# Chapter 2. Dynamic Predicate Logic with Generalized Quantification

## *1. Introduction*

The main goal of this and the following chapter is to situate the present research within the general enterprise of compositional dynamic semantics, in particular:

- to provide the core framework I will build on throughout the dissertation;
- to fix notation;
- to briefly recapitulate the basic empirical generalizations that motivate the dynamic approach to semantics and the basic kinds of semantic analyses that this approach makes possible.

Most of the results reported in these two chapters come from the previous literature and are meant to set the stage for the more complex formal systems presented in the following chapters. The main references are: Kamp (1981), Heim (1982/1988), Kamp & Reyle (1993) for the general dynamic framework, i.e. Discourse Representation Theory (DRT) / File Change Semantics (FCS); Groenendijk & Stokhof (1991) for the way I choose to introduce sentence-level / clause-level compositionality in the framework, i.e. their Dynamic Predicate Logic (DPL); and finally Muskens (1995b, 1996) for the way to go compositional at the sub-sentential level, i.e. his Compositional DRT (CDRT). In particular, the step of going compositional at the sub-sentential level:

> "[…] combine[s] Montague Semantics and Discourse Representation into a formalism that is not only notationally adequate, in the sense that the working linguist need remember only a few rules and notations, but is also mathematically rigorous and based on ordinary type logic. […] DRT's Discourse Representation Structures (DRS's or boxes henceforth) are already present in type logic in the sense that they can simply be viewed as abbreviations of certain first-order terms, provided that some first-order axioms are adopted. […] The presence of boxes in type logic permits us to fuse DRT and Montague Grammar in a rather evenhanded way: both theories will be recognizable in the result. […] With this unification of the theories standard techniques (such as type-shifting) that are used in Montague Grammar become available in DRT." (Muskens (1996): 144-145)

Section **2** introduces DPL (Groenendijk & Stokhof 1991). Sections **4** and **5** introduce the two most straightforward ways to extend DPL to account for generalized quantification in natural language. In particular, section **4** introduces – following a suggestion about adverbs of quantification in Groenendijk & Stokhof (1991) – what can be termed *unselective* generalized quantification – unselective in the sense of Lewis (1975), i.e. generalized quantification relating two sets of information states.

This notion of unselective generalized quantification reproduces in DPL the (somewhat implicit) conception of generalized quantification in Kamp (1981) and Heim (1982/1988).

Defining dynamic generalized quantifiers unselectively fails to account for the weak / strong donkey ambiguity and runs into the proportion problem. Based on Bäuerle & Egli (1985), Root (1986), Rooth (1987), van Eijck & de Vries (1992) and Chierchia (1992, 1995) (see also Heim 1990 and Kamp & Reyle 1993), section **5** extends DPL with selective generalized quantification, which can account for the weak / strong donkey ambiguity and avoids the proportion problem.

The differences between the material in this chapter and the sources mentioned above are for the most part presentational. There are only two novel things. The first is the DPL-style definition of unselective generalized quantification in section **4** that incorporates generalized quantifier *conservativity*.

The second one is the introduction – in section **6** – of the mixed weak & strong donkey sentences, i.e. relative-clause donkey sentences with two donkey indefinites that receive different readings – one strong, the other weak –, e.g. *Every person who buys a book on amazon.com* (strong) *and has a credit card* (weak) *uses it* (the credit card) *to pay for it* (the book); this kind of sentences cannot be accounted for in DRT / FCS / DPL even when they are extended with *selective* generalized quantification. Mixed weak & strong donkey sentences will provide one of the primary motivations for the subsequent revisions and generalizations of CDRT.

I will also introduce several new notational conventions (e.g. the λ-style notation for DPL-style generalized quantification in section **5**) and, on occasion, the departure from the original notation entails minor technical modifications.

But these are basically the only changes I make to the original dynamic systems – and they are made in the interest of clarity: given that the matters under discussion and the technical apparatus devised to handle them become difficult fairly fast, it seems counter-productive to increase the difficulty by repeatedly switching between various notations[1].

## *2. Dynamic Predicate Logic (DPL)*

There are three basic kinds of examples that initially motivated a dynamic approach to the semantics of natural language. First, discourses in which a singular pronoun is anaphoric to an indefinite in a previous sentence, as shown in discourse (1-2) below.

1. A$^{u}$ house-elf fell in love with a$^{u'}$ witch.
2. He$_{u}$ bought her$_{u'}$ an$^{u''}$ alligator purse.

Following the convention in Barwise (1987), antecedents are indexed with superscripts and dependent elements with subscripts. Sentence (2) is interpreted as asserting that the house-elf mentioned in (1), namely $u$, bought an alligator purse to the witch mentioned in (1), i.e. to $u'$. Thus, the pronouns in (2) are interpreted as referring back to the entities evoked in the previous discourse. Heim (1982/1988) argues in detail that such pronouns do refer back to discourse entities and *not* some other entities, e.g. actual individuals that the speaker 'has in mind' when using the indefinites in (1). The hypothesis that the pronominal anaphora in discourse (1-2) is an instance of discourse reference (and not some other kind of reference or covert pronoun binding) is supported by the donkey sentences in (3), (4) and (5), (6) below.

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[1] For example, the non-conservative definition of unselective generalized quantification in section **4** is not given as such in the literature. But the only new thing is the notation – the actual content of the definition is an immediate extension of the analysis of adverbs of quantification in Groenendijk & Stokhof (1991): 81-82 in terms of the 'generalized' implication connectives $\rightarrow_{Q}$.

3. Every farmer who owns a$^{u}$ donkey beats it$_{u}$.
4. Every house-elf who falls in love with a$^{u}$ witch buys her$_{u}$ an$^{u'}$ alligator purse.
5. If a$^{u}$ farmer owns a$^{u'}$ donkey, he$_{u}$ beats it$_{u'}$.
6. If a$^{u}$ house-elf falls in love with a$^{u'}$ witch, he$_{u}$ buys her$_{u'}$ an$^{u''}$ alligator purse.

Sentence (4), for example, cannot be said to make reference to any given witch *u'* that the speaker 'has in mind'; the intuitively most salient interpretation of (4) is that for any pair of individuals *u* and *u'* such that *u* is a house-elf and *u'* is a witch that said elf is in love with, *u* buys *u'* some alligator purse or other. Moreover, the indefinite *a*$^{u'}$ *witch* in (4) cannot bind the pronoun *her*$_{u'}$ because it is not in the required structural position for binding, namely c-command (or some other suitable notion of 'command', depending on the reader's favorite syntactic formalism), e.g. in *Every witch loves herself*, the quantifier *every witch* c-commands and binds the pronoun *herself*. A similar argument can be put forth in the case of donkey anaphora in conditionals, as illustrated by (5) and (6).

The felicity of the discourse reference patterns instantiated by examples (1) through (6) above does not seem to be sensitive to pragmatic factors (e.g. world knowledge, the speaker's communicative intentions etc.), which provides *prima facie* evidence that they should be analyzed in *semantic* terms. However, static formal semantics for natural language of the kind proposed in Montague (1974) cannot account for the cross-sentential scope of the indefinites in discourse (1-2) or for the co-variation without binding that obtains between the pronouns and the indefinites in (3) through (6) above. Hence the move to dynamic semantics.

I will not argue now for the dynamic approach to cross-sentential anaphora and donkey sentences as opposed to the family of D-/E-type approaches. I will compare these two kinds of approaches as I analyze increasingly complex discourses starting with chapter **5**. Anticipating, I will make two main points.

First, as soon as we start examining some of the phenomena that are central to the present investigation, namely donkey sentences with multiple instances of donkey anaphora that receive different readings (i.e. weak and / or strong), e.g. *Every person who buys a book on* *amazon.com* (strong) *and has a credit card* (weak) *uses it to pay for it* (the book)), the D-/E-type approaches that model pronouns as functions of arbitrary arity from individuals to individuals[2] become increasingly complex and counter-intuitive, as opposed to an analysis formulated in a dynamic system formulated in type logic and employing plural info states (i.e. sets of variable assignments).

Second, if we want to extend the other kind of D-/E-type approaches, i.e. the situation-based ones (which model pronouns as functions from (minimal) situations to individuals[3]), to account for such examples, we will very likely end up with a system that is identical in the relevant respects with the dynamic system I propose.

Finally, if we want to extend the account of donkey anaphora in the individual domain to modal anaphora and modal subordination in such a way that we capture the systematic parallels between *modal* and *individual-level* anaphora and quantification – see Geurts (1999), Frank (1996), Stone (1997, 1999), Bittner (2001) and Schlenker (2005) among others for detailed discussion of these parallels –, we can straightforwardly capture the modal phenomena and the cross-domain parallels in a type-logical dynamic semantics system by simply extending it with another basic type for possible worlds, as shown in chapter **7** below (and building on Muskens 1995b and Stone 1999).

It is much less clear how to execute a similar extension for the two kinds of D-/E-type approaches mentioned above (i.e. 'individual'-based and situation-based).

The particular version of dynamic semantics I build on is DPL (Groenendijk & Stokhof 1991) – and for three reasons:

- first, the syntax of the system is the familiar syntax of classical first-order logic (at least in the original notation; in my notation, it is a fairly close variant thereof); this enables us to focus on what is really new, namely the semantics;
- second, the semantics of DPL is minimally different from the standard Tarskian semantics for first-order logic: instead of interpreting a formula as a set of variable assignments (i.e. the set of variable assignments that satisfy the formula

[2] See Chierchia (1995), section **2.5** for a relatively recent example.

[3] See Heim (1990) for the paradigmatic example.

in the given model), we interpret it as a binary relation between assignments[4]; moreover, this minimal semantic modification encodes in a transparent way the core dynamic idea that meaning is not merely *truth-conditional content*, but *context change potential*;

- third, just as classical predicate logic can be straightforwardly generalized to static type logic, DPL can be easily generalized to a dynamic version of type logic, which is what Muskens' Compositional DRT is; and CDRT enables us to introduce compositionality at the sub-sentential/sub-clausal level in the tradition of Montague semantics.

Besides formally defining an intuitive and easily generalizable notion of dynamic semantic value, DPL is able to translate the donkey sentences in (3) through (6) above compositionally, with sentences / clauses as the building blocks (i.e., basically, as compositional as one can get in first-order logic).

For instance, sentences (3) and (5) above are translated as shown in (7) and (8) below and, when interpreted dynamically, the translations capture the intuitively correct truth-conditions.

7. $\forall x(farmer(x) \wedge \exists y(donkey(y) \wedge own(x, y)) \rightarrow beat(x, y))$
8. $\exists x(farmer(x) \wedge \exists y(donkey(y) \wedge own(x, y))) \rightarrow beat(x, y)$

Consider (7) first: as it is customary, *every* is translated as universal quantification plus implication and the indefinite as existential quantification plus conjunction; moreover, the *syntactic* scope of the existential quantification is 'local' (restricted to the antecedent of the implication), but it does *semantically* bind the occurrence of the

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[4] Alternatively, and in certain respects equivalently, we can think of the interpretation of a formula as a function taking as argument a sets of assignments and returning another set of assignments – this is the view underlying FCS, for example. However, in both cases the update is defined pointwise – and a relational view of update reflects this more directly. There are other differences between FCS and DPL (e.g. using partial and total assignments respectively and disallowing vs. allowing reassignment) – see the dynamic cube in Krahmer (1998): 59 for an overview. In particular, the fact that DPL (and CDRT) allows reassignment will be an essential ingredient in accounting for the interaction between anaphora and generalized conjunction (see section **5** of Chapter 1 below). The "destructive reassignment" or "downdate problem" associated with reassignment can be solved using stacks / 'referent systems': see Nouwen (2003) for a recent discussion and Bittner (2006) for a set of 'stack' axioms for dynamic type logic.

variable $y$ in the consequent. Similarly, in (8) the conditional is translated as implication and the indefinites are translated as existentials plus conjunction, again with syntactically 'local' but semantically 'non-local' scope.

As these observations indicate, DPL has two crucial properties that enable it to provide compositional translations for donkey sentences: DPL makes the equivalences in (9) and (10) below valid, so that indefinites can semantically bind outside their syntactic scope and indefinitely to the right, which, in combination with the definition of dynamic implication, allows them to scope out of the antecedent and universally bind in the consequent of the implication.

9. $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$[5]

10. $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$

## 2.1. Definitions and Abbreviations

DPL is a well-known system, so I will provide the definition of the interpretation function without any additional pre-theoretical motivation. The official syntax of DPL (i.e. the one in Groenendijk & Stokhof (1991)) is that of classical first-order logic with identity. However, in view of subsequent developments, I introduce certain modifications: the most salient one is that the symbol for conjunction is ';' (the symbol generally used for dynamic sequencing) and not the usual '$\wedge$'. Moreover, existential and universal quantifications are not officially present in the language; I only define the interpretation of the random assignment to a variable $x$, symbolized as $[x]$ – and the existential and universal quantifiers are defined as abbreviations in terms of $[x]$.

I do not provide the 'official' definition of a well-formed formula (wff) of DPL – it is easily recoverable on the basis of the definition of the interpretation function $\|\cdot\|$ in (11) below. As already indicated, the semantics of DPL interprets formulas as relations between variable assignments, which, for our narrow empirical purposes (i.e. elementary

[5] The symbol '$\Leftrightarrow$' should be interpreted as requiring the identity of the semantic value of two formulas.

aspects of discourse reference to individuals), model the more general dynamic notion of information state in a satisfactory way.

11. **Dynamic Predicate Logic (DPL).** The definition of the DPL interpretation function $\|\phi\|_{DPL}^{\boldsymbol{M}}$ relative to a standard first-order model $\boldsymbol{M}$=<$\boldsymbol{D}^{\boldsymbol{M}}$, $\boldsymbol{I}^{\boldsymbol{M}}$>, where $\boldsymbol{D}$ is the domain of entities and $\boldsymbol{I}$ is the interpretation function which assigns to each $n$-place relation '$R$' a subset of $\boldsymbol{D}^n$. For readability, I drop the subscript and superscript on $\|\cdot\|_{DPL}^{\boldsymbol{M}}$, $\boldsymbol{D}^{\boldsymbol{M}}$ and $\boldsymbol{I}^{\boldsymbol{M}}$. 'T' and 'F' stand for the two truth values.

    For any pair of $\boldsymbol{M}$-variable assignments $<g, h>$:

    **a. Atomic formulas ('lexical' relations and identity):**

    $\|R(x_1, \ldots, x_n)\|<g, h> = \text{T}$

    iff $g$=$h$ and $<g(x_1), \ldots, g(x_n)> \in \boldsymbol{I}(R)$

    $\|x_1=x_2\|<g, h> = \text{T}$

    iff $g$=$h$ and $g(x_1)=g(x_2)$

    **b. Connectives (dynamic conjunction and dynamic negation):**

    $\|\phi; \psi\|<g, h> = \text{T}$

    iff there is a $k$ s.t. $\|\phi\|<g, k> = \text{T}$ and $\|\psi\|<k, h> = \text{T}$

    $\|{\sim}\phi\|<g, h> = \text{T}$

    iff $g$=$h$ and there is no $k$ s.t. $\|\phi\|<g, k> = \text{T}$,

    i.e. $\|{\sim}\phi\|<g, h> = \text{T}$ iff $g$=$h$ and $g \notin \mathbf{Dom}(\|\phi\|)$,

    where $\mathbf{Dom}(\|\phi\|) := \{g$: there is an $h$ s.t. $\|\phi\|<g, h> = \text{T}\}$

    **c. Quantifiers (random assignment of value to variables):**

    $\|[x]\|<g, h> = \text{T}$

    iff for any variable $\upsilon$, if $\upsilon \neq x$ then $g(\upsilon)=h(\upsilon)$

    **d. Truth:** A formula $\phi$ is true with respect to an input assignment $g$ iff there is an output assignment $h$ s.t. $\|\phi\|<g, h> = \text{T}$, i.e. $g \in \mathbf{Dom}(\|\phi\|)$.

Given that variable assignments are functions from variables to entities, if two variable assignments assign identical values to all the variables, they are identical. Hence, based on definition (11c), the formula [ ] defines the 'diagonal' of the product $\boldsymbol{G} \times \boldsymbol{G}$, where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments, as shown in (12).

12. $\|[\ ]\| = \{<g, g>: g \in \boldsymbol{G}\}$,

where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments.

We define the other sentential connectives and the quantifiers as in (13) below.

13. **a. Abbreviations – Connectives (anaphoric closure, disjunction and implication):**

$!\phi := {\sim}{\sim}\phi$ [6],

i.e. $\|!\phi\| = \{<g, h>: g=h \text{ and } g \in \mathbf{Dom}(\|\phi\|)\}$[7]

$\phi \vee \psi := {\sim}({\sim}\phi; {\sim}\psi)$,

i.e. $\|\phi \vee \psi\| = \{<g, h>: g=h \text{ and } g \in \mathbf{Dom}(\|\phi\|) \cup \mathbf{Dom}(\|\psi\|)\}$

$\phi \rightarrow \psi := {\sim}(\phi; {\sim}\psi)$,

i.e. $\|\phi \rightarrow \psi\| = \{<g, h>: g=h$ and for any $k$ s.t. $\|\phi\|<g, k> = \text{T}$,

there is an $l$ s.t. $\|\psi\|<k, l> = \text{T}\}$[8],

i.e. $\|\phi \rightarrow \psi\| = \{<g, h>: g=h \text{ and } (\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)\}$,

where $(\phi)^g := \{h: \|\phi\|<g, h> = \text{T}\}$

**b. Abbreviations – quantifiers (existential, universal, multiple random assignment):**

$\exists x(\phi) := [x]; \phi$

$\forall x(\phi) := {\sim}([x]; {\sim}\phi)$,

i.e. $[x] \rightarrow \phi$ or, equivalently, ${\sim}\exists x({\sim}\phi)$,

i.e. $\|\forall x(\phi)\| = \{<g, h>: g=h$ and

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[6] I use the symbol '!' for closure, as in van den Berg (1996b) and unlike Groenendijk & Stokhof (1991), who use '◊'.

[7] The connective '!' is labeled 'anaphoric closure' because, when applied to a formula $\phi$, it closes off the possibility of subsequent reference to any dref introduced in $\phi$. This is because the input and the output assignments in the denotation of $!\phi$ are identical. The operator '!' is important because $\phi$ and $!\phi$ have the same truth-conditions – see the definition of truth in (11a), i.e. '!' can be said to factor out the truth-conditions of a dynamic formula.

[8] This is shown by the following equivalences:: $\|\phi \rightarrow \psi\|<g, h> = \text{T}$ iff $\|{\sim}(\phi; {\sim}\psi)\|<g, h> = \text{T}$ iff $g=h$ and there is no $k$ s.t. $\|\phi; {\sim}\psi\|<g, k> = \text{T}$ iff $g=h$ and there is no $k$ and no $l$ s.t. $\|\phi\|<g, l> = \text{T}$ and $\|{\sim}\psi\|<l, k> = \text{T}$ iff $g=h$ and there is no $k$ and no $l$ s.t. $\|\phi\|<g, l> = \text{T}$ and $l=k$ and $l \notin \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and there is no $k$ s.t. $\|\phi\|<g, k> = \text{T}$ and $k \notin \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and for any $k$ s.t. $\|\phi\|<g, k> = \text{T}$, we have that $k \in \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and for any $k$ s.t. $\|\phi\|<g, k> = \text{T}$, there is an $l$ s.t. $\|\psi\|<k, l> = \text{T}$. Summarizing: $\|\phi \rightarrow \psi\|<g, h> = \text{T}$ iff $g=h$ and for any $k$ s.t. $\|\phi\|<g, k> = \text{T}$, there is an $l$ s.t. $\|\psi\|<k, l> = \text{T}$.

$$\text{for any } k \text{ s.t. } g[x]k \text{, there is an } l \text{ s.t. } \|\phi\|\langle k, l\rangle = \mathrm{T}\}$$ [9],

$$\text{i.e. } \|\forall x(\phi)\| = \{\langle g, h\rangle: g{=}h \text{ and } ([x])^g \subseteq \mathbf{Dom}(\|\phi\|)\}$$

$$[x_1, \ldots, x_n] := [x_1]; \ldots; [x_n]$$

Given the definitions of dynamic negation '~' and closure '!', the equivalence in (14) below holds; (14) is very useful in proving that many equivalences of interest hold in DPL (e.g. the one in (15) below). Two formulas are equivalent, symbolized as '⇔', iff they denote the same set of variable assignments.

14. $\sim(\phi; \psi) \Leftrightarrow \sim(\phi; !\psi)$[10]

The equivalence in (15) below exhibits the limited extent to which the existential and universal quantifiers are duals[11]; this will prove useful, for example, when we try to determine the DPL translation of the English determiner *no*.

15. $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$[12]

The practice of setting up abbreviations as opposed to directly defining various connectives and quantifiers might seem cumbersome, but it is useful in at least three ways. First, by setting up explicit abbreviations, we see exactly which component of the basic dynamic system does the work, e.g. we see that the universal 'effect' of universal

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[9] This is shown by the following equivalences: $\|\forall x(\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $\|\sim([x]; \sim\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and there is no $k$ s.t. $\|[x]; \sim\phi\|\langle g, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no $k$ and no $l$ s.t. $\|[x]\|\langle g, l\rangle = \mathrm{T}$ and $\|\sim\phi\|\langle l, k\rangle = \mathrm{T}$ iff $g{=}h$ and there is no $k$ and no $l$ s.t. $g[x]l$ and $l{=}k$ and $l \notin \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and there is no $k$ s.t. $g[x]k$ and $k \notin \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and for any $k$ s.t. $g[x]k$, we have that $k \in \mathbf{Dom}(\|\phi\|)$ iff $g{=}h$ and for any $k$ s.t. $g[x]k$, there is an $l$ s.t. $\|\phi\|\langle k, l\rangle = \mathrm{T}$. Summarizing: $\|\forall x(\phi)\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and for any $k$ s.t. $g[x]k$, there is an $l$ s.t. $\|\phi\|\langle k, l\rangle = \mathrm{T}$.

[10] The equivalence holds because the following equalities hold (I use two abbreviations: $(\phi)^g := \{h: \|\phi\|\langle g, h\rangle = \mathrm{T}\}$ and $\mathbf{Dom}(\|\phi\|) := \{g: \text{there is an } h \text{ s.t. } \|\phi\|\langle g, h\rangle = \mathrm{T}\}$):

$\|\sim(\phi; \psi)\| = \{\langle g, h\rangle: g{=}h \text{ and } g \notin \mathbf{Dom}(\|\phi; \psi\|)\} = \{\langle g, h\rangle: g{=}h$ and it is not the case that there is a $k$ s.t. $\|\phi; \psi\|\langle g, k\rangle = \mathrm{T}\} = \{\langle g, h\rangle: g{=}h$ and it is not the case that there is an $l$ and a $k$ s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $\|\psi\|\langle l, k\rangle = \mathrm{T}\} = \{\langle g, h\rangle: g{=}h$ and there is no $l$ s.t. $\|\phi\|\langle g, l\rangle = \mathrm{T}$ and $l \in \mathbf{Dom}(\|\psi\|)\} = \{\langle g, h\rangle: g{=}h \text{ and } (\phi)^g \cap \mathbf{Dom}(\|\psi\|) = \varnothing\} = \{\langle g, h\rangle: g{=}h \text{ and } (\phi)^g \cap \mathbf{Dom}(\|!\psi\|) = \varnothing\} = \{\langle g, h\rangle: g{=}h \text{ and } g \notin \mathbf{Dom}(\|\phi; !\psi\|)\} = \|\sim(\phi; !\psi)\|$.

[11] The other 'half' of the duality, i.e. $\exists x(\sim\phi) \Leftrightarrow \sim\forall x(\phi)$, clearly doesn't hold: using the terminology defined in (16), $\sim\forall x(\phi)$ is a test, while $\exists x(\sim\phi)$ isn't.

[12] $\sim\exists x(\phi) \Leftrightarrow \sim([x]; \phi) \Leftrightarrow$ (given (14)) $\sim([x]; !\phi) \Leftrightarrow \sim([x]; \sim\sim\phi) \Leftrightarrow \forall x(\sim\phi)$.

quantification $\forall x(\phi)$, just as the universal unselective binding 'effect' of implication $\phi \rightarrow \psi$, is in fact due to dynamic negation[13].

Second, distinguishing basic definitions and derived abbreviations will prove useful when we start generalizing the system in various ways. The official definition is the logical 'core' that undergoes modifications when we define extensions of DPL; the system of abbreviations, however, remains more or less constant across extensions. In this way, we are able to exhibit in a transparent way the commonalities between the various systems we consider and also between the analyses of natural language discourses and within these different systems.

Third, the abbreviations indicate explicitly the relation between the 'core' dynamic system and related systems (e.g. DRT). From this perspective, it is useful to add to the core layer of definitions in (11) above and the layer of abbreviations in (13) (which 'recovers' first-order logic) yet another and final layer of abbreviations that 'recovers' DRT (Kamp 1981, Kamp & Reyle 1993).

## 2.2. Discourse Representation Structures (DRS's) in DPL

To this end, I define the semantic notion of *test* and the corresponding syntactic notion of *condition* in (16) and (17) below (see Groenendijk & Stokhof (1991): 57-58, Definitions 11 and 12). The relation between them is stated in (18) (see Groenendijk & Stokhof (1991): 58, Fact 6).

16. A wff $\phi$ is a *test* iff $\|\phi\| \subseteq \{\langle g, g\rangle : g \in \boldsymbol{G}\}$, where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments,

    i.e., in our terms, a wff $\phi$ is a *test* iff $\|\phi\| \subseteq \|[\,]\|$ [14].
17. The set of *conditions* is the smallest set of wff's containing atomic formulas, [ ], negative formulas (i.e. formulas whose main connective is dynamic negation '~'[15]) and closed under dynamic conjunction.

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[13] See the observations in van den Berg (1996b): 6, Section **2.3**.

[14] Note that $\phi \Leftrightarrow\, !\phi$ iff $\phi$ is a test; see Groenendijk & Stokhof (1991): 62.

18. $\phi$ is a test iff $\phi$ is a condition or a contradiction ($\phi$ is a *contradiction* iff $\|\phi\| = \emptyset$)

We indicate that a formula is a condition by placing square brackets around it.

19. **Conditions:**

$[\phi]$ is defined iff $\phi$ is a *condition*; when defined, $[\phi] := \phi$

$[\phi_1, \ldots, \phi_m] := [\phi_1]; \ldots; [\phi_m]$

We can now define a Discourse Representation Structure (DRS) or linearized 'box' as follows:

20. **Discourse Representation Structures (DRS's), a.k.a. linearized 'boxes':**

$[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := [x_1, \ldots, x_n]; [\phi_1, \ldots, \phi_m]$,

equivalently: $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := \exists x_1 \ldots \exists x_n([\phi_1, \ldots, \phi_m])$.

That is, $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m]$ is defined iff $\phi_1, \ldots, \phi_m$ are conditions and, if defined:

$$\|[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m]\| := \{<g, h>: g[x_1, \ldots, x_n]h \text{ and } \|\phi_1\|<h, h> = \mathrm{T} \text{ and } \ldots \|\phi_m\|<h, h> = \mathrm{T}\}$$

## *3. Anaphora in DPL*

The benefit of setting up this system of abbreviations becomes clear as soon as we begin translating natural language discourses into DPL.

### 3.1. Cross-sentential Anaphora

Consider again discourse (1-2) above, repeated in (21-22) below.

21. A$^x$ house-elf fell in love with a$^y$ witch.
22. He$_x$ bought her$_y$ an$^z$ alligator purse.

[15] Note that, given our abbreviations in (13) above, the set of negative formulas includes closed formulas (i.e. formulas of the form '!$\phi$'), disjunctions, implications and universally quantified formulas.

The representation of (21-22) in the unabbreviated system is provided in (23) below; the 'first-order'-style abbreviation is provided in (24) and the DRT-style abbreviation in (25).

23. $[x]$; $house\_elf(x)$; $[y]$; $witch(y)$; $fall\_in\_love(x, y)$;
    $[z]$; $alligator\_purse(z)$; $buy(x, y, z)$
24. $\exists x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)))$;
    $\exists z(alligator\_purse(z); buy(x, y, z))$
25. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$;
    $[z \mid alligator\_purse(z), buy(x, y, z)]$

## 3.2. Relative-clause Donkey Sentences

Consider now the relative-clause donkey sentence in (26) below (repeated from (4) above). The 'first-order'-style translation in terms of universal quantification and implication is provided in (27) and the DRT-style translation in (28). One way to see that the two translations are equivalent is to notice that both of them are equivalent to the formula in (29).

26. Every$^x$ house-elf who falls in love with a$^y$ witch buys her$_y$ an$^z$ alligator purse.
27. $\forall x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y))$
    $\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$
28. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$
    $\rightarrow [z \mid alligator\_purse(z), buy(x, y, z)]$
29. $[x]$; $house\_elf(x)$; $[y]$; $witch(y)$; $fall\_in\_love(x, y)$
    $\rightarrow [z]$; $alligator\_purse(z)$; $buy(x, y, z)$

Moreover, the three translations in (27), (28) and (29) are all equivalent (in DPL) to the formula in (30) below, which is the formula that assigns sentence (26) the intuitively correct truth-conditions when interpreted as in classical first-order logic.

30. $\forall x \forall y(house\_elf(x); witch(y); fall\_in\_love(x, y)$
    $\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$

As already noted, the formulas in (27) through (30) are equivalent because DPL validates the equivalence in (10) above, i.e. $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$[16].

## 3.3. Conditional Donkey Sentences

Finally, the conditional donkey sentence in (31) below (repeated from (6)) is truth-conditionally equivalent to the relative clause donkey sentence in (26), as shown by the fact that they receive the same DRT-style translation, which is provided in (32) below. The 'first-order'-style compositional translation – equivalent to the DRT-style translation and all the other formulas listed above – is given in (33).

31. If $a^x$ house-elf falls in love with $a^y$ witch, $he_x$ buys $her_y$ $an^z$ alligator purse.
32. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$

    $\rightarrow [z \mid alligator\_purse(z), buy(x, y, z)]$
33. $\exists x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)))$

    $\rightarrow \exists z(alligator\_purse(z); buy(x, y, z))$

I conclude this section with the DPL analysis of two negative donkey sentences.

34. $No^x$ house-elf who falls in love with $a^y$ witch buys $her_y$ $an^z$ alligator purse.
35. If $a^x$ house-elf falls in love with $a^y$ witch, $he_x$ never buys $her_y$ $an^z$ alligator purse.

If we follow the canons of classical first-order logic in translating sentence (34), we have a choice between a combination of negation and existential quantification and a combination of negation and universal quantification. But the limited duality exhibited by existential and universal quantification in DPL (see (15) above) is of help here. To see this, note first that the duality can be generalized to the equivalence in (36) below.

36. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$ [17,18]

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[16] $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$ iff $([x]; \phi) \rightarrow \psi \Leftrightarrow \sim([x]; \sim(\phi \rightarrow \psi))$ iff $\sim(([x]; \phi); \sim\psi) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; !(\phi; \sim\psi))$. The last equivalence holds because it is an instance of the more general equivalence $\sim(\phi; \psi) \Leftrightarrow \sim(\phi; !\psi)$ (see (14) above).

[17] The equivalence holds because: $\sim\exists x(\phi; \psi) \Leftrightarrow$ (by (15)) $\forall x(\sim(\phi; \psi)) \Leftrightarrow$ (by (14)) $\forall x(\sim(\phi; !\psi)) \Leftrightarrow \forall x(\sim(\phi; \sim\sim\psi)) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$.

Now, given that the equivalence in (36) holds, we can translate sentence (34) either way, as shown in (37) and (38). Moreover, both translations are equivalent to the formula in (39), which explicitly shows that we quantify universally over all pairs of house-elves and witches standing in the 'fall in love' relation.

37. $\sim\exists x(house\_elf(x);\ \exists y(witch(y);\ fall\_in\_love(x, y));$

$\exists z(alligator\_purse(z);\ buy(x, y, z)))$

38. $\forall x(house\_elf(x);\ \exists y(witch(y);\ fall\_in\_love(x, y))$

$\rightarrow \sim\exists z(alligator\_purse(z);\ buy(x, y, z)))$

39. $\forall x\forall y(house\_elf(x);\ witch(y);\ fall\_in\_love(x, y)$

$\rightarrow \sim\exists z(alligator\_purse(z);\ buy(x, y, z)))$

Consider now sentence (35). There is a compositional DPL translation for it, which becomes apparent as soon as we consider the intuitively equivalent English sentence in (40) below. Both sentence (35) and sentence (40) are compositionally translated as in (41).

40. If a$^x$ house-elf falls in love with a$^y$ witch, he$_x$ doesn't buy her$_y$ an$^z$ alligator purse.

41. $\exists x(house\_elf(x);\ \exists y(witch(y);\ fall\_in\_love(x, y)))$

$\rightarrow \sim\exists z(alligator\_purse(z);\ buy(x, y, z))$

It is easily seen that the DPL translations capture the fact that the English sentences in (34), (35) and (40) are intuitively equivalent.

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[18] The equivalence $\sim\exists x(\phi;\ \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$ in (36) is a generalization of the equivalence $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$ in (15) expressing the partial duality of the two quantifiers because we can obtain (15) from (36) by inserting [ ] in the place of $\phi$ in (36). In particular, the two equivalences in (i) and (ii) below hold:

(i) $[\ ];\ \phi \Leftrightarrow \phi$, hence $\sim\exists x([\ ];\ \phi) \Leftrightarrow \sim\exists x(\phi)$
(ii) $[\ ] \rightarrow \sim\phi \Leftrightarrow \sim([\ ];\ \sim\sim\phi) \Leftrightarrow \sim\sim\sim\phi \Leftrightarrow \sim\phi$, hence $\forall x([\ ] \rightarrow \sim\phi) \Leftrightarrow \forall x(\sim\phi)$

Moreover, we have (by (36)) that $\sim\exists x([\ ];\ \phi) \Leftrightarrow \forall x([\ ] \rightarrow \sim\phi)$; it follows that $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$, i.e. (15), holds.

## 4. Extending DPL with Unselective Generalized Quantification

As the translations of the *every*- and *if*-examples in (26) and (31) above indicate, there is a systematic correspondence in DPL between the generalized quantifier *every* and the unselective implication connective[19]. The same point is established by the equivalence of the DPL translations of the *no*- and *never*-examples in (34) and (35). The correspondence between *every* and implication is concisely captured by the equivalence in (42) (which is none other than the equivalence we mentioned at the beginning of the previous section – see (10) above).

42. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi$ [20]

Moreover, as indicated in (13a) above, when interpreted relative to an input assignment $g$, the implication connective $\phi \rightarrow \psi$ boils down to an inclusion relation between two sets of assignments: $(\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)$, where $(\phi)^g$:={$h$: $\|\phi\|$<$g$, $h$> = T}[21] and $\mathbf{Dom}(\|\psi\|)$:={$h$: there is a $k$ s.t. $\|\phi\|$<$h$, $k$> = T}. The inclusion relation between the two sets is precisely the relation expressed by the static generalized quantifier **EVERY** when applied to the two sets in question, i.e. $\mathbf{EVERY}((\phi)^g, \mathbf{Dom}(\|\psi\|))$. We can therefore give an alternative definition of implication using the static quantifier **EVERY**, as shown in (43) below.

43. $\|\phi \rightarrow \psi\|$ = {<$g$, $h$>: $g$=$h$ and $\mathbf{EVERY}((\phi)^g, \mathbf{Dom}(\|\psi\|))$},
where **EVERY** is the usual static generalized quantifier.

Putting together (42) and (43), we obtain a definition of the natural language quantifier *every* as a binary operator over two DPL formulas:

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[19] Implication is unselective basically because it is a sentential connective.

[20] $\|\forall x(\phi \rightarrow \psi)\|$<$g$, $h$> = T iff $\|[x] \rightarrow (\phi \rightarrow \psi)\|$<$g$, $h$> = T iff $g$=$h$ and for any $k$ s.t. $\|[x]\|$<$g$, $k$> = T, there is an $l$ s.t. $\|\phi \rightarrow \psi\|$<$k$, $l$> = T iff $g$=$h$ and for any $k$ s.t. $g[x]k$, there is an $l$ s.t. $k$=$l$ and for any $k'$ s.t. $\|\phi\|$<$k$, $k'$> = T, there is an $l'$ s.t. $\|\psi\|$<$k'$, $l'$> = T iff $g$=$h$ and for any $k$ and $k'$ s.t. $g[x]k$ and $\|\phi\|$<$k$, $k'$> = T, there is an $l$ s.t. $\|\psi\|$<$k'$, $l$> = T iff $g$=$h$ and for any $k$ s.t. $\|[x]; \phi\|$<$g$, $k$> = T, there is an $l$ s.t. $\|\psi\|$<$k$, $l$> = T iff $\|([x]; \phi) \rightarrow \psi\|$<$g$, $h$> = T.

[21] That is, $(\phi)^g$ is the image of the singleton set {$g$} under the relation $\|\phi\|$.

44. $\|\mathbf{every}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{EVERY}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$

It is easily checked that the equivalence in (42) can be extended as shown in (45) below.

45. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi \Leftrightarrow \mathbf{every}_x(\phi, \psi)$

This equivalence shows that the operator $\mathbf{every}_x(\phi, \psi)$ can be successfully used to translate donkey sentences with *every* and assign them the intuitively correct truth-conditions. The 'in love house-elf' example and its DPL translation are repeated in (46) and (47) below. The equivalent translation based on the binary **every** operator is provided in (48).

46. Every$^x$ house-elf who falls in love with a$^y$ witch buys her$_y$ an$^z$ alligator purse.

47. $\forall x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y))$
$\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$

48. $\mathbf{every}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$
$\exists z(alligator\_purse(z); buy(x, y, z)))$

We can define in a similar way a binary operator over DPL formulas $\mathbf{no}_x(\phi, \psi)$.

49. $\|\mathbf{no}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{NO}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$,
i.e. $\|\mathbf{no}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } ([x]; \phi)^g \cap \mathbf{Dom}(\|\psi\|)=\varnothing\}$

It is easily checked that the equivalence in (36) above extends as shown in (50).

50. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi) \Leftrightarrow \mathbf{no}_x(\phi, \psi)$

Consequently, we can translate sentence (34), repeated as (51), as shown in (52) below.

51. No$^x$ house-elf who falls in love with a$^y$ witch buys her$_y$ an$^z$ alligator purse.

52. $\mathbf{no}_x(house\_elf(x); \exists \mathrm{y}(witch(y); fall\_in\_love(x, y)),$
$\exists \mathrm{z}(alligator\_purse(z); buy(x, y, z)))$

## 4.1. Dynamic Unselective Generalized Quantification

The definitions of **every** and **no** in (44) and (49) and the way in which these operators are used to translate the English sentences in (48) and (51) suggest a way to add generalized quantification to DPL so that we can analyze donkey sentences like (53) and (54) below.

53. Most$^x$ house-elves who fall in love with a$^y$ witch buy her$_y$ an$^z$ alligator purse.
54. Few$^x$ house-elves who fall in love with a$^y$ witch buy her$_y$ an$^z$ alligator purse.

Let's first define the family of unselective binary operators **det** [22].

55. $\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

where **DET** is the corresponding static determiner [23].

The fact that the **det** sentential operators are *unselective* is semantically reflected in the fact that they express generalized quantification between two sets of *info states* (a.k.a. variable assignments), namely $(\phi)^g$ and $\mathbf{Dom}(\|\psi\|)$. And this will bring their downfall: it is their unselectivity (i.e. generalized quantification over info states) that leads them straight into the proportion problem and makes them incapable of accounting for the ambiguity between weak and strong donkey readings. But before we come to that, we need a couple more definitions.

First, note that a formula of the form $\mathbf{det}(\phi, \psi)$ is a test. So, we should also extend our syntactic notion of *condition* defined for DPL in (17) above. The revised definition is:

56. The set of *conditions* is the smallest set of wff's containing atomic formulas, formulas whose main connective is dynamic negation '~' or a **det** operator and closed under dynamic conjunction.

The revised definition in (56) enables us to construct DRS's of the form [… | …, $\mathbf{det}(\phi, \psi)$, …].

[22] Again, note that they are unselective because they are essentially *sentential* operators.

[23] Given that $\mathbf{Dom}(\|\psi\|)=\mathbf{Dom}(\|!\psi\|)$, it is a direct consequence of this definition that $\mathbf{det}(\phi, \psi) \Leftrightarrow \mathbf{det}(\phi, !\psi)$.

The natural language generalized determiners are defined in terms of the unselective **det** operators, as shown in (57) below.

57. $\mathbf{det}_x(\phi, \psi) := \mathbf{det}([x]; \phi, \psi)$

The determiners $\mathbf{every}_x(\phi, \psi)$ and $\mathbf{no}_x(\phi, \psi)$, i.e. the **every** and **no** instances of the general definition in (57), are none other than the determiners directly defined in (44) and (49) above. The generalized determiners defined in this way are still unselective, despite the presence of the variable $x$: the variable $x$ in $\mathbf{det}_x$ is only meant to indicate the presence of the additional update $[x]$, but the basic operator is still the unselective **det**. That is, we still determine the denotation of $\mathbf{det}_x(\phi, \psi)$ by checking whether the static determiner **DET** applies to two sets of info states – and not to two sets of individuals.

Let us make explicit the connections with the previous literature before turning to some examples. First, the definition of $\mathbf{det}(\phi, \psi)$ in (55) above is just the definition of quantificational adverbs in Groenendijk & Stokhof (1991): 81-82, which follows Lewis (1975) in taking adverbs to quantify over *cases* – i.e. information states (in dynamic terms). For example, *never* is translated in Groenendijk & Stokhof (1991): 82 as the binary implication connective $\rightarrow_{no}$ and the definition of $\phi \rightarrow_{no} \psi$ is exactly the definition of $\mathbf{no}(\phi, \psi)$.

The analysis can be extended in the obvious way to other adverbs of quantification, e.g. *always* can be interpreted as $\mathbf{every}(\phi, \psi)$ (just like bare conditionals), *often* and *usually* as $\mathbf{most}(\phi, \psi)$ and *rarely* as $\mathbf{few}(\phi, \psi)$ – where the corresponding static determiners **MOST** and **FEW** are interpreted as more than half and less than half respectively.

Second, the definition of $\mathbf{det}_x(\phi, \psi)$ is actually equivalent to the (implicit) definition of generalized quantification in Kamp (1981) and Heim (1982/1988).

A welcome consequence of defining $\mathbf{det}_x$ in terms of **det** (as in (57) above) is that the systematic natural language correspondence between adverbs of quantification and generalized quantifiers, e.g. the correspondence between *no* and *never* in examples (34) and (35) above, is explicitly captured.

## 4.2. Limitations of Unselectivity: Proportions

Another seemingly welcome consequence is that we can now provide an analysis of donkey sentences with *most* and *few* that can capture the anaphoric connections between the indefinites in the restrictor and the pronouns in the nuclear scope, as shown below in (59) and (62) ('predicate logic'-style) and (60) and (63) (DRT-style).

58. Most$^x$ house-elves who fall in love with a$^y$ witch buy her$_y$ an$^z$ alligator purse.
59. $\mathbf{most}_x(house\_elf(x);\ \exists y(witch(y);\ fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z);\ buy(x, y, z)))$
60. $\mathbf{most}_x([y \mid house\_elf(x),\ witch(y),\ fall\_in\_love(x, y)],$
    $[z \mid alligator\_purse(z),\ buy(x, y, z)])$
61. Few$^x$ house-elves who fall in love with a$^y$ witch buy her$_y$ an$^z$ alligator purse.
62. $\mathbf{few}_x(house\_elf(x);\ \exists y(witch(y);\ fall\_in\_love(x, y)),$
    $\exists z(alligator\_purse(z);\ buy(x, y, z)))$
63. $\mathbf{few}_x([y \mid house\_elf(x),\ witch(y),\ fall\_in\_love(x, y)],$
    $[z \mid alligator\_purse(z),\ buy(x, y, z)])$

The unselective analysis is successful in capturing the donkey anaphoric connections, but it is not successful in capturing the intuitively correct truth-conditions. As shown in Partee (1984)[24], Rooth (1987), Kadmon (1987) and Heim (1990), the analysis has a proportion problem[25].

Consider sentence (1) above and its DRT-style representation in (60). It is easy to see that the representation does not capture the intuitively correct truth-conditions if we examine the equivalent formula in (64) below.

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[24] "[…] when we have to deal with quantification with a complicated and possibly uncertain underlying ontology, we need to specify a 'sort' (for the quantifier to 'live on' in the sense of Barwise & Cooper 1981) separately from whatever further restrictions we want to add (perhaps in terms of 'cases') about which instances of the sort we are quantifying over. In terms of Kamp's framework this means that we have to worry not only about what belongs in the antecedent box but also how to distinguish a substructure within it that plays the role of sortal (the head noun in the NP case)." (Partee 1984: 278).

[25] The 'proportion problem' terminology is due to Kadmon (1987): 312.

64. **most**([$x$, $y$ | *house_elf*($x$), *witch*($y$), *fall_in_love*($x$, $y$)],

[$z$ | *alligator_purse*($z$), *buy*($x$, $y$, $z$)])

The representation in (64) makes clear that we are quantifying over most pairs $<x, y>$ where $x$ is a house-elf that fell in love with a witch $y$. For most such pairs $<x, y>$, the requirement in the nuclear scope, i.e. $x$ bought $y$ some alligator purse $z$, should be satisfied.

However, following Partee (1984), Rooth (1987), Kadmon (1987) and Heim (1990), we can produce a scenario in which the English sentence in (1) is intuitively false while the formula in (64) is true: imagine that there are ten house-elves that fell in love with some witch or other; one of them, call him Dobby, is a Don Juan of sorts, he fell in love with more than one thousand witches[26] and he bought them all alligator purses; the other nine house-elves are less exceptional: they each fell in love with only one witch and they bought them new brooms, not alligator purses.

Sentence (1) is intuitively false in this scenario, while formula (64) is true: all the Dobby-based pairs that satisfy the restrictor also satisfy the nuclear scope – and these pairs are more than half, i.e. *most*, of the pairs under consideration.

### 4.3. Limitations of Unselectivity: Weak / Strong Ambiguities

In addition, the unselective analysis of generalized quantifiers fails to account for the fact that the same donkey sentence can exhibit two different readings, a *strong* one and a *weak* one. Consider again the classical sentence in (65) below.

65. Every$^x$ farmer who owns a$^y$ donkey beats it$_y$.

The most salient reading of this sentence is that every farmer behaves violently towards *each and every* one of his donkeys, i.e. the so-called *strong* reading. The **every**$_x$ operator correctly captures this reading, as shown in (66) below; the equivalent formulas

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[26] To be more precise, one thousand and three witches only in Spain.

in (67) and (68) are provided because they display the 'strength' of the reading in a clearer way.

66. $\mathbf{every}_x([y \mid farmer(x), donkey(y), own(x, y)],\ [beat(x, y)])$
67. $\mathbf{every}([x, y \mid farmer(x), donkey(y), own(x, y)],\ [beat(x, y)])$
68. $\forall x \forall y(farmer(x); donkey(y); own(x, y) \rightarrow beat(x, y))$

However, sentence (65) can receive another, *weak* reading, wherein every farmer beats *some* donkey that he owns, but not necessarily each and every one of them[27]. Chierchia (1995): 64 provides a context in which the most salient reading is the weak one: imagine that the farmers under discussion are all part of an anger management program and they are encouraged by the psychotherapist in charge to channel their aggressiveness towards their donkeys (should they own any) rather than towards each other. The farmers scrupulously follow the psychotherapist's advice – in which case we can assert (65) even if the donkey-owning farmers beat only some of their donkeys.

Furthermore, there are donkey sentences for which the *weak* reading is the most salient one:

69. Every person who has a dime will put it in the meter.
    (Pelletier & Schubert 1989)
70. Yesterday, every person who had a credit card paid his bill with it.
    (R. Cooper, apud Chierchia 1995: 63, (3a))

Thus, both readings seem to be *semantically* available[28] and the unselective analysis of dynamic generalized quantifiers does not allow for both of them.

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[27] Partee (1984) seems to be (one of) the first to notice weak donkey readings: the example in (i) below is from Partee (1984): 280, fn. 12.

(i) If you have a credit card, you should use it here instead of cash.

[28] See for example the discussion in Chierchia (1995): 62-65, in particular the argument that the strong reading is not a conversational implicature triggered in certain contexts.

The weak/strong ambiguity also provides an argument against the unselective analysis of conditionals and adverbs of quantification, as shown, for example, by (71) below.

71. If $a^x$ farmer owns $a^y$ donkey, $he_x$ (always/usually/often/rarely/never) beats $it_y$.

For a detailed discussion of such conditionals, see (among others) Chierchia (1995): 66-69. I will only mention the generalization reached in Kadmon (1987) and summarized in Heim (1990): 153: "Kadmon's generalization is that a multi-case conditional with two indefinites in the antecedent generally allows three interpretations: one where the QAdverb quantifies over pairs, one where it quantifies over instances of the first indefinite and one where it quantifies over instances of the second".

It should be mentioned, however, that a partial solution to the problem posed by the existence of the weak donkey readings is available in classical DRT / FCS (Kamp 1981 and Heim 1982/1988) and DPL: Groenendijk & Stokhof (1991): 89 point out that we can define an alternative implication connective, as shown in (72) below.

72. $\phi \mapsto \psi := {\sim}\phi \vee (\phi;\, \psi)$,

$$\text{i.e. } \|\phi \mapsto \psi\| = \{<g, h>: g{=}h \text{ and } g{\notin}\,\mathbf{Dom}(\|\phi\|) \text{ or } (\phi)^g{\cap}\mathbf{Dom}(\|\psi\|){\neq}\emptyset\}$$

$$\text{i.e. } \|\phi \mapsto \psi\| = \{<g, h>: g{=}h \text{ and } g{\notin}\,\mathbf{Dom}(\|\phi\|) \text{ or } (\phi;\, !\psi)^g{\neq}\emptyset\}^{29}.$$

The weak reading of sentence (73) (repeated from above) is presumably analyzed as shown in (74), which is 'unpacked' in the equivalent (75). The strong reading is given in (76) and (77) for ease of comparison.

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[29] Note that an alternative definition could simply be: $\phi \mapsto \psi := !(\phi;\, !\psi)$, i.e. $\|\phi \mapsto \psi\| = \{<g, h>: g{=}h$ and $(\phi;\, !\psi)^g{\neq}\emptyset\}$. The difference between this definition and the one in (72) is that this one removes the first disjunct $g{\notin}\,\mathbf{Dom}(\|\phi\|)$ (hence, it is more restrictive). Arguably, this is a justified move, since a conditional or a universal quantification have an 'existential' presupposition: there is a presupposition that the antecedent of the conditional, respectively the restrictor of the universal quantification, are *satisfiable* with respect to the current input info state $g$, i.e. that $g{\in}\,\mathbf{Dom}(\|\phi\|)$.

However, given the DPL definition of the universal quantifier $\forall$, this definition would yield the incorrect truth conditions for weak reading of sentence (73) if (73) were represented as in (74): the reading would in fact be a lot 'stronger' than intended – all individuals are required to be farmers and to have some donkey or other that they beat.

73. Every$^x$ farmer who owns a$^y$ donkey beats it$_y$.

74. **weak reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \mapsto beat(x, y))$

75. **weak reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \mapsto [beat(x, y)])$

76. **strong reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \rightarrow beat(x, y))$

77. **strong reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \rightarrow [beat(x, y)])$

However, this analysis of weak implication faces (at least) three problems. First, as we can see from the 'unpacked' formula in (75), we still need the 'strong' implication connective $\rightarrow$ in addition to the 'weak' one $\mapsto$ to capture the correct truth-conditions for the weak reading of sentence (73), i.e. the weak reading is obtained via a combination of 'strong' and 'weak' implication.

Consequently, this solution fails to extend to weak readings of conditionals: as argued by Kadmon, the conditional in (78) below can receive a weak reading that is equivalent to the weak reading of the *every* donkey sentence in (73) above. However, this reading is not captured by the formula in (79), precisely because the equivalence $\exists x(\phi) \mapsto \psi \Leftrightarrow \forall x(\phi \mapsto \psi)$ fails for 'weak' implication – and we *do* want it to fail with respect to the indefinite *a*$^y$ *donkey*, but *not* with respect to the indefinite *a*$^x$ *farmer*.

78. If a$^x$ farmer owns a$^y$ donkey, he$_x$ beats it$_y$.

79. $\exists x(farmer(x); \exists y(donkey(y); own(x, y))) \mapsto beat(x, y)$

Second, the 'weak' implication solution does not generalize to other determiners (consider for example *most*). Third, it does not account for the proportion problem.

In sum, upon closer examination, a donkey sentence turns out to be ambiguous between a weak and a strong reading. The strong reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with an *every* DP. The weak reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with a *some* DP.

Extending DPL with an *unselective* form of generalized quantification fails to account for the weak / strong donkey ambiguity and for the proportion problem – hence the need to further extend DPL with a *selective* form of dynamic generalized quantification.

### 4.4. Conservativity and Unselective Quantification

A final observation before turning to this task: defining dynamic **det**'s in terms of static **DET**'s (as we did in (55) and (57) above) provides us with a version of *unselective dynamic conservativity* that underlies the definition of selective generalized quantification introduced in the next section. Consider again the definition in (55) above: $\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$. Assuming that the static determiner **DET** is conservative, we have that $\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))$ holds iff $\mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|))$ holds.

The latter formula encodes an intuitively appealing meaning for unselective dynamic generalized quantification[30]: a dynamic generalized determiner relates two sets of info states, the first of which is the set of output states compatible with the restrictor, i.e. $(\phi)^g$, while the second one is the set of output states compatible with the restrictor that can be further updated by the nuclear scope, i.e. $(\phi)^g \cap \mathbf{Dom}(\psi)$.

To reformulate this intuition in a more formal way, note that the formula $\mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|))$, which has conservativity built-in, is equivalent to $\mathbf{DET}((\phi)^g, (\phi; !\psi)^g))$. Thus, assuming that all static generalized determiners **DET** are conservative, we can restate the definition in (55) above as follows:

80. **Built-in unselective dynamic conservativity:**

$$\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{DET}((\phi)^g, (\phi; !\psi)^g))\}$$

[30] This has been previously noted with respect to the dynamic definition of *selective* generalized quantification – see for example Chiechia (1992, 1995) and Kamp & Reyle (1993) among others.

Now, putting together the definition of $\mathbf{det}_x(\phi, \psi)$ in (57), i.e. $\mathbf{det}_x(\phi, \psi) := \mathbf{det}([x]; \phi, \psi)$, and the 'conservative' definition in (80), we obtain the following definition of generalized quantification:

81. **Generalized quantification with built-in dynamic conservativity (unselective version):**

$$\|\mathbf{det}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{DET}(([x]; \phi)^g, ([x]; \phi; !\psi)^g))\}$$

The definition of *conservative* unselective quantification in (81) can in fact be thought of as the basis for the definition of selective generalized quantification introduced in Chierchia (1995) among others (see section **5** below): given that we have access to the variable *x* in both the restrictor of the static determiner **DET**, i.e. $[x]; \phi$, and in its nuclear scope, i.e. $[x]; \phi; !\psi$, we can be *selective* and (somehow) λ-abstract over the variable *x* in both formulas. We will consequently obtain two sets of *individuals* and we will require the static determiner **DET** to apply to these two sets individuals and not to the corresponding sets of info states[31].

## 5. Extending DPL with Selective Generalized Quantification (DPL+GQ)

The notion of *selective* generalized quantification introduced in this section has been proposed in various guises by many authors: Bäuerle & Egli (1985), Root (1986) and Rooth (1987) put forth the basic proposal and van Eijck & de Vries (1992) and Chierchia (1992, 1995) were the first to formulate it in DPL terms. The proposal is also adopted in Heim (1990) and Kamp & Reyle (1993)[32].

In defining it, I will use the notation introduced above, i.e. selective dynamic generalized quantification will have the form $\mathbf{det}_x(\phi, \psi)$, where *x* is the bound variable, $\phi$ is the restrictor and $\psi$ is the nuclear scope. Of course, since $\mathbf{det}_x(\phi, \psi)$ is selective, it will

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[31] It might be interesting to pursue in more detail the relation between unselective dynamic conservativity as defined in (80) above and selective dynamic conservativity as defined and argued for in Chierchia (1995): 97 et seqq and Kanazawa (1994a, b).

[32] The particular form of the definition of selective generalized quantification I provide here is based on the one in van den Berg (1994): 4 and van den Berg (1996b): 7.

be directly defined, i.e. it won't be an abbreviation of a formula containing the unselective operator **det**($\phi$, $\psi$), and it will involve a relation between two sets of individuals.

### 5.1. Dynamic Selective Generalized Quantification

The fact that $\mathbf{det}_x(\phi, \psi)$ is defined in terms of sets of *individuals* (and not of info states) will enable us to account for the proportion problem. The weak/strong donkey ambiguity will be attributed to an ambiguity in the interpretation of the selective generalized quantifier, basically following the proposals in Bäuerle & Egli (1985), Rooth (1987), Reinhart (1987), Heim (1990) and Kanazawa (1994a, b)[33].

That is, for each dynamic generalized determiner, we will have a *weak* lexical entry $\mathbf{det}^{wk}{}_x(\phi, \psi)$ and a *strong* lexical entry $\mathbf{det}^{str}{}_x(\phi, \psi)$. An English sentence containing a determiner *det* is ambiguous between the two readings – or, to put it in more appealing terms, any English determiner is underspecified with respect to one of the two readings.

The choice of a particular, fully specified lexical entry for any *det* is determined in each particular instance by a variety of factors, including world-knowledge, information structure, monotonicity of quantifiers etc.

The basic dynamic analysis does not have anything to say about how the choice between the weak and the strong reading depends on such factors – and, arguably, it shouldn't have anything to say about how the choice is made given that:

- which reading is selected in each particular case is influenced by a *diversity* of factors;
- the generalizations correlating these factors and the weak/strong readings have a *defeasible* character typically associated with pragmatic phenomena[34].

The determiners $\mathbf{det}^{wk}$ and $\mathbf{det}^{str}$ are both defined in terms of the corresponding static determiner **DET** as follows[35]:

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[33] Strictly speaking, Kanazawa (1994a, b) does not endorse an *ambiguity* analysis, but a *vagueness* account of dynamic selective generalized quantification. See also the discussion in Geurts (2002): 149 et seqq.

[34] For more details, see the section **6.1** in chapter **5** below.

82. $\| \mathbf{det}^{wk}{}_x(\phi, \psi) \| = \{<g, h>: g=h \text{ and } \mathbf{DET}(\lambda x.\ (\phi)^g,\ \lambda x.\ (\phi; \psi)^g)\}$

$\| \mathbf{det}^{str}{}_x(\phi, \psi) \| = \{<g, h>: g=h \text{ and } \mathbf{DET}(\lambda x.\ (\phi)^g,\ \lambda x.\ (\phi \rightarrow \psi)^g)\}$,

where $(\phi)^g := \{h: \| \phi \| <g, h> = \mathrm{T}\}$

and $\lambda x.\ (\phi)^g := \{h(x): h \in ([x]; \phi)^g\}$[36]

and **DET** is the corresponding static determiner.

Several observations before we turn to an example: first, both lexical entries are selective in the sense that the static determiner **DET** relates two sets of individuals, represented by means of abbreviations of the form $\lambda x.\ (\ldots)^g$. To my knowledge, this abbreviation has not been used in the previous literature despite its rather obvious and intuitive character.

Second, the only difference between the weak and the strong entries has to do with how the nuclear scope of the static quantification is obtained: we employ *dynamic conjunction* $\lambda x.\ (\phi; \psi)^g$ in the *weak* case and *dynamic implication* $\lambda x.\ (\phi \rightarrow \psi)^g$ in the *strong* case.

Dynamic conjunction yields the weak reading because an existential quantifier in the restrictor $\lambda x.\ (\phi)^g$ will still be an existential in the nuclear scope $\lambda x.\ (\phi; \psi)^g$: every farmer that owns *some* donkey beats *some* donkey he owns. Dynamic implication yields the strong reading because it has universal quantification built into it[37]: as we noticed right from the beginning (see (10) above), DPL validates the equivalence $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$, so an indefinite in the restrictor ends up being universally quantified in the nuclear scope: every farmer that owns *some* donkey beats *every* donkey he owns.

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[35] Note the formal similarities between:

- the alternative definition of implication in (72);
- the unselective generalized quantification with built-in conservativity defined in (80);
- the present definition of selective generalized quantification.

[36] Note that the abbreviation $\lambda x.\ (\phi)^g := \{h(x): h \in ([x]; \phi)^g\}$ really boils down to $\lambda$-abstraction in static terms: $\lambda x.\ (\phi)^g$ is the set of entities $a$ s.t. $\| \phi \|_{static}{}^{g[x/a]} = \mathrm{T}$, where $\| \cdot \|_{static}$ is the usual static interpretation function (see for example Gallin 1975).

[37] Incidentally, recall that the universal force of dynamic implication is actually due to dynamic negation '~' since $\phi \rightarrow \psi := \sim(\phi; \sim\psi)$.

Third, note that the unselective conservative entry defined in (81) above provides the basic format for the selective entries. In particular, assuming that, in (82) above, $[x]$ is not reintroduced in $\psi$ (and it cannot be if we want the definitions to work properly), it is always the case that:

83. $\lambda x.\ (\phi;\ \psi)^g = \lambda x.\ (\phi;\ !\psi)^g$ and $\lambda x.\ (\phi \rightarrow \psi)^g = \lambda x.\ (\phi \rightarrow !\psi)^g$ [38].

More generally, the weak and strong selective generalized determiners in (82) above can be defined in terms of generalized quantification over info states if we make use of the closure operator '!' as shown in (84) below. It is easily checked that the two pairs of definitions are equivalent given the fact that there is a bijection between the sets of individuals quantified over in (82) and the set of info states (i.e. variable assignments) quantified over in (84)[39].

84. $\|\mathbf{det}^{wk}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid !(\phi;\ \psi)])^g)\}$

$\|\mathbf{det}^{str}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid !(\phi \rightarrow \psi)])^g)\}$[40],

where $(\phi)^g := \{h: \|\phi\|<g, h> = \text{T}\}$

and **DET** is the corresponding static determiner.

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[38] For dynamic implication $\rightarrow$, we have the more general result that $\phi \rightarrow \psi \Leftrightarrow \phi \rightarrow !\psi$, which follows directly from the equivalence in (14) above.

[39] $\lambda x.\ (\phi)^g := \{h(x): h \in ([x];\ \phi)^g\} = \{a: \text{there is an } h \text{ s.t. } \|[x];\ \phi\|<g, h> = \text{T and } a{=}h(x)\}$

$= \{a: \text{there is a } k \text{ and an } h \text{ s.t. } g[x]k \text{ and } \|\phi\|<k, h> = \text{T and } a{=}h(x)\}$

(since $x$ is not reintroduced in $\phi$, $k(x)=h(x)$)

$= \{a: \text{there is a } k \text{ and an } h \text{ s.t. } g[x]k \text{ and } \|\phi\|<k, h> = \text{T and } a{=}k(x)\}$

$= \{a: \text{there is a } k \text{ s.t. } a{=}k(x) \text{ and } g[x]k \text{ and there is an } h \text{ s.t. } \|\phi\|<k, h> = \text{T}\}$

$= \{a: \text{there is a } k \text{ s.t. } a{=}k(x) \text{ and } g[x]k \text{ and } k \in \mathbf{Dom}(\|\phi\|)\} = \{a: \text{there is a } k \text{ s.t. } k \in ([x];\ !\phi)^g \text{ and } a{=}k(x)\}$.

Thus, $\lambda x.\ (\phi)^g = \{a: \text{there is a } h \text{ s.t. } h \in ([x];\ !\phi)^g \text{ and } a{=}h(x)\}$. Let $f$ be a function from the set of assignments $([x];\ !\phi)^g$ to the set of individuals $\lambda x.\ (\phi)^g$ s.t. $f(h)=h(x)$. By the above equality, $f$ is surjective. Since for any assignment $g$ and individual $a$ there is a unique assignment $h$ s.t. $g[x]h$ and $h(x)=a$, $f$ is injective.

[40] Since $!(\phi \rightarrow \psi) \Leftrightarrow \phi \rightarrow \psi$, the strong determiner can be more simply defined as $\|\mathbf{det}^{str}{}_x(\phi, \psi)\| = \{<g, h>: g{=}h \text{ and } \mathbf{DET}(([x \mid !\phi])^g, ([x \mid \phi \rightarrow \psi])^g)\}$.

Finally, according to definition (82), a formula of the form $\mathbf{det}^{wk}{}_{x}(\phi, \psi)$ or $\mathbf{det}^{str}{}_{x}(\phi, \psi)$ is a test. So, we should further extend the syntactic notion of condition with selective generalized determiners[41]. The new definition is:

85. The set of *conditions* is the smallest set of wff's containing atomic formulas, formulas whose main connective is dynamic negation '~', a **det** operator or a $\mathbf{det}^{wk/str}{}_{\upsilon}$ operator (for any variable $\upsilon$) and closed under dynamic conjunction.

The definition in (85) enables us to construct DRS's of the form $[\ldots \mid \ldots, \mathbf{det}^{wk/str}{}_{x}(\phi, \psi), \ldots]$.

## 5.2. Accounting for Weak / Strong Ambiguities

Let us see how the above definitions derive the weak and strong readings of the classical example in (86) below (repeated from (65)).

86. Every$^{x}$ farmer who owns a$^{y}$ donkey beats it$_{y}$.

The two lexical entries for *every* are given in (87) below and simplified in (88).

87. $\|\mathbf{every}^{wk}{}_{x}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{EVERY}(\lambda x.\ (\phi)^{g}, \lambda x.\ (\phi; \psi)^{g})\}$

    $\|\mathbf{every}^{str}{}_{x}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{EVERY}(\lambda x.\ (\phi)^{g}, \lambda x.\ (\phi \rightarrow \psi)^{g})\}$

88. $\|\mathbf{every}^{wk}{}_{x}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \lambda x.\ (\phi)^{g} \subseteq \lambda x.\ (\phi; \psi)^{g}\}$

    $\|\mathbf{every}^{str}{}_{x}(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \lambda x.\ (\phi)^{g} \subseteq \lambda x.\ (\phi \rightarrow \psi)^{g}\}$

The weak reading of (86) is represented in (89) and simplified in (90)[42].

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[41] Recall that the definition of conditions for DPL was given in (17) above and was extended with unselective determiners in (56).

[42] In more detail, the simplification proceeds as follows:

$\|\mathbf{every}^{wk}{}_{x}(farmer(x); [y]; donkey(y); own(x, y),\ beat(x, y))\| =$

$\{<g,g>: \lambda x.\ (farmer(x); [y]; donkey(y); own(x, y))^{g} \subseteq \lambda x.\ (farmer(x); [y]; donkey(y); own(x, y); beat(x, y))^{g}\} =$

$\{<g, g>: \{h(x): h \in ([x]; farmer(x); [y]; donkey(y); own(x, y))^{g}\} \subseteq$
$\{h(x): h \in ([x]; farmer(x); [y]; donkey(y); own(x, y); beat(x, y))^{g}\}\} =$

$\{<g, g>: \{h(x): g[x, y]h, h(x) \in \boldsymbol{I}(farmer), h(y) \in \boldsymbol{I}(donkey), <h(x), h(y)> \in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): g[x, y]h, h(x) \in \boldsymbol{I}(farmer), h(y) \in \boldsymbol{I}(donkey), <h(x), h(y)> \in (\boldsymbol{I}(own) \cap \boldsymbol{I}(beat)\}\} =$

89. $\mathbf{every}^{wk}{}_{x}(farmer(x);\ [y];\ donkey(y);\ own(x, y),\ \ beat(x, y))$

90. $\|\mathbf{every}^{wk}{}_{x}(farmer(x);\ [y];\ donkey(y);\ own(x, y),\ \ beat(x, y))\| =$

$\{<g, g>: \lambda x.\ (farmer(x);\ [y];\ donkey(y);\ own(x, y))^{g} \subseteq$

$\lambda x.\ (farmer(x);\ [y];\ donkey(y);\ own(x, y);\ beat(x, y))^{g}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$

$\{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in (\boldsymbol{I}(own)\cap \boldsymbol{I}(beat)\}\}=$

$\{<g, g>$: any farmer $a$ who owns a donkey $b$ is s.t. he owns and beats a donkey $b'\}$

As the simplification in (90) shows, the formula in (89) delivers the weak reading because the donkey-owning farmers do not have to beat *all* the donkeys they own – they only have to beat *some* of their donkeys.

The strong reading of (86) is represented in (91) and simplified in (92)[43].

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$\{<g, g>: \{a$: there is a $b$ s.t. $a\in \boldsymbol{I}(farmer), b\in \boldsymbol{I}(donkey), <a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{a$: there is a $b$ s.t. $a\in \boldsymbol{I}(farmer), b\in \boldsymbol{I}(donkey), <a, b>\in (\boldsymbol{I}(own)\cap \boldsymbol{I}(beat)\}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in (\boldsymbol{I}(own)\cap \boldsymbol{I}(beat)\}\} =$

$\{<g, g>$: any farmer $a$ who owns a donkey $b$ is such that he owns and beats a donkey $b'\}$.

[43] In more detail, the simplification proceeds as follows:

$\|\mathbf{every}^{str}{}_{x}(farmer(x);\ [y];\ donkey(y);\ own(x, y),\ \ beat(x, y))\| =$

$\{<g, g>: \lambda x.\ (farmer(x);\ [y];\ donkey(y);\ own(x, y))^{g} \subseteq \lambda x.\ (farmer(x);\ [y];\ donkey(y);\ own(x, y) \rightarrow beat(x, y))^{g}\} =$

$\{<g, g>: \{h(x): h\in ([x]; farmer(x);\ [y];\ donkey(y);\ own(x, y))^{g}\} \subseteq$
$\{h(x): h\in ([x]; (farmer(x);\ [y];\ donkey(y);\ own(x, y) \rightarrow beat(x, y)))^{g}\}\} =$

$\{<g, g>: \{h(x): g[x, y]h, h(x)\in \boldsymbol{I}(farmer), h(y)\in \boldsymbol{I}(donkey), <h(x), h(y)>\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): h\in ([x]; \sim(farmer(x);\ [y];\ donkey(y);\ own(x, y);\ \sim beat(x, y)))^{g}\}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x)$: there is a $k$ s.t. $g[x]k$ and $\|\sim(farmer(x);\ [y];\ donkey(y);\ own(x, y);\ \sim beat(x, y))\|<k, h> = T\}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): g[x]h$ and there is no $l$ s.t. $\|farmer(x);\ [y];\ donkey(y);\ own(x, y);\ \sim beat(x, y)\|<h, l> = T\}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): g[x]h$ and there is no $l$ s.t. $h[y]l, l(x)\in \boldsymbol{I}(farmer), l(y)\in \boldsymbol{I}(donkey), <l(x), l(y)>\in \boldsymbol{I}(own), <l(x), l(y)>\notin \boldsymbol{I}(beat)\}\} =$

$\{<g, g>: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $<a, b>\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): g[x]h$ and for any $l$, if $h[y]l, l(x)\in \boldsymbol{I}(farmer), l(y)\in \boldsymbol{I}(donkey), <l(x), l(y)>\in \boldsymbol{I}(own)$, then $<l(x), l(y)>\in \boldsymbol{I}(beat)\}\} =$

91. $\mathbf{every}^{str}{}_{x}(farmer(x); [y]; donkey(y); own(x, y),\ \ beat(x, y))$

92. $\|\mathbf{every}^{str}{}_{x}(farmer(x); [y]; donkey(y); own(x, y),\ \ beat(x, y))\| =$

$\{\langle g, g\rangle: \lambda x.\ (farmer(x); [y]; donkey(y); own(x, y))^{g} \subseteq$

$\lambda x.\ (farmer(x); [y]; donkey(y); own(x, y) \rightarrow beat(x, y))^{g}\} =$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $\langle a, b\rangle\in \boldsymbol{I}(own)\} \subseteq$

$\{a:$ any $b$ s.t. $a\in \boldsymbol{I}(farmer), b\in \boldsymbol{I}(donkey), \langle a, b\rangle\in \boldsymbol{I}(own)$ is s.t. $\langle a, b\rangle\in \boldsymbol{I}(beat)\}\} =$

$\{\langle g, g\rangle:$ any farmer $a$ who owns a donkey $b$ beats any donkey $b'$ that he owns$\}$

As the simplification in (92) shows, the formula in (91) delivers the strong reading because the donkey-owning farmers have to beat *all* the donkeys they own.

## 5.3. Solving Proportions

Selective generalized quantification also solves the proportion problem. Consider again sentence (1), repeated in (93) below. The most salient reading of this sentence seems to be the strong one, represented in (94), just as the most salient reading of the structurally similar sentence in (95) is the weak one, represented in (96) below.

If the reader's intuitions about the 'strength' of (93) are not very sharp, s/he should consider sentence (97) instead (example (49) in Heim 1990: 162), whose most salient reading is indeed the strong one.

93. Most$^{x}$ house-elves who fall in love with a$^{y}$ witch buy her$_{y}$ an$^{z}$ alligator purse.

94. $\mathbf{most}^{str}{}_{x}(house\_elf(x); [y]; witch(y); fall\_in\_love(x, y),$

$[z]; alligator\_purse(z); buy(x, y, z))$

95. Most$^{x}$ drivers who have a$^{y}$ dime will put it$_{y}$ in the meter.

96. $\mathbf{most}^{wk}{}_{x}(driver(x); [y]; dime(y); have(x, y),\ \ put\_in\_meter(x, y))$

---

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $\langle a, b\rangle\in \boldsymbol{I}(own)\} \subseteq$
$\{h(x): g[x]h$ and for any $b$, if $h(x)\in \boldsymbol{I}(farmer), b\in \boldsymbol{I}(donkey)$ and $\langle h(x), b\rangle\in \boldsymbol{I}(own)$, then $\langle h(x), b\rangle\in \boldsymbol{I}(beat)\}\}$
$=$

$\{\langle g, g\rangle: \{a: a\in \boldsymbol{I}(farmer)$ and there is a $b$ s.t. $b\in \boldsymbol{I}(donkey)$ and $\langle a, b\rangle\in \boldsymbol{I}(own)\} \subseteq$
$\{a:$ any $b$ s.t. $a\in \boldsymbol{I}(farmer), b\in \boldsymbol{I}(donkey)$ and $\langle a, b\rangle\in \boldsymbol{I}(own)$ is s.t. $\langle a, b\rangle\in \boldsymbol{I}(beat)\}\}\} =$

$\{\langle g, g\rangle:$ any farmer $a$ who owns a donkey $b$ beats any donkey $b'$ that he owns$\}$.

97. Most$^{x}$ people that owned a$^{y}$ slave also owned his$_{y}$ offspring.
(Heim 1990: 162, (49))

The formula in (94) is true iff more than half of the house-elves who fall in love with a witch are such that they buy *any* witch that they fall in love with (*strong* reading) some alligator purse or other. This formula is false in the 'Dobby as Don Juan' scenario above, in agreement with our intuitions about the corresponding English sentence in (93).

The formula in (96) makes similarly correct predictions about the truth-conditions of the English sentence in (95): both of them are true in a scenario in which there are ten drivers, each of them has ten dimes in his/her pocket and nine of them put exactly one dime in their respective meters. Out of the one hundred possible pairs of drivers and dimes they have, only nine pairs (far less than half) satisfy the nuclear scope of the quantification, but this is irrelevant as long as a majority of drivers (and not of pairs) satisfies it.

## 6. Limitations of DPL+GQ: Mixed Weak & Strong Donkey Sentences

However, the dynamic notion of selective generalized quantification introduced in the previous section does not offer a completely general account of the weak/strong donkey ambiguity: it fails for more complex weak & strong donkey sentences much as the unselective notion failed for the simplest ones.

Consider again the *dime* example from Pelletier & Schubert (1989), repeated in (98) below. Unselective generalized quantification fails to assign the correct weak interpretation to this example because it cannot distinguish between the various discourse referents (dref's) introduced in the restrictor of the generalized quantifier: *x* (the persons) should be quantified over universally, while *y* (their dimes) should be quantified over existentially.

Selective generalized quantification provides a solution to this problem because it can distinguish between *x*, which is the dref contributed by the generalized determiner, and *y*, which is the dref contributed by the indefinite in the restrictor of the determiner.

98. Every$^x$ person who has a$^y$ dime will put it$_y$ in the meter.

Thus, selective generalized quantification can only distinguish between the 'main' quantified-over dref and the other dref's introduced in the restrictor – it cannot further distinguish between the latter ones, which are collectively interpreted as either weak or strong. Since the decision about the 'strength' of their interpretation is not made on an individual basis, selective generalized quantification as defined in (82) above fails to account for any examples in which two indefinites in the restrictor of a generalized quantifier are not interpreted as both weak or both strong.

Sentences (4) and (5) below are such counter-examples.

99. Every$^x$ person who buys a$^y$ book on amazon.com and has a$^z$ credit card uses it$_z$ to pay for it$_y$.

100. Every$^x$ man who wants to impress a$^y$ woman and who has an$^z$ Arabian horse teaches her$_y$ how to ride it$_z$.

The most salient interpretation of (4) is strong with respect to *a$^y$ book* and weak with respect to *a$^z$ credit card*, i.e. for *every* book bought on amazon.com by any person that is a credit-card owner, the person uses *some* credit card or other to pay for the book. In particular, note that the credit card might vary from book to book, i.e. the strong indefinite *a$^y$ book* seems to be able to 'take scope' over the weak indefinite *a$^z$ credit card*: I can use my Mastercard to buy set theory books and my Visa to buy fantasy novels. This means that, despite the fact that it receives a weak reading, the indefinite *a$^{u'}$ credit card* can introduce a possibly non-singleton set of credit cards.

Similarly, in the case of (5), the indefinite *a$^y$ woman* is interpreted as strong and the indefinite *an$^z$ Arabian horse* as weak; and yet again, the strong indefinite seems to 'take scope' over the weak one: the horse used in the pedagogic activity might vary from female student to female student.

Finally, note that we can easily construct examples of this kind if we are willing to countenance other anaphoric expressions besides pronouns. Sentence (4) for example does not sound clumsy anymore if we replace one of the non-animate pronouns with a definite description – as shown in (101) below[44].

101. Every$^x$ person who buys a$^y$ book on amazon.com and has a$^z$ credit card uses the$_z$ card to pay for it$_y$.

I will not attempt to explicitly extend the DPL-style selective quantification in a way that can discriminate between the dref's introduced by indefinites in the restrictor. The basic idea would be to introduce additional lexical entries for generalized determiners which would bind universally or existentially the indefinites in their restrictor, e.g. the determiner *most* would have a 'single quantifier' entry of the form **every**$_x$, two 'double quantifier' entries of the form **most**$_x\forall_y$ and **most**$_x\exists_y$, four 'triple quantifier' entries of the form **most**$_x\forall_y\forall_z$, **most**$_x\forall_y\exists_z$, **most**$_x\exists_y\forall_z$, **most**$_x\exists_y\exists_z$ etc.[45]

Note that interpreting English sentences in terms of such determiners is not compositional, e.g. to interpret (5), we need a 'triple quantifier' of the form **every**$_x\forall_y\exists_z$, which requires us to look inside the second relative clause, identify the indefinite *an*$^z$ *Arabian horse* and assign it a weak interpretation.

The situation is in fact even more complicated and non-compositional: as already indicated, the indefinites in the restrictor can enter pseudo-scopal relations since the value of the weak indefinite can vary with the value of the strong indefinite, e.g. the same 'triple quantifier' **every**$_x\forall_y\exists_z$ has a choice of scoping $\forall_y$ over $\exists_z$ or the other way around, i.e. **every**$_x\exists_z\forall_y$ [46].

I take these relations to be *pseudo*-scopal because the two donkey indefinites in both (4) and (5) are 'trapped' in a coordination island and none of them can scope out of their

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[44] I substitute a definite description for the pronoun that enters the anaphoric dependency receiving a weak reading; substituting a definite description for the strong pronoun might bring in the additional complexity that the strong reading is in fact due to the use of the (maximal) definite description (see for example the D-/E-type analyses in Neale 1990, Lappin & Francez 1994 and Krifka 1996b).

[45] See for example Heim (1990): 163-164 for sample lexical entries for the 'double' quantifiers formulated both in terms of quantification over individuals and in terms of quantification over minimal situations.

[46] Note that the latter entry **every**$_x\exists_z\forall_y$ is not identical with the other 'triple quantifier' **every**$_x\exists_y\forall_z$.

VP- or CP-conjunct to take scope over the other[47]. Note that the impossibility of scoping out of a coordination structure is not dependent on any particular scoping mechanism; to see this, consider the two sentences in (102) and (103) below showing that a quantifier like *every* cannot scope out of VP- or CP-coordination structures.

102. #Every person who buys every$^x$ *Harry Potter* book on *amazon.com* and gives it$_x$ to a friend must be a *Harry Potter* addict.

103. #Every boy who wanted to impress every$^x$ girl in his class and who planned to buy her$_x$ a fancy Christmas gift asked his best friend for advice.

Quite a few accounts of weak and strong readings – including the dynamic account in van den Berg (1994, 1996) and the hybrid dynamic & D-/E-type approach in Chierchia (1992, 1995) – fail to analyze such conjunction-based, mixed weak & strong donkey sentences: the main difficulty for them is that they cannot allow for the weak 'strong' indefinite to be a (possibly) *non-singleton* set and to co-vary with the value of the strong indefinite despite the fact that the strong donkey indefinite cannot scope over the weak donkey indefinite.

It will be the main goal of chapter **5** to provide an analysis of the weak/strong donkey ambiguity which (i) is completely general in the sense that it is able to discriminate between the indefinites in the restrictor of a dynamic generalized determiner, (ii) does not postulate an ambiguity in the generalized determiner but only in the interpretation of the indefinites and (iii) is compositional.

This completes the review of the DPL (and therefore of classical DRT / FCS) and of the two most straightforward extensions of DPL with generalized quantification. The next two chapters (i.e. chapters **3** and **4**) are dedicated to the reformulation of DPL and of its extensions with generalized quantification in type logic, following Muskens (1995b, 1996).

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[47] Incidentally, note that that any such scope-taking has to ensure that the indefinites still have narrow scope with respect to the quantifiers *every*$^x$ *person* and *every*$^x$ *man*.

Chapter **3** reformulates DPL. The goal is to define an interpretation procedure for English sentences that is both dynamic and compositional at the sub-sentential / sub-clausal level.

Chapter **4** will extend the type-logical formulation of DPL with the notions of unselective and selective quantification defined for DPL in sections **4** and **5** above.