Chapter 5. Structured Nominal Reference: Donkey Anaphora

1. Introduction

This chapter incrementally introduces a new dynamic system that extends CDRT+GQ and within which we can give a compositional account of the multiple donkey sentences in (1) and (2) below. This pair of sentences shows that the analysis of singular donkey anaphora requires a notion of plural discourse reference, i.e. reference to a quantificational dependency between sets of objects (atomic individuals, possible worlds etc.), which is established and subsequently referred to in discourse.

1. Every^{u_1} person who buys a^{u_2} book on amazon.com and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$[1].
2. Every^{u_1} boy who bought a^{u_2} Christmas gift for a^{u_3} girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

Both examples contain multiple instances of singular donkey anaphora that are semantically correlated: (1) shows that singular donkey anaphora can refer to (possibly non-singleton) *sets*, while (2) shows that singular donkey anaphora can refer to a *dependency* between such sets.

Sentence (1) is a *mixed weak & strong* donkey sentence[2]: it is interpreted as asserting that, for *every* book (strong) that any credit-card owner buys on amazon.com,

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[1] Some speakers find the variants in (i) below intuitively more compelling:

(i) Every person who buys a computer / TV and has a credit card uses it to pay for it.

[2] To my knowledge, the existence of mixed reading relative-clause donkey sentences was observed for the first time by van der Does (1993). His example is provided in (i) below – and it is accompanied by the observation that "clear intuitions are absent, but a combined reading in which a whip is used to lash all horses seems available" (van der Does 1993: 18). The intuitions seem much clearer with respect to example (1) above; moreover, it is crucial for our purposes that the weak reading of *a credit card* in (1) does not require the set of credit cards to be a singleton set (that is, some people might use different credit cards to buy different (kinds of) books).

(i) Every farmer who has a horse and a whip in his barn uses it to lash him. (van der Does 1993: 18, (26)) there is *some* credit card (weak) that s/he uses to pay for the book. Note in particular that the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels – which means that even the *weak* indefinite *a^{u_3} credit card* can introduce a (possibly) *non-singleton set*.

For each buyer, the two sets of objects, i.e. all the books purchased on amazon.com and some of the credit cards that the buyer has, are *correlated* and the *dependency* between these sets is specified in the nuclear scope of the quantification: each book is correlated with the credit card that was used to pay for it. The translation of sentence (1) in classical (static) first-order logic is provided in (3) below.

3. $\forall x(person(x) \wedge \exists y(book(y) \wedge buy\_on\_amazon(x, y)) \wedge \exists z(c.card(z) \wedge have(x, z))$
$\rightarrow \forall y'(book(y') \wedge buy\_on\_amazon(x, y')$
$\rightarrow \exists z'(c.card(z') \wedge have(x, z') \wedge use\_to\_pay(x, z', y'))))$

The challenge posed by this sentence is to *compositionally* derive its interpretation while allowing for: (**i**) the fact that the two donkey indefinites in the restrictor of the quantification receive two distinct readings (strong and weak respectively) and (**ii**) the fact that the value of the weak indefinite *a^z credit card* co-varies with / is dependent on the value of the strong indefinite *a^y book* although the strong indefinite cannot syntactically scope over the weak one, since both DP's are trapped in their respective conjuncts.

The *dependency* between the two sets of objects is the most transparent in sentence (2). Both instances of donkey anaphora are strong: we are considering *every* Christmas gift and *every* girl. The restrictor introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. The nuclear scope of the donkey quantification retrieves not only the two sets of objects, but also the *structure* associated with them, i.e. the dependency between them: each gift was wrapped by the

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The existence of mixed reading conditional donkey sentences has been observed at least since Dekker (1993); his example is provided in (ii) below.

(ii) If a man has a dime in his pocket, he throws it in the parking meter. (Dekker 1993: 183, (25)).

deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to *structure* in addition to donkey anaphora to *values*.

Importantly, the structure associated with the two sets, i.e. the dependency between gifts and girls, is *semantically* encoded and not pragmatically inferred: the correlation between the two sets is not left vague / underspecified and subsequently made precise based on various extra-linguistic factors. To see this, consider the following situation. John buys two gifts, one for Mary and the other for Helen. The two girls are deskmates (note that the *deskmate* relation is symmetric). Intuitively, sentence (2) is true if John asked Mary to wrap Helen's gift and Helen to wrap Mary's gift and it is false if John asked each girl to wrap her own gift (i.e. if John asked Mary to wrap the gift bought for her and, similarly, he asked Helen to wrap the gift bought for her). But if the relation between gifts and girls were vague / underspecified, we would predict that sentence (2) should be true even in the second (somewhat odd) situation[3,4].

In sum, we need a *semantic* framework which can account for reference to *non-singleton structured sets*, where the quantificational structure associated with the sets is introduced in a (syntactically) non-local manner – for example, in (1), across a coordination island – and subsequently accessed in a non-local manner – for example, in (2), from outside the relative clause that introduces the structured dependency.

The chapter is structured as follows. Section **2** provides a brief outline of the proposed account. Section **3** introduces an extension of CDRT+GQ with plural info

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[3] Note the similarity between example (2) (which crucially involves the symmetric relation *deskmate*) and the 'indistinguishable participants' examples involving symmetric relations due to Hans Kamp, Jan van Eijck and Irene Heim (see Heim 1990: 147, fn. 6):

(i) If a man shares an apartment with another man, he shares the housework with him. (Heim 1990: 147, (22))

(ii) If a bishop meets a bishop, he blesses him. (Heim 1990: 148, (23)).

[4] The donkey sentence in (2) does not pose problems for CDRT+GQ (or indeed DRT / FCS / DPL) – at least to the extent to which CDRT+GQ can provide a suitable analysis of possessive definite descriptions like *her deskmate*. However, as the remainder of this section will show, the donkey sentence in (2) is an important companion to the mixed reading donkey sentence in (1); it is only together that these two sentences provide an argument for extending CDRT+GQ with plural information states (i.e. the main technical innovation of this chapter) as opposed to a more conservative extension of CDRT+GQ with dref's for sets.

states, which I dub Plural CDRT (PCDRT)[5]. Section **4** shows in detail how PCDRT can be used to compositionally interpret a variety of donkey sentences, including mixed weak & strong relative-clause donkey sentences.

Section **6** compares PCDRT with alternative approaches to donkey anaphora and evaluates how they fare with respect to the proportion problem, the weak/strong donkey ambiguity and mixed reading relative-clause donkey sentences. The appendix contains a summary of the PCDRT system and some of the more technical results about its formal properties.

2. Outline of the Proposed Account

The first issue that we need to address is the weak / strong donkey ambiguity. I will attribute this ambiguity to the donkey indefinites – and not to any other element involved in the donkey anaphora structure, e.g. the generalized determiner, as CDRT+GQ (following Rooth 1987, Heim 1990, Kanazawa 1994a) would have it.

The two basic meanings for the donkey indefinites have the format in (4) below, where the **max** operator taking scope over both the restrictor and the nuclear scope properties delivers the strong (maximal) donkey reading. The **max** operator ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref *u* the maximal set of individuals satisfying both the restrictor dynamic property P' and the nuclear scope dynamic property P.

4. **weak indefinites:** $a^{wk:u} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [u];\ P'(u);\ P(u)$

   strong indefinites: $a^{str:u} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u}(P'(u);\ P(u))$

Attributing the weak / strong ambiguity to the donkey indefinites enables us to give a *compositional* account of the mixed weak & strong donkey sentence in (1) above because we *locally* decide for each indefinite article whether it receives a weak or a

[5] One possible mnemonic for PCDRT is Politically Correct DRT. The author vigorously denies responsibility for any entailments, presuppositions, implicatures or implications of any other kind associated with the use of this mnemonic in any discourse and / or utterance context whatsoever.

strong reading. Moreover, selective generalized determiners like *every*, *no* etc. have the kind of dynamic meaning that we would expect them to have based on their static Montague-style meanings: they are associated with / 'bind' only one dref (their own) and do not need to encode which readings the donkey indefinites in their restrictor have and that is the relative (pseudo-)scope of these indefinites.

Furthermore, this analysis of the weak / strong donkey ambiguity is couched within a framework that enables us to account for the fact that donkey anaphora involves reference to (possibly non-singleton) *structured sets* of individuals. The main innovation (relative to CDRT+GQ) is to minimally complicate the notion of info state: instead of using singular info states consisting of a single 'assignment' i, j, … (type s), I follow the proposal in van den Berg (1994, 1996a) and use *plural* info states I, J, …, consisting of sets of 'assignments' (type st). I will call the resulting system Plural CDRT (PCDRT).

In PCDRT, individual dref's have the same type as in CDRT+GQ, i.e. type se. A dref u (of type se) stores a set of individuals uI with respect to such a plural info state I: as shown in (5) below, the set of individuals uI is the image of the set of 'assignments' I under the function u.

5. **Abbreviation:** $uI := u_{se}[I_{st}] = \{u_{se}i_s\colon i_s{\in}I_{st}\} = \{x_e\colon \exists i_s{\in}I(ui{=}x)\}$

Storing a set of individuals by means of a plural info state and not by means of a dref for sets (its type would be $s(et)$) enables us to access in discourse not only the set of individuals, but also the structure associated with it by the plural info state: for example, two drefs u and u' store two sets of individuals relative to a plural info state I, i.e. $uI = \{ui\colon i{\in}I\}$ and $u'I{=}\{u'i\colon i{\in}I\}$; but the info state I also stores the dependency (i.e. the binary relation) between the two dref's, which is the set of pairs of individuals $\{<ui, u'i>\colon i{\in}I\}$[6].

[6] In DRT / FCS / DPL terminology, we can think of the sets of individuals as being contributed by sets of variable assignments (or sets of embedding functions) G, G' etc. A set of variable assignments introduces both *sets* of individuals, e.g. a variable x is associated with the set of individuals $\{g(x)\colon g{\in}G\}$, and a *relation* between them, e.g. two variables x and y determine the binary relation $\{<g(x), g(y)>\colon g{\in}G\}$ between the two sets of individuals associated with x and y, i.e. between $\{g(x)\colon g{\in}G\}$ and $\{g(y)\colon g{\in}G\}$.

6. Info State I	…	u		u'		…
i_1	…	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	…
i_2	…	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	…
i_3	…	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	…
…	…	…		…		…

Values – sets: $\{x_1, x_2, x_3, \ldots\}$, $\{y_1, y_2, y_3, \ldots\}$ **Structure – relations:** $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \ldots\}$

As (6) above shows, plural info states encode discourse reference to both values and structure. The values are the sets of objects that are stored in the *columns* of the matrix, e.g. a dref u for individuals stores a set of individuals relative to a plural info state, since u is assigned an individual by each assignment (i.e. row). The structure is *distributively* encoded in the *rows* of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment.

Thus, plural info states enable us to capture the structured dependencies between the multiple donkey anaphoric connections in (1) and (2) above. Let us start with the PCDRT analysis of sentence (2): by the time we are done processing the restrictor of the donkey quantification, we will be in an info state I which can be represented as the matrix in (7) below. Note that the strong donkey indefinites introduce both *values*, i.e. the set of gifts $u_2I = \{a_1, a_2, \ldots\}$ and the set of girls $u_3I = \{b_1, b_2, \ldots\}$, and *structure*, i.e. for each 'assignment' $i \in I$, the gift u_2i was bought for girl u_3i.

7. Every^{u_1} boy who bought a$^{\mathbf{str}:u_2}$ Christmas gift for a$^{\mathbf{str}:u_3}$ girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

Info state I	…	u_2 (**all gifts**)	u_3 (**all girls**)	…
i_1	…	a_1 $(=u_2i_1)$	b_1 $(=u_3i_1)$	…
		gift a_1 was bought for girl b_1		
i_2	…	a_2 $(=u_2i_2)$	b_2 $(=u_3i_2)$	…
i_3	…	a_3 $(=u_2i_3)$	b_3 $(=u_3i_3)$	…
…	…	…	…	…

When we process the nuclear scope of the donkey quantification, we are anaphoric to both values and structure: we require each 'assignment' $i \in I$ to be such that the deskmate of girl u_3i was asked to wrap gift u_2i. Thus, the nuclear scope of the donkey quantification elaborates on the structured dependency between the set of gifts u_2I and the set of girls u_3I introduced in the restrictor of the donkey quantification.

The interpretation of sentence (1) is different in two important respects: (**i**) the indefinite *a*^{u_3} *credit card* receives a weak reading and (**ii**) the structural dependency between books and credit cards remains implicit in the restrictor and is explicitly established only in the nuclear scope. That is, by the time we are done processing the restrictor of the donkey quantification in (1), we will be in an info state I like the one in (8) below. We introduce the *maximal* set of books for u_2 (the strong indefinite), we non-deterministically introduce *some* set of credit cards for u_3 (the weak indefinite) and we non-deterministically introduce *some structure* correlating the values of u_2 and u_3.

8. Every^{u_1} person who buys a$^{\textbf{str:}u_2}$ book on amazon.com and has a$^{\textbf{wk:}u_3}$ credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

Info state I	…	u_2 (**all books**)	u_3 (**some credit cards**)	…
i_1	…	a_1 (=u_2i_1)	b_1 (=u_3i_1)	…
		book a_1 is somehow correlated with card b_1		
i_2	…	a_2 (=u_2i_2)	b_2 (=u_3i_2)	…
i_3	…	a_3 (=u_2i_3)	b_3 (=u_3i_3)	…
…	…	…	…	…

The nuclear scope is again anaphoric to both values and structure; in particular, we test that the non-deterministically introduced value for u_3 and the non-deterministically introduced structure associating u_3 and u_2 satisfy the nuclear scope condition, i.e., for each 'assignment' $i \in I$, the credit card u_3i is used to pay for the book u_2i. Yet again, the nuclear scope elaborates on the unspecified dependency between u_3 and u_2 introduced in the restrictor of the donkey quantification. Crucially, the credit cards co-vary with / are dependent on the books and introducing such a dependency does not require the strong indefinite *a*^{u_2} *book* to scope over the weak indefinite *a*^{u_3} *credit card* – which cannot happen because the two DP's are trapped in their respective conjuncts.

As the semi-formal paraphrases above indicate, PCDRT follows CDRT+GQ and interprets a sentence as a DRS, i.e. as a relation between an input and an output info state. The only difference is that the PCDRT info states are plural, hence the type of a DRS is $(st)((st)t)$, i.e. a relation between an input info state I_{st} and an output info state J_{st}. The example in (1) provides the empirical motivation for modeling DRS's as *relations* between plural info states (of type $(st)((st)t)$), i.e. as *non-deterministically* updating a plural info state. We need the non-determinism to introduce both (**i**) the value of the weak indefinite *a*^{u_3} *credit card* and (**ii**) the dependency between the weak indefinite *a*^{u_3} *credit card* and the strong indefinite *a*^{u_2} *book*: both the plural value of dref u_3 and the dependency relative to the dref u_2 are non-deterministically introduced in the restrictor and elaborated upon in the nuclear scope.

The structural non-determinism, i.e. the fact that the dynamics of structural dependencies is essentially the same as the dynamics of values, is a core design feature of PCDRT, which sets it apart from many previous dynamic systems with plural info states (including van den Berg 1996a, Krifka 1996b and Nouwen 2003).

One final observation before turning to the formal development of the account sketched in this section. The hypothesis that singular indefinite articles are ambiguous is not entirely desirable: for one thing, the two readings of the indefinite are always morphologically identical in English; moreover, I do not know of any natural language that would systematically reflect the difference between these two readings in the surface form of the indefinites. Thus, an analysis that would avoid the ambiguity and would derive the two distinct readings solely on the basis of independently motivated semantic and pragmatic factors would be preferable.

However, the proposed analysis of the weak / strong ambiguity gets fairly close to achieving this goal: the only difference between a weak and a strong indefinite article is the presence vs. absence of a maximization operator. We can therefore think of the singular indefinite article as *underspecified* with respect to the presence vs. absence of this maximization operator: the decision to introduce it or not is made online depending on the discourse and utterance context of a particular donkey sentence – much like aspectual coercion[7] or the selection of a particular type for the denotation of an expression[8] are context-driven online processes.

3. CDRT+GQ with Plural Information States: Plural CDRT (PCDRT)

This section incrementally develops Plural CDRT (PCDRT), i.e. the promised extension of CDRT+GQ with plural info states. Section **3.1** gives the new definition of atomic conditions, section **3.2** the definition of new dref introduction, section **3.3** defines

[7] For example, the iterative interpretation of *John sent a letter to the company for years* or of *The light is flashing*.

[8] For example, when proper names are conjoined with generalized quantifiers.

negation, section **3.4** introduces maximization and, finally, section **3.5** defines selective and unselective generalized quantification in PCDRT. I provide the empirical and theoretical motivation for the formal innovations as I introduce them.

3.1. Atomic Conditions

No changes need to be made to our underlying logic Dynamic Ty2, i.e. our 'low-level programming language': we will be working with the same bivalent total logic with no non-atomic individuals. And the changes to our DRT-style abbreviation system, i.e. our 'high-level programming language', are minimal: we introduce plural info states I, J, K, … of type st and we consequently reset the type $\mathbf{t}$ of (saturated) sentences to $(st)((st)t)$: $\mathbf{t}$ is still the type of a binary relation between info states, it's just that the info states themselves are plural[9].

9. Plural info states (type st): H_{st}, I_{st}, J_{st}, K_{st}, H'_{st}, I'_{st}, J'_{st}, K'_{st}, …;
   'Saturated' expressions in PCDRT:
   - sentences (DRSs) – relations between plural info states: $\mathbf{t} := (st)((st)t)$;
   - names (individual dref's) – the same as in CDRT+GQ: $\mathbf{e} := se$.

Just as in CDRT+GQ, the atomic conditions are sets of info states. However, given that we are now working with plural info states, their type will be $(st)t$. Moreover, the atomic conditions will be *unselectively distributive*, where 'unselective' is used in the sense of Lewis (1975), i.e. the atomic conditions are *distributive* over the *plural info states* they accept: they accept a set of 'assignments' iff they accept, in a pointwise manner, every single 'assignment' in the set.

This is implemented by means of universal quantification over the set of assignments in a plural info state I_{st}, as shown in (10) below. The requirement of non-emptiness $I \neq \emptyset$ rules out the 'degenerate' case in which the universal quantification $\forall i_s \in I(\ldots)$ is vacuously satisfied.

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[9] Incidentally, note that $\mathbf{t}$ is the type of generalized determiners over entities of type s, parallel to static (extensional) determiners of type $(et)((et)t)$.

10. **Atomic conditions** – type $(st)t$.

$R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in} I(R(u_1 i, \ldots, u_n i))$,

for any non-logical constant R of type $e^n t$,

where $e^n t$ is defined as follows: $e^0 t := t$ and $e^{m+1} t := e(e^m t)$.

$u_1{=}u_2 := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in} I(u_1 i{=}u_2 i)$

As already suggested, the requirement enforced by an atomic condition can be intuitively depicted by means of a matrix, as shown in (11) below.

11. Info state I	…	u_1	…	u_n	…
i	…	$a_1\ (=u_1 i)$	…	$a_n\ (=u_n i)$	…
		$\underbrace{\qquad\qquad\qquad}$ $R(u_1 i, \ldots, u_n i)$, **i.e.** $R(a_1, \ldots, a_n)$			
i'	…	$a_1'\ (=u_1 i')$	…	$a_n'\ (=u_n i')$	…
i''	…	$a_1''\ (=u_1 i'')$	…	$a_n''\ (=u_n i'')$	…
…	…	…	…	…	…

The unselectively distributive structure of the atomic conditions endows the set of plural information states characterized by them with a lattice-theoretic ideal structure.

12. $\Im$ is an *ideal* with respect to the partial order induced by set inclusion $\subseteq$ on the power set of the domain of 'assignments' $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$ (i.e. $<\wp(\boldsymbol{D}_s^{\boldsymbol{M}}), \subseteq>$) iff:

a. $\Im \subseteq \wp(\boldsymbol{D}_s^{\boldsymbol{M}})$;

b. for any I_{st} and J_{st}, if $I{\in}\Im$ and $J{\subseteq}I$, then $J{\in}\Im$ (closure under subsets);

c. for any I_{st} and J_{st}, if $I{\in}\Im$ and $J{\in}\Im$, then $(I{\cup}J){\in}\Im$ (closure under finite unions).

$\Im$ is a *complete ideal* iff (**a**) and (**b**) are as above and, instead of (**c**), we require closure under arbitrary unions.

A complete ideal $\Im$ has a supremum, namely $\cup\Im$. Given the requirement of closure under subsets and closure under arbitrary unions, a complete ideal $\Im$ is a complete Boolean algebra, as stated in (13) below.

13. $\Im = \wp(\cup\Im)$, for any complete ideal $\Im$ (in the atomic lattice $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$).

We introduced the notation in (14) below to handle the non-emptiness requirement in the definition of atomic conditions.

14. Let $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$ be the power set of the domain of 'assignments' without the empty set Ø. A *(complete) ideal without a bottom element* is defined just as in (12) above, except that, instead of (12a), we require inclusion in $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$ and, instead of (12b), we require closure under non-empty subsets.

Since we are concerned here only with complete ideals without a bottom element, I will henceforth use "c-ideal" instead of the longer "complete ideal without a bottom element". The most important fact is that, for any c-ideal $\Im$, we have that $\Im = \wp^+(\cup\Im)$, i.e. c-ideals are complete Boolean algebras without a bottom element.

The definition of atomic conditions in (10) above ensures that they always denote c-ideals (in the atomic lattice $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$). We can in fact characterize them in terms of the supremum of their denotation.

15. **Atomic Conditions as C-Ideals.**
For any non-logical constant R of type e^nt and sequence of unspecific[10] dref's $<u_1, \ldots, u_n>$, let $\mathbb{I}(R, <u_1, \ldots, u_n>) := \lambda i_s.\ R(u_1 i, \ldots, u_n i)$, abbreviated $\mathbb{I}^R$ whenever the sequence of dref's can be recovered from context. Then, $R\{u_1, \ldots, u_n\} = \wp^+(\mathbb{I}^R)$ [11].

The fact that atomic conditions denote c-ideals will be useful in showing that PCDRT has a range of desirable properties and it will guide several design choices we have to make on the way.

3.2. New Discourse Referents

We turn now to defining the introduction of new dref's in PCDRT. I will consider only two candidate definitions, both given in (16) below, and I will argue that the first

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[10] For the notion of unspecific dref, see definition **4** in section **2.2** of chapter **3** above.

[11] Convention: $\wp^+(\emptyset_{st}) = \emptyset_{(st)t}$.

one, namely (16a), is the empirically and theoretically better choice. Both definitions relate two plural info states I_{st} and J_{st} in terms of the pointwise relation $i_s[u]j_s$.

16. **Introducing new dref's in PCDRT – two candidate definitions**:

a. $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[u]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[u]j))$

b. $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et}{\neq}\varnothing(J{=}\bigcup_{i_s\in I}\{j_s : i[u]j \ \wedge\ uj\in X\}),$

equivalently: $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et}{\neq}\varnothing(J{=}\{j_s{:}\ \exists i_s{\in}I(i[u]j \wedge uj{\in}X)\})$.

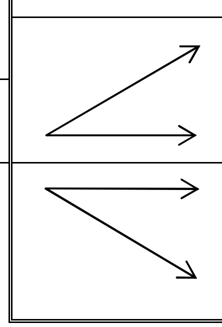
Definition (16a) is the more general and logically weaker one: it simply requires any 'assignment' i in the input info state I to have a successor 'assignment' j in the output state J and, similarly, any 'assignment' j in the output info state J should have an ancestor 'assignment' i in the input state I. In this way, we will necessarily preserve all the discourse information[12] in the input state I when we non-deterministically update it and obtain the output state J.

Definition (16b) has an extra-requirement over and above definition (16a): we need to *uniformly* reassign the value of the dref u for all the 'assignments' i_s in the input info state I_{st}, i.e. there is some random set X_{et} of new values for u and each input 'assignment' i is updated (relative to u) with each and every single value in X. The effect of definition (16b) is shown in (17) below: the input state I_{st} contains two 'assignments' i and i' and the set X_{et} of new values for u contains two individuals a and b.

17. ***I*{*u*}*J*, where X_{et}={*a*, *b*}**

Input state I_{st}	…
i	…
i'	…

$I\{u\}J$



Output state J_{st}	…	u	…
i_a	…	$a\ (=u(i_a))$	…
i_b	…	$b\ (=u(i_b))$	…
i'_a	…	$a\ (=u(i'_a))$	…
i'_b	…	$b\ (=u(i'_b))$	…

[12] Recall that, in PCDRT, the preserved discourse information consists of: (i) the previously established *values* for all the dref's other than u and (ii) the previously established *structured dependencies* between the dref's other than u.

The choice between the two definitions in (16)[13,14] boils down to how we want to handle the new component of our information states, i.e. the *structure* associated with the values of the dref's. The singular info states of CDRT+GQ encode only *values* – and we non-deterministically assign new values to a particular dref. Thus, for each particular info state, the value of the dref is *determined*, but throughout the entire discourse context, i.e. throughout the space of all possible output info states for the random assignment [u], the value of the dref is *not determined*: for every possible value that the dref u can take, there will be some output info state that assigns that value to u.

The plural info states of PCDRT encode *values* and, in addition, *structure*, i.e. they encode dependencies between the values of the dref's in a pointwise manner ('assignment' by 'assignment'). Our first definition $I[u]J$ treats the structural component in parallel to the value component of the info state: we non-deterministically introduce both new *values* for u and new *structure*, as the values for u in the output state can be stored in a particular configuration of pointwise associations with the other dref's.

Thus, in each info state, the value and the structure of dref u are *determined*, but throughout the entire discourse context, i.e. throughout the space of all possible plural output states, the value and the structure of dref u are *not determined*: for every possible non-empty set of values, for every possible structure (i.e. pointwise distribution) of that set, there is some plural output state that assigns to u that particular value with that particular associated structure.

The second definition $I\{u\}J$ does not treat the two components of a plural info state, i.e. value and structure, in a parallel way: we are still non-deterministic with respect to the value, but we are *deterministic* with respect to the structure – for any set of

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[13] Both definitions appear in van den Berg's work: an equivalent of (16a) is used in van den Berg (1994): 15, fn 12 and in van den Berg (1996b): 18, (49), while van den Berg (1996a): 134-135, (2.7) & (2.8) uses a version of (16b). The two definitions I consider differ from van den Berg's definitions in several respects: first, (16a) and (16b) are formulated in type logic, unlike van den Berg's, which are formulated in DPL terms; second, the definitions of random assignment in van den Berg are more complex because he works with a three-valued logic and also countenances a dummy / 'undefined' individual $\star$. To my knowledge, there is no comparison of the two alternative definitions in van den Berg's work.

[14] Nouwen (2003) follows van den Berg (1996a) and assumes the definition of $\{u\}$ in (16b); the alternative option is not mentioned.

individuals that is randomly assigned as a value, there is only one possible structure (i.e. pointwise distribution) of that set throughtout the output discourse context (i.e. throughout the space of output info states).

This choice seems to be preferable if we want to make the system computationally more efficient because it would significantly cut down the number of possible output info states for any given instance of new dref introduction (a.k.a. plural random assignment). Moreover, a more constrained system (presumably) runs a lower risk of over-generation. Finally, the structure we choose for every random value is the least 'biased' one: we introduce the entire set assigned to u with respect to each input 'assignment' i, so there is no 'biased' correspondence / dependency between the values of some other dref u' and the values newly assigned to u. That is, although the update is structurally deterministic, it always associates *the least possible amount of structural information* with each new value.

Despite the fact that the second definition $\{u\}$ is more constrained (hence, *ceteris paribus*, more desirable), I will provide three reasons, one empirical and two theoretical, for preferring the first definition, namely $[u]$. The first, empirical reason is provided by our mixed weak & strong donkey sentences, repeated below for convenience.

18. Every^{u_1} person who buys a^{u_2} book on amazon.com and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$.
19. Every^{u_1} man who wants to impress a^{u_2} woman and who has an^{u_3} Arabian horse teaches her$_{u_2}$ how to ride it$_{u_3}$.

Recall that, intuitively, we want to allow for credit cards that vary from book to book and also for Arabian horses that vary from woman to woman. Consider now the definition in (16a), i.e. $[u]$, and its effect on the interpretation of the quantification in (19) (the same reasoning applies to (18)). By the time we process the second conjunct in the restrictor, i.e. *who has an^{u_3} Arabian horse*, we have already processed the first one *who wants to impress a^{u_2} woman* and, therefore, the dref u_2 has already been introduced and was assigned appropriate womanly values. Now we introduce u_3 by means of the update

[u_3] and we *non-deterministically* assign it a set of equine values and *non-deterministically* associate a structure with this set of values, i.e. we non-deterministically associate each u_3-horse with some u_2-woman.

The nuclear scope subsequently filters the non-deterministically assigned values and structure: we require the u_3-horses to stand in the 'u_2 *rides* u_3' relation to the u_2-set of women and this requirement has to be satisfied in a pointwise manner, i.e. relative to each individual 'assignment' in the plural info state.

In contrast, the definition of random assignment in (16b), i.e. $\{u_3\}$, requires us to introduce the *same* set of horses with respect to each and every u_2-woman. This yields intuitively incorrect, overly strong truth-conditions since, for sentence (19) to be intuitively true, we do not have to require each and every woman to ride the same horse or the same set of horses as the other women.

Thus, the structural non-determinism built into the definition of random assignment in (16a) allows us to introduce a value and a structure for u_3 that can verify sentence (19) without imposing overly strong truth-conditions.

The second, theoretical reason in favor of $I[u]J$ and against $I\{u\}J$ is that $I[u]J$ preserves the formally desirable properties of the pointwise relation $i[u]j$, while $I\{u\}J$ doesn't. More exactly, $I[u]J$ is an equivalence relation[15], just as $i[u]j$, while the relation $I\{u\}J$ is neither reflexive nor symmetric (as the reader can easily check).

The third and final reason in favor of $I[u]J$ and against $I\{u\}J$ is that the relation $[u]$, but not the relation $\{u\}$, preserves the c-ideal structure that the atomic conditions have[16]

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[15] The reflexivity, symmetry and transitivity of the relation $I[u]J$ follow from the reflexivity, symmetry and transitivity of $i[u]j$ in a straightforward way.

[16] A relation $\mathbb{R}$ between plural info states (of type $\mathbf{t} := (st)((st)t)$) preserves c-ideals under images iff if $\mathfrak{I}$ is a c-ideal, then $\mathfrak{I}'=\{J_{st}: \exists I_{st}(\mathbb{R}IJ \wedge I\in\mathfrak{I}\}$ is a c-ideal. A relation $\mathbb{R}$ between plural info states preserves c-ideals under pre-images iff if $\mathfrak{I}'$ is a c-ideal, then $\mathfrak{I}=\{I_{st}: \exists J_{st}(\mathbb{R}IJ \wedge J\in\mathfrak{I}'\}$ is a c-ideal. The relation $[u]$ preserves c-ideals under both images and pre-images.

(again, the reader can easily verify this statement). I conclude that the relation $I[u]J$ is the empirically most adequate and theoretically most natural generalization of $i[u]j$ [17].

20. **Introducing new dref's in PCDRT:**

$$[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[u]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[u]j))$$

Introducing new dref's by means of $[u]$ has an immediate benefit. We now have a clear understanding of the denotation of a DRS D containing only atomic conditions or of arbitrary dynamic conjunctions of such DRS's. The relevant definitions are provided in (21) below.

21. **Atomic DRS's (DRS's containing only one atomic condition)** – type $(st)((st)t)$.

$$[R\{u_1, \ldots, u_n\}] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge R\{u_1, \ldots, u_n\}J$$

$$[u_1{=}u_2] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge (u_1{=}u_2)J$$

DRS-level connectives (dynamic conjunction):

$$D_1; D_2 := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(D_1IH \wedge D_2HJ),$$

where D_1 and D_2 are DRSs (type $(st)((st)t)$)

Tests (generalizing atomic DRS's):

$$[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge C_1J \wedge \ldots \wedge C_mJ$$ [18],

where $C_1, \ldots, C_m$ are conditions (atomic or not) of type $(st)t$.

We know that the domain and the range of any atomic DRS are c-ideals. We also know that the domain and the range of an arbitrary dynamic conjunction of atomic DRSs

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[17] We can in fact define $\{u\}$ in terms of $[u]$ and the closure condition **enough_assignments** defined in (i) below. The name of the condition indicates the formal similarity between this PCDRT condition and Axiom **4** ("Enough 'assignments'") of Dynamic Ty2, repeated in (ii) below. The definition $\{u\}$ in terms of $[u]$ is provided in (iii).

(i) **enough_assignments**$\{u\} := \lambda I_{st}.\ \forall x_e{\in}uI\forall i_s{\in}I(\exists i'_s{\in}I(i[u]i' \wedge ui'{=}x))$

(ii) **Axiom4**: $\forall i_s\forall v_{s\tau}\forall f_\tau(\mathbf{udref}(v) \rightarrow \exists j_s(i[v]j \wedge vj{=}f))$, for any type $\tau\in$ **STyp**.

(iii) $\{u\} := \lambda I_{st}J_{st}.\ I[u]J \wedge$ **enough_assignments**$\{u\}J$,
i.e. $[u \mid$ **enough_assignments**$\{u\}]$ in DRT-style abbreviation.

[18] Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction as follows:

$[C_1, \ldots, C_m] := \lambda I_{st}J_{st}.\ ([C_1]; \ldots; [C_m])IJ$, where $[C] := \lambda I_{st}J_{st}.\ I{=}J \wedge CJ$.

are c-ideals because the intersection of a set of c-ideals is a c-ideal (assuming that the intersection is non-empty). This is summarized in (22) below.

22. $\mathbf{Dom}([C]) = \mathbf{Ran}([C]) = C = \wp^{+}(\cup C)$, for any condition C that is a c-ideal.

$\mathbf{Dom}([C_1, \ldots, C_m]) = \mathbf{Ran}([C_1, \ldots, C_m]) = C_1 \cap \ldots \cap C_m$

$= \wp^{+}((\cup C_1) \cap \ldots \cap (\cup C_m))$,

for any conditions $C_1, \ldots, C_m$ that are c-ideals.

These results are generalized to DRS's in which new dref's are introduced: they are defined in (23) below and the general form of their denotation is provided in (24).

23. **Multiple random assignment**.

$[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$

DRS's with new dref's – type $(st)((st)t)$.

$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])IJ$,

where $C_1, \ldots, C_m$ are conditions,

i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I[u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J$.

24. **DRS's in terms of C-Ideals over Relations.**

Given a DRS D of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, we have that:

$\mathbf{Ran}(D) = C_1 \cap \ldots \cap C_m = \wp^{+}((\cup C_1) \cap \ldots \cap (\cup C_m))$;

$\mathbf{Dom}(D) = \wp^{+}(\{i_s\colon \exists j_s(i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m))\})$.

Note that, since $i[u_1, \ldots, u_n]j$ is reflexive, $\mathbf{Ran}(D) \subseteq \mathbf{Dom}(D)$.

Let $\mathbb{R}^{D} := \{<i_s, j_s>\colon i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)\}$. Then:

$D = \{<I_{st}, J_{st}>\colon \exists \mathbb{R}_{s(st)}(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \in \wp^{+}(\mathbb{R}^{D}))\}$,

i.e. $D = \{<I_{st}, J_{st}>\colon \exists \mathbb{R}_{s(st)} \neq \emptyset(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^{D})\}$.

That is:

$\mathbb{R}^{D} := \lambda i_s.\lambda j_s.\ i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)$

$D := \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R}_{s(st)} \in \wp^{+}(\mathbb{R}^{D})(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}))$.

The properties of DRS denotations identified in (22) and (24) above will prove useful when we decide how to define negation in PCDRT. Two final observations before we address negation. First, just as in CDRT+GQ, the existential force of the random assignment $[u]$ (see (20) above) is an automatic consequence of the way it is defined when coupled with the PCDRT definition of *truth* for DRS's, provided in (25) below.

25. **Truth:** A DRS D (type $(st)((st)t)$) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$, i.e. iff $I \in \mathbf{Dom}(D)$, where $\mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}$.

Second, note that we can already translate discourse (7-8) below in PCDRT (assuming that all the indefinites are weak). Given the definition of truth for DRS's in (25) above, the translation in (10) below derives the intuitively correct truth-conditions, as shown in (29).

26. A$^{\mathbf{wk}:u_1}$ house-elf fell in love with a$^{\mathbf{wk}:u_2}$ witch.
27. He$_{u_1}$ bought her$_{u_2}$ an$^{\mathbf{wk}:u_3}$ alligator purse.
28. $[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}];$
    $[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]$
29. $\lambda I_{st}.\ \exists J_{st}(([u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}];$
    $[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}])IJ)$, i.e.
    $\lambda I_{st}.\ I \neq \emptyset \wedge \exists x_e \exists y_e \exists z_e(house\_elf(x) \wedge witch(y) \wedge fall\_in\_love(x, y) \wedge$
    $alligator\_purse(z) \wedge buy(x, y, z))$

3.3. Negation

Let us turn now to the definition of negation in PCDRT. The fact that plural info states encode both *values* and *structure* makes the issue non-trivial. A first attempt would be to simply import the CDRT+GQ definition, which is basically the DRT / FCS / DPL one, as shown in (30) below [19].

30. **Negation – first attempt**:

$$\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \neg \exists K_{st}(DIK), \quad \text{where } D \text{ is a DRS (type } (st)((st)t)),$$

$$\text{i.e. } \sim D := \lambda I_{st}.\ I \neq \emptyset \wedge I \notin \mathbf{Dom}(D), \quad \text{where } \mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}.$$

[19] Factoring out various complications, i.e. the fact that van den Berg's Dynamic Plural Logic is intended to handle anaphora to dref's introduced within the scope of negation and the fact that it is a partial logic, the DPL-style definition in (30) is the one used in van den Berg's Dynamic Plural Logic – see van den Berg (1994): 10, (27), van den Berg (1996a): 136, (6), van den Berg (1996b): 18, Definition D, (e).

However, given the PCDRT definition of atomic conditions, the definition in (30) yields incorrect truth-conditions for the negation example in (31) below.

31. Every^{u_1} farmer who owns a$^{\mathbf{str}:u_2}$ donkey doesn't feed it$_{u_2}$ properly. [20]

Consider example (31) more closely: intuitively, the indefinite *a*$^{\mathbf{str}:u_2}$ *donkey* is strong (hence the notation *a*$^{\mathbf{str}:u_2}$) and the interpretation of (31) is that no donkey-owning farmer feeds *any* of his donkeys properly. Thus, by the time we process the restrictor of the quantification in (31), we have a plural information state I of the form shown in (32) below in which, for a given donkey-owning farmer a, every 'assignment' $i \in I$ stores some donkey d_1, d_2 etc. that a owns.

32. Info state I	…	u_1 (**one farmer**)		u_2 (**all donkeys**)	…
i_1	…	a (=$u i_1$)	$\xrightarrow{a \text{ owns } d_1}$	d_1 (=$u' i_1$)	…
i_2	…	a (=$u i_2$)	$\xrightarrow{a \text{ owns } d_2}$	d_2 (=$u' i_2$)	…
i_3	…	a (=$u i_3$)	$\xrightarrow{a \text{ owns } d_3}$	d_3 (=$u' i_3$)	…
…	…	…		…	…

Now, we reach the nuclear scope condition in (33) below, interpreted according to the definition of negation in (30) above.

33. $(\sim[feed\_proper\{u_1, u_2\}])I \ = \ I \neq \emptyset \wedge \exists i_s \in I(\neg feed\_proper(u_1 i, u_2 i))$

The truth-conditions derived by (33) are too weak: they only require farmer a to feed *some* donkey he owns poorly and they allow for cases in which he feeds properly all his other donkeys – while intuitively we should require him to feed *all* his donkeys poorly. We see that the DPL-style definition of negation in conjunction with the PCDRT definition of atomic conditions, which is unselectively distributive, yields overly weak

[20] See also the example in (i) below from van der Does (1993): 18, (27c).

(i) A$^{\mathbf{wk/str}:u}$ boy who had an$^{\mathbf{str}:u'}$ apple in his rucksack didn't give it$_{u'}$ to his sister.

truth-conditions. I will therefore give a stronger definition for negation, provided in (34) below.

34. **Negation in PCDRT.**

$$\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st}(H \neq \emptyset \wedge H \subseteq I \rightarrow \neg\exists K_{st}(DHK)),$$

where D is a DRS (type $(st)((st)t)$),

$$\text{i.e. } \sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st} \neq \emptyset(H \subseteq I \rightarrow H \notin \mathbf{Dom}(D)).$$

The PCDRT definition of negation in (34) requires that:

I is not in $\mathbf{Dom}(D)$ – just as the DPL-style definition (30);

no singleton subset of I is in $\mathbf{Dom}(D)$ – which enables us to account for the donkey sentence in (31) above, since the nuclear scope condition $(\sim[feed\_proper\{u_1, u_2\}])I$ is 'unpacked' as $I \neq \emptyset \wedge \forall i_s \in I(\neg feed\_proper(u_1 i, u_2 i))$, which yields the intuitively correct, strong truth-conditions;

all the other non-empty subsets of I are not in $\mathbf{Dom}(D)$.

The third and final requirement ensures that the denotation of a negative condition preserves the c-ideal structure of the negated DRS. For example, if the negated DRS D is of the form given in (23) above, its domain $\mathbf{Dom}(D)$ is a c-ideal and, if $\mathbf{Dom}(D)$ is a c-ideal, $\sim D$ is the maximal c-ideal disjoint from $\mathbf{Dom}(D)$. This is stated in (35) below.

35. If $\mathbf{Dom}(D)$ is a c-ideal, $\sim D$ is the unique maximal c-ideal disjoint from $\mathbf{Dom}(D)$[21]. That is, $\sim D = \wp^+(\boldsymbol{D}_s^{\boldsymbol{M}} \setminus \cup \mathbf{Dom}(D))$ if $\mathbf{Dom}(D) = \wp^+(\cup \mathbf{Dom}(D))$.

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[21] **$\sim D$ is a c-ideal if Dom(D) is a c-ideal.**

Proof: **(i)** $\sim D \subseteq \wp^+(D_s^{M})$; **(ii)** for any I_{st} and J_{st}, if $I \in \sim D$ and $J \subseteq I$ and $J \neq \emptyset$, then $J \in \sim D$ (this follows directly from definition (34)); **(iii)** if $\Upsilon \subseteq \sim D$, then $\cup\Upsilon \in \sim D$. (Proof: suppose (iii) doesn't hold, i.e. $\Upsilon \subseteq \sim D$ and $\cup\Upsilon \notin \sim D$. Then, there is an H s.t. $H \neq \emptyset$ and $H \subseteq \cup\Upsilon$ and $H \in \mathbf{Dom}(D)$. Since $H \subseteq \cup\Upsilon$ and $H \neq \emptyset$, there must be at least one $I \in \Upsilon$ s.t. $H \cap I \neq \emptyset$. Let $I' = H \cap I$. Since $I' \subseteq H$ and $H \in \mathbf{Dom}(D)$ and $\mathbf{Dom}(D)$ is a c-ideal, we have that $I' \in \mathbf{Dom}(D)$. But $I' \subseteq I$ and $I \in \Upsilon \subseteq \sim D$, so, by definition (34), $I' \notin \mathbf{Dom}(D)$. Contradiction. □). □

$\sim D$ is maximal.

Proof: Suppose $\sim D$ is not maximal. Then, there is a c-ideal $\Im$ s.t. $\Im \cap \mathbf{Dom}(D) = \emptyset$ and $\sim D \subset \Im$. Then, there is some $I \in \Im$ s.t. $I \notin \sim D$; hence, there is an H s.t. $H \neq \emptyset$ and $H \subseteq I$ and $H \in \mathbf{Dom}(D)$. Since $\Im$ is a c-ideal, $I \in \Im$ and $H \subseteq I$, we have that $H \in \Im$. Hence, $\Im \cap \mathbf{Dom}(D) \neq \emptyset$. Contradiction. □.

$\sim D$ is unique.

In sum, given the properties of the denotations of DRS's in PCDRT, the dynamic negation defined in (34) above is as well-behaved as possible[22].

We can now represent the discourse in (36-37) below. The representation, provided in (38), derives the intuitively correct truth-conditions, given in (39): there is a house-elf that fell in love with some witch and that bought her no alligator purse.

36. A $^{\mathbf{wk}:u_1}$ house-elf fell in love with a $^{\mathbf{wk}:u_2}$ witch.

37. (Surprisingly) He $_{u_1}$ didn't buy her $_{u_2}$ an $^{\mathbf{wk}:u_3}$ alligator purse.

38. $[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}];$

$[\sim[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

39. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e \exists y_e(house\_elf(x) \wedge witch(y) \wedge fall\_in\_love(x, y) \wedge$

$\neg\exists z_e(alligator\_purse(z) \wedge buy(x, y, z)))$

3.4. Maximization

Now that the core part of PCDRT is in place, we can turn to the maximization operator, which is the essential ingredient in the analysis of the weak / strong donkey ambiguity. The definition of the **max** operator is provided in (40) below; **max** is an operator over DRS's: its argument is a DRS, i.e. a term of type $\mathbf{t} := (st)((st)t)$, and its value is another DRS, i.e. another term of type **t**. Note that we actually define a family of maximization operators, each one specified for the dref u over which we maximize.

Proof: Suppose $\sim D$ is not unique. Then, there is a maximal c-ideal $\Im$ s.t. $\Im \cap \mathbf{Dom}(D)=\emptyset$ and $\sim D \neq \Im$. Since both $\sim D$ and $\Im$ are maximal, there is some $I \in \Im$ s.t. $I \notin \sim D$ and some $J \in \sim D$ s.t. $I \notin \Im$. The reasoning is now similar to the maximality proof: since $I \notin \sim D$, there must be an H s.t. $H \neq \emptyset$ and $H \subseteq I$ and $H \in \mathbf{Dom}(D)$. Since $\Im$ is a c-ideal, $I \in \Im$ and $H \subseteq I$, we have that $H \in \Im$. Hence, $\Im \cap \mathbf{Dom}(D) \neq \emptyset$. Contradiction. □.

[22] For completeness, I provide the definitions of anaphoric closure, disjunction and implication in PCDRT.

(i) **Anaphoric closure:** $!D := \lambda I_{st}.\ \exists K_{st}(DIK)$, i.e. $!D := \mathbf{Dom}(D)$

(ii) **Disjunction:** $D_1 \vee D_2 := \lambda I_{st}.\ \exists K_{st}(D_1 IK \vee D_2 IK)$, i.e. $D_1 \vee D_2 := \mathbf{Dom}(D_1) \cup \mathbf{Dom}(D_2)$

(iii) **Implication:** $D_1 \rightarrow D_2 := \lambda I_{st}.\ \forall H_{st}(D_1 IH \rightarrow \exists K_{st}(D_2 HK))$,
i.e. $D_1 \rightarrow D_2 := \lambda I_{st}.\ D_1 I \subseteq \mathbf{Dom}(D_2)$, where $DI := \{J_{st}: DIJ\}$,
i.e. $D_1 \rightarrow D_2 := (\wp^+(D_s^{\mathbf{M}}) \backslash \mathbf{Dom}(D_1)) \cup \{I \in \mathbf{Dom}(D_1): D_1 I \subseteq \mathbf{Dom}(D_2)\}$.

40. $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ \exists H_{st}(I[u]H \wedge DHJ) \wedge \forall K_{st}(\exists H'_{st}(I[u]H' \wedge DH'K) \rightarrow uK \subseteq uJ)$,

where D is a DRS, i.e. a term of type $\mathbf{t} := (st)((st)t)$,

i.e. $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ)$.

The first conjunct in (40) introduces u as a new dref (i.e. $I[u]H$) and makes sure (by DHJ) that each individual in uJ 'satisfies' D, i.e. we store *only* individuals that 'satisfy' D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e. any other set of individuals that 'satisfies' D) is included in uJ, i.e. we store *all* the individuals that satisfy D.

Note that, because of its maximality requirement, the **max** operator does not preserve the c-ideal structure of the range of the DRS over which it scopes. To see this, consider the second, shorter formulation of the definition in (40). This formulation explicitly shows that the relation between info states denoted by the maximized DRS $\mathbf{max}^u(D)$ is always a subset of the relation denoted by $[u]; D$, i.e. we 'strengthen' the DRS $[u]; D$ by ruling out the output info states J that assign to u strict subsets of maximal set that is assigned to u throughout $\mathbf{Ran}([u]; D)$ [23].

The DRS $\mathbf{max}^u(D)$ can be thought of as dynamic λ-abstraction over individuals: the 'abstracted variable' is the individual dref u, the 'scope' is the DRS D and the result of the 'abstraction' is a set of individuals uJ (where J is the output info state) containing *all* and *only* the individuals that 'satisfy' D. Thus, *maximization* together with *plural info states* and the *unselective distributivity* built into the definition of atomic conditions enables us to 'dynamize' λ-abstraction: (**i**) the maximization operator stores the λ-abstracted set in a dref, so that we can access it in discourse; (**ii**) unselective distributivity enables us to λ-abstract one value at a time; (**iii**) finally, plural info states enable us to store the dependency structure associated with each λ-abstracted value.

The empirical motivation for the *selectivity* of the $\mathbf{max}^u$ operator (as definition (40) shows, $\mathbf{max}^u$ *selectively* maximizes over the dref u) is provided by the mixed weak &

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[23] The update $\mathbf{max}^u(D)$ fails if such a supremum set does not exist, i.e. $\mathbf{max}^u(D)$ fails for an input info state I if the family of sets $\{uJ: ([u]; D)IJ\}$ does not have a supremum.

strong donkey sentences: we do not want to indiscriminately maximize over all donkey indefinites, but only over those that receive a strong reading. So, the selective $\mathbf{max}^u$ operator enables us to define the strong meaning for donkey indefinites in such a way that it is minimally different from the weak meaning. Both basic meanings are provided in (41) below[24].

41. **weak indefinites:** $a^{wk:u} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [u];\ P'(u);\ P(u)$

strong indefinites: $a^{str:u} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u);\ P(u))$,

where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$.

Note that it is the compositional system that makes sure we have the correct 'configuration' within the scope of $\mathbf{max}^u$, i.e. that the DRS $P'(u)$ over which we maximize has the dref u in the appropriate (argument) places. This is very much like the technique employed in static semantics: it is the compositional system that ensures that the λ-abstraction over the variable x takes scope over a formula that has x in the appropriate 'slots'.

The definition of $\mathbf{max}^u$ and the way it is used in the analysis of strong donkey anaphora will become clearer if we look at an example. Consider (42) below and assume that it is uttered in a context in which there is some unique salient boy with apples in his rucksack. For example, twenty children (ten brother-sister pairs) travel by bus and the bus passes an apple orchard; as the story goes, the girls are overwhelmed with desire for the fruit, but none of them gets it because no one on the bus has any apples – except for one boy, but he doesn't care about anyone's plea, not even his sister's. In this context, we can felicitously utter that the other boys would have given an apple to their sisters if they had one, but:

[24] Note the similarity between the PCDRT representation of weak indefinites and the representation of indefinites in CDRT+GQ.

42. The$_{u_1}$ (one) boy who had an$^{\mathbf{str}:u_2}$ apple in his rucksack didn't give it$_{u_2}$ to his sister [25].

In this context, (42) is interpreted as: the boy who had (some) apples in his rucksack didn't give any to his sister. I will assume that the definite article the$_{u_1}$ functions as an anaphor, i.e. it simply tests that some contextually salient dref u_1 satisfies both its restrictor and its nuclear scope, as shown in (43) below. For simplicity, the restrictor $P'(u)$ in (43) is not represented as a presupposition, but as part of the assertion[26].

43. **Definite articles as anaphors:**

$$the_u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ P'(u);\ P(u),$$

where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$.

44. $\mathbf{unique}\{u\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(ui = ui')$,

i.e. $\mathbf{unique}\{u\} := \lambda I_{st}.\ |uI| = 1$,

where $|uI|$ is the cardinality of the set uI.

I take the definite article to contribute an atomic condition $\mathbf{unique}\{u\}$, defined in (44), which encodes a *weak* form of uniqueness: it requires that the dref anaphorically retrieved by the definite article has a unique value with respect to the current plural info state, i.e. it requires the set uI stored by the dref to be a singleton. This kind of uniqueness is *weak* because it is relativized to the current info state (i.e. it is salience-dependent uniqueness); I take *strong* uniqueness to be uniqueness relative to the entire model. As we will see in (51) below, strong uniqueness can be obtained by combining weak uniqueness, i.e. the condition $\mathbf{unique}\{u\}$, and the $\mathbf{max}^u$ operator.

Sentence (42) is represented as shown in (45) below.

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[25] I ignore throughout most of this chapter the uniqueness implications sometimes associated with donkey anaphora, e.g., in example (42), the intuition that the apple is unique. For more discussion about the uniqueness effects associated with singular anaphora in quantificational subordination and donkey anaphora, see sections **6.1** and **6.2** of chapter 6 below.

I am indebted to Roger Schwarzschild (p.c.) for suggesting the sentence in (i) below as an alternative example that does not exhibit uniqueness effects.

(i) A / The boy who had a^u pen in his backpack didn't give it$_u$ to his sister.

[26] See Muskens (1995b): 165 for a similar lexical entry in a CDRT kind of system.

45. [**unique**$\{u_1\}$, $boy\{u_1\}$]; $\mathbf{max}^{u_2}$([$apple\{u_2\}$, $have\_in\_rucksack\{u_1, u_2\}$]);

[~[$give\_to\_sister\{u_1, u_2\}$]]

By the end of the $\mathbf{max}^{u_2}$ update, we are in a plural information state I like the one in (46) below. The dref u_1 stores the same boy b throughout the info state I (due to **unique**$\{u\}$) and the dref u_2 stores all the apples a_1, a_2, a_3 etc. that boy b has in his rucksack (due to $\mathbf{max}^{u_2}$).

46. Info state I	…	u_1 (**the boy**)		u_2 (**all apples**)	…
i_1	…	b $(=u_1i_1)$	$\xrightarrow{b\ has\ a_1}$	a_1 $(=u_2i_1)$	…
i_2	…	b $(=u_1i_2)$	$\xrightarrow{b\ has\ a_2}$	a_2 $(=u_2i_2)$	…
i_3	…	b $(=u_1i_3)$	$\xrightarrow{b\ has\ a_3}$	a_3 $(=u_2i_3)$	…
…	…	…		…	…

Given the PCDRT definition of negation, the translation in (45) derives the intuitively correct truth-conditions: the formula in (47) below is true iff there is exactly one contextually salient boy that has some apples and gives none of them to his sister.

47. $\lambda I.\ \exists J$([**unique**$\{u_1\}$, $boy\{u_1\}$]; $\mathbf{max}^{u_2}$([$apple\{u_2\}$, $have\_in\_rucksack\{u_1, u_2\}$]);

[~[$give\_to\_sister\{u_1, u_2\}$]])$IJ$) =

$\lambda I.\ \exists x_e(u_1I=\{x\} \wedge boy(x) \wedge \exists Y_{et}\neq\emptyset(\forall y_e(apple(y) \wedge h.i.r(x, y) \leftrightarrow y\in Y) \wedge$

$\forall y_e\in Y(\neg g.t.s(x, y))))$

This example makes clear that the $\mathbf{max}^u$ operator defined in (40) is *selective* in exactly the sense in which the dynamic quantification in CDRT+GQ is selective: the set of output states that are in the range of a $\mathbf{max}^u$ DRS is determined based on the set of *individuals* that such an output state stores with respect to the dref u. However, in view of the fact that donkey conditionals seem to exhibit *unselectively* strong readings, e.g. the conditional in (48) below, I will define an unselective form of maximization – as shown in (49).

48. If a$^{\mathbf{str}:u_1}$ house-elf borrows a$^{\mathbf{str}:u_2}$ broom from a$^{\mathbf{str}:u_3}$ witch, he$_{u_1}$ (always) gives it$_{u_2}$ back to her$_{u_3}$ the next day.

49. **unselective maximization:**

$$\mathbf{max}(D) := \lambda I_{st}J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K\subseteq J)$$ [27]

The unselective **max** operator in (49) retrieves the supremum in an inclusion partial order over sets of info states and not over sets of individuals (i.e. it is unselective in the sense of Lewis 1975). This operator will be used to defined unselective generalized quantification in PCDRT.

I conclude the section with two observations about selective maximization, one empirical and the other theoretical. First, note that selective maximization seems to be independently motivated by the Russellian uses of definite descriptions in natural language, i.e. the definite descriptions that intuitively require strong uniqueness (uniqueness relative to the entire model). The definite DP in (50) below exemplifies the Russellian kind of definite descriptions, i.e. definite descriptions that are non-anaphoric and that require existence and strong uniqueness.

50. Hagrid fell in love with the^{u} tallest witch in the world.

In PCDRT, we can analyze Russellian definite descriptions by suitably combining weak uniqueness, i.e. the condition **unique**$\{u\}$, and the $\mathbf{max}^{u}$ operator. In fact, PCDRT can analyze definite articles in any of the four ways listed in (51) below; deciding which one (if any) is the right meaning falls outside the scope of the current investigation.

51. **The definite article – possible meanings in PCDRT.**

a. anaphoric and weakly unique:

$$the_u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ P'(u);\ P(u),$$

where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$

and $\mathbf{unique}\{u\} := \lambda I_{st}.\ \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$.

b. anaphoric, no uniqueness:

[27] Note that, for any I_{st}, the set $\{J_{st}\text{: } \mathbf{max}(D)IJ\}$ is either empty or a singleton set.

the$_u$ ⇝ $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ P'(u); P(u)$

c. existence and strong uniqueness, non-anaphoric (the Russellian analysis):

the^u ⇝ $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u)); [\mathbf{unique}\{u\}]; P(u)$

d. existence and maximality (no uniqueness), non-anaphoric:

the^u ⇝ $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u)); P(u)$ [28]

I conclude this section with the examination of DRS's in which one $\mathbf{max}^u$ operator is embedded within the scope of another, as schematically shown in (52) below.

52. $\mathbf{max}^u(D; \mathbf{max}^{u'}(D'))$

Such structures occur fairly frequently in the PCDRT translations of natural language discourses and they are difficult to grasp at an intuitive level. To simplify derivations and make translations more transparent, I show that the values assigned to multiply embedded $\mathbf{max}^u$ operators are often reducible to non-embedded ones.

The main result is stated in the corollary in (53) below – see section **0** of the Appendix to this chapter for its proof.

53. **Simplifying 'max-under-max' representations (corollary):**

$$\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D'),$$

if the following three conditions obtain:

a. u is not reintroduced in D';

b. $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

If $C_1, \ldots, C_m$ are c-ideals, condition (53b) follows from (53c) [29].

[28] Note that this meaning is different from the strong meaning of the indefinite article with respect to the scope of the $\mathbf{max}^u$ operator: in the case of the definite, this operator has scope only over the restrictor DRS, i.e. $\mathbf{max}^u(P'(u))$, while in the case of the indefinite, it has scope over both the restrictor and nuclear scope DRS's, i.e. $\mathbf{max}^u(P'(u); P(u))$.

[29] **If $C_1, \ldots, C_m$ are c-ideals, condition (53b) follows from (53c).**

Proof: In general, we have that $\mathbf{Dom}(\mathbf{max}^{u'}(D')) \subseteq \mathbf{Dom}([u']; D')$, so we only have to prove that $\mathbf{Dom}([u']; D') \subseteq \mathbf{Dom}(\mathbf{max}^{u'}(D'))$. But an info state $I \in \mathbf{Dom}([u']; D')$ fails to be in $\mathbf{Dom}(\mathbf{max}^{u'}(D'))$ iff the family of sets $\{u'J: ([u']; D')IJ\}$ does not have a supremum. And the existence of the supremum follows by an application

Let us reanalyze the example in (42) above, repeated in (54), in terms of the Russellian analysis of definite descriptions, i.e. letting the definite article the^{u_1} contribute existence and uniqueness as in (51c) above: $the^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u))$; [$\mathbf{unique}\{u\}$]; $P(u)$. The example is translated as shown in (55).

54. The^{u_1} (one) boy who had an$^{\mathbf{str}:u_2}$ apple in his rucksack didn't give it$_{u_2}$ to his sister.

55. $\mathbf{max}^{u_1}([boy\{u_1\}];\ \mathbf{max}^{u_2}([apple\{u_2\},\ have\_in\_rucksack\{u_1, u_2\}]))$;
[$\mathbf{unique}\{u_1\}$]; [~[$give\_to\_sister\{u_1, u_2\}$]]

The representation in (55) gives us the opportunity to apply the corollary in (53) above. Conditions (53a) and (53c) are clearly satisfied; checking that condition (53b) holds is also straightforward: given that both conditions $apple\{u_2\}$ and $have\_in\_rucksack\{u_1, u_2\}$ are c-ideals, (53b) follows from (53c).

Thus, the translation in (55) is equivalent to the one in (56) below. The truth-conditions, provided in (57), are the intuitively correct ones (assuming that the definite article should indeed receive the Russellian analysis): sentence (54) is true iff there is a unique boy with some apples in his rucksack such that he didn't give any of his apples to his sister.

56. $\mathbf{max}^{u_1}([boy\{u_1\}];\ [u_2 \mid apple\{u_2\},\ have\_in\_rucksack\{u_1, u_2\}])$;
$\mathbf{max}^{u_2}([apple\{u_2\},\ have\_in\_rucksack\{u_1, u_2\}])$;
[$\mathbf{unique}\{u_1\}$]; [~[$give\_to\_sister\{u_1, u_2\}$]]

57. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e(\forall z_e(boy(z) \wedge \exists y_e(apple(y) \wedge have\_in\_rucksack(z, y)) \leftrightarrow z{=}x) \wedge$
$\exists Y_{et}{\neq}\emptyset(\forall y_e(apple(y) \wedge have\_in\_rucksack(x, y) \leftrightarrow y{\in}Y) \wedge$
$\forall y_e{\in}Y(\neg give\_to\_sister(x, y))))$

---

of the result stated in (24) above: just take the image of the info state I under the relation $\mathbb{R}^{[u']; D'}=\{<i, j>: i[u', u_1, \ldots, u_n]j \wedge j{\in}(\cup C_1)\cap \ldots \cap(\cup C_m)\}$, i.e. $\mathbb{J}=\{j: \exists i{\in}I(\mathbb{R}^{[u']; D'}ij)\}$ and note that $u'\mathbb{J}$ is the supremum of $\{u'J: ([u']; D')IJ\}$. □

To see more clearly that the truth-conditions enforced by PCDRT formulas are of the form "there are *some values* and there is *some structure* associated with those values such that…", I will rewrite the formula in (57) as shown in (58) below, i.e. by using of a relation $R_{e(et)}$ between individuals which encodes the *structure* associated with the values in question (i.e. with the unique boy and his apples). **Dom**(R) and **Ran**(R) are defined as usual, i.e. **Dom**(R) := $\{x_e: \exists y_e(Rxy)\}$ and **Ran**(R) := $\{y_e: \exists x_e(Rxy)\}$

58. $\lambda I_{st}.\ I \neq \emptyset \wedge \exists R_{e(et)}(\mathbf{Dom}(R) \neq \emptyset \wedge \mathbf{Ran}(R) \neq \emptyset \wedge |\mathbf{Dom}(R)|=1 \wedge$
$\mathbf{Dom}(R) = \{x_e: boy(x) \wedge \exists y_e(apple(y) \wedge have\_in\_rucksack(x, y))\} \wedge$
$\mathbf{Ran}(R) = \{y_e: apple(y) \wedge \exists x_e \in \mathbf{Dom}(R)(have\_in\_rucksack(x, y))\} \wedge$
$\forall x_e \forall y_e(Rxy \rightarrow have\_in\_rucksack(x, y)) \wedge$
$\forall x_e \forall y_e(Rxy \rightarrow \neg give\_to\_sister(x, y)))$

3.5. Generalized Quantification

The only thing left to define in PCDRT is generalized quantification. We start with selective generalized quantification.

Selective generalized determiners are relations between two dynamic properties $P'_{\mathbf{et}}$ (the restrictor) and $P_{\mathbf{et}}$ (the nuclear scope), i.e. their denotations are of the expected type (**et**)((**et**)**t**). The PCDRT definition of selective generalized determiners has to be formulated in such a way that:

- on the one hand, we capture the fact that anaphors in the nuclear scope can have antecedents in the restrictor;
- on the other hand, we avoid the proportion problem and, at the same time, allow for the weak / strong donkey ambiguity.

To avoid the proportion problem, a selective generalized determiner has to relate sets of individuals and not sets of 'assignments'. Thus, the main problem in a dynamic system is to find an appropriate way to extract the two sets of individuals, i.e. the restrictor set and the nuclear scope set, based on the restrictor and the nuclear scope dynamic properties.

The proposed ways to solve this problem fall into two broad categories. The first category of solutions is the one exemplified by CDRT+GQ (following DRT / FCS / DPL): we employ a dynamic framework based on singular info states and we analyze generalized quantification as internally dynamic and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property, while the nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope dynamic property, so that the anaphoric connections between them are captured.

The second category of solutions employs a dynamic framework based on plural information states and it analyzes generalized quantification as both internally and externally dynamic. The main reference for this kind of solution is van den Berg (1994, 1996a) (but see also Krifka (1996b) and Nouwen (2003) among others). The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property and, then, the nuclear scope set of individuals is the maximal subset of the restrictor set of individuals that satisfies the nuclear scope dynamic property. The restrictor and the nuclear scope sets are stored in the output plural info state and are available for anaphoric retrieval, e.g. *Every*^{u} *man saw a*$^{u'}$ *woman / two*$^{u'}$ *women. They*$_{u}$ *greeted them*$_{u'}$.

Given that the notion of a dref being a *subset* of another required for van den Berg's definition of quantification involves non-trivial complexities[30] that are largely orthogonal to the donkey issues we are interested in, I will analyze selective generalized quantification following the format of the CDRT+GQ definition[31].

However, since PCDRT is a system based on *plural* info states, the definition of selective generalized determiners I will provide is novel. This definition is intermediate between the above two strategies of defining selective dynamic quantification and, as

[30] E.g., it requires the introduction of a dummy / 'undefined' / exception individual # – see chapter **6**. For the corresponding notion of dummy / 'undefined' / exception possible world, see the analysis of structured discourse reference to propositions in chapter **7**.

[31] But see chapter **6** for a van den Berg-style definition of generalized quantification in PCDRT which is used in the analysis of quantificational subordination.

such, it is useful in exhibiting the commonalities and differences between them in a formally explicit way.

The generalized quantifiers we will be considering throughout the present investigation are domain-level and discourse-level distributive in the sense that they relate two sets of *atomic* individuals (i.e. domain-level distributivity) and these sets of atomic individuals are required to satisfy the restrictor and nuclear scope dynamic properties *one individual at a time* (i.e. discourse-level distributivity). We enforce the first kind of distributivity (i.e. domain-level) by restricting our domain of individuals $\boldsymbol{D}_e$ to atomic individuals (there are no non-atomic individuals in the sense of Link 1983). We enforce the second kind of distributivity by making use of the dynamic condition **unique**$\{u\}$, which was introduced in the previous chapter for the analysis of definite descriptions. The definition of selective quantification is provided in (59) below.

59. **Selective Generalized Determiners** ($\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$).

$$\mathsf{det}^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{det}_u(P'(u), P(u))],$$

where $\mathbf{det}_u(D, D') := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[DI],\ u[(D; D')I])$

and $u[DI] := \cup\{uJ\text{: } ([u \mid \mathbf{unique}\{u\}]; D)IJ\}$

and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$

and **DET** is the corresponding static determiner.

Intuitively, the definition $u[DI] := \cup\{uJ\text{: } ([u \mid \mathbf{unique}\{u\}]; D)IJ\}$ above instructs us to do the following 'operations' to an input matrix I: add a column u to matrix I, fill it up with only one individual x and then check that the resulting matrix satisfies D (the resulting matrix satisfies D iff it can be updated with D, i.e. iff it has at least one output state J relative to D). If this matrix satisfies D, then x is in the set $u[DI]$, otherwise not.

The definition of generalized quantification in (59) is *selective* because the static determiner **DET** relates sets of individuals. The individuals in these sets are obtained in basically the same way as they are obtained in the case of the CDRT+GQ weak generalized determiners. The restrictor set contains all the individuals which 'satisfy' the restrictor DRS D when plugged in one atomic individual at a time, i.e. $[u \mid \mathbf{unique}\{u\}]$. The nuclear scope contains all the individuals which satisfy both the restrictor DRS D

and the nuclear scope DRS D' when plugged in one atomic individual at a time. Clearly, this definition of selective quantification enables us to avoid the proportion problem just as the corresponding CDRT+GQ definition does.

The definition of unselective generalized quantification is provided in (60) below.

60. **Unselective generalized determiners (in terms of unselective maximization):**

$$\textbf{det} \rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{det}(D', D)],$$

where $\mathbf{det}(D, D') := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(\mathbf{max}[DI],\ \mathbf{max}[(D; [!D'])I])$

and $\mathbf{max}[DI] := \cup\{J_{st}: \mathbf{max}(D)IJ\}$ and $!D' := \mathbf{Dom}(D')$

and **DET** is the corresponding static determiner.

This definition is unselective because the static determiner **DET** relates sets of 'assignments', i.e. sets of cases in the terminology of Lewis (1975). The info states in these sets are obtained much as they are obtained in the case of CDRT+GQ unselective determiners: the restrictor is the set of all the 'assignments' / cases that 'satisfy' the restrictor DRS D relative to input info state I and the nuclear scope is the set of all the 'assignments' / cases that 'satisfy' both the restrictor DRS D and the nuclear scope DRS D'.

The unselective **max** operator functions as a dynamic λ-abstraction over 'assignments', i.e. over the cases of Lewis (1975) – much like the condition **unique**$\{u\}$ together with the union of sets of individuals in the definition of selective generalized quantification in (59) above functions as dynamic λ-abstraction over individuals[32].

4. Solutions to Donkey Problems

In this section, we see in detail how the PCDRT system introduced in the preceding section can be used to compositionally interpret a variety of donkey sentences, including mixed weak & strong relative-clause donkey sentences.

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[32] Recall that, since $\mathbf{max}(D) := \lambda I_{st}J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K \subseteq J)$, the set $\{J_{st}: \mathbf{max}(D)IJ\}$ is either empty or a singleton set, so the union over unselectively maximized info states is basically vacuous and needed here only for technical reasons: we want to access the maximal plural info state and not the singleton set whose only member is the maximal plural info state.

As we have already observed in chapter **3**, the compositional aspect of the interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences, which we abbreviate as **e** and **t**. An extensional static logic without pluralities (i.e. the static component of our Dynamic Ty2) identifies **e** and *e* (atomic entities) and also **t** and *t* (truth-values). CDRT+GQ complicates this setup by interpreting a sentence as a relation between an input and an output 'assignment', hence $\mathbf{t} := (s(st))$, and a name as an individual dref, i.e. as a function from 'assignments' to individuals, hence $\mathbf{e} := (se)$.

In PCDRT, names are interpreted just as in CDRT+GQ, but sentences are interpreted as relations between plural info states, i.e. as relations between an input *set* of 'assignments' and an output *set* of 'assignments', hence $\mathbf{t} := (st)((st)t)$. Everything else in our definition of type-driven translation remains the same. In particular, the only translation rule we need to change is **TR0**, i.e. the translation rule for the basic meanings – and even here, the modifications are minimal, as the table in (45) below shows.

61. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[sleep]_{V_{in}}$	$\leadsto \lambda v_{\mathbf{e}}.\ [sleep_{et}\{v\}]$	$\mathbf{et}$
$[own]_{V_{tr}}$	$\leadsto \lambda Q_{\mathbf{(et)t}}.\ \lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [own_{e(et)}\{v, v'\}])$	$((\mathbf{et})\mathbf{t})(\mathbf{et})$
$[buy]_{V_{di}}$	$\leadsto \lambda Q'_{\mathbf{(et)t}}.\ \lambda Q_{\mathbf{(et)t}}.\ \lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v''_{\mathbf{e}}.\ [buy_{e(e(et))}\{v, v', v''\}]))$	$(\mathbf{ett})((\mathbf{ett})(\mathbf{et}))$
$[house\text{-}elf]_{N}$	$\leadsto \lambda v_{\mathbf{e}}.\ [house\_elf_{et}\{v\}]$	$\mathbf{et}$
$[he_u]_{DP}$	$\leadsto \lambda P_{\mathbf{et}}.\ P(u_{\mathbf{e}})$	$(\mathbf{et})\mathbf{t}$
$[the_u]_{D}$	$\leadsto \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ P'(u);\ P(u)$, where $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$, i.e. anaphoric and 'weakly' unique. $\leadsto \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ P'(u);\ P(u)$, i.e. anaphoric.	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[t_v]_{DP}$	$\leadsto \lambda P_{\mathbf{et}}.\ P(v_{\mathbf{e}})$	$(\mathbf{et})\mathbf{t}$
$[he_{Dobby}]_{DP}$	$\leadsto \lambda P_{\mathbf{et}}.\ P(Dobby_{\mathbf{e}})$	$(\mathbf{et})\mathbf{t}$

61. TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[Dobby^u]_{DP}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [u \mid u{=}Dobby];\ P(u)$	$(\mathbf{et})\mathbf{t}$
$[who]_{DP}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ P$	$(\mathbf{et})(\mathbf{et})$
$[\emptyset]_I$ / $[\text{-}ed]_I$ / $[\text{-}s]_I$	$\rightsquigarrow \lambda D_{\mathbf{t}}.\ D$	$\mathbf{tt}$
$[doesn't]_I$ / $[didn't]_I$	$\rightsquigarrow \lambda D_{\mathbf{t}}.\ [{\sim}D]$	$\mathbf{tt}$
$[a^{\mathbf{wk}:u}]_D$	$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [u];\ P'(u);\ P(u)$, i.e. $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \exists u(P'(u);\ P(u))$, where $\exists u(D) := [u];\ D$	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[a^{\mathbf{str}:u}]_D$	$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u);\ P(u))$, where $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$, i.e. $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(P'(u);\ P(u))$, where $\exists^{\mathbf{m}}u(D) := \mathbf{max}^u(D)$	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[the^u]_D$	$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u));\ [\mathbf{unique}\{u\}];\ P(u)$, where $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui')$ and $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$, i.e. $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(P'(u));\ [\mathbf{unique}\{u\}];\ P(u)$, i.e. existence and uniqueness – the Russellian analysis $\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u));\ P(u)$, where $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$, i.e. $\lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(P'(u));\ P(u)$, i.e. existence and maximality	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[det^u]_D$ e.g. *every*^{u}, *no*^{u}, *most*^{u}… (but not $a^{\mathbf{wk}:u}$, $a^{\mathbf{str}:u}$, the_u or the^u)	$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{det}_u(P'(u),\ P(u))]$, where: $\mathbf{det}_u(D_1,\ D_2) := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[D_1I],\ u[(D_1;\ D_2)I])$, where $u[DI] := \cup\{uJ\colon ([u \mid \mathbf{unique}\{u\}];\ D)IJ\}$ and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui')$ and **DET** is the corresponding static determiner	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[if\ (\text{+}adv.\ of\ quant.)]_C$	$\rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{det}(D',\ D)]$, where: $\mathbf{det}(D_1,\ D_2) := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(\mathbf{max}[D_1I],\ \mathbf{max}[(D_1;\ [!D_2])I])$, where $\mathbf{max}[DI] := \cup\{J_{st}\colon \mathbf{max}(D)IJ\}$ and $\mathbf{max}(D) := \lambda I_{st}J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K{\subseteq}J)$ and $!D := \mathbf{Dom}(D)$ and **DET** is the corresponding static determiner	$\mathbf{t}(\mathbf{tt})$
$[if]_C$ (i.e. bare *if*)	$\rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{every}(D',\ D)]$	$\mathbf{t}(\mathbf{tt})$

61. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	**Translation**	**Type** **e** := *se* **t** := $(st)((st)t)$
$[\textit{and}]_{\text{Conj}}$	$\leadsto \lambda v_1. \ldots \lambda v_n. v_1 \sqcap \ldots \sqcap v_n$	$\tau(\ldots(\tau\tau)\ldots)$
$[\textit{or}]_{\text{Conj}}$	$\leadsto \lambda v_1. \ldots \lambda v_n. v_1 \sqcup \ldots \sqcup v_n$	$\tau(\ldots(\tau\tau)\ldots)$

The definition of dynamically conjoinable types (**DCTyp**) is the same as in CDRT+GQ modulo the fact that we reset **t** to $(st)((st)t)$, as shown in (62) below.

62. **PCDRT Dynamically Conjoinable Types (DCTyp).**
The set of PCDRT dynamically conjoinable types **DCTyp** is the smallest subset of **Typ** s.t. $\mathbf{t} \in \mathbf{DCTyp}$ ($\mathbf{t} := (st)((st)t)$) and, if $\tau \in \mathbf{DCTyp}$, then $(\sigma\tau) \in \mathbf{DCTyp}$ for any $\sigma \in \mathbf{Typ}$.

We define generalized (pointwise) dynamic conjunction and disjunction as shown in (43) below (the same as the CDRT+GQ definition) – and thereby complete the definition of *and* and *or* in table (45) above.

63. **Generalized Pointwise Dynamic Conjunction $\sqcap$ and Disjunction $\sqcup$.**
For any two terms α and β of type τ, for any $\tau \in \mathbf{DCTyp}$:

$$\alpha \sqcap \beta := (\alpha; \beta) \text{ if } \tau = \mathbf{t} \quad \text{and} \quad \alpha \sqcap \beta := \lambda v_\sigma. \alpha(v) \sqcap \beta(v) \text{ if } \tau = (\sigma\rho);$$

$$\alpha \sqcup \beta := [\alpha \vee \beta] \text{ if } \tau = \mathbf{t} \quad \text{and} \quad \alpha \sqcup \beta := \lambda v_\sigma. \alpha(v) \sqcup \beta(v) \text{ if } \tau = (\sigma\rho).$$

Abbreviation. $\alpha_1 \sqcap \alpha_2 \sqcap \ldots \sqcap \alpha_n := (\ldots(\alpha_1 \sqcap \alpha_2) \sqcap \ldots \sqcap \alpha_n)$

$$\alpha_1 \sqcup \alpha_2 \sqcup \ldots \sqcup \alpha_n := (\ldots(\alpha_1 \sqcup \alpha_2) \sqcup \ldots \sqcup \alpha_n).$$

We are now ready to analyze the donkey examples we have introduced in the preceding sections and chapters.

4.1. Bound Variable Anaphora

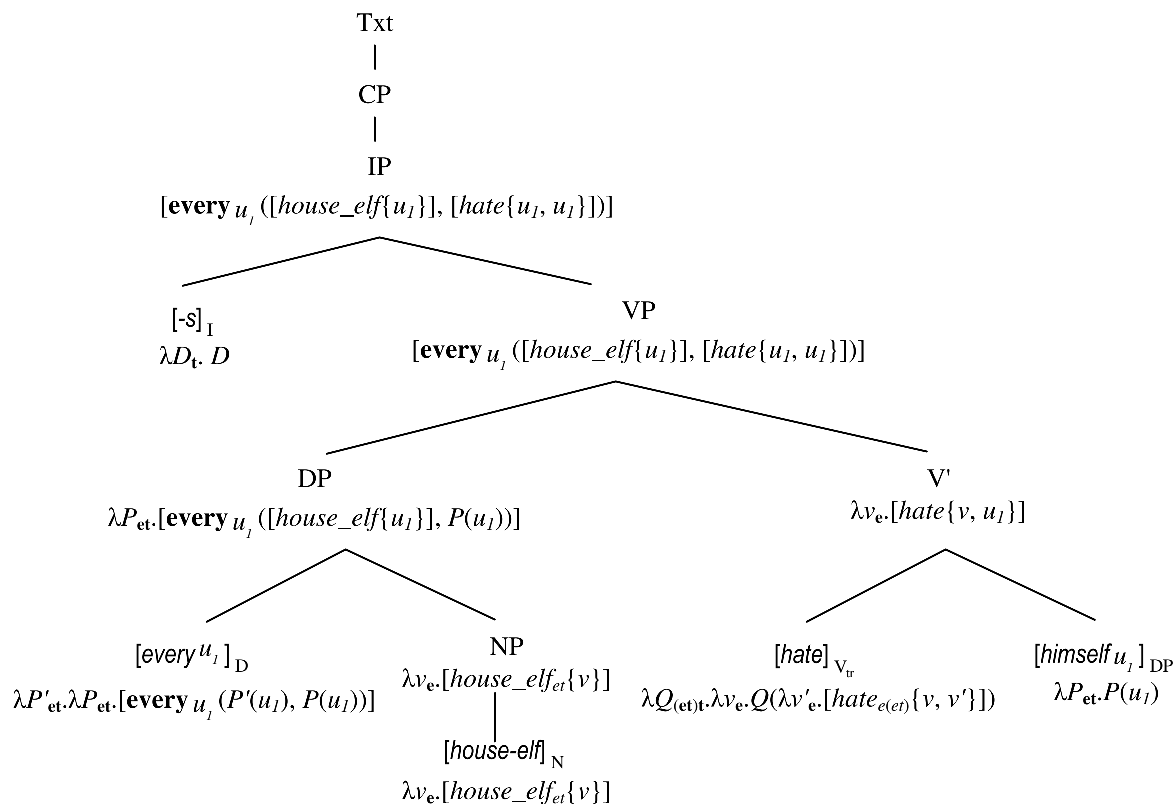
First, we show that PCDRT preserves the compositional CDRT+GQ account of the basic kinds of examples. Let's start with the bound anaphora example in (51) below.

64. Every ^{u_1} house-elf hates himself $_{u_1}$.

Just as CDRT+GQ, PCDRT can compositionally account for bound anaphora without Quantifier Raising and Quantifying-In: co-indexation is enough for binding because the meaning of the determiner *every* dynamically conjoins the restrictor and the nuclear scope DRS's to determine the set of individuals in its nuclear scope – thus, *every* quantifies over 'assignments' in a selective way.

Sentence (55) is compositionally translated as shown in (65) below. The PCDRT representation derives the intuitively correct truth-conditions, provided in (66).

65. Every ^{u_1} house-elf hates himself $_{u_1}$.



66. $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e(house\_elf(x) \rightarrow hate(x, x))$

4.2. Quantifier Scope Ambiguities

Let us turn now to the example in (67) below exhibiting quantifier scope ambiguities over and above the lexical ambiguity of the indefinite. We start with the weak reading of the indefinite – and we assign intuitively correct truth-conditions to both LFs, as shown in (69) and (71) below.

67. Every^{u_1} house-elf adores a$^{\mathbf{wk}:u_2}$ witch.

68. ***every***^{u_1}***>>a***$^{\mathbf{wk}:u_2}$: $[\mathbf{every}_{u_1}([house\_elf\{u_1\}],\ [u_2 \mid witch\{u_2\}, adore\{u_1, u_2\}])]$

69. ***every***^{u_1}***>>a***$^{\mathbf{wk}:u_2}$: $\lambda I_{st}.\ I{\neq}\emptyset \wedge$

$$\forall x_e(house\_elf(x) \rightarrow \exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(witch(y) \wedge adore(x, y))))$$ [33]

70. ***a***$^{\mathbf{wk}:u_2}$***>>every***^{u_1}: $[u_2 \mid witch\{u_2\},\ \mathbf{every}_{u_1}([house\_elf\{u_1\}],\ [adore\{u_1, u_2\}])]$

71. ***a***$^{\mathbf{wk}:u_2}$***>>every***^{u_1}: $\lambda I_{st}.\ I{\neq}\emptyset \wedge$

$$\exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y))))$$ [34]

Take the update in (70), for instance. Intuitively, this update instructs us to do the following 'operations' on an input matrix I: fill column u_2 only with witches; then, check that each way of filling column u_1 with a single elf x is a way of filling column u_1 with the elf x such that x adores every (corresponding) u_2-witch.

The LF's for the two readings are provided in (72) and (73) below.

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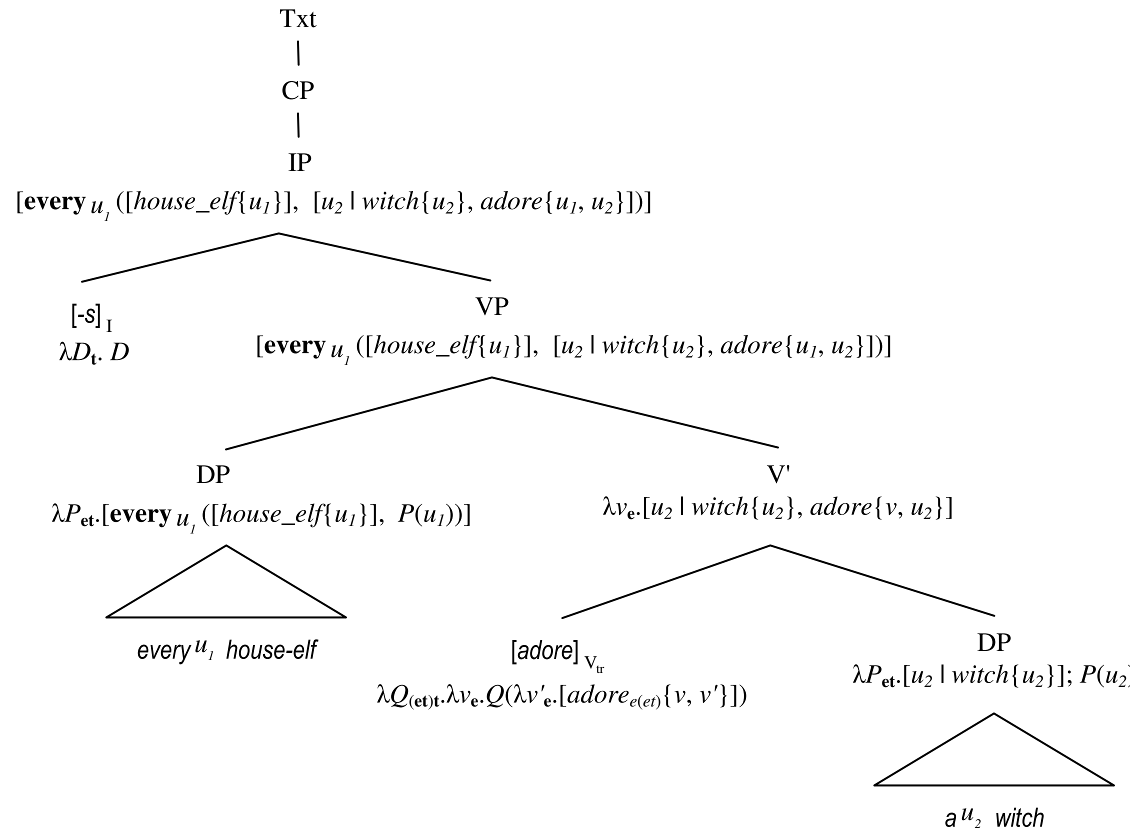
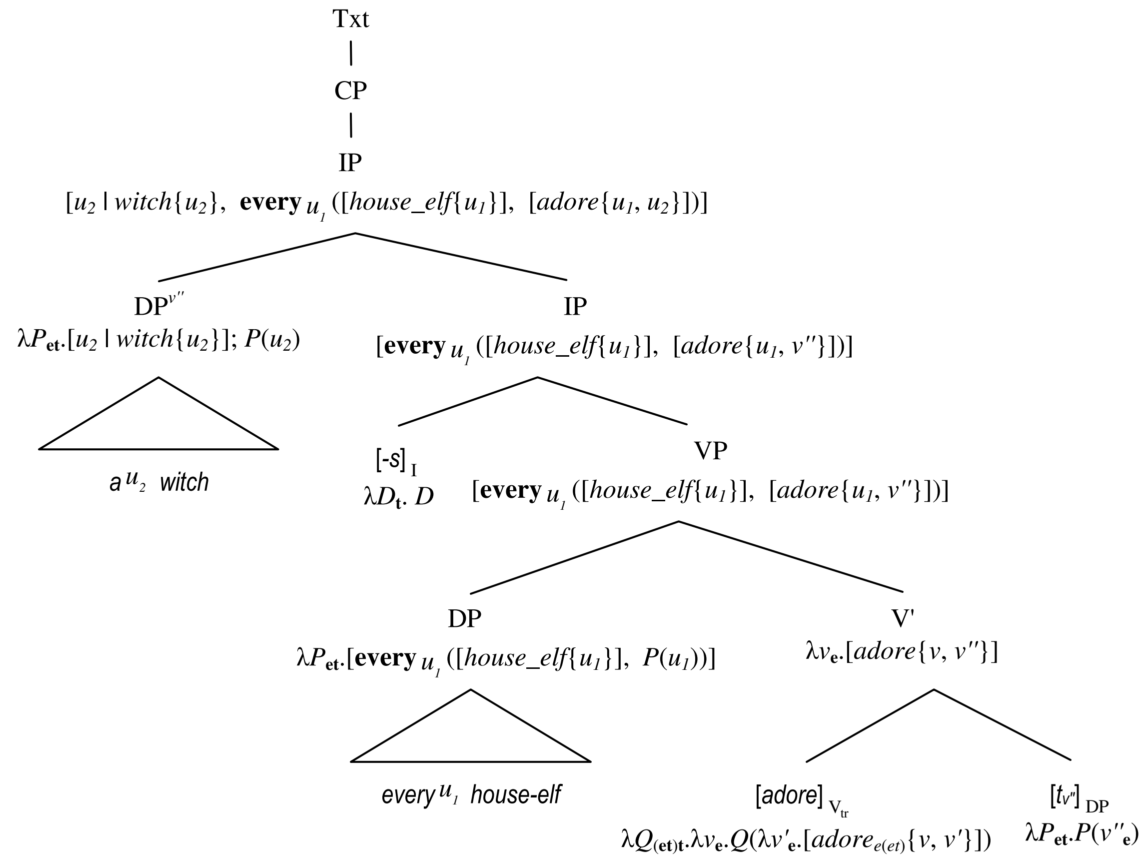
[33] I use quantification over sets $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(witch(y) \wedge adore(x, y)))$ in (69) only to make more explicit the relation between truth-conditions and *plural* info states in PCDRT (which plural info states store possibly non-singleton sets of individuals). In this particular case, quantification over sets is clearly not essential since $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(witch(y) \wedge adore(x, y)))$ is equivalent to the first order formula $\exists y_e(witch(y) \wedge adore(x, y))$.

[34] Just as before (see fn. **33** above), note that quantification over sets is not essential: the formula $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y))))$ is equivalent to the first-order formula $\exists y_e(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y)))$.

72. ***every***^{u_1} ***>>a***$^{\mathbf{wk:}u_2}$: Every^{u_1} house-elf adores a$^{\mathbf{wk:}u_2}$ witch.



73. ***a*** $^{\mathbf{wk}:u_2}$***>>every*** ^{u_1}: Every ^{u_1} house-elf adores a $^{\mathbf{wk}:u_2}$ witch.



If the indefinite is strong – as shown in (74) below –, we have two more LF's with the same structure as (72) and (73) above. Yet again, we assign intuitively correct truth-conditions to both LF's, as shown in (76) and (78) below.

74. Every ^{u_1} house-elf adores a $^{\mathbf{str}:u_2}$ witch.

75. ***every*** ^{u_1}***>>a*** $^{\mathbf{str}:u_2}$: $[\mathbf{every}_{u_1}([h.elf\{u_1\}],\ \mathbf{max}^{u_2}([witch\{u_2\}, adore\{u_1, u_2\}]))]$

76. ***every*** ^{u_1}***>>a*** $^{\mathbf{str}:u_2}$: $\lambda I_{st}.\ I{\neq}\emptyset \wedge$

$$\forall x_e(h.elf(x) \rightarrow \exists Y_{et}{\neq}\emptyset(\forall y_e(witch(y) \wedge adore(x, y) \leftrightarrow y{\in}Y)))$$ [35]

77. ***a*** $^{\mathbf{str}:u_2}$***>>every*** ^{u_1}: $\mathbf{max}^{u_2}([witch\{u_2\},\ \mathbf{every}_{u_1}([h.elf\{u_1\}],\ [adore\{u_1, u_2\}])])$

[35] Yet again, we can do away with quantification over sets since, for our purposes (i.e. the interpretation of (74)), we can substitute $\exists y_e(\boldsymbol{F}y)$ for $\exists Y_{et}{\neq}\emptyset(\forall y_e(\boldsymbol{F}y \leftrightarrow y{\in}Y))$ in both (76) and (78) – where $\boldsymbol{F}$ stands for the predicate that is appropriate in each of the two cases.

78. $\textbf{\textit{a}}^{\mathbf{str:}u_2}$**>>*every***^{u_1}: $\lambda I_{st}.\ I{\neq}\emptyset \wedge$

$$\exists Y_{et}{\neq}\emptyset(\forall y_e(witch(y) \wedge \forall x_e(h.elf(x) \rightarrow adore(x, y)) \leftrightarrow y{\in}Y))$$

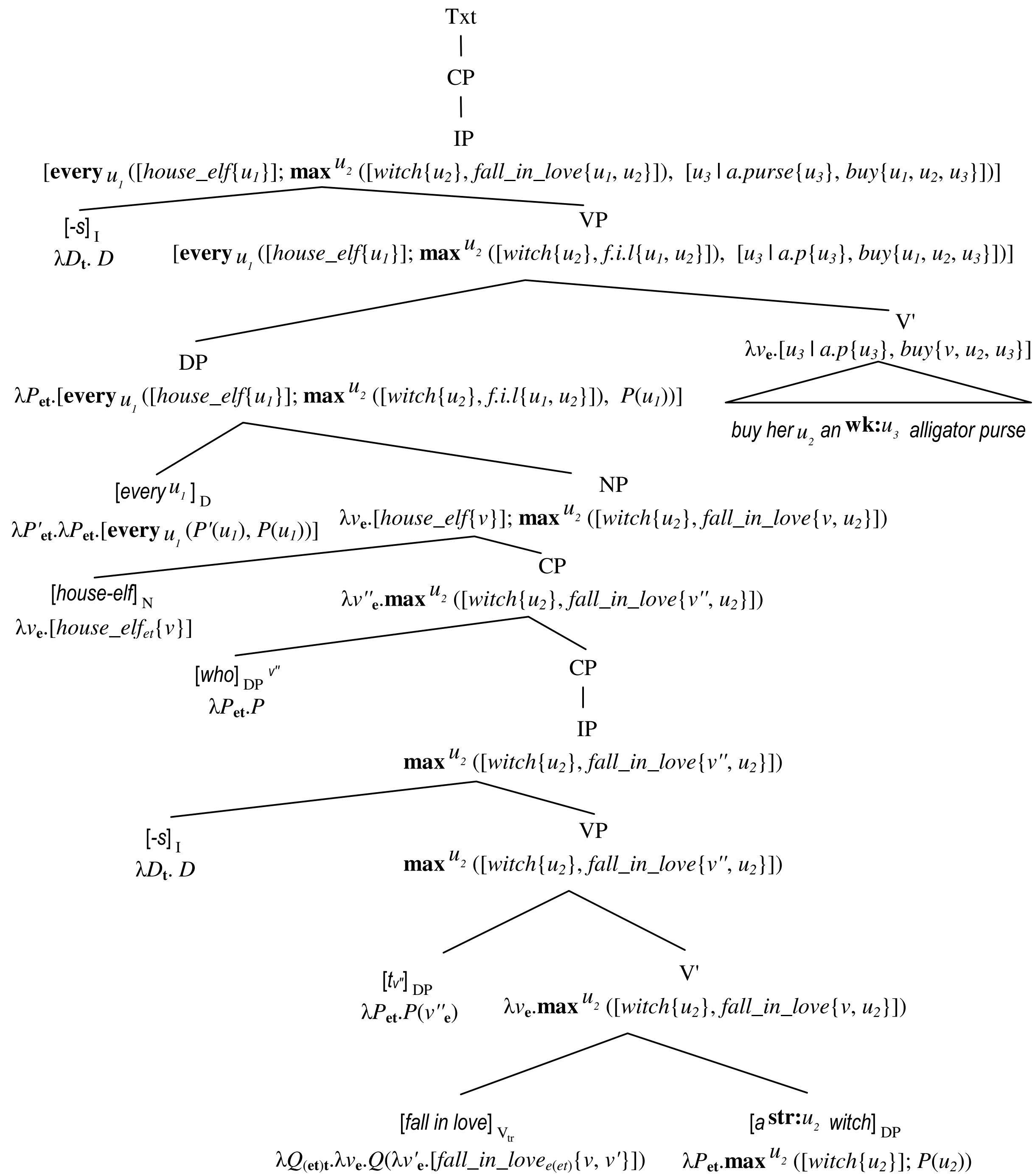
4.3. Weak / Strong Ambiguities

Consider first the strong donkey example in (79) below, which is most readily understood as a generalization about the habits of house-elves that are in love – this being the reason for the fact that the donkey indefinite receives a strong reading. The LF of the sentence and the main steps of its compositional translation are provided in (79).

Intuitively, the translation in (79), i.e. the update associated with the Txt / CP / IP node, instructs us to check that, for any given matrix *I*, each way of pairing up a witch-loving elf with each of the witches he loves is a way of pairing up a witch-loving elf with each of the witches he loves and with some purse he bought her.

The translation derives the intuitively correct truth-conditions, given in (80).

79. Every^{u_1} house-elf who falls in love with a$^{\mathbf{str:}u_2}$ witch buys her$_{u_2}$ an$^{\mathbf{wk:}u_3}$ alligator purse. [36]



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[36] For more discussion of the particular interpretation of *who* in (79), see section **5** of chapter **3**.

80. $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e \forall y_e(house\_elf(x) \wedge witch(y) \wedge adore(x, y)$

$$\rightarrow \exists z_e(alligator\_purse(z) \wedge buy(x, y, z)))$$

The analysis of the classical weak donkey sentence in (81) below proceeds as expected – see the PCDRT translation in (82) and the truth-conditions in (83).

Intuitively, the update in (82) instructs us to check the following, for any given matrix I: for each person x, if you can form a matrix based on I which stores x in column u_1 and which stores some non-empty set of dimes that x has in column u_2, then you should be able to form a (possibly different) matrix based on I which stores x in column u_1 and some non-empty set of dimes that x has and puts in the meter in column u_2.

81. Every^{u_1} person who has a$^{\mathbf{wk}:u_2}$ dime will put it$_{u_2}$ in the meter.

82. $[\mathbf{every}_{u_1}([u_2 \mid person\{u_1\}, dime\{u_2\}, have\{u_1, u_2\}],\ [put\_in\_meter\{u_1, u_2\}])]$

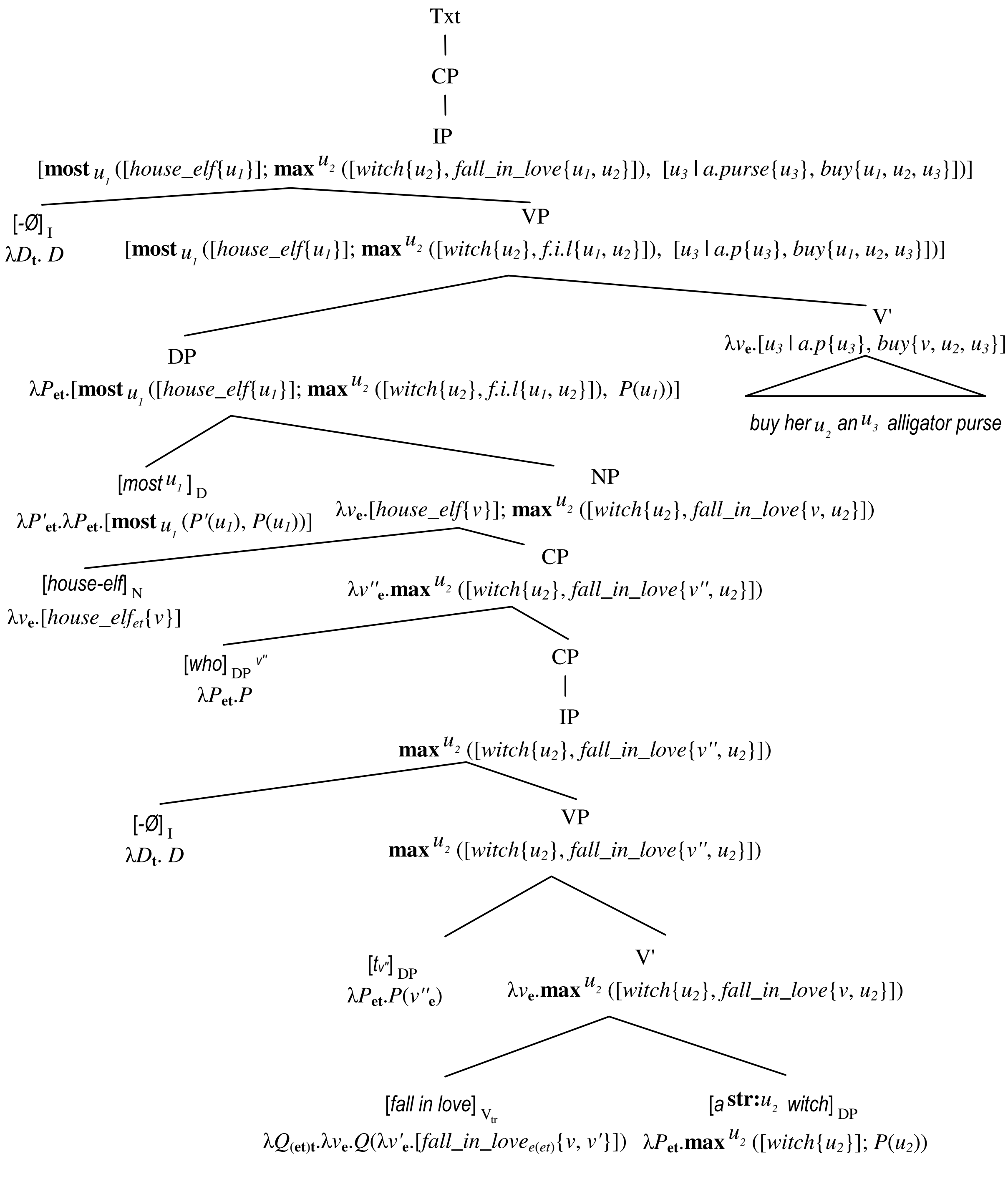
83. $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e(person(x) \wedge \exists y_e(dime(y) \wedge have(x, y))$

$$\rightarrow \exists z_e(dime(z) \wedge have(x, z) \wedge put\_in\_meter(x, z)))$$

4.4. Proportions

The proportion problem is solved because we work with a *selective* form of generalized quantification – as exemplified by the analysis of sentence (84) below. This sentence is most readily understood as a generalization about the behavior of most house-elves that are in love with a witch: *every* such witch ends up getting an alligator purse from the house-elf that is in love with her. Thus, the donkey indefinite *a witch* receives a strong reading. The PCDRT translation derives the intuitively correct truth-conditions (identical to the CDRT+GQ truth-conditions), provided in (85) below; note in particular that the PCDRT representation is false in the "Dobby as Don Juan" scenario, as desired.

84. Most^{u_1} house-elves who fall in love with a$^{\mathbf{str:}u_2}$ witch buy her$_{u_2}$ an$^{\mathbf{wk:}u_3}$ alligator purse.



85. $\lambda I_{st}.\ I \neq \emptyset \wedge |\{x_e: h.elf(x) \wedge \exists y_e(witch(y) \wedge f.i.l(x, y)) \wedge$

$\forall y_e(witch(y) \wedge f.i.l(x, y) \rightarrow \exists z_e(a.p(z) \wedge buy(x, y, z)))\}| >$

$$|\{x_e\text{: } house\_elf(x) \wedge \exists y_e(witch(y) \wedge fall\_in\_lovel(x, y) \wedge \neg\exists z_e(a.purse(z) \wedge buy(x, y, z)))\}|$$

4.5. Mixed Weak & Strong Sentences

The PCDRT definition of selective generalized quantification enables us to assign intuitively correct interpretations to our mixed weak & strong donkey sentences, repeated in (86) and (87) below.

86. Every^{u_1} person who buys a$^{\mathbf{str}:u_2}$ book on amazon.com and has a$^{\mathbf{wk}:u_3}$ credit card uses it$_{u_3}$ to pay for it$_{u_2}$.
87. Every^{u_1} man who wants to impress a$^{\mathbf{str}:u_2}$ woman and who has an$^{\mathbf{wk}:u_3}$ Arabian horse teaches her$_{u_2}$ how to ride it$_{u_3}$.

Given that their PCDRT analyses are basically identical, I will analyze only sentence (86). Its PCDRT translation – obtained compositionally in much the same way as the translations for the donkey sentences we have just examined – is provided in (88).

88. $[\mathbf{every}_{u_1}([pers\{u_1\}]; \mathbf{max}^{u_2}([bk\{u_2\}, buy\{u_1, u_2\}]); [u_3 \mid c.card\{u_3\}, hv\{u_1, u_3\}], [use\_to\_pay\{u_1, u_2, u_3\}])]$

The PCDRT translation in (88) derives the intuitively correct truth-conditions, provided in (89) below.

89. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(person(x) \wedge \exists R_{e(et)}{\neq}\emptyset(\forall y_e(book(y) \wedge buy(x, y) \leftrightarrow y{\in}\mathbf{Dom}(R)) \wedge \forall z_e{\in}\mathbf{Ran}(R)(c.card(z) \wedge have(x, z)))$
$\rightarrow \exists R_{e(et)}{\neq}\emptyset(\forall y_e(book(y) \wedge buy(x, y) \leftrightarrow y{\in}\mathbf{Dom}(R)) \wedge \forall z_e{\in}\mathbf{Ran}(R)(c.card(z) \wedge have(x, z))$
$\forall y_e\forall z_e(Ryz \rightarrow use\_to\_pay(x, y, z))))$, i.e.
$\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(person(x) \wedge \exists y_e(bk(y) \wedge buy(x, y)) \wedge \exists z_e(c.card(z) \wedge hv(x, z))$
$\rightarrow \forall y_e(bk(y) \wedge buy(x, y) \rightarrow \exists z_e(c.card(z) \wedge hv(x, z) \wedge u.t.p(x, y, z))))$

4.6. Donkey Anaphora to Structure

Let us turn to an example that involves *structured* donkey anaphora, i.e. the nuclear scope anaphorically retrieves not only the values of the donkey indefinites, but also the relational structure associated with those values.

Consider (90) below: as we have already noticed, both indefinites, i.e. *a*$^{\mathbf{str}:u_2}$ *Christmas gift* and *a*$^{\mathbf{str}:u_3}$ *girl in his class*, receive a strong reading, i.e. for each u_1-boy, we consider the set of *all* gifts that he bought for some girl in his class and the set of *all* girls that said u_1-boy bought a gift for. However, we need to store not only the sets, but also the correspondences between them established by the buying events, so that we can retrieve this correspondence in the nuclear scope, where we assert that, for each u_3-girl, her deskmate was asked to wrap the u_2-gift that was bought for said u_3-girl.

90. Every^{u_1} boy who bought a$^{\mathbf{str}:u_2}$ Christmas gift for a$^{\mathbf{str}:u_3}$ girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

I will analyze *her*$_{u_3}$ *deskmate* as *the*^{u_4} *deskmate of her*$_{u_3}$ and give a Russellian translation for the definite description, i.e. I assume it contributes existence and uniqueness. Since the uniqueness of the u_4-deskmate needs to be relativized to the u_3-girl, I will use an anaphoric uniqueness condition of the form $\mathbf{unique}_{u'}\{u\}$, as shown in (91) below.

91. *her*$_{u_3}$ ^{u_4} *deskmate* (i.e. *the*$_{u_3}$ ^{u_4} *deskmate of her*$_{u_3}$)

$$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u_4}([deskmate\{u_4\}, of\{u_4, u_3\}]); [\mathbf{unique}_{u_3}\{u_4\}]; P(u),$$

where $\mathbf{unique}_{u_3}\{u_4\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(u_3i{=}u_3i' \rightarrow u_4i{=}u_4i')$

The PCDRT translation of sentence (90) is provided in (92) below. The translation derives the intuitively correct truth-conditions, given in (93).

92. $[\textbf{every}_{u_1}([boy\{u_1\}]; \textbf{max}^{u_2}([gift\{u_2\}]; \textbf{max}^{u_3}([girl\{u_3\}, buy\{u_1, u_2, u_3\}])),$

$\textbf{max}^{u_4}([deskmate\{u_4\}, of\{u_4, u_3\}]); [\textbf{unique}_{u_3}\{u_4\}]; [a.t.w\{u_1, u_4, u_2\}])]$[37]

93. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(boy(x) \wedge$

$\exists R_{e(et)}{\neq}\emptyset(\textbf{Dom}(R) = \{y_e\!: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\textbf{Ran}(R) = \{z_e\!: girl(z) \wedge \exists y_e{\in}\textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\forall y_e \forall z_e(Ryz \to buy(x, y, z)))$

$\to \exists R_{e(et)}{\neq}\emptyset(\textbf{Dom}(R) = \{y_e\!: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\textbf{Ran}(R) = \{z_e\!: girl(z) \wedge \exists y_e{\in}\textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\forall y_e \forall z_e(Ryz \to buy(x, y, z)) \wedge$

$\forall y_e \forall z_e(Ryz \to$

$\exists z'_e(\forall z''_e(d.m(z'') \wedge of(z'', z) \leftrightarrow z''{=}z') \wedge a.t.w(x, z', y))))),$

i.e. given the natural assumption that no boy bought the same gift for two distinct girls, so that there is only one relation *R* with the required properties,

$\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e \forall R_{e(et)}{\neq}\emptyset(boy(x) \wedge$

$\textbf{Dom}(R) = \{y_e\!: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\textbf{Ran}(R) = \{z_e\!: girl(z) \wedge \exists y_e{\in}\textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\forall y_e \forall z_e(Ryz \to buy(x, y, z))$

$\to \forall y_e \forall z_e(Ryz \to$

$\exists z'_e(\forall z''_e(d.m(z'') \wedge of(z'', z) \leftrightarrow z''{=}z') \wedge a.t.w(x, z', y))))$

5. Summary

The main goal of this chapter was to give a compositional account of weak / strong ambiguities that generalizes to mixed reading relative-clause donkey sentences like the one in (1) above. The main proposal is that the weak / strong donkey ambiguity is located at the level of the indefinite article, which is ambiguous (or underspecified) between a weak and a strong / maximal reading.

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[37] Intuitively, uniqueness needs to be relativized to u_3 in $\textbf{unique}_{u_3}\{u_4\}$ because, otherwise, we would require every u_4-individual to be the same, i.e. there would have to be just one deskmate over all.

The two crucial ingredients of the analysis are: (**i**) plural information states (modeled as sets of 'variable assignments', which can be represented as matrices with 'assignments' as rows) and (**ii**) a maximization operator use to specify the meaning of strong indefinite articles. The resulting system is dubbed Plural Compositional DRT (PCDRT). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

In PCDRT, sentences denote relations between an input and an output plural info state, i.e. sentences non-deterministically update a plural info state. Indefinites non-deterministically introduce both values and structure, i.e. they introduce structured sets of individuals, and pronouns are anaphoric to such structured sets. Quantification over individuals is defined in terms of matrices (i.e. plural info states) instead of single 'assignments' and the semantics of the non-quantificational part becomes rules for how to fill out a matrix.

PCDRT enables us to give a compositional account of a variety of phenomena, including mixed reading relative-clause donkey sentences, while keeping the dynamic meanings of generalized determiners, pronouns and indefinite articles very close to their static, Montagovian counterparts.

6. Comparison with Alternative Approaches

To my knowledge, the existence of mixed reading donkey sentences was observed for the first time by van der Does (1993) for relative-clause donkey sentences and by Dekker (1993) for conditional donkey sentences. Their examples are provided in (94) and (95) below.

94. Every farmer who has a horse and a whip in his barn uses it to lash him.
    (van der Does 1993: 18, (26))
95. If a man has a dime in his pocket, he throws it in the parking meter.
    (Dekker 1993: 183, (25)).

The example in (1) above (*Every*^{u_1} *person who buys a*^{u_2} *book on* *amazon.com* *and has a*^{u_3} *credit card uses it*$_{u_3}$ *to pay for it*$_{u_2}$) makes one additional point that is obscured by the examples in (94) and (95), namely that the weak reading of the indefinite *a credit card* in (1) is compatible with the set of credit cards being a non-singleton set – since I could use different credit cards to buy different (kinds of) books.

As already remarked in the previous chapters, weak / strong donkey ambiguities in general and mixed weak & strong relative-clause donkey sentences in particular pose problems for many influential dynamic theories of donkey sentences, including Heim (1982/1988), Kamp & Reyle (1993), Dekker (1993), Kanazawa (1994a) and Chierchia (1995). The main reason is that, in these dynamic theories, donkey indefinites do not have any quantificational force whatsoever, so all the truth-conditional effects associated with donkey anaphora have to be built into whatever element in the environment gives the quantificational force of the indefinite.

In the case of the mixed reading example in (1), this requires us to pack an entire logical form into the meaning of the generalized determiner *every*. As shown explicitly by the classical first-order translation of example (1), repeated in (96) below, the generalized determiner *every* needs to specify three things: (**i**) the fact that the indefinite *a book* is strong; (ii) the fact that the indefinite *a credit card* is weak and (**iii**) the fact that the strong indefinite *a book* can take scope over the weak indefinite *a credit card*, since I can use different cards to buy different (kinds of) books.

96. $\forall x(person(x) \wedge \exists y(book(y) \wedge buy\_on\_amazon(x, y)) \wedge \exists z(c.card(z) \wedge have(x, z))$
$\rightarrow \forall y'(book(y') \wedge buy\_on\_amazon(x, y')$
$\rightarrow \exists z'(c.card(z') \wedge have(x, z') \wedge use\_to\_pay(x, z', y'))))$

Thus, dynamic approaches of this kind are forced to give increasingly complex and stipulative meanings for selective generalized determiners. In contrast, the proposal I have pursued in this chapter is that indefinites should be endowed with a minimal quantificational force of their own: (**i**) just as in DPL, I let them contribute an existential quantification; (**ii**) what is new is that I also let them specifiy whether the existential quantification they introduce is maximal or not, i.e. whether they introduce in discourse *some* witness set or the *maximal* witness set that satisfies the nuclear scope update[38].

The pseudo-scopal relation between the strong indefinite *a book* and the weak indefinite *a credit card* in (1) above ("pseudo" because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite) arises as a consequence of the fact that PCDRT uses plural information states, which store and pass on information about both the sets of objects and the dependencies between these objects that are introduced and elaborated upon in discourse.

Before examining alternative approaches in more detail, I want to indicate three respects in which PCDRT differs from most previous dynamic approaches (irrespective of whether or how they analyze weak / strong ambiguities).

The first difference is conceptual: PCDRT explicitly embodies the idea that reference to *structure* is as important as reference to value and that the two should be treated in parallel (see the definition of dref introduction and its justification in section **3.2** above).

Capturing reference to structure as *discourse* reference to structure, i.e. by means of plural information states rather than by means of choice and / or Skolem functions (or dref's for such functions), is preferable for the following reason: such functions can in principle be used to capture donkey anaphora to structure, but they have to have variable arity depending on how many simultaneous donkey anaphoric connections there are, i.e. the arity of the functions is determined by the discourse context. It is therefore more desirable to encode this context dependency in the database that stores discourse information, i.e. the info state, and not in the representation of a lexical item (the donkey pronoun and / or the donkey indefinite); for a related argument, see also section **7.2** in chapter **7** below.

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[38] A witness set for a static quantifier **DET**(A) (where **DET** is a static determiner and A is a set of individuals) is any set of individuals B such that $B \subseteq A$ and **DET**(A)(B). See Barwise & Cooper 1981: 103 (page references to Portner & Partee 2002).

The second difference is empirical: the motivation for *plural* information states is provided by singular and intra-sentential donkey anaphora, in contrast to the previous literature which relies on plural and cross-sentential anaphora (see van den Berg 1994, 1996a, b, Krifka 1996b, and Nouwen 2003 among others).

Importantly, donkey anaphora to structure provides a much stronger argument for the idea that plural info states are *semantically* necessary. To see this, consider anaphora to value first: a pragmatic account is plausible for cases of cross-sentential anaphora (e.g. in *A man came in. He sat down*, the pronoun *he* can be taken to refer to whatever man is pragmatically brought to salience by the use of an indefinite in the first sentence), but less plausible for cases of intra-sentential donkey anaphora (no single donkey is brought to salience in *Every farmer who owns a donkey beats it*).

Similarly, a pragmatic account of anaphora to structure is plausible for cases of cross-sentential anaphora like *Every man saw a woman. They greeted them.* This discourse asserts that every man greeted the woman / women that he saw, i.e. the greeting structure is the same as the seeing structure – but the identity of structure might be a pragmatic addition to semantic values that are *unspecified* for structure (e.g. the second sentence *They greeted them* could be interpreted cumulatively in the sense of Scha 1981). However, a pragmatic approach is much less plausible for cases of intra-sentential donkey anaphora to structure instantiated by sentence (2) above.

Third, PCDRT takes the research program in Muskens (1996) of unifying different semantic frameworks, i.e. Montague semantics and dynamic semantics, one step further: PCDRT unifies in *classical type logic* the static, compositional analysis of generalized quantification in Montague semantics and van den Berg's Dynamic Plural Logic. The unification is not a trivial task, given certain peculiarities of Dynamic Plural Logic, e.g. the fact that its underlying logic is partial and the fact that discourse-level plurality (i.e. the use of plural information states) and domain-level plurality (i.e. non-atomic individuals) are conflated[39].

[39] For more on the distinction between discourse-level and domain-level plurality, see chapter **8** below and Brasoveanu (2006c).

One of the advantages of the resulting type-logical framework is that it can be extended in the usual way with additional sorts for eventualities, times and possible worlds, which enables PCDRT to account for *temporal* and *modal* anaphora and quantification in a way that is *parallel* to the account of individual-level anaphora and quantification. The modal extension is worked out in chapter **7** below.

The previous accounts of weak / strong donkey sentences fall (roughly) into three categories:

- accounts that locate the ambiguity at the level of the generalized determiner (e.g. the determiner *every* in the classic example *Every farmer who owns a donkey beats it*); most dynamic accounts fall into this category, including Rooth (1987), Van Eijck & de Vries (1992), Dekker (1993), Kanazawa (1994a, b), but also the D-/E-type approach in Heim (1990); these approaches will be discussed in section **6.1**;
- accounts that locate the ambiguity at the level of the donkey pronoun, e.g. the D-/E-type approaches in van der Does (1993) and Lappin & Francez (1994); these approaches will be discussed in section **6.2**;
- accounts that locate the ambiguity at the level of the indefinite article; this is the approach pursued in this chapter and in van den Berg (1994, 1996a); van den Berg's approach will be discussed in section **6.3**.

In addition, there is also the hybrid dynamic/E-type approach pursued in Chierchia (1995). This approach will be discussed in section **6.2**.

6.1. Weak / Strong Determiners

I can see two reasons for locating the weak / strong ambiguity in the donkey indefinites and not in the dynamic meaning of generalized determiners.

The first one – already presented above – has to do with the syntax/semantics aspect of the interpretation of donkey sentences, in particular, with the requirement of (strict) compositionality. If we attribute the weak / strong ambiguity to the determiner and we want to derive the intuitively correct truth-conditions for the mixed reading donkey sentence in (1), we basically need to pack an entire logical form into the meaning of the generalized determiner *every*, which needs to non-locally / non-compositionally determine both the readings associated with different donkey indefinites and their relative (pseudo-)scope.

The other reason for locating the weak / strong ambiguity in the indefinites is concerned with the semantics/pragmatics side of the interpretation of donkey sentences, namely the *variety of factors* that influence which reading is selected in any given instance of donkey anaphora and the *defeasible character* of the generalizations correlating these factors and the resulting readings.

Some of these factors are:

- *the logical properties of the determiners* – see Kanazawa (1994a, b);
- *world-knowledge* – see the 'dime' example in Pelletier & Schubert (1989) and, also, the examples and discussion in Geurts (2002);
- the *information* (focus-topic-background) *structure* of the sentence – see Kadmon (1987), Heim (1990);
- the kind of *predicates* that are used, i.e. total vs. partial predicates – see Krifka (1996a) and references therein;
- whether the donkey indefinite is referred back to by a donkey pronoun – see Bäuerle & Egli (1985)[40].

Given the variety of factors that influence which reading is selected in any given instance of donkey anaphora and also the defeasible character of the generalizations correlating these factors and the resulting readings, I think that the most conservative hypothesis is to locate the weak / strong ambiguity at the level of the donkey indefinites themselves, i.e. to make the donkey items ambiguous between a weak and a strong meaning[41], and let more general and defeasible pragmatic mechanisms decide which meaning is selected in any particular case.

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[40] Apud Heim (1990).

[41] Ambiguous between a weak and a strong reading or, alternatively, underspecified for weak / strong readings (like quantifier scope, for example, is underspecified) or vague (like adjectives).

One of the most theoretically appealing accounts of the weak / strong donkey ambiguity is due to Kanazawa (1994a, b), which locates the ambiguity in the meaning of the generalized determiners. I will therefore dedicate the remainder of this section to making two more points that seem to favor the PCDRT indefinite-based theory – or, at least, to give it sufficient initial plausibility.

First, Kanazawa's account is ultimately pragmatic, just like the account I am suggesting. In fact, except for the fact that he chooses to make the dynamic generalized determiner – and not the indefinite – underspecified, I think that all the observations below also apply to the PCDRT account.

> "The primary assumption I make is the following: […] The grammar rules in general underspecify the interpretation of a donkey sentence.
> Thus, I assume that, for any donkey sentence, the grammar only partially characterizes its meaning, with which a range of specific interpretations are compatible. So the truth value of donkey sentences in particular situations may be left undecided by the grammar. This may not be such an outrageous idea; it may explain the lack of robust intuitions about donkey sentences.
> For the sake of concreteness, I assume that the underspecified interpretation of a donkey sentence Det N' VP assigned to by the grammar can be represented using an indeterminate dynamic generalized determiner *Q* which is related to the static generalized determiner Q denoted by Det and which satisfies certain natural properties. […]
> Even if its interpretation is underspecified, a sentence may be assigned a definite truth-value in special circumstances. […] It is not unreasonable to suppose that people are capable of assessing the truth value of a donkey sentence without resolving the 'vagueness' of the meaning given by the grammar when there is no need to do so. […] underspecification causes no problems for people in assigning a truth value to a donkey sentence in situations where the uniqueness condition for the donkey pronoun is met."
> (Kanazawa 1994a: 151-152)

Note in particular the situations in which the "uniqueness condition" is met are precisely the situations in which the PCDRT weak and strong meanings for the indefinite article are conflated; for more discussion about uniqueness effects in donkey sentences, see section **6.2** of chapter **6** below.

Thus, both accounts of the weak / strong donkey ambiguity defer the task of disambiguation to pragmatics – which brings me to the second, empirical point. The hypothesis that the weak / strong ambiguity (or underspecification) should be located in the generalized determiner has more plausibility than the PCDRT hypothesis only if we observe that the logical properties of the determiners are, consistently, the main deciding disambiguation factor. This is clearly not true for the determiner *every*: its monotonicity-based bias for strong readings is easily trumped by world knowledge (as shown by the 'dime' example; see also the discussion in Kanazawa 1994a: 122-124 and Geurts 2002).

I will now point out that the monotonicity-based bias can be systematically overridden for most other determiners in a particular kind of construction that involves nuclear scope negation. This observation – together with the above list of five unrelated factors that influence the choice between weak and strong readings – provides support for the conservative hypothesis that the source of the weak / strong donkey ambiguity should be located in the donkey indefinites and not in some other element in their environment.

Donkey Readings and Nuclear Scope Negation[42]

I use "nuclear scope negation" as a cover term for negative items, e.g. sentential negation or negative verbs like *fail*, *forget* and *refuse*, that occur within the nuclear scope of a quantification and that semantically take scope over the other elements in the nuclear scope. To my knowledge, the only examples of nuclear scope negation discussed in the previous literature are the ones provided in (97), (98), (99) and (100) below[43].

97. A boy who had an^u apple in his rucksack didn't give it$_u$ to his sister.
    (van der Does 1993: 18, (27c))
98. No man who had a^u credit card failed to use it$_u$.
    (Kanazawa 1994a: 117, fn. 16)
99. Every person who had a^u dime in his pocket did not put it$_u$ into the meter.
    (Lappin & Francez 1994: 401, (22a))
100. Every person who had a^u dime in his pocket refused to put it$_u$ into the meter.
    (Lappin & Francez 1994: 401, (22a))

[42] I am grateful to Hans Kamp (p.c.) for pointing out to me that there seems to be a systematic correlation between sentential negation and donkey readings. Most of the empirical observations in this sub-section emerged during or as a result of our conversations.

[43] Geurts (2002) also mentions the examples due to van der Does (1993) and Kanazawa (1994a), but he believes that "such examples are hard to find" (Geurts (2002): 131).

The generalization that emerges based in these examples and which trumps the monotonicity-based bias observed in Kanazawa 1994a is that nuclear scope negation generally requires the strong reading for donkey sentences; see also Lappin & Francez (1994) for observations that point towards the same generalization (p. 408 in particular) and for a critique of Kanazawa (1994a) based on sentences (99) and (100) (pp. 410-411). Sentence (97) is interpreted as asserting that there is some boy such that, for *every* apple in his rucksack, he didn't give that apple to his sister. Sentence (98) is interpreted as asserting that no man is such that, for *every* credit card of his, he failed to use that card, i.e. no man failed to use every credit card of his – or, equivalently, every man used some credit card or other.

The examples in (97) and (98) form minimal pairs with sentences (101) and (102) below, where there is no nuclear scope negation and where the most salient donkey reading is the weak one (just as Kanazawa 1994a predicts they should).

101. A boy who had an^{u} apple in his rucksack gave it$_{u}$ to his sister.
102. No man who had a^{u} credit card used it$_{u}$ (to pay the bill).

We can observe a similar contrast for non-monotone intersective determiners of the form *exactly n*, also predicted by Kanazawa (1994a) to favor the weak reading (just as the intersective but monotone determiners *a* and *no* do). The most salient reading of (103) below is the strong donkey reading: exactly two men are such that, for *every* credit card they had, they failed to use that card. The most salient reading of (104) is the weak one: exactly two men used *some* credit card they had.

103. Exactly two men who had a^{u} credit card failed to use it$_{u}$ / didn't use it$_{u}$ / forgot to use it$_{u}$.
104. Exactly two men who had a^{u} credit card used it$_{u}$.

The same applies to the *only*-based donkey examples in (105) and (106) below.

105. Only two men who had a^{u} credit card failed to use it$_{u}$ / didn't use it$_{u}$ / forgot to use it$_{u}$.
106. Only two men who had a^{u} credit card used it$_{u}$.

As the examples (99) and (100) above show, even the classical weak reading example in (107) below becomes strong under the influence of nuclear scope negation: the example in (108) below is interpreted as asserting that every man who had a quarter was such that, for *every* quarter of his, he refused to put that quarter in the meter. The pairs of *at least n*-, *at most n*- and *most*-sentences in (109)-(110), (111)-(112) and (113)-(114) below instantiate the same kind of contrast.

107. Every man who had a^u quarter put it_u in the meter.
108. Every man who had a^u quarter refused to put it_u in the meter / forgot to put it_u in the meter.
109. At least two men who had a^u quarter put it_u in the meter.
110. At least two men who had a^u quarter refused to put it_u in the meter / forgot to put it_u in the meter.
111. At most two men who had a^u quarter put it_u in the meter.
112. At most two men who had a^u quarter refused to put it_u in the meter / forgot to put it_u in the meter.
113. Most men who had a^u nice suit wore it_u at the town meeting.
(based on Kanazawa 2001: 386, (17))
114. Most men who had a^u nice suit refused to wear it_u at the town meeting / forgot to wear it_u at the town meeting / didn't wear it_u at the town meeting.

In contrast, note that negation with scope over the entire donkey quantification does not have a similar 'strengthening' effect, as the examples in (115), (116) and (117) below show. Consider (116) for example: its strong reading is that not every man who had a credit card is such that, for *every* credit card he had, he used that card to pay the bill – an assertion that borders on triviality. Intuitively, sentence (116) asserts that not every man who had a credit card used *some* credit card of his to pay the bill – or, equivalently, that there is a man who had a credit card and who didn't use any of his cards to pay, i.e. the weak donkey reading.

115. Not every man who had a^u quarter put it_u in the meter.
116. Not every man who had a^u credit card used it_u to pay the bill.

117. Not every person who buys a^{u} book on amazon.com and who has $a^{u'}$ credit card uses $it_{u'}$ to pay for it_{u}.

However, just like the other generalizations about the distribution of weak vs. strong donkey readings, the correlation between nuclear scope negation and the strong donkey reading is not without exception. A top-level negation cancels the 'strengthening' effect of the nuclear scope negation, as the examples in (118) and (119) below show.

Incidentally, note that the weak donkey sentences in (118) and (119) and the ones in (115), (116) and (117) above show that ↑MON↓ determiners like *not every* and *not all* reliably tolerate weak readings, contra Kanazawa (1994a): 118 et seqq.

118. Not every man who had a^{u} credit card failed to use it_{u}.

119. Not every man who had a nice suit refused to wear it_{u} at the town meeting / forgot to wear it_{u} at the town meeting.

Sentences (118) and (119) indicate that, if there is any correlation between negation, the monotonicity properties of the generalized determiners and the choice between weak and strong donkey readings, this correlation cannot be locally and deterministically established by taking into account only some particular item in the context of the donkey indefinites, be it the generalized determiner or the nuclear scope negation – we need to take into account the whole quantification and, on top of that, factors of a different nature, e.g. world knowledge about how credit card owners normally behave (they don't pay with all their credit cards) or about how people normally wear their suits (not all of them at the same time, even if they are very nice).

I conclude with the example in (120) below, which provides one more exception to the correlation between nuclear scope negation and strong donkey readings. The most salient reading of (120) is that every man who placed a suitcase on the belt took back every suitcase after it was X-rayed, i.e. no man who placed a suitcase on the belt failed, for *some* such suitcase, to take it back, i.e. the weak donkey reading.

120. (At the airport "self check-in", where customers place their suitcase / suitcases on the belt to have them X-rayed:)

No man who placed a^{u} suitcase on the belt forgot to take it$_{u}$ back after it$_{u}$ was X-rayed / failed to take it$_{u}$ back after it$_{u}$ was X-rayed.

I leave the analysis of the above generalizations for future research – but I hope to have established that the volatile nature of the weak / strong donkey ambiguity makes the PCDRT account at least as plausible as the alternative dynamic strategy of locating the source of the ambiguity in the selective generalized determiners.

6.2. Weak / Strong Pronouns

D-/E-type accounts of donkey anaphora fall into two categories with respect to the problem posed by weak / strong ambiguities. If they address the problem (e.g. Neale 1990 and Elbourne 2005 do not), they either locate the weak / strong ambiguity in the meaning of the generalized determiner, e.g. Heim (1990), or in the meaning of the donkey pronoun, e.g. van der Does (1993) and Lappin & Francez (1994).

Given that the strategy in Heim (1990) is basically the same as the one pursued by the dynamic accounts discussed in the previous section, the resulting analysis faces the same kind of problems (*mutatis mutandis*).

In this section, I will focus on accounts that take the donkey pronoun to be the source of the weak / strong ambiguity; in particular, I will focus on the account in Lappin & Francez (1994), but the general argument also applies to van der Does (1993).

Lappin & Francez (1994) assume the ontology in Link (1983), which countenances both (atomic) individuals and individual sums thereof – or *i*-sums. Lappin & Francez (1994): 403 propose to analyze donkey pronouns as functions from individuals to *i*-sums, e.g., in the classical donkey example *Every farmer who owns a donkey beats it*, the pronoun *it* denotes a function f that, for every donkey-owning farmer x, returns some *i*-sum $f(x)$ of donkeys that x owns, i.e. the sum of some subset of the donkeys that x owns.

Strong donkey readings are obtained by placing a maximality constraint on the function f, which requires f to select, for each x in its domain, the supremum of its possible values, i.e., in the case at hand, the maximal *i*-sum of donkeys that x owns. Weak donkey readings are obtained by suspending the maximality constraint, i.e. f is a choice function from x to one of the i-sums of donkeys that x owns.

I will use DP-conjunction donkey sentences of the kind analyzed in section **5.6** of chapter **4** above to distinguish between the D-/E-type strategy of locating the weak / strong ambiguity in the meaning of the donkey pronoun and the PCDRT strategy of locating it in the meaning of the donkey indefinite.

DP-Conjunction Donkey Sentences with Mixed Readings

Consider the mixed weak & strong donkey sentences in (121) below, whose subjects is a conjunction of two DP's.

121. (Today's newspaper claims that, based on the most recent statistics:)
Every^{u_1} company who hired a^{u_2} Moldavian man, but no^{u_3} company who hired a^{u_2} Transylvanian man promoted him$_{u_2}$ within two weeks of hiring.

Intuitively, the sentence asserts that every company who hired a Moldavian promoted *every* Moldavian it hired within two weeks, while there is no company who hired some Transylvanian and promoted *some* Transylvanian it hired within two weeks – that is, the donkey anaphora to *a Moldavian man* is strong and the donkey anaphora to *a Transylvanian man* is weak.

Crucially, the very same pronon *it* is intuitively anaphoric to both indefinites. Example (121) poses a problem for approaches like Lappin & Francez (1994) and van der Does (1993), which locate the weak / strong ambiguity in the donkey pronouns, because there is only one pronoun in (121), but two distinct donkey readings.

Note that there is no immediately obvious was in which covert syntactic operations could 'reconstruct' two pronouns in the case of (121) – or in the case of the similar example in (122) below. Examples (121) and (122) do not seem to be instances of ellipsis or Right Node Raising, in which case we could have assumed that the pronoun is covertly duplicated at the level of LF. Also, covertly duplicating at LF the pronoun in (121) (or (122)) by rightward Across-the-Board (ATB) movement of the VP does not seem to be an independently motivated syntactic operation in English. And, even if rightward ATB movement of the VP is possible, one still needs to reconstruct the VP in both places to get two pronouns and, presumably, assign the reconstructed pronouns two different indices.

Sentence (122) below makes the same point as (121) – the only difference is that, in (122), we conjoin two DP's headed by the same generalized determiner[44,45]. The sentence in (122) can be felicitously uttered in the following context: there is this Sunday fair where, among other things, people come to sell their young puppies – and they do want to get rid of all of them before they are too old. Also, the fair entrance fee is one dollar. Now, the fair rules are strict: all the puppies need to be checked for fleas at the gate and, at the same time, the one dollar bills also need to be checked for authenticity because of the many faux-monnayeurs in the area. So:

122. Everyone^{u_1} who has a^{u_2} puppy (to sell) and everyone^{u_3} who has a^{u_2} dollar (to pay the fee) brings it$_{u_2}$ to the gate to be checked.

The most salient interpretation of sentence (122) is that every potential seller brings *all* her or his puppies to the gate to be checked, while every potential buyer needs to bring only *one* of her or his dollars, i.e. anaphora to *a*^{u_2} *puppy* is strong, while anaphora to *a*^{u_2} *dollar* is weak.

Thus, I assume that, in the case of both (121) and (122) above, what one sees is what one gets: two donkey indefinites, one donkey pronoun and two donkey readings. These mixed weak & strong donkey sentences pose problems for the approach in Lappin

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[44] Note also that the intonational tune in example (122) is the same as the one associated with declarative sentences like *Every student and every professor was invited to the party*, the LF of which is not derived by ellipsis and / or Right Node Raising.

[45] Variants of the mixed reading example in (122) are given in (i) and (ii) below; note that, in all cases, the context needs to be tweaked in a way that prevents the default parallel interpretation of the two conjuncts (i.e. both donkey indefinites are strong or both are weak).

The example in example (122) is the refinement of (ii), due to Sam Cumming, following Klaus von Heusinger's and Hans Kamp's suggestions (p.c.).

(i) (There aren't that many ambulant theater troupes anymore in Romania. This is because of the following custom:) At the end of a play, every person that liked the play and has a^{u} dime and every person that didn't like the play and has a^{u} rotten tomato throws it$_{u}$ at the actors.

(ii) (It's market day. So:) Every farmer who owns a^{u} donkey and every spectator who has a^{u} dollar – for entry – brings it$_{u}$ to the saleyard.

& Francez (1994) because either the donkey pronouns him_{u_2} in (121) and it_{u_2} in (122) are subject to the maximality constraint and therefore can deliver only strong donkey readings or the maximality constraint is suspended and the donkey pronouns can deliver only the weak reading.

These sentences pose an even more severe problem for the hybrid approach to weak / strong ambiguities proposed in Chierchia (1995), where the weak reading is derived within a dynamic framework and the strong reading is attributed to a D-/E-type reading of the donkey pronoun. Given that Chierchia (1995) agrees with the observation that examples like (121) and (122) above involve a single pronoun (he actually uses examples of the same form to argue for a semantic as opposed to a syntactic approach to donkey anaphora), his approach is faced with the problem of deriving, by means of a single pronoun, two different donkey readings which are furthermore claimed to involve two different kinds of semantic representations for the pronoun.

One more move seems to still be open possible for the D-/E-type approach in Lappin & Francez (1994); following a suggestion from Chierchia (1995): 116-117, the donkey pronouns him_{u_2} in (121) and it_{u_2} in (122) could be interpreted as denoting the union of two different functions, a maximal one that is contributed by the first DP in their respective sentences and a non-maximal, choice-based one that is contributed by the second DP. Note, however, that this strategy does not work in general because the union of two functions is not necessarily a function. In particular, suppose that, in (121), the very same company *x* hired both a Moldavian man and a Transylvanian man; the first function will return the Moldavian man as value for the argument *x*, while the second function will return the Transylvanian man, so the result of their union is not function and, therefore, not a suitable kind of meaning for a donkey pronoun.

Finally, suppose that we take the function union approach one step further and assume that, when we take the union of two functions *f* and *f'*, we require the resulting function to return, for any *x* that is in the domain of both *f* and *f'*, the sum of the individuals *f*(*x*) and *f'*(*x*). This "union & sum" strategy could yield the correct truth-conditions for example (122) where, for a person *x*, *x* brings to the gate to be checked every individual in the *i*-sum formed out of *x*'s puppies and one of *x*'s dollar bills – but it will not yield the intuitively correct truth-conditions for (121).

Moreover, the "union & sum" strategy (and D-/E-type approaches in general) predict that the sum should be available for subsequent *singular* cross-sentential anaphora – if the function that provides the meaning of the pronoun is salient enough the first time around, it should still be salient enough immediately afterwards. However, subsequent singular anaphora to puppy-dollar sums is unacceptable, as shown in (123) below[46].

123. **a.** Everyone who has a^u puppy and everyone who has a^u dollar brings it_u to the gate to be checked.
b. #They do so because the rules of the fair require that it_u (should) be checked.

PCDRT, on the other hand, can account for this kind of examples without any additional stipulations: their analysis is parallel to the CDRT+GQ analysis of the example *Every*^{u_1} *boy who has a*^{u_2} *dog and every*^{u_3} *girl who has a*^{u_2} *cat must feed it*$_{u_2}$ from Chierchia (1995) (see section **5.6** of chapter **4** above). Sentences (121) and (122) receive the readings in (124) and (125) below.

124. Every^{u_1} company who hired a$^{\mathbf{str:}u_2}$ Moldavian man, but no^{u_3} company who hired a$^{\mathbf{wk:}u_2}$ Transylvanian man promoted him$_{u_2}$ within two weeks of hiring.

125. Everyone^{u_1} who has a$^{\mathbf{str:}u_2}$ puppy and everyone^{u_3} who has a$^{\mathbf{wk:}u_2}$ dollar brings it$_{u_2}$ to the gate to be checked.

I will only analyze (124), since the analysis of (125) is parallel. The PCDRT translation is given in (126) below the derived truth-conditions, which are intuitively correct, are provided in (127).

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[46] Plural anaphora is, however, possible, as shown by (i) below. But D-/E-type approaches cannot offer any explanation for this asymmetry. I believe that PCDRT can and that the explanation would be similar to the account of the infelicitous telescoping cases in section **6.3** of chapter **6** below.

(i) **a.** Everyone who has a^u puppy and everyone who has a^u dollar brings it_u to the gate to be checked. **b.** They do so because the rules of the fair require that $they_u$ (should) be checked.

126. every^{u_1} company who hired a$^{\mathbf{str}:u_2}$ Moldavian man

$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{every}_{u_1}([company\{u_1\}];\ \mathbf{max}^{u_2}([mold\{u_2\}, hire\{u_1, u_2\}]),\ P(u_1))]$

no^{u_3} company who hired a$^{\mathbf{wk}:u_2}$ Transylvanian man

$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{no}_{u_3}([company\{u_3\}];\ [u_2 \mid trans\{u_2\}, hire\{u_3, u_2\}],\ P(u_3))]$

every^{u_1} company who hired a$^{\mathbf{str}:u_2}$ M.man, but no^{u_3} company who hired a$^{\mathbf{wk}:u_2}$ T.man

$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{every}_{u_1}([company\{u_1\}];\ \mathbf{max}^{u_2}([mold\{u_2\}, hire\{u_1, u_2\}]),\ P(u_1)),$

$\mathbf{no}_{u_3}([company\{u_3\}];\ [u_2 \mid trans\{u_2\}, hire\{u_3, u_2\}],\ P(u_3))]$

promoted him$_{u_2}$ within two weeks of hiring $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [promote\{v, u_2\}]$

every^{u_1} company who hired a$^{\mathbf{str}:u_2}$ Moldavian man, but no^{u_3} company who hired a$^{\mathbf{wk}:u_2}$ Transylvanian man promoted him$_{u_2}$ within two weeks of hiring

$\rightsquigarrow [\mathbf{every}_{u_1}([company\{u_1\}];\ \mathbf{max}^{u_2}([mold\{u_2\}, hire\{u_1, u_2\}]),\ [prom\{u_1, u_2\}]),$

$\mathbf{no}_{u_3}([company\{u_3\}];\ [u_2 \mid trans\{u_2\}, hire\{u_3, u_2\}],\ [prom\{u_3, u_2\}])]$

127. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e \forall y_e(company(x) \wedge mold(y) \wedge hire(x, y) \rightarrow promote(x, y)) \wedge$
$\forall x'_e \forall y'_e(company(x') \wedge trans(y') \wedge hire(x', y') \rightarrow \neg promote(x', y'))$

To conclude, note that the PCDRT account of mixed reading donkey sentences (including the DP-conjunction examples above) predicts that the same indefinite cannot be interpreted as strong with respect to one pronoun (or any other kind of anaphor, e.g. a definite) and weak with respect to another pronoun. This prediction seems to be borne out[47]. By the same token, the D-/E-type analysis in Lappin & Francez (1994) (the points also applies to the hybrid approach in Chierchia 1995), which locates the weak / strong ambiguity at the level of the pronoun (or anaphor, in the general case), predicts the exact opposite – and, it seems, incorrectly so. That is, according to the D-/E-type analysis, the same indefinite should be able to be interpreted as strong with respect to one pronoun and as weak with respect to another. I am not aware of any example of this form.

[47] I am indebted to Roger Schwarzschild (p.c.) for emphasizing this point.

Unifying Dynamic Semantics and Situation Semantics

In this sub-section, I want to suggest that PCDRT effectively unifies dynamic and situation-based D-/E-type approaches of the kind proposed in Heim (1990) (among others) in a way that remains faithful to many of their respective goals and underlying intuitions.

In particular, the type *s* in PCDRT can be taken to be the type of partial situations as they are used in Heim (1990) – with the added advantage that PCDRT does not have the problem of indistinguishable participants (a.k.a. Kamp's 'bishop' problem) and does not need to address the issues raised by the 'formal link' condition.

Moreover, two major differences between dynamic and D-/E-type approaches to anaphora mentioned in Heim (1990): 137 are effectively invalidated by PCDRT. These differences (see the contrasting items (ii)-(iii) and (ii')-(iii') in (Heim 1990: 137) concern:

- the treatment of anaphoric pronouns: they are "plain bound variables" in dynamic approaches, while D-/E-type approaches analyze them as "semantically equivalent to (possibly complex) definite descriptions" (Heim 1990: 137);
- the treatment of quantificational determiners: they are "capable of binding multiple variables" in dynamic approaches, while they "bind just one variable each" (Heim 1990: 137) in D-/E-type approaches.

In PCDRT, anaphoric pronouns are basically analyzed as individual-level dref's, i.e. as functions from entities of type *s* to individuals (type *e*). Depending on how we prefer to intuitively think about the entities of type *s*, i.e. as 'variable assignments' or 'partial situations', the anaphoric pronouns are bound variables, i.e. they are the equivalent of projection functions on variable assignments (type *s*), or definite descriptions characterizing a unique individual in a given partial situation (again, type *s*).

Similarly, quantificational structures contributed by determiners or the generic operator in conditionals are analyzed as having the general form in (59) above (see section **3.5** of the present chapter), i.e. $\mathbf{det}_u(D, D')$. Insofar as these quantificational structures operate over the DRS's *D* and *D'*, hence over relations between info states, they are capable of binding multiple variables, but insofar as they contribute a particular dref *u* that is crucial in relating the two updates *D* and *D'*, they bind one variable each.

Finally, it seems to me that, if situation-based D-/E-type approaches are to be extended to account for mixed weak & strong donkey sentences like (1) above, they will have to introduce mechanisms that involve quantification over sets of partial situations and, also, updates of such sets that will be very similar to the notions of plural info state, quantification and info state update in PCDRT. I leave a more thorough investigation and comparison between PCDRT and situation-based D-/E-type approaches for future research.

6.3. Weak / Strong Indefinites

I will conclude with a brief examination of the approach in van den Berg (1994, 1996a), which, just as PCDRT, locates the weak / strong donkey ambiguity in the meaning of the donkey indefinites.

The first thing we need to do is to introduce van den Berg's notion of dynamic maximization. Abstracting away from the fact that it is formulated in a three-valued logic, the definition in van den Berg (1994): 15, (45) is different from the PCDRT definition in only one respect: it is is a weaker version of the $\mathbf{max}^u$ operator insofar as it does not require the existence of a supremum – it simply requires an output state to non-deterministically store a (locally) maximal set[48]. A PCDRT definition that is as close as possible to the maximization operator in van den Berg (1994) is given in (128) below, where '⊂' stands for strict inclusion. This operator and the corresponding one in PCDRT stand in the relation shown in (129) below.

128. $\mathbf{max\text{-}wk}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];\ D)IJ \wedge \neg\exists K_{st}(([u];\ D)IK \wedge uJ \subset uK)$

129. $\mathbf{max}^u(D) \subseteq \mathbf{max\text{-}wk}^u(D)$

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[48] For example, assume that if we update a given input info state *I* with a DRS of the form [*u*]; *D*, we get three possible output states J_1, J_2 and J_3 such that $uJ_1=\{a\}$, $uJ_2=\{a, b\}$ and $uJ_3=\{a, c\}$. The PCDRT supremum-based form of maximization will simply discard the input info state *I* altogether because there is no supremum in the set $\{uJ_1, uJ_2, uJ_3\}$. The weak, maxima-based form of maximization will retain the input info state *I* and the corresponding output states J_2 and J_3, but not J_1.

Van den Berg (1994, 1996a) crucially needs the weaker form of maximization **max-wk**^{u} (as opposed to the PCDRT one) to be able to account for weak / strong ambiguities. The reason for this is that he takes indefinites to be generalized quantifiers and, in his framework, generalized quantifiers are defined in terms of maximization[49]. He, therefore, uses a maximization operator to give the meaning of both weak and strong donkey indefinites[50].

In the case of the weak indefinites, however, van den Berg needs to neutralize the maximization effect (since people usually do not put *all* their dimes in the meter), so he adds an additional *singular* condition (basically the same as the **unique**$\{u\}$ condition defined in (44) above), which requires the weak indefinite dref to store a singleton set relative to a plural info state. Obviously, this can work only in tandem with weak maximization: as we saw in section **3.4** above (see definition (51) in particular), strong maximization plus a singular condition **unique**$\{u\}$ requires model-level uniqueness and yields the Russellian analysis of definite description – and not the desired weak donkey indefinites. Van den Berg's meanings for weak and strong indefinites are provided in (130) below, rendered in a compositional PCDRT format for ease of comparison.

130. **Van den Berg's weak indefinites in PCDRT format:**

$$a^{\mathsf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max\text{-}wk}^u([\mathbf{unique}\{u\}];\ P(u);\ P'(u)),$$

where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$

and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$.

Van den Berg's strong indefinites in PCDRT format:

$$a^{\mathsf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max\text{-}wk}^u(P(u);\ P'(u))$$

Van den Berg's analysis can account for simple instances of weak / strong donkey ambiguities, but it does not generalize to the mixed weak & strong donkey sentences analyzed in this chapter – and repeated in (131) and (132) below for convenience. The

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[49] For a similar definition of generalized quantification in PCDRT – which, crucially, does not include indefinites – see chapter **6** below.

[50] Analyzed in terms of his "collective" and "distributive" existential quantification respectively: see van den Berg (1994): 18-19 and van den Berg (1996a): 163-164.

reason is that van den Berg's weak donkey indefinites always introduce singleton sets, while the sentences in (131) and (132) are compatible with situations in which the value of the weak indefinites (*a*$^{\mathbf{wk:}u_3}$ *credit card* and *an*$^{\mathbf{wk:}u_3}$ *Arabian horse* respectively) is different for different values of the strong indefinites (*a*$^{\mathbf{str:}u_2}$ *book* and *a*$^{\mathbf{str:}u_2}$ *woman* respectively), i.e. in situations in which the credit cards vary from book to book and the horses from woman to woman. In the case of (131), for example, Van den Berg's analysis incorrectly pairs all the u_2-books with the same u_3-credit card, as shown in (133) below.

131. Every^{u_1} person who buys a$^{\mathbf{str:}u_2}$ book on amazon.com and has a$^{\mathbf{wk:}u_3}$ credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

132. Every^{u_1} man who wants to impress a$^{\mathbf{str:}u_2}$ woman and who has an$^{\mathbf{wk:}u_3}$ Arabian horse teaches her$_{u_2}$ how to ride it$_{u_3}$.

133. $[person\{u_1\}]$; $\mathbf{max\text{-}wk}^{u_2}([book\{u_2\}, buy\_on\_amazon\{u_1, u_2\}])$;

$$\mathbf{max\text{-}wk}^{u_3}([\mathbf{unique}\{u_3\}, credit\_card\{u_3\}, have\{u_1, u_3\}])$$

Moreover, extracting the strong indefinite out of its VP-conjunct and scoping it over the weak one is not possible because the resulting syntactic structure violates the Coordinate Structure Constraint[51]. As far as the analysis of the weak / strong donkey ambiguity is concerned, the definition of maximization in van den Berg (1996a): 139, (3.1)[52] is the same as the definition in van den Berg (1994)[53], so the above observations apply to it too.

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[51] That the Coordinate Structure Constraint does indeed apply to this kind of examples is shown by the two sentences in (i) and (ii) below, where the *every*-quantifiers cannot scope out of their own conjuncts to bind pronouns.

(i) #Every person who buys every^{u} *Harry Potter* book on amazon.com and gives it$_{u}$ to a friend must be a *Harry Potter* addict.

(ii) #Every boy who wanted to impress every^{u} girl in his class and who planned to buy her$_{u}$ a fancy Christmas gift asked his best friend for advice.

[52] See also the alternative formulation in van den Berg (1996a): 141, (3.2) and Lemma (3.3) for the relation between the two.

[53] Although it is not relevant for the weak / strong ambiguity problem, it is interesting to compare the two definitions. The definition of maximization in van den Berg (1996a): 139, (3.1) is different from the definition in van den Berg (1994) in two respect. First, the way in which new dref's are introduced is

In closing, note that van den Berg's system could in principle provide an alternative analysis of mixed weak & strong donkey sentences if it is extended with a form of anaphoric / relativized uniqueness of the kind defined in (134) below. If the uniqueness condition contributed by the weak indefinite is anaphoric / relativized to the strong indefinite, the value of the weak indefinite will be able to vary with the value of the strong indefinite; we will, therefore, be able to adequately translate the quantifier restrictor of sentence (131), as shown in (135) below.

134. $\mathbf{unique}_{u'}\{u\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(u'i = u'i' \rightarrow ui = ui')$

135. $[person\{u_1\}];\ \mathbf{max\text{-}wk}^{u_2}([book\{u_2\}, buy\_on\_amazon\{u_1, u_2\}]);$

$\mathbf{max\text{-}wk}^{u_3}([\mathbf{unique}_{u_2}\{u_3\}, credit\_card\{u_3\}, have\{u_1, u_3\}])$

Such an analysis, however, is more complex than the PCDRT one: the meaning of the weak indefinites involves a maximization operator, just like the meaning of the strong indefinites, and, in addition, the weak indefinites involve a relativized uniqueness condition that effectively neutralizes their maximization operator. Moreover, the **max-**

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different from the one we have chosen in PCDRT: it is the relation $I\{u\}J$ defined in (16b) above. As argued in section **3.2**, introducing new dref's by means of $\{u\}$ makes incorrect (overly strong) predictions with respect to mixed reading donkey sentences, so this does not amend the incorrect predictions made by the notion of maximization in van den Berg (1994).

The second difference, however, provides us with an interesting notion of pseudo-selective maximization. As the definition in van den Berg (1996a): 133, (2.6) shows, he requires maximality not only with respect to output sets of individuals, but also with respect to the output sets of info states, which results in a notion of maximization that is intermediate between selective and unselective maximization. The definition of $\mathbf{max\text{-}unsel}^u$ in (i) below is the PCDRT correspondent of this notion – compare it with the PCDRT definitions of selective (strong) maximization in (ii) below and unselective (strong) maximization in (iii).

(i) $\mathbf{max\text{-}unsel}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ \wedge K \subseteq J)$

(ii) $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ)$

(iii) $\mathbf{max}(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K \subseteq J)$

However, the $\mathbf{max\text{-}unsel}^u$ operator does not add to the expressive power of the PCDRT system: as the identity in (iv) below shows, it can be defined in terms of unselective maximization, much as we were able to define the DRT/FCS/DPL version of pseudo-selective generalized quantification (repeated in (vi) below) in terms of unselective generalized quantification (repeated in (v)) (see section **2** of chapter **4**).

(iv) $\mathbf{max\text{-}unsel}^u(D) = \mathbf{max}([u]; D)$

(v) $\mathbf{det}(D, D') := \lambda i_s.\ \mathbf{DET}(Di, \mathbf{Dom}(D'))$,

where **DET** is the corresponding static determiner, $Di = \{j_s: Dij\}$ and $\mathbf{Dom}(D') := \{i_s: \exists j_s(Dij)\}$

(vi) $\mathbf{det}_u(D, D') := \mathbf{det}([u]; D, D')$

$\mathbf{wk}^u$ operator is more complex than the PCDRT $\mathbf{max}^u$ operator and its added complexity (i.e. the fact that it is maxima-based and not supremum-based) obscures the correspondence between dynamic maximization and static λ-abstraction. Therefore, I believe that the PCDRT account is theoretically preferable.

Moreover, empirically, it is not clear how to independently motivate the fact that the run-of-the-mill indefinite $a^{\mathbf{wk}:u_3}$ *credit card* in sentence (131) above contributes an anaphoric condition $\mathbf{unique}_{u_2}\{u_3\}$, since it is not anaphorically dependent in any obvious way on the strong indefine $a^{\mathbf{str}:u_2}$ *book*.

Appendix

A1. Plural CDRT (PCDRT): The Formal System

136. **PCDRT** (subscripts on terms represent their types)**.**

a. Atomic conditions – type $(st)t$:

$R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I\neq\emptyset \wedge \forall i_s\in I(R(u_1i, \ldots, u_ni))$,

for any non-logical constant R of type e^nt,

where e^nt is defined as follows: $e^0t := t$ and $e^{m+1}t := e(e^mt)$.

$u_1=u_2 := \lambda I_{st}.\ I\neq\emptyset \wedge \forall i_s\in I(u_1i=u_2i)$

All atomic conditions are c-ideals.

b. Atomic DRS's (DRS's containing exactly one atomic condition) – type $(st)((st)t)$

$[R\{u_1, \ldots, u_n\}] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge R\{u_1, \ldots, u_n\}J$

$[u_1=u_2] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge (u_1=u_2)J$

The domain **Dom**(D) and range **Ran**(D) of an atomic DRS D are c-ideals, where **Dom**(D) := $\{I_{st}: \exists J_{st}(DIJ)\}$ and **Ran**(D) := $\{J_{st}: \exists I_{st}(DIJ)\}$.

c. Condition-level connectives (negation, closure, disjunction, implication), i.e. non-atomic conditions:

$\sim D := \lambda I_{st}.\ I\neq\emptyset \wedge \forall H_{st}(H\neq\emptyset \wedge H\subseteq I \rightarrow \neg\exists K_{st}(DHK))$,

where D is a DRS (type $(st)((st)t)$),

i.e. $\sim D := \lambda I_{st}.\ I\neq\emptyset \wedge \forall H_{st}\neq\emptyset(H\subseteq I \rightarrow H\notin \mathbf{Dom}(D))$.

If **Dom**(D) is a c-ideal (hence **Dom**$(D) = \wp^+(\cup\mathbf{Dom}(D))$), $\sim D$ is the unique maximal c-ideal disjoint from **Dom**(D): $\sim D = \wp^+(\boldsymbol{D}_s^{\boldsymbol{M}} \backslash \cup\mathbf{Dom}(D))$.

$!D := \lambda I_{st}.\ \exists K_{st}(DIK)$,

i.e. $!D :=$ **Dom**(D).

If **Dom**(D) is a c-ideal, $\sim[\sim D] = !D$.

$D_1 \vee D_2 := \lambda I_{st}.\ \exists K_{st}(D_1IK \vee D_2IK)$,

i.e. $D_1 \vee D_2 := \mathbf{Dom}(D_1) \cup \mathbf{Dom}(D_2)$.

$D_1 \rightarrow D_2 := \lambda I_{st}.\ \forall H_{st}(D_1IH \rightarrow \exists K_{st}(D_2HK))$,

i.e. $D_1 \rightarrow D_2 := \lambda I_{st}.\ D_1I \subseteq \mathbf{Dom}(D_2)$, where $DI := \{J_{st}: DIJ\}$,

i.e. $D_1 \rightarrow D_2 := (\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}}) \backslash \mathbf{Dom}(D_1)) \cup \{I_{st} \in \mathbf{Dom}(D_1): D_1I \subseteq \mathbf{Dom}(D_2)\}$.

d. Tests (generalizing 'atomic' DRS's):

$[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge C_1J \wedge \ldots \wedge C_mJ$ [54],

where $C_1, \ldots, C_m$ are conditions (atomic or not) of type $(st)t$.

The domain **Dom**(D) and range **Ran**(D) of any test D is a c-ideal if all the conditions are c-ideals.

e. DRS-level connectives (dynamic conjunction):

$D_1; D_2 := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(D_1IH \wedge D_2HJ)$,

where D_1 and D_2 are DRSs (type $(st)((st)t)$)

f. Quantifiers (random assignment of value to a dref):

$[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

If a DRS D has the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, we have that:

i. $\mathbf{Ran}(D) = C_1 \cap \ldots \cap C_m = \wp^+((\cup C_1) \cap \ldots \cap (\cup C_m))$;

ii. $\mathbf{Dom}(D) = \wp^+(\{i_s: \exists j_s(i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m))\})$.

Since $i[u_1, \ldots, u_n]j$ is reflexive, $\mathbf{Ran}(D) \subseteq \mathbf{Dom}(D)$.

g. Selective m**aximization:**

$\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(I[u]H \wedge DHJ) \wedge \forall K_{st}(\exists H_{st}(I[u]H \wedge DHK) \rightarrow uK \subseteq uJ)$,

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[54] Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction as follows:
$[C_1, \ldots, C_m] := \lambda I_{st}J_{st}.\ ([C_1]; \ldots; [C_m])IJ$, where $[C] := \lambda I_{st}J_{st}.\ I=J \wedge CJ$.

where D is a DRS of type $(st)((st)t)$,

i.e. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ)$

The $\mathbf{max}^u$ operator does not preserve the c-ideal structure of the domain or range of the embedded DRS.

Multiply embedded $\mathbf{max}^u$ operators can be reduced as follows:

$\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D')$,

if: i. u is not reintroduced in D';

ii. $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

iii. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

h. Unselective maximization:

$\mathbf{max}(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ \ \&\ \forall K_{st}(DIK \rightarrow K \subseteq J)$

i. Selective Generalized Determiners (non-atomic conditions):

$\mathbf{det}_u(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(u[D_1I],\ u[(D_1; D_2)I])$,

where $u[DI] := \cup\{uJ\!: ([u \mid \mathbf{unique}\{u\}]; D)IJ\}$

and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(ui = ui')$

and **DET** is the corresponding static determiner.

The lexical entries for selective generalized determiners are:

$\mathbf{det}^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{det}_u(P'(u), P(u))]$, where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$

j. Unselective Generalized Determiners (non-atomic conditions):

$\mathbf{det}(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(\mathbf{max}[D_1I],\ \mathbf{max}[(D_1; [!D_2])I])$,

where $\mathbf{max}[DI] := \cup\{J_{st}\!: \mathbf{max}(D)IJ\}$

and **DET** is the corresponding static determiner.

The lexical entries for unselective generalized determiners are:

$\mathbf{det} \rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{det}(D', D)]$, where $\mathbf{t} := (st)((st)t)$.

k. Truth: A DRS D (type $(st)((st)t)$) is true with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$, i.e. iff $I \in \mathbf{Dom}(D)$ (or, equivalently, $I \in\ !D$).

We supplement the definition of basic PCDRT in (5) with the list of abbreviations in (137) below.

137. **a. Additional abbreviations – DRS-level quantifiers (multiple random assignment, existential quantification, maximal existential quantification)**:

$[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$

$\exists u(D) := [u]; D$

$\exists^{\mathbf{m}} u(D) := \mathbf{max}^u(D)$

b. Additional abbreviations – condition-level quantifiers (universal quantification):

$\forall u(D) := \sim([u]; \sim D)$,

i.e. $\forall u(D) := \sim\exists u(\sim D)$.

c. Additional abbreviations – DRS's (a.k.a. linearized 'boxes'):

$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])IJ$,

where $C_1, \ldots, C_m$ are conditions (atomic or not),

i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. I[u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J$.

d. Additional abbreviations – negation based condition-level connectives (N-closure, N-disjunction, N-implication):

N-Closure: $!D := \sim[\sim D]$

N-Disjunction: $D_1 \vee D_2 := \sim[\sim D_1, \sim D_2]$

If $\mathbf{Dom}(D_1)$ and $\mathbf{Dom}(D_2)$ are c-ideals, then $D_1 \vee D_2 = \wp^+(\cup(\mathbf{Dom}(D_1) \cup \mathbf{Dom}(D_2)))$. Therefore, if $\mathbf{Dom}(D_1)$ and $\mathbf{Dom}(D_2)$ are c-ideals, we have that $D_1 \vee D_2 \subseteq D_1 \vee D_2$.

N-Implication: $D_1 \Rightarrow D_2 := \sim(D_1; [\sim D_2])$

Note that $\forall u(D) = [u] \Rightarrow D$.

If $D_1 = [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ and $C_1, \ldots, C_m$, $\mathbf{Dom}(D_2)$ are c-ideals, then $D_1 \Rightarrow D_2 = \wp^+(\{i_s: \forall j_s(i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m) \rightarrow j \in (\cup \mathbf{Dom}(D_2))\})$. Therefore, if $D_1 = [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ and $C_1, \ldots, C_m$, $\mathbf{Dom}(D_2)$ are c-ideals, we have that $D_1 \Rightarrow D_2 \subseteq D_1 \rightarrow D_2$.

If, in addition, we can establish that $\forall i_s j_s(i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m) \rightarrow j \in (\cup \mathbf{Dom}(D_2))$, then $D_1 \rightarrow D_2 = D_1 \Rightarrow D_2 = \wp^+(\mathbf{D}_s^{\mathbf{M}})$.

The definitions of the dynamic universal and existential quantifiers in (137b-c) above preserve their DPL / CDRT+GQ partial duality if we quantify over DRS's whose domains are c-ideals.

138. $\sim\exists u(D) = \forall u(\sim D)$,

if $\mathbf{Dom}(D)$ is a c-ideal (hence $\mathbf{Dom}(D) = \mathbf{Dom}([\sim[\sim D]])$).

Just as in CDRT+GQ, the partial duality in (138) can be generalized by means of N-implication as shown in (139) below.

139. $\sim\exists u(D; D') = \forall u(D \Rightarrow [\sim D'])$,

if: **a.** $\mathbf{Dom}(D')$ is a c-ideal (hence $\mathbf{Dom}(D) = \mathbf{Dom}([\sim[\sim D]])$);

b. D preserves c-ideals under pre-images [55,56].

A2. Simplifying 'Max-under-Max' Representations

The general version of the theorem is stated in (140) below.

140. **Simplifying 'max-under-max' representations:**

$\mathbf{max}^{u}(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^{u}(D; [u']; D'); \mathbf{max}^{u'}(D')$,

if the following three conditions obtain:

a. u is not reintroduced in D';

b. $\forall I_{st}\forall X_{et}\ (\ \exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ)\ )$;

c. $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$[57].

---

[55] D preserves c-ideals under pre-images iff if $\mathfrak{I}'$ is a c-ideal, then $\mathfrak{I}=\{I_{st}: \exists J_{st}(DIJ \wedge J\in\mathfrak{I}')\}$ is a c-ideal.

[56] Proof: The reader can easily check that the following identities hold: $\forall u([D \Rightarrow [\sim D']) = \forall u([\sim(D; [\sim[\sim D']]) = \forall u([\sim(D; D')]) = \sim([u]; [\sim[\sim(D; D')]]) = \sim([u]; D; D') = \sim\exists u(D; D')$. □

[57] Proof:

$\mathbf{max}^{u}(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ) \wedge \forall K(\exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HK) \rightarrow uK\subseteq uJ)$

We have that $\forall I_{st}\forall X_{et}(\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ))$ (condition (140b)). Hence:

$\mathbf{max}^{u}(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ) \wedge \forall K(\exists H(([u]; D)IH \wedge ([u']; D')HK) \rightarrow uK\subseteq uJ)$

$= \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ) \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK\subseteq uJ)$.

We have that $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$ (condition (140c)). Hence:

Given that, by condition (140a), u is not reintroduced in D', the second condition (140b) can be further reduced to the condition in (141) below.

141. Given (140a), condition (140b) is equivalent to:

$$\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D')).$$

Moreover, based on the two facts in (142) below, we can further simplify condition (140c).

142. a. If D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$,

then $\forall I_{st} \forall J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$[58].

b. If $\forall I_{st} \forall J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$,

then $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$[59].

$\mathbf{max}^{u}(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge ([u']; D'; \mathbf{max}^{u'}(D'))HJ) \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ)$

$= \exists H(([u]; D; [u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ) \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ)$

$= \exists H(([u]; D; [u']; D')IH \wedge \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ) \wedge \mathbf{max}^{u'}(D')HJ)$

Since u is not reintroduced in D' (condition (140a)), we have that $uJ=uH$. Hence:

$\mathbf{max}^{u}(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D; [u']; D')IH \wedge \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uH) \wedge \mathbf{max}^{u'}(D')HJ)$

$= \exists H(\mathbf{max}^{u}(D; [u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ) = (\mathbf{max}^{u}(D; [u']; D'); \mathbf{max}^{u'}(D'))IJ$. □

[58] Proof: $([u']; D') := \lambda I_{st} J_{st}.\ I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J$. Therefore:

$([u']; D')IJ \wedge ([u']; D')JK$ iff $I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J \wedge J[u', u_1, \ldots, u_n]K \wedge C_1 K \wedge \ldots \wedge C_m K$

iff $I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J \wedge I[u', u_1, \ldots, u_n]K \wedge C_1 K \wedge \ldots \wedge C_m K$ iff $([u']; D')IJ \wedge ([u']; D')IK$. □

[59] Proof:

Claim1: If $\forall I_{st} J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$, then $[u']; D' = [u']; D'; [u']; D'$ (note that the premise ensures that the relation denoted by $[u']; D'$ is a KD45 kind of accessibility relation).

Proof of Claim1: $([u']; D'; [u']; D')IJ$ iff $\exists H(([u']; D')IH \wedge ([u']; D')HJ)$ iff (by the premise)

$\exists H(([u']; D')IH \wedge ([u']; D')IJ)$ iff $\exists H(([u']; D')IH) \wedge ([u']; D')IJ$ iff $([u']; D')IJ$. □

$([u']; D'; \mathbf{max}^{u'}(D'))IJ$ iff $\exists H(([u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ)$ iff

$\exists H(([u']; D')IH \wedge ([u']; D')HJ \wedge \forall K(([u']; D')HK) \rightarrow u'K \subseteq u'J))$ iff (by the premise)

$\exists H(([u']; D')IH \wedge ([u']; D')HJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff

$([u']; D'; [u']; D')IJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff (by **Claim1**)

$([u']; D')IJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff $\mathbf{max}^{u'}(D')IJ$. □

Thus, we obtain the corollary given in the main text of the chapter, repeated in (143) below.

143. **Simplifying 'max-under-max' representations (corollary):**

$$\mathbf{max}^{u}(D;\ \mathbf{max}^{u'}(D')) = \mathbf{max}^{u}(D;\ [u'];\ D');\ \mathbf{max}^{u'}(D'),$$

if the following three conditions obtain:

a. u is not reintroduced in D';

b. $\mathbf{Dom}([u'];\ D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.