accounts, the IP-CDRT dref's for possibilities have static objects as values – in particular, they are of type $s\mathbf{w}$, storing a possible world w relative to each 'assignment' i. The dref's for individuals have the usual type se. But, unlike in encapsulated quantification accounts, the quantificational dependencies between possibilities and individuals are stored in the plural info states that are incrementally updated in discourse and not in the static objects that the modal and individual-level dref's have as values.

For example, in a sentence like *A wolf might come in*, the modal *might* introduces a dref p of type $s\mathbf{w}$ which, with respect to a plural info state I_{st}, stores the set of worlds $pI := \{pi: i \in I\}$ in which a wolf comes in. The indefinite *a wolf* introduces a dref u of type se which, with respect to each world w in which a wolf comes in, stores the wolf or wolves that come in in w. That is, for every world w, the sub-state $I_{p=w} := \{i \in I: pi=w\}$ (which stores only the world w relative to p) stores the corresponding wolf or wolves relative to u, i.e. the set of wolves associated with w is $uI_{p=w} := \{ui: i \in I_{p=w}\}$. Thus, the dependency between worlds and wolves is stored in the plural info state I_{st} in a pointwise manner: for each $i_s \in I$, the wolf ui comes in in world pi.

The subset of the p-worlds that are epistemically accessible from the actual world w^* are also accessed via the the quantificational dependencies stored in the plural info state I_{st}. First, we have that the dref for the actual world p^* stores only the actual world w^* relative to the entire plural info state I_{st}, i.e. we have that $p^*I=\{w^*\}$ – consequently, the plural info state I is the same as $I_{p^*=w^*}$. Second, following the proposal in Kratzer (1981), IP-CDRT assumes that an epistemic modal base μ is contextually supplied: μ is a dref of type $s(\mathbf{w}t)$[36] and the dref μ stores a set of propositions $\mu I_{p^*=w^*} := \{\mu i: i \in I_{p^*=w^*}\}$ relative to the current plural info state $I_{p^*=w^*}$, hence relative to the actual world w^*.

The differences between IP-CDRT and previous approaches stem from the two main features of its account of modal subordination: (**i**) the use of modal dref's that have static objects as values; (**ii**) the use of plural info states to encode quantificational dependencies.

7.1. Statically vs. Dynamically Valued Modal Dref's

The first feature, namely using modal dref's with static objects as values, is shared with encapsulated quantification accounts. Using modal dref's with static objects as

[36] Note the simplification in type relative to the modal bases in Kratzer (1981), which have type $\mathbf{w}((\mathbf{w}t)t)$.

values has several advantages relative to the first two categories of approaches, i.e. accommodation approaches and approaches that use dref's with information states as values. Stone (1999) (see pp. 5-11 in particular) provides a lucid review of these two categories of approaches and a persuasive argument for using modal dref's with static objects as values, which I will not iterate here. I will only summarize the two main arguments – the first one is empirical, while the second is more theoretical in nature.

Empirically, the first two categories of approaches to modal subordination have difficulties accounting for discourses that involve multiple inter-related possible scenarios like the one in (109) below.

109. **a.** A^{u} wolf might walk in.
b. We would be safe because John has a$^{u'}$ gun.
c. He would use it$_{u'}$ to shoot it$_{u}$.
(Stone 1999: 8, (18))

As Stone (1999) puts it:

> "[The discourse in (109)] describes two situations: an actual present situation, in which John has a gun; and a possible future development of that situation, in which a wolf walks in. The last sentence of [(109)] illustrates that the speaker may refer both to the possible wolf and to John's gun in a description of that possible future. […] In previous dynamic approaches, scenarios are interpreted as sets of DYNAMIC objects, in which possible worlds are paired with assignments that indicate what entities are available for reference there. (Entities are introduced into a sequence of evolving SCENARIOS rather than into evolving representations of the DISCOURSE.)
> Because scenario referents explicitly inventory available referents, we can only refer to a gun in a scenario in which a gun has been explicitly added. This is incompatible with the pattern of reference in [(109)]. First, the discourse describes a possible elaboration of what we know, where a wolf comes in (and we are safe). Then the discourse evokes a further elaboration of our information which includes a gun. Although this elaboration describes reality, it nevertheless leaves the original hypothetical scenario unchanged. There is therefore no gun to refer to in the wolf-scenario."
> (Stone 1999: 8-9)

For more details, see Stone (1999): 9-11.

In contrast, encapsulated quantification approaches and IP-CDRT (see in particular the account of *de re* readings in section **6.6** above) can account for such discourses because they model possible scenarios as ordinary static objects and can relate multiple scenarios and the individuals featured in them in very much the same way as classical DRT / FCS / DPL approaches introduce and relate multiple individual dref's.

The theoretical argument in Stone (1999) against the first two kinds of approaches to modal subordination is that they fail to capture the anaphoric and quantificational parallels between the individual and modal domains argued for in Stone (1997, 1999), Bittner (2001, 2006) and Schlenker (2003, 2005b) among others. In contrast (as shown by the parallel analysis of quantificational and modal subordination in section **6** above), the theoretical architecture of IP-CDRT enables us to give a point-for-point parallel account of anaphora and quantification in the individual and modal domains, from the types assigned to the modal and individual dref's to the translations compositionally associated with anaphoric and quantificational expressions.

7.2. Plural Info States vs. Encapsulated Quantification

Let us turn now to the second feature of the IP-CDRT account of modal subordination, namely the use of plural info states to capture quantificational dependencies. This is the feature that distinguishes IP-CDRT from encapsulated quantification accounts (e.g. Stone 1997, 1999 and Bittner 2001, 2006).

There is one argument that seems to recommend the use of plural info states to encode quantificational dependencies as opposed to the use of encapsulated quantification: encapsulated quantification approaches (which, in a broad sense, include approaches that make use of choice functions and / or Skolem functions to account for donkey anaphora and quantificational subordination – see section **6** of chapter **5** above) do not store quantificational dependencies introduced in discourse in the database that is meant to store discourse-related information, i.e. in the information states, but in the meaning of the lexical items, be they the indefinite-like items that introduce new dref's or the pronoun-like items that retrieve them.

The point (already made in van den Berg 1994, 1996a with respect to individual-level plural anaphora) can be more easily clarified if we consider the quantificational subordination examples in (110) and (111) below. The modal subordination based argument is similar.

110. **a.** Every^{u} man loves a$^{u'}$ woman.
b. They$_{u}$ bring them$_{u'}$ flowers to prove this.
(van den Berg 1996a: 168, (16))

111. **a.** Every^{u} boy bought a$^{u'}$ flower and gave it$_{u'}$ to a$^{u''}$ girl.
b. They$_{u''}$ thanked them$_{u}$ for the$_{u'}$ very nice gifts.

Consider (110) first. Sentence (110a) establishes a twofold dependency between men and the women that they love and sentence (110b) further elaborates on this dependency. Encapsulated quantification approaches have to make use of functions from individuals to individuals of type *ee* (or relations between individuals of type *e*(*et*)) to capture the intuition that sentence (110b) elaborates on the dependency introduced in sentence (110a). That is, either the quantifiers (*every^{u} man* and *a$^{u'}$ woman*) or the pronouns (*they$_{u}$* and *them$_{u'}$*) – or both – have to have such functions as (part of) their semantic value.

Now consider discourse (111). Sentence (111a) establishes a threefold dependency between boys, flowers and girls and sentence (111b) further elaborates on this dependency. In this case, encapsulated quantification approaches need to make use of functions and / or relations that are more complex than the ones needed for discourse (110). Therefore, the semantic values assigned to quantifiers and / or pronouns will have to be more complex in the case of (111), despite the fact that the very same lexical items are used.

That is, quantifiers and / or pronouns denote functions / relations of different arities depending on the discourse context, i.e. depending on how many simultaneous anaphoric connections are established in a particular discourse. And these functions / relations become a lot more complex as soon as we start to explicitly represent anaphora to and quantification over possible worlds, times, locations, eventualities, degrees etc.

Summarizing, the (mostly theoretical) argument for plural info states as opposed to encapsulated quantification approaches is the following: since the arity of the functions / relations denoted by pronouns and / or quantifiers is determined by the discourse context, we should encode this context dependency in the info state (the purpose of which is to store precisely this kind of discourse information) and not in the representation of the lexical items themselves.

Turning now to more empirical considerations, IP-CDRT and encapsulated quantification approaches seem to have a similar empirical coverage as far as the English phenomena considered in this chapter are concerned (although see the observations in sections **7.4** and **7.5** below). However, only future research will decide if IP-CDRT based approaches can also scale up to account for typologically different languages (e.g. Kalaallisut), which have been successfully analyzed in an encapsulated quantification dynamic framework (see for example Bittner 2006).

Note, however, that the two frameworks are not incompatible, since IP-CDRT can also make use of dref's that have more complex modal objects as values, e.g. the dref's for modal bases and ordering sources used in this chapter. But, even in such cases, the use of plural info states enables us to simplify the types of such dref's – much like the types of modal and individual-level dref's in Stone (1999) are simplified in IP-CDRT: we only need to use dref's for possible worlds of type s**w** in IP-CDRT as opposed to the dref's for accessibility relations of type $s(\mathbf{w}(\mathbf{w}t))$ in Stone (1999); also, we only need to use dref's for individuals of type *se* in IP-CDRT instead of the dref's for individual concepts of type $s(\mathbf{w}e)$ used in Stone (1999).

I will conclude this section with three more observations about the differences between IP-CDRT and the encapsulated quantification system in Stone (1999).

First, for simplicity, Stone (1999) treats modal bases and ordering sources as static objects (see the definitions for necessity and possibility in Stone 1999: 27, (47)). IP-CDRT introduces dref's for modal bases and ordering sources, thus providing a dynamic treatment for all the contextually dependent components of modal quantification argued for in Kratzer (1981).

Second, IP-CDRT employs maximal *unparametrized* restrictor and nuclear scope sets in the definition of modal quantification, in contrast to Stone (1999), who introduces restrictor and nuclear scope sets for modal quantifiers by means an ***if***-update with a Lewis-style similarity ordering source built in (see Stone 1999: 17, (34)). To see that the built-in parametrization is too restrictive, consider the deontic conditional in (112) below (based on Kratzer 1981): (112) does not seem to involve a similarity ordering source because the conditional simply states that, according to the law, the deontically ideal worlds among the set of worlds where there is a murder are such that the murderer goes to jail. The deontic quantification is not restricted to the set of worlds where there is a murder and which are as similar as possible to the actual world since many of the facts in the actual world are orthogonal to the legal requirement specified by (112).

112. If there is a^u murder, the_u murderer must go to jail.

Finally, in contrast to the IP-CDRT definitions, the definitions of necessity and possibility in Stone 1999: 27, (47) do not associate contents with modal quantifications, so they cannot account for the *therefore* discourses in (1/2) and (9) above, in which *therefore* relates contents and not meanings (i.e. context-change potentials); for more discussion, see section **2.2** of the present chapter.

7.3. Conjunctions under Modals

Roberts (1996) (see also Roberts 1995) presents the following challenge for dynamic / anaphoric accounts of modal subordination. Consider the two discourses in (113) and (114) below (example (19) in Roberts 1996: 224 and example (3) in Roberts 1996: 216 respectively).

113. **a.** You should buy a^u lottery ticket and put it_u in a safe place.
[**b.** You're a person with good luck.]
c. It_u might be worth millions.

114. **a.** You should buy a^u lottery ticket and put it_u in a safe place.
b. #It_u's worth a million dollars.

Note that the *might* modal quantification in (113c) is restricted by the content of the first conjunct below the modal *should* in (113a), i.e. it is interpreted as asserting that, given that you're a generally lucky person, *if you buy a lottery ticket*, it might be worth millions. Crucially, (113c) is not restricted by the content of both conjuncts in (113a) or by the set of deontically ideal worlds contributed by *should*.

The challenge for dynamic approaches is to show that they do not under-generate, i.e. that they can account for the felicitous discourse in (113), and that they do not over-generate, i.e. that they can account for the infelicitous discourse in (114). In this section, I will briefly sketch how IP-CDRT can account for the first, felicitous example and derive the infelicity of the second. In the process, we will see that example (113) provides another empirical argument for the explicit introduction of contents in discourse.

The discourse in (113) is analyzed like the Aquinas discourse in (1/2) above, i.e. in terms of structured anaphora to propositions. The only component we need to add is a translation for *and* that introduces and relates the contents of its conjuncts, much like the analysis of conjunction in classical modal logic. A suitable translation is provided in (115) below, which, just as the translation for modal quantifiers in section **6.1** above, relies on structured inclusion to capture the anaphoric connections between the first and the second conjunct in (113a) above. Also, note that the conjunction *and* relates two maximal *unparametrized* sets of possible worlds – again, just like the definition of modal quantification in section **6.1** above.

115. $and^{p,p'\sqsubseteq p} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{p'}(\mathbb{P}'(p')));\ [q\sqsubseteq p']$

The translation for the modal *should* in (113a) is provided in (116) below; it is the expected one, modulo the fact that we omit the distributivity operator ${}_{p'}(\ldots)$ over the nuclear scope update[37].

[37] This is needed to ensure that the structural dependencies introduced within the two conjuncts are properly inherited by the nuclear scope dref p' – and it can be seen as the limit case (no distributivity operator at all) of the variability with respect to nuclear scope distributivity operators argued for in section **6.2** of chapter **6** above.

116. $if^{p} + should_{\mu,\omega}{}^{p'\sqsubseteq p} \rightsquigarrow$

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(\mathbb{P}'(p'));\ [\mathbf{NEC}_{q,\mu,\omega}\{p, p'\}]$$

In (116), μ is an epistemic modal base, ω is a deontic ordering source, the antecedent $\mathbb{P}$ is accommodated as $\lambda q_{\mathbf{s}}.\ [q\Subset q]$ (due to the fact that (113a) is discourse initial – see section **6.5** above for more discussion) and the consequent $\mathbb{P}'$ is the conjunction *you buy a lottery ticket and you put it in a safe place*, i.e. the dynamic proposition in (117) below. The final representation of (113a) has the form given in (118) below, which can be simplified as shown in (119), i.e. by omitting the dref p_2.

117. $\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p_1}(\,{}_{p_1}(\mathbb{P}_1(p_1)));\ \mathbf{max}^{p_2 \hat{o}\ p_1}(\,{}_{p_2}(\mathbb{P}_2(p_2)));\ [q\sqsubseteq p_2]$,

where $\mathbb{P}_1$ is "you buy a^{u} lottery ticket" and $\mathbb{P}_2$ is "you put it$_{u}$ in a safe place".

118. $\mathbf{max}^{p}([p\Subset p]);\ \mathbf{max}^{p'\sqsubseteq p}(\mathbf{max}^{p_1}(\,{}_{p_1}([u \mid lottery\_ticket_{\,p_1}\{u\},\ you\_buy_{\,p_1}\{u\}]));$

$\mathbf{max}^{p_2 \hat{o}\ p_1}(\,{}_{p_2}([you\_put\_in\_safe\_place_{\,p_2}\{u\}]));\ [p'\sqsubseteq p_2]);$

$[\mathbf{NEC}_{p*,\mu,\omega}\{p, p'\}]$,

where $p*$ is the dref for the actual world.

119. $\mathbf{max}^{p}([p\Subset p]);\ \mathbf{max}^{p_1 \hat{o}\ p}(\,{}_{p_1}([u \mid lottery\_ticket_{\,p_1}\{u\},\ you\_buy_{\,p_1}\{u\}]));$

$\mathbf{max}^{p' \hat{o}\ p_1}({}_{p'}([you\_put\_in\_safe\_place_{p'}\{u\}]));\ [\mathbf{NEC}_{p*,\mu,\omega}\{p, p'\}]$

Informally, the update in (119) instructs us to do the following operations on the default input info state $\{i_{\#}\}$. First, given that the modal verb is contextually dependent (much like deictic pronouns), we need to accommodate an update that introduces the dref for the actual world $p*$, the epistemic modal base μ and the deontic ordering source ω. Then, we process the first update in (119), namely $\mathbf{max}^{p}([p\Subset p])$, which instructs us to add a p column to the input info state and store in it the set of all possible worlds.

The next update instructs us to add a p_1 column and store in it all the p worlds in which you buy a lottery ticket; also, we add a u column and store in it the lottery ticket(s) that you buy in each corresponding p_1-world. Then, we add a p' column and store in it all the p_1-worlds in which you put in a safe place the corresponding u-lottery ticket(s).

Finally, we test that all the ω-deontically ideal worlds among the μ-epistemically accessible p-worlds are included in p'. That is, since p stores the set of all possible worlds, we simply test that all the ω-deontically ideal worlds among the μ-epistemically accessible worlds are such that you buy a lottery ticket and put it in a safe place.

Crucially, at the end of the update contributed by sentence (113a), we have access to the set of p_1-worlds satisfying the first conjunct below the modal *should*, i.e. we have access to all the worlds in which you buy a lottery ticket. We will therefore be able to interpret sentence (113c) in the usual way, i.e. as simultaneously anaphoric to the modal dref p_1 and the individual-level dref u. Thus, IP-CDRT is able to capture all the structured anaphoric connections established in discourse (113) and derive the intuitively correct truth-conditions associated with it.

The IP-CDRT account of the infelicitous discourse in (114) is basically the same as the account of the infelicitous discourse in (101) above (see section **6.6** of the present chapter).

7.4. Weak / Strong Ambiguities under Modals

Donkey anaphora in modalized conditionals exhibits weak / strong ambiguities just as it does in (extensional) relative-clause donkey sentences. In particular, the conditional in (2a) above, repeated in (120) below, provides an instance of strong donkey anaphora, while the conditional in (121) below, due to Partee (1984), provides an instance of weak donkey anaphora.

120. If a^u man is alive, he_u must find something pleasurable.
121. If you have a^u credit card, you should use it_u here instead of cash.
(Partee (1984): 280, fn. 12)

Given the analysis of the weak / strong ambiguity in chapter **5** above, it should be clear that IP-CDRT can account for both examples: the indefinite *a^u man* in (120) receives a strong reading (see section **4.1** of the present chapter), while the indefinite *a^u credit card* in (121) receives a weak reading. The intuitively correct truth-conditions for both discourses are derived in the usual way.

Weak / strong donkey ambiguities pose problems for all three categories of alternative approaches mentioned above. Accommodation-based approaches like Roberts (1987, 1989) can account only for strong donkey readings – a feature they inherit from the underlying classical DRT framework.

Approaches that use dref's for information states can also account only for strong readings. For example, the definitions of info state dref update in Frank (1996): 98, (36) and Geurts (1905/1999): 154, (43b) update a set F of <world, assignment> pairs with a DRS K (the denotation of which is a binary relation between <world, assignment> pairs) by taking the image of the set F under the relation denoted by K. That is, the output set G of <world, assignment> pairs obtained after updating F with K is the set $G = \{<w', g'>: \exists <w, g> \in F(<w, g>K<w', g'>)\}$. This kind of update predicts that, by the time we have processed the antecedent of the conditional in (121), the output set of <world, assignment> pairs will contain all the credit cards that you have, which in turn predicts that the conditional in (121) counter-intuitively requires you to use all your credit cards.

Finally, the encapsulated quantification approach in Stone (1999) can account only for weak donkey readings because indefinites introduce dref's for individual concepts (they are functions of type $s(\mathbf{w}e)$), hence, for each possible world, the dref will store exactly one individual. Such dref's are, basically, dref's for choice functions: given a world w, the individual concept will choose a particular entity that is a credit card you have in that world.

Thus, Stone (1999) can account for the weak reading conditional in (121) as follows: the indefinite in the antecedent (arbitrarily) chooses a credit card relative to each world w in which you have a non-empty set of credit cards; the consequent elaborates on this by requiring all the deontically ideal worlds w to be such that you use the corresponding card instead of cash.

By the same token, Stone (1999) cannot account for the strong reading conditional in (120), where the indefinite in the antecedent needs to introduce *all* the men that are alive in any given world w. An easy fix that would enable Stone (1999) to account for the strong donkey conditional in (120) would be to introduce dref's for properties, i.e. dref's of type $s(\mathbf{w}(et))$ which, relative to a given world w, would store the set of all men that are alive in w. However, for the reasons mentioned in section **1** of chapter **5** above, this strategy would fail for more complex examples involving multiple strong indefinites.

7.5. Uniqueness Effects under Modals

Modal subordination discourses exhibit the same kind of uniqueness effects (and variability thereof) as quantificational subordination discourses. Consider again examples (2) (*If* a^u *man is alive,* he_u *must find something pleasurable. Therefore, if* he_u *doesn't have any spiritual pleasure, he must have a carnal pleasure*), (53) (A^u *wolf might come in.* It_u *would attack Harvey first*), and (86) (*If* a^u *wolf came in, John could not legally kill* it_u*. But he still would have to*) above.

Discourse (53) seems to exhibit relativized uniqueness effects: it is (preferably) understood as talking about epistemic possibilities featuring a unique wolf coming in. In contrast, discourses (2) and (86) do not exhibit any uniqueness effects: (2) is not talking only about worlds / possibilities in which exactly one man is alive and (86) is interpreted as asserting that, if he wants to obey the law, John cannot kill any wolf or wolves that come in and, in addition, if he wants to survive, John has to kill any wolf or wolves that come in – neither the law, nor John's survival instinct are particularly geared towards possible scenarios in which a unique wolf comes in.

IP-CDRT can capture the relativized uniqueness effects associated with discourse (53) in much the same way as it captures the relativized uniqueness effects associated with quantificational subordination (see section **6.1** of chapter **6**). That is, if we use a strong / maximized indefinite article, the (relativized) uniqueness emerges from the interaction between the $\mathbf{max}^u$ operator contributed by the strong indefinite $a^{str:u}$ *wolf* and the $\mathbf{unique}\{u\}$ condition contributed by the singular pronoun it_u.

The lack of uniqueness effects associated with discourses (2) and (86) can be captured in the same way as the lack of uniqueness effects associated with donkey anaphora (see section **6.2** of chapter **6** for details), i.e. by means of suitable distributivity operators that neutralize / vacuously satisfy the **unique** conditions contributed by singular pronouns.

Thus, IP-CDRT can capture the wavering nature of the uniqueness implications associated with modal subordination in much the same way as it captures the wavering nature of the uniqueness effects associated with quantificational subordination and donkey anaphora.

It is not obvious to me how the alternative approaches mentioned above can capture the behavior of uniqueness effects in modal subordination discourses – so, I will leave this issue as a topic for future investigation and discussion.

Appendix

A1. Intensional PCDRT: Definitions and Translations

122. **New Dref's, Structured Inclusion, Maximization and Distributivity.**

a. $[p] := \lambda I_{st}J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[p]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[p]j)))$

b. $p'{\Subset}p := \lambda I_{st}.\ \forall i_s{\in}I(p'i{=}pi \vee p'i{=}\#)$

c. $p'{\sqsubseteq}p := \lambda I_{st}.\ (p'{\Subset}p)I \wedge \forall i_s{\in}I(pi{\in}p'I_{p'\neq\#} \rightarrow pi{=}p'i)$

d. $\mathbf{max}^{p}(D) := \lambda I_{st}.\lambda J_{st}.\ ([p];\, D)IJ \wedge \forall K_{st}(([p];\, D)IK \rightarrow pK_{p\neq\#}{\subseteq}pJ_{p\neq\#})$

e. $\mathbf{max}^{p'\sqsubseteq p}(D) := \mathbf{max}^{p'}([p'{\sqsubseteq}p];\, D)$

f. $\mathbf{max}^{p\Subset p'}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H([p \mid p{\Subset}p']IH \wedge DHJ \wedge$

$\forall K_{st}([p \mid p{\Subset}p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p\neq\#}{\subseteq}H_{p\neq\#}))$ [38]

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[38] This operator, more precisely $\mathbf{max}^{u'\Subset u}$, is independently required to analyze the example in (i) below within the revised PCDRT system of chapter **6**. This example can be easily analyzed within the system of chapter **5** (the only difference is that, to obtain the intuitively correct truth-conditions, we need the indefinite *a son* to be weak, not strong), but (i) poses problems for the revised definition of generalized quantification in chapter **6**, repeated in (ii) below for convenience. The problem is that (i) is falsified by any parent who has a son in high school and who has lent him the car on a weeknight even if said parent has another son who never got the car. This problem is posed by any determiner that is downward monotonic in his right argument, e.g. *Few parents with a son still in high school lend him the car on weekends* is intuitively falsified if most parents are such that they have a son in high school and they lent him the car on a weeknight even if, at the same time, all parents have at least one son who never got the car.

(i) No$^{u,u'\Subset u}$ parent with a$^{\mathbf{str}:u''}$ son still in high school has ever lent him$_{u''}$ the car on a weeknight.
(Rooth 1987: 256, (48))

(ii) $det^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}(_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

g. $\mathbf{dist}_p(D) := \lambda I_{st}.\lambda J_{st}.\ \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \leftrightarrow J_{p=w}\neq\emptyset) \wedge \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \rightarrow DI_{p=w}J_{p=w})$,

i.e. $\mathbf{dist}_p(D) := \lambda I_{st}J_{st}.\ pI=pJ \wedge \forall w_{\mathbf{w}}\in pI(DI_{p=w}J_{p=w})$

h. $_{p}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$

i. $_{\langle p\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge (I_{p\neq\#}=\emptyset \rightarrow I=J) \wedge (I_{p\neq\#}\neq\emptyset \rightarrow \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#})$

j. $_{p}(C) := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall w\in pI_{p\neq\#}(CI_{p=w})$, where C is a condition (of type $(st)t$)

k. $_{p}(\alpha_1, \ldots, \alpha_n) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}[\alpha_1, \ldots, \alpha_n]J_{p\neq\#}$,

where $p\notin\{\alpha_1, ..., \alpha_n\}$ and $[\alpha_1, ..., \alpha_n] := [\alpha_1]; ...; [\alpha_n]$

The definition in (ii) is problematic for the following reason. First, note that, if the indefinite *a son* is weak, we obtain intuitively incorrect truth-conditions for (i) because, if the indefinite introduces only the u''-son who never got the car relative to the corresponding u-parent, the $\mathbf{NO}\{u, u'\}$ condition is verified and we incorrectly predict that (i) is true in such a situation. Second, note that, if the indefinite *a son* is strong, i.e. we introduce both the u''-son that got the car and the u''-son that didn't get it with respect to the corresponding u-parent, then the $\mathbf{max}^{u'\sqsubseteq u}$ operator used to extract the nuclear scope will discard this parent, i.e. this u-parent will not be stored in u', because it is not the case that this u-parent lends the car to all the corresponding u''-sons. Hence, yet again, the $\mathbf{NO}\{u, u'\}$ condition is verified and we incorrectly predict that (i) is true in such a situation.

However, using the $\mathbf{max}^{u'\Subset u}$ operator to provide the alternative translation in (iii) below for (certain occurrences of) determiners that are downward monotonic in their right argument yields the intuitively correct truth-conditions for example (i) if the indefinite *a son* is strong. The reason is that the $\mathbf{max}^{u'\Subset u}$ update will retain any u-parent that lent the car to at least one son – and the the $\mathbf{NO}\{u, u'\}$ condition (or the $\mathbf{FEW}\{u, u'\}$ condition etc.) will not be verified anymore.

(iii) $det^{u,u'\Subset u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\Subset u}(_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

Given that the $\mathbf{max}^{p'\Subset p}$ operator is associated, in the modal domain, with conditional antecedents, which are also downward monotonic, a fairly general procedure for translating individual-level and modal determiners seems to emerge: the right upward monotone determiners $det\uparrow$ (*every*, *most* etc.) should receive the $det^{\alpha,\alpha'\sqsubseteq\alpha}$ type of translation in (ii), while the right downward determiners $det\downarrow$ (*no*, *few* etc.) should receive the $det^{\alpha,\alpha'\Subset\alpha}$ type of translation in (iii). Also, if the restrictor of a determiner is anaphoric to another dref α'', then, for left upward determiners $\uparrow det$, they should be translated as $det^{\alpha\sqsubseteq\alpha'',\alpha'\sqsubseteq\alpha}$ (if they are right upward monotone) or $det^{\alpha\sqsubseteq\alpha'',\alpha'\Subset\alpha}$ (if they are right downward monotone). If the determiners are left downward monotone, i.e. $\downarrow det$ (*every*, *if* etc.) and their restrictor is anaphoric to a dref α'', they should be translated as $det^{\alpha\Subset\alpha'',\alpha'\sqsubseteq\alpha}$ (if they are right upward monotone) or $det^{\alpha\Subset\alpha'',\alpha'\Subset\alpha}$ (if they are right downward monotone). For instance, the *if+must* determiner in the second conditional (i.e. the conclusion) of the Aquinas argument in (1/2) receives the $det^{p\Subset p'',p'\sqsubseteq p}$ translation in (iv) below.

(iv) $det^{p\Subset p'',p'\Subset p} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.\lambda \mathbb{P}'_{\mathbf{st}}.\ \mathbf{max}^{p\Subset p''}(_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{DET}\{p, p'\}]$.

I leave the investigation of this suggestion – as well as the problem posed by the translation of non-monotonic determiners (e.g. *exactly n*) – for future research.

123. **Distributivity-based Equivalences.**

a. $_p([C_1, \ldots, C_m]) = [_p(C_1), \ldots, _p(C_m)]$

b. $_p([\alpha_1, \ldots, \alpha_n \mid C_1, \ldots, C_m]) = [_p(\alpha_1, \ldots, \alpha_n) \mid _p(C_1), \ldots, _p(C_m)])$

124. **Atomic Conditions.**

a. $R_p\{u_1, ..., u_n\} := \lambda I_{st}.\ I_{p \neq \#, u_1 \neq \#, ..., u_n \neq \#} \neq \emptyset \wedge$

$$\forall i_s \in I_{p \neq \#, u_1 \neq \#, ..., u_n \neq \#}(R_{pi}(u_1 i, \ldots, u_n i))$$

b. $\mathbf{unique}\{p\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s, i'_s \in I_{p\neq\#}(pi=pi')$

c. $\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge$

$$\mathbf{MODAL}_{\mu I\neq\{\#\}, \omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\},$$

where μ (modal based dref) and ω (ordering source dref) are of type $s(\mathbf{w}t)$

d. $\mathbf{NEC}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{NEC}_{\mu I\neq\{\#\}, \omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

where $\mathbf{NEC}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M)\cap W_1)\subseteq W_2$,

where W_1 and W_2 are of type $\mathbf{w}t$

and M (modal base) and O (ordering source) are of type $(\mathbf{w}t)t$

e. $\mathbf{POS}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{POS}_{\mu I\neq\{\#\}, \omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

where $\mathbf{POS}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M)\cap W_1)\cap W_2\neq\emptyset$,

where W_1 and W_2 are of type $\mathbf{w}t$

and M (modal base) and O (ordering source) are of type $(\mathbf{w}t)t$

f. Generalized Limit Assumption.

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\forall w \in W \exists w' \in W((w' <_O w \vee w'=w) \wedge \neg\exists w'' \in W(w'' <_O w'))$$

g. The Ideal function.

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\mathbf{Ideal}_O(W) := \{w \in W: \neg\exists w' \in W(w' <_O w)\}$$

125. **Translations.**

a. if^p + $modal_{\mu,\omega}{}^{p'\sqsubseteq p}$ $\rightsquigarrow$

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^p(_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$$

b. $modal_{\mu,\omega,p}{}^{p'\sqsubseteq p}$ $\rightsquigarrow$ $\lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

c. $if^{p\Subset p''}$ + $modal_{\mu,\omega}{}^{p'\sqsubseteq p}$ $\rightsquigarrow$

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p\Subset p''}(_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$$

d. indicative$_{p^*}$ $\rightsquigarrow$ $\lambda\mathbb{P}_{\mathbf{st}}.\ [\mathbf{unique}\{p^*\}];\ _{p^*}(\mathbb{P}(p^*))$,

where p^* is the dref for the actual world

e. *girl* $\rightsquigarrow$ $\lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [girl_q\{v\}]$

f. det$^{u,u'\sqsubseteq u}$ $\rightsquigarrow$

$$\lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{u}(_{\langle u\rangle}(P(u)(q)));\ \mathbf{max}^{u'\sqsubseteq u}(_{\langle u'\rangle}(P'(u')(q)));\ [\mathbf{DET}\{u, u'\}]$$

g. *a*$^{\mathbf{wk}:u}$ $\rightsquigarrow$ $\lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u];\ _u(P(u)(q));\ _u(P'(u)(q))$

h. *a*$^{\mathbf{str}:u}$ $\rightsquigarrow$ $\lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{u}(_u(P(u)(q));\ _u(P'(u)(q)))$

i. *he*$_u$ $\rightsquigarrow$ $\lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\mathbf{unique}\{u\}];\ _u(P(u)(q))$

j. *Harvey*^{u} $\rightsquigarrow$ $\lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u \mid u\Subset Harvey];\ _u(P(u)(q))$,

where $Harvey := \lambda i_s.\ harvey_e$

k. *might*$_{\mu,\omega}{}^{p,p'\sqsubseteq p}$ $\rightsquigarrow$

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}(_p(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{p'}(\mathbb{P}'(p')));\ [\mathbf{POS}_{q,\mu,\omega}\{p, p'\}]$$

l. *would*$_{\mu,\omega,p'}{}^{p''\sqsubseteq p'}$ $\rightsquigarrow$ $\lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p''\sqsubseteq p'}(_{p''}(\mathbb{P}(p'')));\ [\mathbf{NEC}_{q,\mu,\omega}\{p', p''\}]$