STRUCTURED NOMINAL AND MODAL REFERENCE

by

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ABSTRACT OF THE DISSERTATION

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The dissertation argues that discourse reference in natural language involves two equally important components with essentially the same interpretive dynamics, namely reference to values, i.e. non-singleton sets of objects (individuals and possible worlds), and reference to structure, i.e. the correlation / dependency between such sets, which is introduced and incrementally elaborated upon in discourse.

To define and investigate structured discourse reference, a new dynamic system couched in classical (many-sorted) type logic is introduced which extends Compositional DRT (CDRT, Muskens 1996) with plural information states, i.e. information states are modeled as sets of variable assignments (following van den Berg 1996a), which can be can be represented as matrices with assignments (sequences) as rows. A plural info state encodes both values (the columns of the matrix store sets of objects) and structure (each row of the matrix encodes a correlation / dependency between the objects stored in it).

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

The idea that plural info states are semantically necessary is motivated by examples with morphologically singular anaphors, in contrast to the previous literature that argues for plural info states based on plural anaphora.

Plural Compositional DRT (PCDRT) enables us compositionally account for a variety of phenomena, including: (*i*) mixed weak & strong donkey anaphora, e.g. *Every person who buys a^u computer and has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_u$*, (*ii*) quantificational subordination, e.g. *Harvey courts a^u girl at every$^{u'}$ convention. She$_u$ always$_{u'}$ comes to the banquet with him* (Karttunen 1976), (*iii*) modal anaphora and modal subordination, e.g. *A^u wolf might^p come in. It$_u$ would$_p$ eat Harvey first* (based on Roberts 1989) and (*iv*) naturally-occurring discourses exhibiting complex interactions between modalized conditionals, donkey anaphora, modal subordination and the entailment particle *therefore*, e.g. *[A] man cannot live without joy. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures* (Thomas Aquinas).

The PCDRT account of these phenomena explicitly and systematically captures the anaphoric and quantificational parallels between the individual and modal domains.

Acknowledgements

The first thing I wanted these acknowledgements to show is that they are not just another one of those "thank you and good night" things that people sometimes say on their way out mostly because that is what they are expected to say. I wanted them to show that they really are a heartfelt thank you addressed to everyone that made this dissertation possible and the last five years and a half of my (academic) life so fruitful and enjoyable.

So, I tried to make them look different and fancy and, after many unsuccessful attempts, I realized (duh!) that the canons of this literary species have the particular form that they have for a reason. I will therefore follow the canons and – like everyone else, I'm sure – mean each and every word. Here it goes.

This dissertation would not have been possible without my advisors Maria Bittner, Hans Kamp, Roger Schwarzschild and Matthew Stone.

Maria Bittner has been my semantics teacher ever since I came to Rutgers and my main advisor for the last three years and a half. She taught an "Introduction to Dynamic Semantics" topics course during my very first semester at Rutgers and a "Compositional Dynamic Semantics and Centering" seminar the following one. By the end of that year, I knew that dynamic semantics was what I wanted to do. Maria taught me and helped me so much over the years that it is pointless to try and say how much I owe her: I am her student through and through. I'll only mention here that less than four months ago, I returned to Rutgers from Stuttgart with several (quite sizeable) drafts but, technically speaking, no dissertation. It took only a couple of meetings with Maria and, lo and behold, where there was only a pile of drafts, a dissertation started to take shape.

Roger Schwarzschild is the other semantics teacher I had the good fortune of having during my first semester at Rutgers – and my friend ever since. His way of thinking about semantics and its relationships to the other sub-disciplines of linguistics and about how data and theory (should) come together and illuminate each other in any given analysis pervade my entire education and research. His insight and originality of thought have many a time cleared a path from a vague "this is kind of interesting" to a real, incisive question and from a seemingly unassailable problem to a surprisingly simple solution.

Matthew Stone has provided constant and very generous academic guidance and moral support for the last three years and a half of my doctorate. My intellectual debt to him becomes obvious upon the most cursory examination of the contents of this dissertation; in fact, one only needs to read the title. From our very first conversation, he taught me that semantics research receives its full significance only when properly located within the larger field of cognitive science and only when the detailed investigation of this or that phenomenon or formal system is systematically brought to bear on broader cognitive issues. His very generous help and constant encouragement were, especially over the last two years, one of the main forces that kept me going.

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My five months in Stuttgart, especially, felt like paradise: there was nothing else to do except to think about my research, write things up and then have another one of those amazing meetings with him that would go on for several hours. That time is for me the purest instance of what it is to do research – and how meaningful and gratifying that can be. I have been interested in the topics treated in this dissertation for several years, but it was only during those five months of interaction with him that most of the material finally crystallized and was written in more or less its present form.

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The academic environment at Rutgers, with its lively interaction between linguistic sub-disciplines on the one hand and between linguistics and other cognitive science disciplines on the other hand, provided the perfect opportunity to acquire a well-rounded education and was the ideal medium for the development of my ideas.

The support of a Deutscher Akademischer Austausch Dienst (DAAD) Graduate Scholarship grant, which made it possible for me to be in Germany for the academic year of 2005-2006, thereby financially supporting an important part of the present research, is gratefully acknowledged.

Finally, my parents, my sister Irina, her husband Mihai and my six oldest friends Codrin, Cristi, George, Micky, Stefan and Teo have always been there for me, in every way they could, through good and bad times.

It was my parents' love, their unlimited support and faith in me and their sometimes almost daily advice that helped me find the resolve to do everything I needed to do over the last eighteen months. This dissertation is dedicated to them.

As wiser people reportedly said (I am indebted to Jane Grimshaw for passing on these words of wisdom), one never finishes writing a dissertation – one only stops. It is with great reluctance that I have to stop now and I hope the reader will be able to forgive the many ways in which the present dissertation is still a working draft and the remaining errors of form and content. These errors are, of course, solely my own responsibility.

Dedication

Din ceas, dedus, adîncul acestei calme creste,

Intrată prin oglindă în mîntuit azur,

Tăind pe înecarea cirezilor agreste

În grupurile apei un joc secund, mai pur.

(Ion Barbu)

Table of Contents

6. STRUCTURED NOMINAL REFERENCE: QUANTIFICATIONAL SUBORDINATION 202

Chapter 1. Introduction

Karttunen's seminal paper on *Discourse Referents* begins with the following exhortation:

> "Consider a device designed to read a text in some natural language, interpret it and store the content in some manner, say, for the purpose of being able to answer questions about it. To accomplish this task, the machine will have to fulfill at least the following basic requirement. It has to be able to build a file that consists of records of all the individuals, that is, events, objects, etc., mentioned in the text and, for each individual, record whatever is said about it."
> (Karttunen 1976: 364)

The abstract study of such a text interpreter ultimately comes down to a reconsideration of the nature of the (literal) meaning of natural language expressions. What is being reconsidered is the truth-conditional, referential theory of meaning, aptly summarized in Hintikka (1969) as follows:

> "[…] it seems to me in any case completely hopeless to try to divorce the idea of the meaning of a sentence from the idea of the *information* that that the sentence can convey to a hearer or reader, should someone truthfully address it to him. Now what is this information? Clearly it is just information to the effect that the sentence is true, that the world is such as to meet the truth-conditions of the sentence."
> (Hintikka 1969: 146)

Moreover, the truth-conditions are determined in terms of the reference (denotation) relations that hold between linguistic expressions and independent, extra-linguistic entities.

Karttunen's exhortation shifts the classical perspective on meaning in two ways. First, the central problem is not the interpretation of sentences in isolation, but the interpretation of texts, i.e. of discourse. As Kamp (2001) puts it, "discourse meanings are more than plain conjunctions of sentence meanings. And this 'more' is often the effect of interpretation principles that are an integral part of linguistic knowledge, and thus legitimate objects of linguistic study" (Kamp 2001: 57).

Thus, over and above the investigation of how natural language interpretation is context dependent, we also need to investigate how the interpretation of a natural language expression changes the context, i.e. it creates a new context out of the old one, and thereby affects how subsequent expressions are interpreted. As soon as we turn to discourse interpretation, the dynamics of meaning comes into focus and we shift from a static truth-conditional theory to a theory of meaning as information update.

The second way in which the perspective on meaning shifts is reflected in the title of Karttunen's paper: natural language interpretation crucially involves a notion of *discourse reference* which mediates between linguistic expressions and their reference in the classical sense. This is the basic requirement put forth for our abstract text interpreter: the interpreter incrementally builds a file that contains records of the individuals mentioned in the text. At any given point, the file encodes the current information state of the interpreter, i.e. the current discourse context, and we refer to actual individuals via this information state: reference in natural language is inextricably discourse reference.

The present work is part of the general project of investigating the notions of discourse reference and information state involved in natural language interpretation. In particular, I will argue that over and above the basic requirement that the information state should be able to detect when a novel individual is mentioned in discourse and "store it along with its characterization for future reference" (Karttunen 1976: 364), the information state should also be able to encode dependencies between individuals (or sets thereof) that are established and subsequently referred to in discourse.

The main proposal is that nominal and modal expressions introduce (non-singleton) sets of objects, i.e. individuals and possible worlds respectively, and that these sets are correlated in discourse: discourse reference involves two equally important components with essentially the same interpretive dynamics, namely reference to values, i.e. sets of objects, and reference to structure, i.e. the correlation / dependency between such sets, which is introduced and incrementally elaborated upon in discourse. Hence the title: *Structured Nominal and Modal Reference*.

The dissertation focuses on the development of a new dynamic system couched in classical type logic that formalizes this notion of discourse reference and in which natural language discourses involving complex descriptions of multiple related objects can be compositionally translated.

I will, therefore, have little to say about the problem of interfacing the resulting system with a more general theory of discourse structure and anaphora resolution of the kind pursued in Hobbs (1979, 1990, 1993), Kameyama (1994), Grosz et al (1995), Kehler (1995, 2002) and Asher & Lascarides (2003) among others. In particular, one of the most important assumptions I will make throughout the dissertation is that the logical form of sentences and discourses comes with the 'intended' anaphoric indexation.

The dissertation consists of eight chapters, the first one (i.e. the current one) being the introduction and last one the conclusion. Chapters **2**, **3** and **4** mostly review the previous literature and introduce the basic type-logical dynamic system that underlies the entire present investigation. The remaining chapters introduce the new contributions: chapters **5** and **6** are dedicated to the study of donkey anaphora and quantificational subordination respectively (i.e. structured reference in the nominal domain), while chapter **7** is dedicated to the study of modal anaphora and modal subordination (i.e. structured reference in the modal domain). A more detailed description follows.

Chapter 2. Dynamic Predicate Logic with Generalized Quantification

Chapter **2** formally explicates Karttunen's basic requirement for discourse reference. The three most important formal systems modeling the notion of discourse reference in Karttunen (1976) are Discourse Representation Theory (DRT, Kamp 1981), File Change Semantics (FCS, Heim 1982/1988) and Dynamic Predicate Logic (DPL, Groenendijk & Stokhof 1991). The empirical coverage of these classical systems is roughly the same and, notwithstanding several non-trivial differences between them, they all follow the insight in Lewis (1975) that a dynamic information state (in terms of which discourse reference is to be defined) is a *case*, which is modeled as an *assignment of values to variables*.

"What is a case? […] [A] case may be regarded as the 'tuple of its participants; and these participants are values of the variables that occur free in the open sentence modified by the adverb. In other words, we are taking the cases to be the admissible assignments of values to these variables."
(Lewis 1975: 5-7)

The particular version of dynamic semantics surveyed in chapter **2** is DPL – and for three reasons. First, the syntax of the system is the familiar syntax of classical first-order logic (at least in the original notation; the notation in chapter **2** is a close variant thereof); this enables us to focus on what is really new, namely the semantics.

Second, the semantics of DPL is minimally different from the standard Tarskian semantics for first-order logic: instead of interpreting a formula as a set of variable assignments (i.e. the set of variable assignments that satisfy the formula in the given model), we interpret it as a binary relation between assignments; moreover, this minimal semantic modification encodes in a transparent way the core dynamic idea that meaning is not merely truth-conditional content, but context change potential.

Third, just as classical first-order logic can be generalized to static type logic, DPL can be generalized to a dynamic version of type logic and we can thereby introduce compositionality at the sub-sentential / sub-clausal level – to which chapter **3** is dedicated.

Besides introducing it, chapter **2** extends DPL with a dynamic notion of selective generalized quantification (as opposed to the unselective generalized quantification of Lewis 1975). This dynamic notion of selective generalized quantification has been proposed in various guises by many authors: Bäuerle & Egli (1985), Root (1986) and Rooth (1987) put forth the basic proposal and van Eijck & de Vries (1992) and Chierchia (1992, 1995) were the first to formulate it in DPL terms; the proposal is also adopted in Heim (1990) and Kamp & Reyle (1993).

Selective generalized quantification enables us to solve the proportion problem and account for weak / strong donkey readings.

The proportion problem is exemplified by sentence (1) below, where, intuitively, we do not quantify over most pairs $\langle x, y \rangle$ such that x is a house-elf that falls in love with a

witch *y* – as unselective quantification would have it – but only over most house-elves *x* that fall in love with some witch or other.

1. Most house-elves who fall in love with a witch buy her an alligator purse.

The weak / strong ambiguity problem is posed by the donkey sentence in (2) below, which has a weak reading, in contrast to the classical donkey sentence in (3), which has a strong reading. Sentence (2) exhibits a weak reading in the following sense: its most salient interpretation is that every person who has a dime will put *some* dime s/he has in the meter (and not all her / his dimes). Sentence (3) exhibits a strong reading in the sense that its most salient interpretation is that every farmer beats *every* donkey s/he owns.

2. Every person who has a dime will put it in the meter. (Pelletier & Schubert 1989)
3. Every farmer who owns a donkey beats it.

However, the notion of selective generalized quantification introduced to account for the weak / strong donkey examples in (2) and (3) above cannot *compositionally* account for the mixed weak & strong relative-clause donkey sentences in (4) and (5) below.

4. Every person who buys a book on amazon.com and has a credit card uses it to pay for it.
5. Every man who wants to impress a woman and who has an Arabian horse teaches her how to ride it.

Consider sentence (4): it is interpreted as asserting that, for *every* book (strong) that any credit-card owner buys on amazon.com, there is *some* credit card (weak) that s/he uses to pay for the book. Note, in particular, that the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels, which means that even the weak indefinite *a credit card* can introduce a (possibly) non-singleton set.

For each buyer, the two sets of objects, i.e. all the books purchased on amazon.com and some of the credit cards that the buyer has, are correlated and the dependency between these sets – left implicit in the restrictor of the quantification – is specified in the

nuclear scope: each book is correlated with the credit card that was used to pay for it. The above paraphrase of the meaning of sentence (4) is formalized in classical (static) first-order logic as shown in (6) below.

6. $\forall x(person(x) \land \exists y(book(y) \land buy\_on\_amazon(x, y)) \land \exists z(c.card(z) \land have(x, z))$

$\rightarrow \forall y'(book(y') \land buy\_on\_amazon(x, y')$

$\rightarrow \exists z'(c.card(z') \land have(x, z') \land use\_to\_pay(x, z', y'))))$

As the first-order translation in (6) explicitly shows, the challenge posed by sentence (4) is to *compositionally* derive its interpretation while allowing for: (**i**) the fact that the two donkey indefinites in the restrictor of the quantification receive two distinct readings (strong and weak respectively) and (**ii**) the fact that the value of the weak indefinite *a credit card* co-varies with / is dependent on the value of the strong indefinite *a book* although, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically scope over the weak one since both DP's are trapped in their respective VP-conjuncts. Example (5) is a variation on the same theme.

The fact that DPL with selective quantification cannot compositionally account for the mixed reading relative-clause donkey sentences in (4) and (5) above provides the basic plot and motivation for the next three chapters of the dissertation, namely chapters **3**, **4** and **5**. In particular, chapters **3** and **4** endeavor to recast DPL and its extension with selective generalized quantification in classical (many-sorted) type logic, which will automatically enable us to define a compositional interpretation procedure of the kind available in Montague semantics.

Chapter 3. Compositional DRT

Chapter **3** is the last chapter that is almost entirely a review of the previous literature. In particular, I review Compositional DRT (CDRT, Muskens 1996), which generalizes DPL to a dynamic version of type logic (just as static type logic generalizes classical first-order logic). This move enables us to introduce compositionality at the sub-sentential / sub-clausal level in the tradition of Montague semantics.

The first part of the chapter is dedicated to the definition of Dynamic Ty2 (which is Muskens' Logic of Change with negligible modifications) and to the translation of DPL in Dynamic Ty2. The Dynamic Ty2 translation is shown to preserve the DPL account of cross-sentential anaphora, relative-clause donkey sentences and conditional donkey sentences.

The second part defines a type-drive translation procedure based on a rough-and-ready syntax for a fragment of English. The resulting CDRT system effectively unifies DPL and Montague semantics and enables us to compositionally account for a variety of anaphoric and quantificational phenomena, including bound variable anaphora, quantifier scope ambiguities and donkey anaphora.

Chapter 4. Compositional DRT with Generalized Quantification

Chapter **4** is the first one that adds to the previous literature in a more substantial way by translating in Dynamic Ty2 the DPL-style definitions of unselective and selective generalized quantification introduced in chapter **2**.

CDRT is then extended with these two notions of dynamic generalized quantification. The resulting system, which I label CDRT+GQ, provides a fully compositional account of the proportion problem and of the simple (non-mixed) examples of weak / strong donkey sentences.

The chapter also introduces the analysis of the interaction between anaphora and generalized coordination in Muskens (1996). I show that this analysis successfully generalizes to account for DP-conjunction donkey sentences like *Every boy who has a dog and every girl who has a cat must feed it* due to Chierchia (1995).

Such examples are interesting for two reasons. First, Chierchia (1995) uses DP-conjunction donkey sentences of this kind to argue in favor of an approach to natural language interpretation that builds (part of) the dynamics into the semantic value of natural language expressions and against approaches that build the dynamics of the interpretation into syntactic operations at the level of Logical Form (LF).

The argument is, in a nutshell, that the same donkey pronoun is semantically bound by two distinct donkey indefinites, which can be naturally accounted for in a dynamic type-logical system with generalized conjunction (generalized to arbitrary types in the sense of Partee & Rooth 1983 among others). This kind of 'double binding', however, presents difficulties for approaches that require a particular syntactic configuration at the level of LF for the donkey pronouns to be semantically bound – because the same pronoun cannot enter two such distinct syntactic configurations.

The second reason for examining DP-conjunction donkey sentences is that, in the following chapter (chapter **5**), they will help us distinguish between different accounts of mixed reading donkey sentences.

Chapter **4** concludes with the somewhat surprising observation that CDRT with generalized quantification (CDRT+GQ) inherits the problem of DPL with generalized quantification: CDRT+GQ is not compositional enough to account for the mixed weak & strong relative-clause donkey sentences in (4) (and (5)) above. The main difficulty is due to the fact that, in (4), the weak indefinite *a credit card* co-varies with / is dependent on the value of the strong indefinite *a book*, although the strong indefinite cannot syntactically scope over the weak one (recall that they are both trapped in a coordination island).

It will be the task of the following chapter to modify the notion of information state employed in CDRT+GQ (and inherited from DRT / FCS / DPL) and thereby provide a compositional account of mixed weak & strong relative-clause donkey sentences.

Chapter 5. Structured Nominal Reference: Donkey Anaphora

This chapter incrementally introduces a new dynamic system that extends CDRT+GQ and within which we can give a compositional account of the donkey sentences in (4) and (5) above.

The main proposal is that discourse reference involves two equally important components with essentially the same interpretive dynamics, namely reference to values, i.e. sets of objects, and reference to structure, i.e. the correlation / dependency between such sets, which is introduced and incrementally elaborated upon in discourse.

The main technical innovation is that, just as in the Dynamic Plural Logic of van den Berg (1994, 1996a), information states are modeled as *sets* of variable assignments. Such *plural* info states can be represented as matrices with assignments (sequences) as rows, as shown in (7) below.

Plural info states encode discourse reference to both values and structure: the values are the sets of objects that are stored in the *columns* of the matrix, e.g. a discourse referent (dref) *u* for individuals stores a set of individuals relative to a plural info state, since *u* is assigned an individual by each assignment (i.e. row). The structure is *distributively* encoded in the *rows* of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref *u* by that assignment is structurally correlated with the individual assigned to some other dref *u'* by the same assignment. The resulting system is dubbed Plural CDRT.

7. Info State I	…	u		u'		…
i_1	…	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	…
i_2	…	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	…
i_3	…	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	…
…	…	…		…		…

Values – sets: $\{x_1, x_2, x_3, …\}$, $\{y_1, y_2, y_3, …\}$ **Structure – relations:** $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, …\}$

In Plural CDRT (PCDRT), sentences denote *relations* between an input and an output plural info state. Indefinites non-deterministically introduce both values and structure, i.e. they introduce *structured sets* of individuals; pronouns are anaphoric to such structured sets. Quantification is defined in terms of matrices instead of single assignments and the semantics of the non-quantificational part becomes rules for how to fill out a matrix.

The PCDRT analysis of sentence (4) is as follows. First, the weak / strong donkey ambiguity is attributed to the indefinite articles. This is the first step towards a compositional account because we *locally* decide for each indefinite article whether it receives a weak or a strong reading. The two basic meanings have the format provided in (8) below.

8. **weak indefinites:** $a^{\text{wk:}u} \leadsto \lambda P'_{\text{et}}.\ \lambda P_{\text{et}}.\ [u];\ P'(u);\ P(u)$

   strong indefinites: $a^{\text{str:}u} \leadsto \lambda P'_{\text{et}}.\ \lambda P_{\text{et}}.\ \textbf{max}^{u}(P'(u);\ P(u))$

The only difference between a weak and a strong indefinite article is the presence vs. absence of a maximization (**max**) operator. The **max** operator ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref u the *maximal* set of individuals satisfying both the restrictor dynamic property P' and the nuclear scope dynamic property P. In contrast, a weak indefinite will non-deterministically store *some* set of individuals satisfying its restrictor and nuclear scope.

In sentence (4), the indefinite $a^{\text{str:}u_2}$ *book* in is strong and the indefinite $a^{\text{wk:}u_3}$ *credit card* is weak. Thus, by the time we are done processing the restrictor of the quantification in (4), we will be in an info state that stores: (**i**) the *maximal set* of books with respect to the dref u_2; (**ii**) *some* (non-deterministically introduced) *set* of credit cards with respect to the dref u_3 (the weak indefinite) and (**iii**) *some* (non-deterministically introduced) *structure* correlating the values of u_2 and u_3.

The nuclear scope of the quantification in (4) is anaphoric to both values and structure: we test that the non-deterministically introduced value for u_3 and the non-deterministically introduced structure associating u_3 and u_2 satisfy the nuclear scope condition, i.e., for each assignment in the info state, the u_3-card stored in that assignment is used to pay for the u_2-book stored in the same assignment. That is, the nuclear scope elaborates on the unspecified dependency between u_3 and u_2 introduced in the restrictor of the quantification – and, crucially, introducing such a dependency does not require the strong indefinite to take scope over the weak one.

The PCDRT account successfully generalizes to the mixed reading DP-conjunction donkey sentences in (9) and (10) below, where the same pronoun is intuitively interpreted as having two distinct indefinites as antecedents – and the two indefinites have different readings (one is weak and the other is strong). These examples will be used to distinguish between PCDRT and D-/E-type approaches to donkey anaphora.

9. (Today's newspaper claims that, based on the most recent statistics:)

   Every^{u_1} company who hired a$^{\mathbf{str}:u_2}$ Moldavian man, but no^{u_3} company who hired a$^{\mathbf{wk}:u_2}$ Transylvanian man promoted him$_{u_2}$ within two weeks of hiring.

10. (Imagine a Sunday fair where people come to sell their young puppies before they get too old and where the entrance fee is one dollar. The fair has two strict rules: all the puppies need to be checked for fleas at the gate and, at the same time, the one dollar bills also need to be checked for authenticity because of the many faux-monnayeurs in the area. So:)

    Everyone^{u_1} who has a$^{\mathbf{str}:u_2}$ puppy and everyone^{u_3} who has a$^{\mathbf{wk}:u_2}$ dollar brings it$_{u_2}$ to the gate to be checked.

Finally, chapter **5** shows that PCDRT preserves all the previously obtained results, including the analysis of bound variable anaphora, the analysis of quantifier scope ambiguities and the compositional account of the proportion problem and of the simple (non-mixed) examples of weak / strong ambiguities.

Chapter 6. Structured Nominal Reference: Quantificational Subordination

Chapter **6** extends the PCDRT system introduced in chapter **5** with a notion of dynamic generalized quantification that enables us to give a compositional account of quantificational subordination, specifically of the contrast between the interpretations of the following two discourses from Karttunen (1976):

11. **a.** Harvey courts a^u girl at every convention.
    b. She$_u$ is very prett*y*.
12. **a.** Harvey courts a^u girl at every convention.
    b. She$_u$ always comes to the banquet with him.

The initial sentence (11a/12a) is ambiguous between two quantifier scopings: Harvey courts the same girl at every convention (*a^u girl*>>every convention) vs. at every convention, Harvey courts a (possibly) different girl (*every convention*>>*a^u girl*).

However, discourse (11) as a whole allows only for the "same girl" reading, while discourse (12) allows for both readings.

Using plural information states, we can capture the cross-sentential interaction between quantifier scope and anaphora exhibited by (11), in particular, the fact that the *singular* pronoun in sentence (11b) can disambiguate between the two readings of sentence (11a) (to see that the disambiguation is due to the singular pronoun, replace (11b) with *They$_u$ are very pretty*).

The basic idea is that plural info states enable us to store both quantifier domains (i.e. values) and quantificational dependencies (i.e. structure), pass them across sentential boundaries and further elaborate on them, e.g. by letting a pronoun constrain the cardinality of a previously introduced quantifier domain. More precisely, after processing the update contributed by sentence (11a), the dref *u* will store the set of all girls that Harvey courts at some convention or other. The singular pronoun *it$_u$* in (11b) will then constrain this set to be a singleton set; hence, the only available reading for discourse (11) as a whole is "wide-scope indefinite" reading.

The fact that discourse (12) is also compatible with the "narrow-scope indefinite" reading is attributed to the presence of the quantificational adverb *always* in (12b), which can take scope over the singular pronoun *she$_u$* and thereby neutralize the effect that singular number morphology has on the cardinality of the previously introduced set of girls.

PCDRT derives the contrast between the two Karttunen examples with minimal stipulations: the dynamic meanings for generalized quantifiers and singular number morphology are basically reformulations of their independently motivated static meanings that incorporate the notion of structured discourse reference argued for in the previous chapters.

Chapter 7. Structured Modal Reference: Modal Anaphora and Subordination

Chapter **7** shows that PCDRT successfully generalizes to other phenomena independent of (yet interacting with) structured donkey anaphora and quantificational subordination, while still preserving both the intuitive appeal of the DRT account of anaphora and the compositional (Montagovian) character of the analyses.

In particular, the fact that we work with finer-grained meanings (given our plural info states) enables us to analyze complex interactions between individual-level and modal anaphora by simply adding discourse referents (dref's) p, p' etc. for possible worlds: a dref p stores a set of worlds, i.e. a *proposition*, relative to a plural info state and it can be structurally correlated with some other modal dref p' and / or with individual-level dref's u, u' etc.

The resulting Intensional PCDRT (IP-CDRT) system generalizes the notion of dynamic quantification introduced in chapter **6** to the modal domain, thereby enabling us to provide an account of the modal subordination that is completely parallel to the account of quantificational subordination. In particular, the analysis of the modal subordination discourse in (13) below is point-for-point parallel to the analysis of the quantificational subordination discourse in (12) above (provided in chapter **6**).

13. **a.** A^u wolf might come in. **b.** It$_u$ would eat Harvey first.

Thus, IP-CDRT allows us to systematically capture the anaphoric and quantificational parallels between the individual and modal domains argued for in Stone (1999), Bittner (2001) and Schlenker (2005b) among others.

Moreover, chapter **7** also shows that IP-CDRT successfully generalizes to more complex interactions between modal and invidual-level anaphora exhibited by naturally occurring discourses like (14) below.

14. **a.** [A] man cannot live without joy. **b.** Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures. (Thomas Aquinas)

In particular, we are interested in the entailment relation established by *therefore* between the modal premise in (14a) and the modal conclusion in (14b) – and, to capture this, we need to account for several interrelated phenomena.

First, we want to capture the meaning of the entailment particle *therefore*, which relates the content of the premise (14a) and the content of the conclusion (14b) and requires the latter to be entailed by the former. I take the content of a sentence to be truth-

conditional in nature, i.e. to be the set of possible worlds in which the sentence is true, and entailment to be content inclusion, i.e. (14a) entails (14b) iff for any world w, if (14a) is true in w, so is (14b));

Second, we want to capture the meanings of (14a) and (14b). I take meaning to be context-change potential, i.e. to encode both content (truth-conditions) and anaphoric potential.

Thus, on the one hand, we are interested in the contents of (14a) and (14b). They are both modal quantifications: (14a) involves a circumstantial modal base (to use the terminology introduced in Kratzer 1981) and asserts that, in view of the circumstances, i.e. given that God created man in a particular way, as long as a man is alive, he must find some thing or other pleasurable; (14b) involves the same modal base and elaborates on the preceding modal quantification: in view of the circumstances, if a man is alive and has no spiritual pleasure, he must have a carnal pleasure. Note that we need to make the contents of (14a) and (14b) accessible in discourse so that the entailment particle *therefore* can relate them.

On the other hand, we are interested in the anaphoric potential of (14a) and (14b), i.e. in the anaphoric connections between them. These connections are explicitly represented in discourse (15) below, which is intuitively equivalent to (14) albeit more awkwardly phrased.

15. **a.** If a^{u_1} man is alive, he$_{u_1}$ must find something^{u_2} pleasurable / he$_{u_1}$ must have a^{u_2} pleasure.
    b. Therefore, if he$_{u_1}$ doesn't have any^{u_3} spiritual pleasure, he$_{u_1}$ must have a^{u_4} carnal pleasure.

Discourse (14/15) is analyzed in Intensional PCDRT (IP-CDRT) as a network of structured anaphoric connections and the meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora.

In particular, note that the conditional in (15b) is modally subordinated to the antecedent of the conditional in (15a), i.e. (15b) is interpreted as: if *a man is alive and* he

doesn't have any spiritual pleasure, he must have a carnal pleasure. Modal subordination is analyzed as simultaneous modal and individual-level anaphora, i.e. the morpheme *if* in (15b) is anaphoric to the modal dref introduced by the antecedent of (15a) and the pronoun *he*$_{u_1}$ is anaphoric to the individual-level dref introduced by the indefinite *a*^{u_1} *man* in the antecedent of (15a).

The IP-CDRT account of discourse (14/15) brings further support to the idea that the dynamic turn in natural language semantics does not require us to abandon the classical approach to meaning and reference. In fact, the analysis of (14/15) does the exact opposite: the introduction of propositional dref's in IP-CDRT enables us to recover the classical notion of truth-conditional content, which in turn enables us to analyze the Aquinas discourse in (14/15) as involving structured discourse reference to the propositional contents contributed by the premise and the conclusion of the argument.

Chapter 8. Conclusion

The last chapter contains a summary of the main results and briefly presents two future extensions of Intensional PCDRT, namely: *de se* attitudes (focusing on the Romanian subjunctive B mood) and plural anaphora and quantification.

The dissertation is located at the intersection of two major research programs in semantics that have gained substantial momentum in the last fifteen years: (**i**) the development of theories and formal systems that unify different semantic frameworks and (**ii**) the investigation of the semantic parallels between the individual, temporal and modal domains. As the dissertation shows, one of the outcomes of bringing together these two research programs is a novel compositional account of non-local (modal and individual-level) quantificational dependencies as *anaphora to structure*.

Thus, on the one hand, the present investigation takes the program in Muskens (1996) (see also Janssen 1986 and Groenendijk & Stokhof 1990 among others) of

unifying Montague semantics and dynamic semantics one step further[1]: Intensional PCDRT unifies – in classical type logic – the static Lewis (1973) / Kratzer (1981) analysis of modal quantification and van den Berg's Dynamic Plural Logic.

On the other hand, Intensional PCDRT enables us to explicitly and systematically capture the anaphoric and quantificational parallels between the individual and modal domains, in particular, between quantificational and modal subordination, thus bringing further support to the conjecture that our semantic competence is domain neutral, first put forth in Partee (1973, 1984) for the individual and temporal domains and extended to the modal domain by Stone (1997, 1999), Bittner (2001) and Schlenker (2005b) among others.

Summarizing, the dissertation can be seen as an extended investigation of a central issue raised by the dynamic turn in natural language semantics, namely: what kind of information is stored in an information state and how is this information updated in discourse? And, in particular: what can anaphora and quantification in both the nominal and the modal domain tell us about this?

The main result of the investigation is a new representation language, i.e. Intensional PCDRT, which is couched in classical type logic and in which natural language discourses involving complex descriptions of multiple related objects can be compositionally translated. Intensional PCDRT formalizes the idea that information states involve two equally important components with essentially the same interpretive dynamics, namely discourse information about values (sets of objects: individuals and possible worlds, but also times, eventualities etc.) and discourse information about structure (the correlations / quantificational dependencies between sets of objects that are introduced and elaborated upon in discourse).

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[1] The research program of unifying Montague semantics and dynamic semantics goes back at least to Partee (1984): "I don't see how to incorporate Montague's elegant treatment of compositionality into the framework followed in this paper, nor do I see how to reproduce within Montague's theory the unified and explanatory account of nominal and temporal anaphora provided by these extensions of Kamp's, Heim's and Hinrichs' work. So the next task is to try to construct a theoretical framework which incorporates the insights of both approaches." (Partee 1984: 279).

Chapter 2. Dynamic Predicate Logic with Generalized Quantification

1. Introduction

The main goal of this and the following chapter is to situate the present research within the general enterprise of compositional dynamic semantics, in particular:

- to provide the core framework I will build on throughout the dissertation;
- to fix notation;
- to briefly recapitulate the basic empirical generalizations that motivate the dynamic approach to semantics and the basic kinds of semantic analyses that this approach makes possible.

Most of the results reported in these two chapters come from the previous literature and are meant to set the stage for the more complex formal systems presented in the following chapters. The main references are: Kamp (1981), Heim (1982/1988), Kamp & Reyle (1993) for the general dynamic framework, i.e. Discourse Representation Theory (DRT) / File Change Semantics (FCS); Groenendijk & Stokhof (1991) for the way I choose to introduce sentence-level / clause-level compositionality in the framework, i.e. their Dynamic Predicate Logic (DPL); and finally Muskens (1995b, 1996) for the way to go compositional at the sub-sentential level, i.e. his Compositional DRT (CDRT). In particular, the step of going compositional at the sub-sentential level:

> "[…] combine[s] Montague Semantics and Discourse Representation into a formalism that is not only notationally adequate, in the sense that the working linguist need remember only a few rules and notations, but is also mathematically rigorous and based on ordinary type logic. […] DRT's Discourse Representation Structures (DRS's or boxes henceforth) are already present in type logic in the sense that they can simply be viewed as abbreviations of certain first-order terms, provided that some first-order axioms are adopted. […] The presence of boxes in type logic permits us to fuse DRT and Montague Grammar in a rather evenhanded way: both theories will be recognizable in the result. […] With this unification of the theories standard techniques (such as type-shifting) that are used in Montague Grammar become available in DRT." (Muskens (1996): 144-145)

Section **2** introduces DPL (Groenendijk & Stokhof 1991). Sections **4** and **5** introduce the two most straightforward ways to extend DPL to account for generalized quantification in natural language. In particular, section **4** introduces – following a suggestion about adverbs of quantification in Groenendijk & Stokhof (1991) – what can be termed *unselective* generalized quantification – unselective in the sense of Lewis (1975), i.e. generalized quantification relating two sets of information states.

This notion of unselective generalized quantification reproduces in DPL the (somewhat implicit) conception of generalized quantification in Kamp (1981) and Heim (1982/1988).

Defining dynamic generalized quantifiers unselectively fails to account for the weak / strong donkey ambiguity and runs into the proportion problem. Based on Bäuerle & Egli (1985), Root (1986), Rooth (1987), van Eijck & de Vries (1992) and Chierchia (1992, 1995) (see also Heim 1990 and Kamp & Reyle 1993), section **5** extends DPL with selective generalized quantification, which can account for the weak / strong donkey ambiguity and avoids the proportion problem.

 The differences between the material in this chapter and the sources mentioned above are for the most part presentational. There are only two novel things. The first is the DPL-style definition of unselective generalized quantification in section **4** that incorporates generalized quantifier *conservativity*.

The second one is the introduction – in section **6** – of the mixed weak & strong donkey sentences, i.e. relative-clause donkey sentences with two donkey indefinites that receive different readings – one strong, the other weak –, e.g. *Every person who buys a book on amazon.com* (strong) *and has a credit card* (weak) *uses it* (the credit card) *to pay for it* (the book); this kind of sentences cannot be accounted for in DRT / FCS / DPL even when they are extended with *selective* generalized quantification. Mixed weak & strong donkey sentences will provide one of the primary motivations for the subsequent revisions and generalizations of CDRT.

I will also introduce several new notational conventions (e.g. the λ-style notation for DPL-style generalized quantification in section **5**) and, on occasion, the departure from the original notation entails minor technical modifications.

But these are basically the only changes I make to the original dynamic systems – and they are made in the interest of clarity: given that the matters under discussion and the technical apparatus devised to handle them become difficult fairly fast, it seems counter-productive to increase the difficulty by repeatedly switching between various notations[1].

2. Dynamic Predicate Logic (DPL)

There are three basic kinds of examples that initially motivated a dynamic approach to the semantics of natural language. First, discourses in which a singular pronoun is anaphoric to an indefinite in a previous sentence, as shown in discourse (1-2) below.

1. A^u house-elf fell in love with $a^{u'}$ witch.
2. He_u bought $her_{u'}$ $an^{u''}$ alligator purse.

Following the convention in Barwise (1987), antecedents are indexed with superscripts and dependent elements with subscripts. Sentence (2) is interpreted as asserting that the house-elf mentioned in (1), namely u, bought an alligator purse to the witch mentioned in (1), i.e. to u'. Thus, the pronouns in (2) are interpreted as referring back to the entities evoked in the previous discourse. Heim (1982/1988) argues in detail that such pronouns do refer back to discourse entities and *not* some other entities, e.g. actual individuals that the speaker 'has in mind' when using the indefinites in (1). The hypothesis that the pronominal anaphora in discourse (1-2) is an instance of discourse reference (and not some other kind of reference or covert pronoun binding) is supported by the donkey sentences in (3), (4) and (5), (6) below.

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[1] For example, the non-conservative definition of unselective generalized quantification in section **4** is not given as such in the literature. But the only new thing is the notation – the actual content of the definition is an immediate extension of the analysis of adverbs of quantification in Groenendijk & Stokhof (1991): 81-82 in terms of the 'generalized' implication connectives $\rightarrow_Q$.

3. Every farmer who owns a^u donkey beats it$_u$.

4. Every house-elf who falls in love with a^u witch buys her$_u$ an$^{u'}$ alligator purse.

5. If a^u farmer owns a$^{u'}$ donkey, he$_u$ beats it$_{u'}$.

6. If a^u house-elf falls in love with a$^{u'}$ witch, he$_u$ buys her$_{u'}$ an$^{u''}$ alligator purse.

Sentence (4), for example, cannot be said to make reference to any given witch u' that the speaker 'has in mind'; the intuitively most salient interpretation of (4) is that for any pair of individuals u and u' such that u is a house-elf and u' is a witch that said elf is in love with, u buys u' some alligator purse or other. Moreover, the indefinite $a^{u'}$ *witch* in (4) cannot bind the pronoun *her*$_{u'}$ because it is not in the required structural position for binding, namely c-command (or some other suitable notion of 'command', depending on the reader's favorite syntactic formalism), e.g. in *Every witch loves herself*, the quantifier *every witch* c-commands and binds the pronoun *herself*. A similar argument can be put forth in the case of donkey anaphora in conditionals, as illustrated by (5) and (6).

The felicity of the discourse reference patterns instantiated by examples (1) through (6) above does not seem to be sensitive to pragmatic factors (e.g. world knowledge, the speaker's communicative intentions etc.), which provides *prima facie* evidence that they should be analyzed in *semantic* terms. However, static formal semantics for natural language of the kind proposed in Montague (1974) cannot account for the cross-sentential scope of the indefinites in discourse (1-2) or for the co-variation without binding that obtains between the pronouns and the indefinites in (3) through (6) above. Hence the move to dynamic semantics.

I will not argue now for the dynamic approach to cross-sentential anaphora and donkey sentences as opposed to the family of D-/E-type approaches. I will compare these two kinds of approaches as I analyze increasingly complex discourses starting with chapter **5**. Anticipating, I will make two main points.

First, as soon as we start examining some of the phenomena that are central to the present investigation, namely donkey sentences with multiple instances of donkey anaphora that receive different readings (i.e. weak and / or strong), e.g. *Every person who buys a book on amazon.com* (strong) *and has a credit card* (weak) *uses it to pay for it*

(the book)), the D-/E-type approaches that model pronouns as functions of arbitrary arity from individuals to individuals[2] become increasingly complex and counter-intuitive, as opposed to an analysis formulated in a dynamic system formulated in type logic and employing plural info states (i.e. sets of variable assignments).

Second, if we want to extend the other kind of D-/E-type approaches, i.e. the situation-based ones (which model pronouns as functions from (minimal) situations to individuals[3]), to account for such examples, we will very likely end up with a system that is identical in the relevant respects with the dynamic system I propose.

Finally, if we want to extend the account of donkey anaphora in the individual domain to modal anaphora and modal subordination in such a way that we capture the systematic parallels between *modal* and *individual-level* anaphora and quantification – see Geurts (1999), Frank (1996), Stone (1997, 1999), Bittner (2001) and Schlenker (2005) among others for detailed discussion of these parallels –, we can straightforwardly capture the modal phenomena and the cross-domain parallels in a type-logical dynamic semantics system by simply extending it with another basic type for possible worlds, as shown in chapter **7** below (and building on Muskens 1995b and Stone 1999).

It is much less clear how to execute a similar extension for the two kinds of D-/E-type approaches mentioned above (i.e. 'individual'-based and situation-based).

The particular version of dynamic semantics I build on is DPL (Groenendijk & Stokhof 1991) – and for three reasons:

- first, the syntax of the system is the familiar syntax of classical first-order logic (at least in the original notation; in my notation, it is a fairly close variant thereof); this enables us to focus on what is really new, namely the semantics;
- second, the semantics of DPL is minimally different from the standard Tarskian semantics for first-order logic: instead of interpreting a formula as a set of variable assignments (i.e. the set of variable assignments that satisfy the formula

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[2] See Chierchia (1995), section **2.5** for a relatively recent example.

[3] See Heim (1990) for the paradigmatic example.

in the given model), we interpret it as a binary relation between assignments[4]; moreover, this minimal semantic modification encodes in a transparent way the core dynamic idea that meaning is not merely *truth-conditional content*, but *context change potential*;

- third, just as classical predicate logic can be straightforwardly generalized to static type logic, DPL can be easily generalized to a dynamic version of type logic, which is what Muskens' Compositional DRT is; and CDRT enables us to introduce compositionality at the sub-sentential/sub-clausal level in the tradition of Montague semantics.

Besides formally defining an intuitive and easily generalizable notion of dynamic semantic value, DPL is able to translate the donkey sentences in (3) through (6) above compositionally, with sentences / clauses as the building blocks (i.e., basically, as compositional as one can get in first-order logic).

For instance, sentences (3) and (5) above are translated as shown in (7) and (8) below and, when interpreted dynamically, the translations capture the intuitively correct truth-conditions.

7. $\forall x(farmer(x) \land \exists y(donkey(y) \land own(x, y)) \rightarrow beat(x, y))$
8. $\exists x(farmer(x) \land \exists y(donkey(y) \land own(x, y))) \rightarrow beat(x, y)$

Consider (7) first: as it is customary, *every* is translated as universal quantification plus implication and the indefinite as existential quantification plus conjunction; moreover, the *syntactic* scope of the existential quantification is 'local' (restricted to the antecedent of the implication), but it does *semantically* bind the occurrence of the

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[4] Alternatively, and in certain respects equivalently, we can think of the interpretation of a formula as a function taking as argument a sets of assignments and returning another set of assignments – this is the view underlying FCS, for example. However, in both cases the update is defined pointwise – and a relational view of update reflects this more directly. There are other differences between FCS and DPL (e.g. using partial and total assignments respectively and disallowing vs. allowing reassignment) – see the dynamic cube in Krahmer (1998): 59 for an overview. In particular, the fact that DPL (and CDRT) allows reassignment will be an essential ingredient in accounting for the interaction between anaphora and generalized conjunction (see section **5** of Chapter 1 below). The "destructive reassignment" or "downdate problem" associated with reassignment can be solved using stacks / 'referent systems': see Nouwen (2003) for a recent discussion and Bittner (2006) for a set of 'stack' axioms for dynamic type logic.

variable *y* in the consequent. Similarly, in (8) the conditional is translated as implication and the indefinites are translated as existentials plus conjunction, again with syntactically 'local' but semantically 'non-local' scope.

As these observations indicate, DPL has two crucial properties that enable it to provide compositional translations for donkey sentences: DPL makes the equivalences in (9) and (10) below valid, so that indefinites can semantically bind outside their syntactic scope and indefinitely to the right, which, in combination with the definition of dynamic implication, allows them to scope out of the antecedent and universally bind in the consequent of the implication.

9. $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$[5]

10. $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$

2.1. Definitions and Abbreviations

DPL is a well-known system, so I will provide the definition of the interpretation function without any additional pre-theoretical motivation. The official syntax of DPL (i.e. the one in Groenendijk & Stokhof (1991)) is that of classical first-order logic with identity. However, in view of subsequent developments, I introduce certain modifications: the most salient one is that the symbol for conjunction is ';' (the symbol generally used for dynamic sequencing) and not the usual '$\wedge$'. Moreover, existential and universal quantifications are not officially present in the language; I only define the interpretation of the random assignment to a variable *x*, symbolized as [*x*] – and the existential and universal quantifiers are defined as abbreviations in terms of [*x*].

I do not provide the 'official' definition of a well-formed formula (wff) of DPL – it is easily recoverable on the basis of the definition of the interpretation function $\|\cdot\|$ in (11) below. As already indicated, the semantics of DPL interprets formulas as relations between variable assignments, which, for our narrow empirical purposes (i.e. elementary

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[5] The symbol '$\Leftrightarrow$' should be interpreted as requiring the identity of the semantic value of two formulas.

aspects of discourse reference to individuals), model the more general dynamic notion of information state in a satisfactory way.

11. **Dynamic Predicate Logic (DPL).** The definition of the DPL interpretation function $\|\phi\|_{DPL}^{M}$ relative to a standard first-order model $M=<D^{M}, I^{M}>$, where D is the domain of entities and I is the interpretation function which assigns to each n-place relation 'R' a subset of D^{n}. For readability, I drop the subscript and superscript on $\|\cdot\|_{DPL}^{M}$, D^{M} and I^{M}. 'T' and 'F' stand for the two truth values. For any pair of M-variable assignments $<g, h>$:

a. Atomic formulas ('lexical' relations and identity):

$\|R(x_1, \ldots, x_n)\|<g, h> = T$

iff $g=h$ and $<g(x_1), \ldots, g(x_n)> \in I(R)$

$\|x_1=x_2\|<g, h> = T$

iff $g=h$ and $g(x_1)=g(x_2)$

b. Connectives (dynamic conjunction and dynamic negation):

$\|\phi; \psi\|<g, h> = T$

iff there is a k s.t. $\|\phi\|<g, k> = T$ and $\|\psi\|<k, h> = T$

$\|\sim\phi\|<g, h> = T$

iff $g=h$ and there is no k s.t. $\|\phi\|<g, k> = T$,

i.e. $\|\sim\phi\|<g, h> = T$ iff $g=h$ and $g \notin \mathbf{Dom}(\|\phi\|)$,

where $\mathbf{Dom}(\|\phi\|) := \{g$: there is an h s.t. $\|\phi\|<g, h> = T\}$

c. Quantifiers (random assignment of value to variables):

$\|[x]\|<g, h> = T$

iff for any variable v, if $v \neq x$ then $g(v)=h(v)$

d. Truth: A formula ϕ is true with respect to an input assignment g iff there is an output assignment h s.t. $\|\phi\|<g, h> = T$, i.e. $g \in \mathbf{Dom}(\|\phi\|)$.

Given that variable assignments are functions from variables to entities, if two variable assignments assign identical values to all the variables, they are identical. Hence, based on definition (11c), the formula [ ] defines the 'diagonal' of the product $G \times G$, where G is the set of all M-variable assignments, as shown in (12).

12. $\|[\ ]\| = \{<g,\ g>:\ g\in G\}$,

> where G is the set of all M-variable assignments.

We define the other sentential connectives and the quantifiers as in (13) below.

13. **a. Abbreviations – Connectives (anaphoric closure, disjunction and implication):**

$!\phi := \sim\sim\phi$ [6],

> i.e. $\|!\phi\| = \{<g,\ h>:\ g=h\ \text{and}\ g\in \mathbf{Dom}(\|\phi\|)\}$ [7]

$\phi \vee \psi := \sim(\sim\phi;\ \sim\psi)$,

> i.e. $\|\phi \vee \psi\| = \{<g,\ h>:\ g=h\ \text{and}\ g\in \mathbf{Dom}(\|\phi\|)\cup\mathbf{Dom}(\|\psi\|)\}$

$\phi \rightarrow \psi := \sim(\phi;\ \sim\psi)$,

> i.e. $\|\phi \rightarrow \psi\| = \{<g,\ h>:\ g=h\ \text{and for any}\ k\ \text{s.t.}\ \|\phi\|<g,\ k> = T,$
> there is an l s.t. $\|\psi\|<k,\ l> = T\}$ [8],

> i.e. $\|\phi \rightarrow \psi\| = \{<g,\ h>:\ g=h\ \text{and}\ (\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)\}$,
> where $(\phi)^g := \{h:\ \|\phi\|<g,\ h> = T\}$

b. Abbreviations – quantifiers (existential, universal, multiple random assignment):

$\exists x(\phi) := [x];\ \phi$

$\forall x(\phi) := \sim([x];\ \sim\phi)$,

> i.e. $[x] \rightarrow \phi$ or, equivalently, $\sim\exists x(\sim\phi)$,

> i.e. $\|\forall x(\phi)\| = \{<g,\ h>:\ g=h\ \text{and}$

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[6] I use the symbol '!' for closure, as in van den Berg (1996b) and unlike Groenendijk & Stokhof (1991), who use '◊'.

[7] The connective '!' is labeled 'anaphoric closure' because, when applied to a formula ϕ, it closes off the possibility of subsequent reference to any dref introduced in ϕ. This is because the input and the output assignments in the denotation of $!\phi$ are identical. The operator '!' is important because ϕ and $!\phi$ have the same truth-conditions – see the definition of truth in (11a), i.e. '!' can be said to factor out the truth-conditions of a dynamic formula.

[8] This is shown by the following equivalences:: $\|\phi \rightarrow \psi\|<g,\ h> = T$ iff $\|\sim(\phi;\ \sim\psi)\|<g,\ h> = T$ iff $g=h$ and there is no k s.t. $\|\phi;\ \sim\psi\|<g,\ k> = T$ iff $g=h$ and there is no k and no l s.t. $\|\phi\|<g,\ l> = T$ and $\|\sim\psi\|<l,\ k> = T$ iff $g=h$ and there is no k and no l s.t. $\|\phi\|<g,\ l> = T$ and $l=k$ and $l\notin \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and there is no k s.t. $\|\phi\|<g,\ k> = T$ and $k\notin \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and for any k s.t. $\|\phi\|<g,\ k> = T$, we have that $k\in \mathbf{Dom}(\|\psi\|)$ iff $g=h$ and for any k s.t. $\|\phi\|<g,\ k> = T$, there is an l s.t. $\|\psi\|<k,\ l> = T$. Summarizing: $\|\phi \rightarrow \psi\|<g,\ h> = T$ iff $g=h$ and for any k s.t. $\|\phi\|<g,\ k> = T$, there is an l s.t. $\|\psi\|<k,\ l> = T$.

for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|<k, l> = \text{T}\}$[9],

i.e. $\|\forall x(\phi)\| = \{<g, h>: g=h \text{ and } ([x])^g \subseteq \textbf{Dom}(\|\phi\|)\}$

$$[x_1, …, x_n] := [x_1]; …; [x_n]$$

Given the definitions of dynamic negation '~' and closure '!', the equivalence in (14) below holds; (14) is very useful in proving that many equivalences of interest hold in DPL (e.g. the one in (15) below). Two formulas are equivalent, symbolized as '⇔', iff they denote the same set of variable assignments.

14. $\sim(\phi; \psi) \Leftrightarrow \sim(\phi; !\psi)$[10]

The equivalence in (15) below exhibits the limited extent to which the existential and universal quantifiers are duals[11]; this will prove useful, for example, when we try to determine the DPL translation of the English determiner *no*.

15. $\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$[12]

The practice of setting up abbreviations as opposed to directly defining various connectives and quantifiers might seem cumbersome, but it is useful in at least three ways. First, by setting up explicit abbreviations, we see exactly which component of the basic dynamic system does the work, e.g. we see that the universal 'effect' of universal

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[9] This is shown by the following equivalences: $\|\forall x(\phi)\|<g, h> = \text{T}$ iff $\|\sim([x]; \sim\phi)\|<g, h> = \text{T}$ iff $g=h$ and there is no k s.t. $\|[x]; \sim\phi\|<g, k> = \text{T}$ iff $g=h$ and there is no k and no l s.t. $\|[x]\|<g, l> = \text{T}$ and $\|\sim\phi\|<l, k>$ $= \text{T}$ iff $g=h$ and there is no k and no l s.t. $g[x]l$ and $l=k$ and $l\notin\textbf{Dom}(\|\phi\|)$ iff $g=h$ and there is no k s.t. $g[x]k$ and $k\in\textbf{Dom}(\|\phi\|)$ iff $g=h$ and for any k s.t. $g[x]k$, we have that $k\in\textbf{Dom}(\|\phi\|)$ iff $g=h$ and for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|<k, l> = \text{T}$. Summarizing: $\|\forall x(\phi)\|<g, h> = \text{T}$ iff $g=h$ and for any k s.t. $g[x]k$, there is an l s.t. $\|\phi\|<k, l> = \text{T}$.

[10] The equivalence holds because the following equalities hold (I use two abbreviations: $(\phi)^g := \{h: \|\phi\|<g, h> = \text{T}\}$ and $\textbf{Dom}(\|\phi\|) := \{g: \text{there is an } h \text{ s.t. } \|\phi\|<g, h> = \text{T}\})$:

$\|\sim(\phi; \psi)\| = \{<g, h>: g=h \text{ and } g\notin\textbf{Dom}(\|\phi; \psi\|)\} = \{<g, h>: g=h \text{ and it is not the case that there is a } k \text{ s.t. } \|\phi; \psi\|<g, k> = \text{T}\} = \{<g, h>: g=h \text{ and it is not the case that there is an } l \text{ and a } k \text{ s.t. } \|\phi\|<g, l> = \text{T} \text{ and } \|\psi\|<l, k> = \text{T}\} = \{<g, h>: g=h \text{ and there is no } l \text{ s.t. } \|\phi\|<g, l> = \text{T} \text{ and } l\in\textbf{Dom}(\|\psi\|)\} = \{<g, h>: g=h \text{ and } (\phi)^g\cap\textbf{Dom}(\|\psi\|)=\emptyset\} = \{<g, h>: g=h \text{ and } (\phi)^g\cap\textbf{Dom}(\|!\psi\|)=\emptyset\} = \{<g, h>: g=h \text{ and } g\notin\textbf{Dom}(\|\phi; !\psi\|)\} = \|\sim(\phi; !\psi)\|$.

[11] The other 'half' of the duality, i.e. $\exists x(\sim\phi) \Leftrightarrow \sim\forall x(\phi)$, clearly doesn't hold: using the terminology defined in (16), $\sim\forall x(\phi)$ is a test, while $\exists x(\sim\phi)$ isn't.

[12] $\sim\exists x(\phi) \Leftrightarrow \sim([x]; \phi) \Leftrightarrow$ (given (14)) $\sim([x]; !\phi) \Leftrightarrow \sim([x]; \sim\sim\phi) \Leftrightarrow \forall x(\sim\phi)$.

quantification $\forall x(\phi)$, just as the universal unselective binding 'effect' of implication $\phi \rightarrow \psi$, is in fact due to dynamic negation[13].

Second, distinguishing basic definitions and derived abbreviations will prove useful when we start generalizing the system in various ways. The official definition is the logical 'core' that undergoes modifications when we define extensions of DPL; the system of abbreviations, however, remains more or less constant across extensions. In this way, we are able to exhibit in a transparent way the commonalities between the various systems we consider and also between the analyses of natural language discourses and within these different systems.

Third, the abbreviations indicate explicitly the relation between the 'core' dynamic system and related systems (e.g. DRT). From this perspective, it is useful to add to the core layer of definitions in (11) above and the layer of abbreviations in (13) (which 'recovers' first-order logic) yet another and final layer of abbreviations that 'recovers' DRT (Kamp 1981, Kamp & Reyle 1993).

2.2.   Discourse Representation Structures (DRS's) in DPL

To this end, I define the semantic notion of *test* and the corresponding syntactic notion of *condition* in (16) and (17) below (see Groenendijk & Stokhof (1991): 57-58, Definitions 11 and 12). The relation between them is stated in (18) (see Groenendijk & Stokhof (1991): 58, Fact 6).

16. A wff ϕ is a *test* iff $\|\phi\| \subseteq \{<g, g>: g \in G\}$, where G is the set of all M-variable assignments,

    i.e., in our terms, a wff ϕ is a *test* iff $\|\phi\| \subseteq \|[\ ]\|$ [14].

17. The set of *conditions* is the smallest set of wff's containing atomic formulas, [ ], negative formulas (i.e. formulas whose main connective is dynamic negation '$\sim$'[15]) and closed under dynamic conjunction.

---

[13] See the observations in van den Berg (1996b): 6, Section **2.3**.

[14] Note that $\phi \Leftrightarrow !\phi$ iff ϕ is a test; see Groenendijk & Stokhof (1991): 62.

18. ϕ is a test iff ϕ is a condition or a contradiction (ϕ is a *contradiction* iff $\|\phi\| = \emptyset$)

We indicate that a formula is a condition by placing square brackets around it.

19. **Conditions:**

> $[\phi]$ is defined iff ϕ is a *condition*; when defined, $[\phi] := \phi$
>
> $[\phi_1, \ldots, \phi_m] := [\phi_1]; \ldots; [\phi_m]$

We can now define a Discourse Representation Structure (DRS) or linearized 'box' as follows:

20. **Discourse Representation Structures (DRS's), a.k.a. linearized 'boxes':**

> $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := [x_1, \ldots, x_n]; [\phi_1, \ldots, \phi_m]$,
>
> equivalently: $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := \exists x_1 \ldots \exists x_n([\phi_1, \ldots, \phi_m])$.

That is, $[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m]$ is defined iff $\phi_1, \ldots, \phi_m$ are conditions and, if defined:

$$\| [x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] \| := \{ <g, h>: g[x_1, \ldots, x_n]h \text{ and}$$
$$\| \phi_1 \| <h, h> = T \text{ and } \ldots \| \phi_m \| <h, h> = T \}$$

3. Anaphora in DPL

The benefit of setting up this system of abbreviations becomes clear as soon as we begin translating natural language discourses into DPL.

3.1. Cross-sentential Anaphora

Consider again discourse (1-2) above, repeated in (21-22) below.

21. A^x house-elf fell in love with a^y witch.
22. He$_x$ bought her$_y$ an^z alligator purse.

---

[15] Note that, given our abbreviations in (13) above, the set of negative formulas includes closed formulas (i.e. formulas of the form '!ϕ'), disjunctions, implications and universally quantified formulas.

The representation of (21-22) in the unabbreviated system is provided in (23) below; the 'first-order'-style abbreviation is provided in (24) and the DRT-style abbreviation in (25).

23. [x]; *house_elf*(x); [y]; *witch*(y); *fall_in_love*(x, y);

    [z]; *alligator_purse*(z); *buy*(x, y, z)

24. $\exists x$(*house_elf*(x); $\exists y$(*witch*(y); *fall_in_love*(x, y)));

    $\exists z$(*alligator_purse*(z); *buy*(x, y, z))

25. [x, y | *house_elf*(x), *witch*(y), *fall_in_love*(x, y)];

    [z | *alligator_purse*(z), *buy*(x, y, z)]

3.2. Relative-clause Donkey Sentences

Consider now the relative-clause donkey sentence in (26) below (repeated from (4) above). The 'first-order'-style translation in terms of universal quantification and implication is provided in (27) and the DRT-style translation in (28). One way to see that the two translations are equivalent is to notice that both of them are equivalent to the formula in (29).

26. Every^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.

27. $\forall x$(*house_elf*(x); $\exists y$(*witch*(y); *fall_in_love*(x, y))

        $\rightarrow \exists z$(*alligator_purse*(z); *buy*(x, y, z)))

28. [x, y | *house_elf*(x), *witch*(y), *fall_in_love*(x, y)]

        $\rightarrow$ [z | *alligator_purse*(z), *buy*(x, y, z)]

29. [x]; *house_elf*(x); [y]; *witch*(y); *fall_in_love*(x, y)

        $\rightarrow$ [z]; *alligator_purse*(z); *buy*(x, y, z)

Moreover, the three translations in (27), (28) and (29) are all equivalent (in DPL) to the formula in (30) below, which is the formula that assigns sentence (26) the intuitively correct truth-conditions when interpreted as in classical first-order logic.

30. $\forall x \forall y$(*house_elf*(x); *witch*(y); *fall_in_love*(x, y)

        $\rightarrow \exists z$(*alligator_purse*(z); *buy*(x, y, z)))

As already noted, the formulas in (27) through (30) are equivalent because DPL validates the equivalence in (10) above, i.e. $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$[16].

3.3. Conditional Donkey Sentences

Finally, the conditional donkey sentence in (31) below (repeated from (6)) is truth-conditionally equivalent to the relative clause donkey sentence in (26), as shown by the fact that they receive the same DRT-style translation, which is provided in (32) below. The 'first-order'-style compositional translation – equivalent to the DRT-style translation and all the other formulas listed above – is given in (33).

31. If a^x house-elf falls in love with a^y witch, he$_x$ buys her$_y$ an^z alligator purse.

32. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$

$\rightarrow [z \mid alligator\_purse(z), buy(x, y, z)]$

33. $\exists x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)))$

$\rightarrow \exists z(alligator\_purse(z); buy(x, y, z))$

I conclude this section with the DPL analysis of two negative donkey sentences.

34. No^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.

35. If a^x house-elf falls in love with a^y witch, he$_x$ never buys her$_y$ an^z alligator purse.

If we follow the canons of classical first-order logic in translating sentence (34), we have a choice between a combination of negation and existential quantification and a combination of negation and universal quantification. But the limited duality exhibited by existential and universal quantification in DPL (see (15) above) is of help here. To see this, note first that the duality can be generalized to the equivalence in (36) below.

36. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$ [17,18]

---

[16] $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$ iff $([x]; \phi) \rightarrow \psi \Leftrightarrow \sim([x]; \sim(\phi \rightarrow \psi))$ iff $\sim(([x]; \phi); \sim\psi) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; \sim\sim(\phi; \sim\psi))$ iff $\sim([x]; (\phi; \sim\psi)) \Leftrightarrow \sim([x]; !(\phi; \sim\psi))$. The last equivalence holds because it is an instance of the more general equivalence $\sim(\phi; \psi) \Leftrightarrow \sim(\phi; !\psi)$ (see (14) above).

[17] The equivalence holds because: $\sim\exists x(\phi; \psi) \Leftrightarrow$ (by (15)) $\forall x(\sim(\phi; \psi)) \Leftrightarrow$ (by (14)) $\forall x(\sim(\phi; !\psi)) \Leftrightarrow \forall x(\sim(\phi; \sim\sim\psi)) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi)$.

Now, given that the equivalence in (36) holds, we can translate sentence (34) either way, as shown in (37) and (38). Moreover, both translations are equivalent to the formula in (39), which explicitly shows that we quantify universally over all pairs of house-elves and witches standing in the 'fall in love' relation.

37. ~∃*x*(*house_elf*(*x*); ∃*y*(*witch*(*y*); *fall_in_love*(*x*, *y*));

      ∃*z*(*alligator_purse*(*z*); *buy*(*x*, *y*, *z*)))

38. ∀*x*(*house_elf*(*x*); ∃*y*(*witch*(*y*); *fall_in_love*(*x*, *y*))

      → ~∃*z*(*alligator_purse*(*z*); *buy*(*x*, *y*, *z*)))

39. ∀*x*∀*y*(*house_elf*(*x*); *witch*(*y*); *fall_in_love*(*x*, *y*)

      → ~∃*z*(*alligator_purse*(*z*); *buy*(*x*, *y*, *z*)))

Consider now sentence (35). There is a compositional DPL translation for it, which becomes apparent as soon as we consider the intuitively equivalent English sentence in (40) below. Both sentence (35) and sentence (40) are compositionally translated as in (41).

40. If a^x house-elf falls in love with a^y witch, he$_x$ doesn't buy her$_y$ an^z alligator purse.

41. ∃*x*(*house_elf*(*x*); ∃*y*(*witch*(*y*); *fall_in_love*(*x*, *y*)))

      → ~∃*z*(*alligator_purse*(*z*); *buy*(*x*, *y*, *z*))

It is easily seen that the DPL translations capture the fact that the English sentences in (34), (35) and (40) are intuitively equivalent.

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[18] The equivalence ~∃*x*(ϕ; ψ) ⇔ ∀*x*(ϕ → ~ψ) in (36) is a generalization of the equivalence ~∃*x*(ϕ) ⇔ ∀*x*(~ϕ) in (15) expressing the partial duality of the two quantifiers because we can obtain (15) from (36) by inserting [ ] in the place of ϕ in (36). In particular, the two equivalences in (i) and (ii) below hold:

(i) [ ]; ϕ ⇔ ϕ,   hence ~∃*x*([ ]; ϕ) ⇔ ~∃*x*(ϕ)
(ii) [ ] → ~ϕ ⇔ ~([ ]; ~~ϕ) ⇔ ~~~ϕ ⇔ ~ϕ,   hence ∀*x*([ ] → ~ϕ) ⇔ ∀*x*(~ϕ)

Moreover, we have (by (36)) that ~∃*x*([ ]; ϕ) ⇔ ∀*x*([ ] → ~ϕ); it follows that ~∃*x*(ϕ) ⇔ ∀*x*(~ϕ), i.e. (15), holds.

4. Extending DPL with Unselective Generalized Quantification

As the translations of the *every-* and *if-*examples in (26) and (31) above indicate, there is a systematic correspondence in DPL between the generalized quantifier *every* and the unselective implication connective[19]. The same point is established by the equivalence of the DPL translations of the *no-* and *never-*examples in (34) and (35). The correspondence between *every* and implication is concisely captured by the equivalence in (42) (which is none other than the equivalence we mentioned at the beginning of the previous section – see (10) above).

42. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi$ [20]

Moreover, as indicated in (13a) above, when interpreted relative to an input assignment g, the implication connective $\phi \rightarrow \psi$ boils down to an inclusion relation between two sets of assignments: $(\phi)^g \subseteq \mathbf{Dom}(\|\psi\|)$, where $(\phi)^g := \{h: \|\phi\| <g, h> = T\}$[21] and $\mathbf{Dom}(\|\psi\|) := \{h: \text{ there is a } k \text{ s.t. } \|\phi\| <h, k> = T\}$. The inclusion relation between the two sets is precisely the relation expressed by the static generalized quantifier **EVERY** when applied to the two sets in question, i.e. **EVERY**$((\phi)^g, \mathbf{Dom}(\|\psi\|))$. We can therefore give an alternative definition of implication using the static quantifier **EVERY**, as shown in (43) below.

43. $\|\phi \rightarrow \psi\| = \{<g, h>: g=h \text{ and } \mathbf{EVERY}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

  where **EVERY** is the usual static generalized quantifier.

Putting together (42) and (43), we obtain a definition of the natural language quantifier *every* as a binary operator over two DPL formulas:

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[19] Implication is unselective basically because it is a sentential connective.

[20] $\|\forall x(\phi \rightarrow \psi)\| <g, h> = T$ iff $\|[x] \rightarrow (\phi \rightarrow \psi)\| <g, h> = T$ iff $g=h$ and for any k s.t. $\|[x]\| <g, k> = T$, there is an l s.t. $\|\phi \rightarrow \psi\| <k, l> = T$ iff $g=h$ and for any k s.t. $g[x]k$, there is an l s.t. $k=l$ and for any k' s.t. $\|\phi\| <k, k'> = T$, there is an l' s.t. $\|\psi\| <k', l'> = T$ iff $g=h$ and for any k and k' s.t. $g[x]k$ and $\|\phi\| <k, k'> = T$, there is an l s.t. $\|\psi\| <k', l> = T$ iff $g=h$ and for any k s.t. $\|[x]; \phi\| <g, k> = T$, there is an l s.t. $\|\psi\| <k, l> = T$ iff $\|([x]; \phi) \rightarrow \psi\| <g, h> = T$.

[21] That is, $(\phi)^g$ is the image of the singleton set $\{g\}$ under the relation $\|\phi\|$.

44. $\lVert \mathbf{every}_x(\phi, \psi) \rVert = \{<g, h>: g=h \text{ and } \mathbf{EVERY}(([x]; \phi)^g, \mathbf{Dom}(\lVert \psi \rVert))\}$

It is easily checked that the equivalence in (42) can be extended as shown in (45) below.

45. $\forall x(\phi \rightarrow \psi) \Leftrightarrow ([x]; \phi) \rightarrow \psi \Leftrightarrow \mathbf{every}_x(\phi, \psi)$

This equivalence shows that the operator $\mathbf{every}_x(\phi, \psi)$ can be successfully used to translate donkey sentences with *every* and assign them the intuitively correct truth-conditions. The 'in love house-elf' example and its DPL translation are repeated in (46) and (47) below. The equivalent translation based on the binary **every** operator is provided in (48).

46. Every^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.

47. $\forall x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y))$

$\rightarrow \exists z(alligator\_purse(z); buy(x, y, z)))$

48. $\mathbf{every}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$

$\exists z(alligator\_purse(z); buy(x, y, z)))$

We can define in a similar way a binary operator over DPL formulas $\mathbf{no}_x(\phi, \psi)$.

49. $\lVert \mathbf{no}_x(\phi, \psi) \rVert = \{<g, h>: g=h \text{ and } \mathbf{NO}(([x]; \phi)^g, \mathbf{Dom}(\lVert \psi \rVert))\}$,

i.e. $\lVert \mathbf{no}_x(\phi, \psi) \rVert = \{<g, h>: g=h \text{ and } ([x]; \phi)^g \cap \mathbf{Dom}(\lVert \psi \rVert)=\varnothing\}$

It is easily checked that the equivalence in (36) above extends as shown in (50).

50. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi) \Leftrightarrow \mathbf{no}_x(\phi, \psi)$

Consequently, we can translate sentence (34), repeated as (51), as shown in (52) below.

51. No^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.

52. $\mathbf{no}_x(house\_elf(x); \exists y(witch(y); fall\_in\_love(x, y)),$

$\exists z(alligator\_purse(z); buy(x, y, z)))$

4.1. Dynamic Unselective Generalized Quantification

The definitions of **every** and **no** in (44) and (49) and the way in which these operators are used to translate the English sentences in (48) and (51) suggest a way to add generalized quantification to DPL so that we can analyze donkey sentences like (53) and (54) below.

53. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.
54. Few^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.

Let's first define the family of unselective binary operators **det** [22].

55. $\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g=h$ and $\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

where **DET** is the corresponding static determiner [23].

The fact that the **det** sentential operators are *unselective* is semantically reflected in the fact that they express generalized quantification between two sets of *info states* (a.k.a. variable assignments), namely $(\phi)^g$ and $\mathbf{Dom}(\|\psi\|)$. And this will bring their downfall: it is their unselectivity (i.e. generalized quantification over info states) that leads them straight into the proportion problem and makes them incapable of accounting for the ambiguity between weak and strong donkey readings. But before we come to that, we need a couple more definitions.

First, note that a formula of the form **det**(ϕ, ψ) is a test. So, we should also extend our syntactic notion of *condition* defined for DPL in (17) above. The revised definition is:

56. The set of *conditions* is the smallest set of wff's containing atomic formulas, formulas whose main connective is dynamic negation '~' or a **det** operator and closed under dynamic conjunction.

The revised definition in (56) enables us to construct DRS's of the form [… | …, **det**(ϕ, ψ), …].

---

[22] Again, note that they are unselective because they are essentially *sentential* operators.

[23] Given that $\mathbf{Dom}(\|\psi\|)=\mathbf{Dom}(\|!\psi\|)$, it is a direct consequence of this definition that $\mathbf{det}(\phi, \psi) \Leftrightarrow \mathbf{det}(\phi, !\psi)$.

The natural language generalized determiners are defined in terms of the unselective **det** operators, as shown in (57) below.

57. **det**$_x(\phi, \psi)$:= **det**$([x]; \phi, \psi)$

The determiners **every**$_x(\phi, \psi)$ and **no**$_x(\phi, \psi)$, i.e. the **every** and **no** instances of the general definition in (57), are none other than the determiners directly defined in (44) and (49) above. The generalized determiners defined in this way are still unselective, despite the presence of the variable x: the variable x in **det**$_x$ is only meant to indicate the presence of the additional update $[x]$, but the basic operator is still the unselective **det**. That is, we still determine the denotation of **det**$_x(\phi, \psi)$ by checking whether the static determiner **DET** applies to two sets of info states – and not to two sets of individuals.

Let us make explicit the connections with the previous literature before turning to some examples. First, the definition of **det**(ϕ, ψ) in (55) above is just the definition of quantificational adverbs in Groenendijk & Stokhof (1991): 81-82, which follows Lewis (1975) in taking adverbs to quantify over *cases* – i.e. information states (in dynamic terms). For example, *never* is translated in Groenendijk & Stokhof (1991): 82 as the binary implication connective $\rightarrow_{no}$ and the definition of $\phi \rightarrow_{no} \psi$ is exactly the definition of **no**(ϕ, ψ).

The analysis can be extended in the obvious way to other adverbs of quantification, e.g. *always* can be interpreted as **every**(ϕ, ψ) (just like bare conditionals), *often* and *usually* as **most**(ϕ, ψ) and *rarely* as **few**(ϕ, ψ) – where the corresponding static determiners **MOST** and **FEW** are interpreted as more than half and less than half respectively.

Second, the definition of **det**$_x(\phi, \psi)$ is actually equivalent to the (implicit) definition of generalized quantification in Kamp (1981) and Heim (1982/1988).

A welcome consequence of defining **det**$_x$ in terms of **det** (as in (57) above) is that the systematic natural language correspondence between adverbs of quantification and generalized quantifiers, e.g. the correspondence between *no* and *never* in examples (34) and (35) above, is explicitly captured.

4.2.　Limitations of Unselectivity: Proportions

Another seemingly welcome consequence is that we can now provide an analysis of donkey sentences with *most* and *few* that can capture the anaphoric connections between the indefinites in the restrictor and the pronouns in the nuclear scope, as shown below in (59) and (62) ('predicate logic'-style) and (60) and (63) (DRT-style).

58. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.

59. **most**$_x$($house\_elf(x)$; $\exists y(witch(y)$; $fall\_in\_love(x, y))$,

　　　　$\exists z(alligator\_purse(z)$; $buy(x, y, z)))$

60. **most**$_x$($[y \mid house\_elf(x)$, $witch(y)$, $fall\_in\_love(x, y)]$,

　　　　$[z \mid alligator\_purse(z)$, $buy(x, y, z)])$

61. Few^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.

62. **few**$_x$($house\_elf(x)$; $\exists y(witch(y)$; $fall\_in\_love(x, y))$,

　　　　$\exists z(alligator\_purse(z)$; $buy(x, y, z)))$

63. **few**$_x$($[y \mid house\_elf(x)$, $witch(y)$, $fall\_in\_love(x, y)]$,

　　　　$[z \mid alligator\_purse(z)$, $buy(x, y, z)])$

The unselective analysis is successful in capturing the donkey anaphoric connections, but it is not successful in capturing the intuitively correct truth-conditions. As shown in Partee (1984)[24], Rooth (1987), Kadmon (1987) and Heim (1990), the analysis has a proportion problem[25].

Consider sentence (1) above and its DRT-style representation in (60). It is easy to see that the representation does not capture the intuitively correct truth-conditions if we examine the equivalent formula in (64) below.

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[24] "[…] when we have to deal with quantification with a complicated and possibly uncertain underlying ontology, we need to specify a 'sort' (for the quantifier to 'live on' in the sense of Barwise & Cooper 1981) separately from whatever further restrictions we want to add (perhaps in terms of 'cases') about which instances of the sort we are quantifying over. In terms of Kamp's framework this means that we have to worry not only about what belongs in the antecedent box but also how to distinguish a substructure within it that plays the role of sortal (the head noun in the NP case)." (Partee 1984: 278).

[25] The 'proportion problem' terminology is due to Kadmon (1987): 312.

64. **most**([*x, y | house_elf*(*x*), *witch*(*y*), *fall_in_love*(*x, y*)],

     [*z | alligator_purse*(*z*), *buy*(*x, y, z*)]])

The representation in (64) makes clear that we are quantifying over most pairs <*x, y*> where *x* is a house-elf that fell in love with a witch *y*. For most such pairs <*x, y*>, the requirement in the nuclear scope, i.e. *x* bought *y* some alligator purse *z*, should be satisfied.

However, following Partee (1984), Rooth (1987), Kadmon (1987) and Heim (1990), we can produce a scenario in which the English sentence in (1) is intuitively false while the formula in (64) is true: imagine that there are ten house-elves that fell in love with some witch or other; one of them, call him Dobby, is a Don Juan of sorts, he fell in love with more than one thousand witches[26] and he bought them all alligator purses; the other nine house-elves are less exceptional: they each fell in love with only one witch and they bought them new brooms, not alligator purses.

Sentence (1) is intuitively false in this scenario, while formula (64) is true: all the Dobby-based pairs that satisfy the restrictor also satisfy the nuclear scope – and these pairs are more than half, i.e. *most*, of the pairs under consideration.

4.3. Limitations of Unselectivity: Weak / Strong Ambiguities

In addition, the unselective analysis of generalized quantifiers fails to account for the fact that the same donkey sentence can exhibit two different readings, a *strong* one and a *weak* one. Consider again the classical sentence in (65) below.

65. Every^x farmer who owns a^y donkey beats it$_y$.

The most salient reading of this sentence is that every farmer behaves violently towards *each and every* one of his donkeys, i.e. the so-called *strong* reading. The **every**$_x$ operator correctly captures this reading, as shown in (66) below; the equivalent formulas

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[26] To be more precise, one thousand and three witches only in Spain.

in (67) and (68) are provided because they display the 'strength' of the reading in a clearer way.

66. **every**$_x$([y | *farmer*(x), *donkey*(y), *own*(x, y)], [*beat*(x, y)])

67. **every**([x, y | *farmer*(x), *donkey*(y), *own*(x, y)], [*beat*(x, y)])

68. $\forall x \forall y$(*farmer*(x); *donkey*(y); *own*(x, y) $\rightarrow$ *beat*(x, y))

However, sentence (65) can receive another, *weak* reading, wherein every farmer beats *some* donkey that he owns, but not necessarily each and every one of them[27]. Chierchia (1995): 64 provides a context in which the most salient reading is the weak one: imagine that the farmers under discussion are all part of an anger management program and they are encouraged by the psychotherapist in charge to channel their aggressiveness towards their donkeys (should they own any) rather than towards each other. The farmers scrupulously follow the psychotherapist's advice – in which case we can assert (65) even if the donkey-owning farmers beat only some of their donkeys.

Furthermore, there are donkey sentences for which the *weak* reading is the most salient one:

69. Every person who has a dime will put it in the meter.
    (Pelletier & Schubert 1989)

70. Yesterday, every person who had a credit card paid his bill with it.
    (R. Cooper, apud Chierchia 1995: 63, (3a))

Thus, both readings seem to be *semantically* available[28] and the unselective analysis of dynamic generalized quantifiers does not allow for both of them.

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[27] Partee (1984) seems to be (one of) the first to notice weak donkey readings: the example in (i) below is from Partee (1984): 280, fn. 12.

(i) If you have a credit card, you should use it here instead of cash.

[28] See for example the discussion in Chierchia (1995): 62-65, in particular the argument that the strong reading is not a conversational implicature triggered in certain contexts.

The weak/strong ambiguity also provides an argument against the unselective analysis of conditionals and adverbs of quantification, as shown, for example, by (71) below.

71. If a^x farmer owns a^y donkey, he$_x$ (always/usually/often/rarely/never) beats it$_y$.

For a detailed discussion of such conditionals, see (among others) Chierchia (1995): 66-69. I will only mention the generalization reached in Kadmon (1987) and summarized in Heim (1990): 153: "Kadmon's generalization is that a multi-case conditional with two indefinites in the antecedent generally allows three interpretations: one where the QAdverb quantifies over pairs, one where it quantifies over instances of the first indefinite and one where it quantifies over instances of the second".

It should be mentioned, however, that a partial solution to the problem posed by the existence of the weak donkey readings is available in classical DRT / FCS (Kamp 1981 and Heim 1982/1988) and DPL: Groenendijk & Stokhof (1991): 89 point out that we can define an alternative implication connective, as shown in (72) below.

72. $\phi \mapsto \psi := \sim\phi \vee (\phi; \psi)$,

i.e. $\|\phi \mapsto \psi\| = \{<g, h>: g=h$ and $g \notin \mathbf{Dom}(\|\phi\|)$ or $(\phi)^g \cap \mathbf{Dom}(\|\psi\|) \neq \emptyset\}$

i.e. $\|\phi \mapsto \psi\| = \{<g, h>: g=h$ and $g \notin \mathbf{Dom}(\|\phi\|)$ or $(\phi; !\psi)^g \neq \emptyset\}$[29].

The weak reading of sentence (73) (repeated from above) is presumably analyzed as shown in (74), which is 'unpacked' in the equivalent (75). The strong reading is given in (76) and (77) for ease of comparison.

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[29] Note that an alternative definition could simply be: $\phi \mapsto \psi := !(\phi; !\psi)$, i.e. $\|\phi \mapsto \psi\| = \{<g, h>: g=h$ and $(\phi; !\psi)^g \neq \emptyset\}$. The difference between this definition and the one in (72) is that this one removes the first disjunct $g \notin \mathbf{Dom}(\|\phi\|)$ (hence, it is more restrictive). Arguably, this is a justified move, since a conditional or a universal quantification have an 'existential' presupposition: there is a presupposition that the antecedent of the conditional, respectively the restrictor of the universal quantification, are *satisfiable* with respect to the current input info state g, i.e. that $g \in \mathbf{Dom}(\|\phi\|)$.

However, given the DPL definition of the universal quantifier $\forall$, this definition would yield the incorrect truth conditions for weak reading of sentence (73) if (73) were represented as in (74): the reading would in fact be a lot 'stronger' than intended – all individuals are required to be farmers and to have some donkey or other that they beat.

73. Every^x farmer who owns a^y donkey beats it$_y$.

74. **weak reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \mapsto beat(x, y))$

75. **weak reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \mapsto [beat(x, y)])$

76. **strong reading**: $\forall x(farmer(x); \exists y(donkey(y); own(x, y)) \rightarrow beat(x, y))$

77. **strong reading**: $[x] \rightarrow ([y \mid farmer(x), donkey(y), own(x, y)] \rightarrow [beat(x, y)])$

However, this analysis of weak implication faces (at least) three problems. First, as we can see from the 'unpacked' formula in (75), we still need the 'strong' implication connective $\rightarrow$ in addition to the 'weak' one $\mapsto$ to capture the correct truth-conditions for the weak reading of sentence (73), i.e. the weak reading is obtained via a combination of 'strong' and 'weak' implication.

Consequently, this solution fails to extend to weak readings of conditionals: as argued by Kadmon, the conditional in (78) below can receive a weak reading that is equivalent to the weak reading of the *every* donkey sentence in (73) above. However, this reading is not captured by the formula in (79), precisely because the equivalence $\exists x(\phi) \mapsto \psi \Leftrightarrow \forall x(\phi \mapsto \psi)$ fails for 'weak' implication – and we *do* want it to fail with respect to the indefinite a^y *donkey*, but *not* with respect to the indefinite a^x *farmer*.

78. If a^x farmer owns a^y donkey, he$_x$ beats it$_y$.

79. $\exists x(farmer(x); \exists y(donkey(y); own(x, y))) \mapsto beat(x, y)$

Second, the 'weak' implication solution does not generalize to other determiners (consider for example *most*). Third, it does not account for the proportion problem.

In sum, upon closer examination, a donkey sentence turns out to be ambiguous between a weak and a strong reading. The strong reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with an *every* DP. The weak reading is intuitively paraphrasable by replacing the donkey pronoun in the nuclear scope of the donkey quantification with a *some* DP.

Extending DPL with an *unselective* form of generalized quantification fails to account for the weak / strong donkey ambiguity and for the proportion problem – hence the need to further extend DPL with a *selective* form of dynamic generalized quantification.

4.4. Conservativity and Unselective Quantification

A final observation before turning to this task: defining dynamic **det**'s in terms of static **DET**'s (as we did in (55) and (57) above) provides us with a version of *unselective dynamic conservativity* that underlies the definition of selective generalized quantification introduced in the next section. Consider again the definition in (55) above: $\|\mathbf{det}(\phi, \psi)\|$ = {<g, h>: $g=h$ and $\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))$}. Assuming that the static determiner **DET** is conservative, we have that $\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))$ holds iff $\mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|))$ holds.

The latter formula encodes an intuitively appealing meaning for unselective dynamic generalized quantification[30]: a dynamic generalized determiner relates two sets of info states, the first of which is the set of output states compatible with the restrictor, i.e. $(\phi)^g$, while the second one is the set of output states compatible with the restrictor that can be further updated by the nuclear scope, i.e. $(\phi)^g \cap \mathbf{Dom}(\psi)$.

To reformulate this intuition in a more formal way, note that the formula $\mathbf{DET}((\phi)^g, (\phi)^g \cap \mathbf{Dom}(\|\psi\|))$, which has conservativity built-in, is equivalent to $\mathbf{DET}((\phi)^g, (\phi; !\psi)^g))$. Thus, assuming that all static generalized determiners **DET** are conservative, we can restate the definition in (55) above as follows:

80. **Built-in unselective dynamic conservativity:**

$\|\mathbf{det}(\phi, \psi)\|$ = {<g, h>: $g=h$ and $\mathbf{DET}((\phi)^g, (\phi; !\psi)^g))$}

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[30] This has been previously noted with respect to the dynamic definition of *selective* generalized quantification – see for example Chiechia (1992, 1995) and Kamp & Reyle (1993) among others.

Now, putting together the definition of $\mathbf{det}_x(\phi, \psi)$ in (57), i.e. $\mathbf{det}_x(\phi, \psi):=\mathbf{det}([x]; \phi, \psi)$, and the 'conservative' definition in (80), we obtain the following definition of generalized quantification:

81. **Generalized quantification with built-in dynamic conservativity (unselective version):**

$$\|\mathbf{det}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{DET}(([x]; \phi)^g, ([x]; \phi; !\psi)^g))\}$$

The definition of *conservative* unselective quantification in (81) can in fact be thought of as the basis for the definition of selective generalized quantification introduced in Chierchia (1995) among others (see section **5** below): given that we have access to the variable x in both the restrictor of the static determiner **DET**, i.e. $[x]; \phi$, and in its nuclear scope, i.e. $[x]; \phi; !\psi$, we can be *selective* and (somehow) λ-abstract over the variable x in both formulas. We will consequently obtain two sets of *individuals* and we will require the static determiner **DET** to apply to these two sets individuals and not to the corresponding sets of info states[31].

5. Extending DPL with Selective Generalized Quantification (DPL+GQ)

The notion of *selective* generalized quantification introduced in this section has been proposed in various guises by many authors: Bäuerle & Egli (1985), Root (1986) and Rooth (1987) put forth the basic proposal and van Eijck & de Vries (1992) and Chierchia (1992, 1995) were the first to formulate it in DPL terms. The proposal is also adopted in Heim (1990) and Kamp & Reyle (1993)[32].

In defining it, I will use the notation introduced above, i.e. selective dynamic generalized quantification will have the form $\mathbf{det}_x(\phi, \psi)$, where x is the bound variable, ϕ is the restrictor and ψ is the nuclear scope. Of course, since $\mathbf{det}_x(\phi, \psi)$ is selective, it will

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[31] It might be interesting to pursue in more detail the relation between unselective dynamic conservativity as defined in (80) above and selective dynamic conservativity as defined and argued for in Chierchia (1995): 97 et seqq and Kanazawa (1994a, b).

[32] The particular form of the definition of selective generalized quantification I provide here is based on the one in van den Berg (1994): 4 and van den Berg (1996b): 7.

be directly defined, i.e. it won't be an abbreviation of a formula containing the unselective operator **det**(ϕ, ψ), and it will involve a relation between two sets of individuals.

5.1. Dynamic Selective Generalized Quantification

The fact that **det**$_x$(ϕ, ψ) is defined in terms of sets of *individuals* (and not of info states) will enable us to account for the proportion problem. The weak/strong donkey ambiguity will be attributed to an ambiguity in the interpretation of the selective generalized quantifier, basically following the proposals in Bäuerle & Egli (1985), Rooth (1987), Reinhart (1987), Heim (1990) and Kanazawa (1994a, b)[33].

That is, for each dynamic generalized determiner, we will have a *weak* lexical entry **det**$^{wk}_x$(ϕ, ψ) and a *strong* lexical entry **det**$^{str}_x$(ϕ, ψ). An English sentence containing a determiner *det* is ambiguous between the two readings – or, to put it in more appealing terms, any English determiner is underspecified with respect to one of the two readings.

The choice of a particular, fully specified lexical entry for any *det* is determined in each particular instance by a variety of factors, including world-knowledge, information structure, monotonicity of quantifiers etc.

The basic dynamic analysis does not have anything to say about how the choice between the weak and the strong reading depends on such factors – and, arguably, it shouldn't have anything to say about how the choice is made given that:

- which reading is selected in each particular case is influenced by a *diversity* of factors;
- the generalizations correlating these factors and the weak/strong readings have a *defeasible* character typically associated with pragmatic phenomena[34].

The determiners **det**^{wk} and **det**^{str} are both defined in terms of the corresponding static determiner **DET** as follows[35]:

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[33] Strictly speaking, Kanazawa (1994a, b) does not endorse an *ambiguity* analysis, but a *vagueness* account of dynamic selective generalized quantification. See also the discussion in Geurts (2002): 149 et seqq.

[34] For more details, see the section **6.1** in chapter **5** below.

82. $\| \mathbf{det}^{wk}_x(\phi, \psi) \| = \{<g, h>: g=h \text{ and } \mathbf{DET}(\lambda x. (\phi)^g, \ \lambda x. (\phi; \psi)^g)\}$

$\| \mathbf{det}^{str}_x(\phi, \psi) \| = \{<g, h>: g=h \text{ and } \mathbf{DET}(\lambda x. (\phi)^g, \ \lambda x. (\phi \to \psi)^g)\},$

where $(\phi)^g := \{h: \| \phi \| <g, h> = T\}$

and $\lambda x. (\phi)^g := \{h(x): h \in ([x]; \phi)^g\}$[36]

and **DET** is the corresponding static determiner.

Several observations before we turn to an example: first, both lexical entries are selective in the sense that the static determiner **DET** relates two sets of individuals, represented by means of abbreviations of the form $\lambda x. (...)^g$. To my knowledge, this abbreviation has not been used in the previous literature despite its rather obvious and intuitive character.

Second, the only difference between the weak and the strong entries has to do with how the nuclear scope of the static quantification is obtained: we employ *dynamic conjunction* $\lambda x. (\phi; \psi)^g$ in the *weak* case and *dynamic implication* $\lambda x. (\phi \to \psi)^g$ in the *strong* case.

Dynamic conjunction yields the weak reading because an existential quantifier in the restrictor $\lambda x. (\phi)^g$ will still be an existential in the nuclear scope $\lambda x. (\phi; \psi)^g$: every farmer that owns *some* donkey beats *some* donkey he owns. Dynamic implication yields the strong reading because it has universal quantification built into it[37]: as we noticed right from the beginning (see (10) above), DPL validates the equivalence $\exists x(\phi) \to \psi \Leftrightarrow \forall x(\phi \to \psi)$, so an indefinite in the restrictor ends up being universally quantified in the nuclear scope: every farmer that owns *some* donkey beats *every* donkey he owns.

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[35] Note the formal similarities between:

- the alternative definition of implication in (72);

- the unselective generalized quantification with built-in conservativity defined in (80);

- the present definition of selective generalized quantification.

[36] Note that the abbreviation $\lambda x. (\phi)^g := \{h(x): h \in ([x]; \phi)^g\}$ really boils down to λ-abstraction in static terms: $\lambda x. (\phi)^g$ is the set of entities a s.t. $\| \phi \|_{static}^{g[x/a]} = T$, where $\| \cdot \|_{static}$ is the usual static interpretation function (see for example Gallin 1975).

[37] Incidentally, recall that the universal force of dynamic implication is actually due to dynamic negation '~' since $\phi \to \psi := {\sim}(\phi; {\sim}\psi)$.

Third, note that the unselective conservative entry defined in (81) above provides the basic format for the selective entries. In particular, assuming that, in (82) above, [x] is not reintroduced in ψ (and it cannot be if we want the definitions to work properly), it is always the case that:

83. $\lambda x.\,(\phi;\,\psi)^g = \lambda x.\,(\phi;\,!\psi)^g$ and $\lambda x.\,(\phi \to \psi)^g = \lambda x.\,(\phi \to !\psi)^{g\ 38}$.

More generally, the weak and strong selective generalized determiners in (82) above can be defined in terms of generalized quantification over info states if we make use of the closure operator '!' as shown in (84) below. It is easily checked that the two pairs of definitions are equivalent given the fact that there is a bijection between the sets of individuals quantified over in (82) and the set of info states (i.e. variable assignments) quantified over in (84)^{39}.

84. $\left\| \mathbf{det}^{wk}_x(\phi,\,\psi) \right\| = \{<g,\,h>\colon g=h \text{ and } \mathbf{DET}(([x\mid !\phi])^g,\,([x\mid !(\phi;\,\psi)])^g)\}$

$\left\| \mathbf{det}^{str}_x(\phi,\,\psi) \right\| = \{<g,\,h>\colon g=h \text{ and } \mathbf{DET}(([x\mid !\phi])^g,\,([x\mid !(\phi \to \psi)])^g)\}^{40}$,

where $(\phi)^g := \{h\colon \left\|\phi\right\|<g,\,h> = \mathrm{T}\}$

and $\mathbf{DET}$ is the corresponding static determiner.

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^{38} For dynamic implication $\to$, we have the more general result that $\phi \to \psi \Leftrightarrow \phi \to !\psi$, which follows directly from the equivalence in (14) above.

^{39} $\lambda x.\,(\phi)^g := \{h(x)\colon h\in ([x];\,\phi)^g\}$ = $\{a\colon$ there is an h s.t. $\left\|[x];\,\phi\right\|<g,\,h> = \mathrm{T}$ and $a=h(x)\}$

= $\{a\colon$ there is a k and an h s.t. $g[x]k$ and $\left\|\phi\right\|<k,\,h> = \mathrm{T}$ and $a=h(x)\}$

(since x is not reintroduced in ϕ, $k(x)=h(x)$)

= $\{a\colon$ there is a k and an h s.t. $g[x]k$ and $\left\|\phi\right\|<k,\,h> = \mathrm{T}$ and $a=k(x)\}$

= $\{a\colon$ there is a k s.t. $a=k(x)$ and $g[x]k$ and there is an h s.t. $\left\|\phi\right\|<k,\,h> = \mathrm{T}\ \}$

= $\{a\colon$ there is a k s.t. $a=k(x)$ and $g[x]k$ and $k\in \mathbf{Dom}(\left\|\phi\right\|)\}$ = $\{a\colon$ there is a k s.t. $k\in ([x];\,!\phi)^g$ and $a=k(x)\}$.

Thus, $\lambda x.\,(\phi)^g = \{a\colon$ there is a h s.t. $h\in ([x];\,!\phi)^g$ and $a=h(x)\}$. Let f be a function from the set of assignments $([x];\,!\phi)^g$ to the set of individuals $\lambda x.\,(\phi)^g$ s.t. $f(h)=h(x)$. By the above equality, f is surjective. Since for any assignment g and individual a there is a unique assignment h s.t. $g[x]h$ and $h(x)=a$, f is injective.

^{40} Since $!(\phi \to \psi) \Leftrightarrow \phi \to \psi$, the strong determiner can be more simply defined as $\left\| \mathbf{det}^{str}_x(\phi,\,\psi) \right\| = \{<g,\,h>\colon g=h \text{ and } \mathbf{DET}(([x\mid !\phi])^g,\,([x\mid \phi \to \psi])^g)\}$.

Finally, according to definition (82), a formula of the form $\mathbf{det}^{wk}_x(\phi, \psi)$ or $\mathbf{det}^{str}_x(\phi, \psi)$ is a test. So, we should further extend the syntactic notion of condition with selective generalized determiners[41]. The new definition is:

85. The set of *conditions* is the smallest set of wff's containing atomic formulas, formulas whose main connective is dynamic negation '~', a **det** operator or a $\mathbf{det}^{wk/str}_v$ operator (for any variable v) and closed under dynamic conjunction.

The definition in (85) enables us to construct DRS's of the form $[\ldots \mid \ldots, \mathbf{det}^{wk/str}_x(\phi, \psi), \ldots]$.

5.2. Accounting for Weak / Strong Ambiguities

Let us see how the above definitions derive the weak and strong readings of the classical example in (86) below (repeated from (65)).

86. Every^x farmer who owns a^y donkey beats it$_y$.

The two lexical entries for *every* are given in (87) below and simplified in (88).

87. $\|\mathbf{every}^{wk}_x(\phi, \psi)\| = \{<g, h>: g=h$ and $\mathbf{EVERY}(\lambda x. (\phi)^g, \lambda x. (\phi; \psi)^g)\}$

$\|\mathbf{every}^{str}_x(\phi, \psi)\| = \{<g, h>: g=h$ and $\mathbf{EVERY}(\lambda x. (\phi)^g, \lambda x. (\phi \to \psi)^g)\}$

88. $\|\mathbf{every}^{wk}_x(\phi, \psi)\| = \{<g, h>: g=h$ and $\lambda x. (\phi)^g \subseteq \lambda x. (\phi; \psi)^g\}$

$\|\mathbf{every}^{str}_x(\phi, \psi)\| = \{<g, h>: g=h$ and $\lambda x. (\phi)^g \subseteq \lambda x. (\phi \to \psi)^g\}$

The weak reading of (86) is represented in (89) and simplified in (90)[42].

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[41] Recall that the definition of conditions for DPL was given in (17) above and was extended with unselective determiners in (56).

[42] In more detail, the simplification proceeds as follows:

$\|\mathbf{every}^{wk}_x(farmer(x); [y]; donkey(y); own(x, y),\ beat(x, y))\| =$

$\{<g,g>: \lambda x. (farmer(x); [y]; donkey(y); own(x, y))^g \subseteq \lambda x. (farmer(x); [y]; donkey(y); own(x, y); beat(x, y))^g\}=$

$\{<g, g>: \{h(x): h \in ([x]; farmer(x); [y]; donkey(y); own(x, y))^g\} \subseteq$
$\quad\quad\quad \{h(x): h \in ([x]; farmer(x); [y]; donkey(y); own(x, y); beat(x, y))^g\}\} =$

$\{<g, g>: \{h(x): g[x, y]h, h(x) \in \mathbf{I}(farmer), h(y) \in \mathbf{I}(donkey), <h(x), h(y)> \in \mathbf{I}(own)\} \subseteq$
$\quad\quad\quad \{h(x): g[x, y]h, h(x) \in \mathbf{I}(farmer), h(y) \in \mathbf{I}(donkey), <h(x), h(y)> \in (\mathbf{I}(own) \cap \mathbf{I}(beat))\}\} =$

89. **every**$^{wk}_{x}$(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*), *beat*(*x*, *y*))

90. ‖**every**$^{wk}_{x}$(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*), *beat*(*x*, *y*))‖ =

{<*g*, *g*>: λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*))^{g} ⊆

λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*); *beat*(*x*, *y*))^{g}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆

{*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈ (*I*(*own*)∩*I*(*beat*))}}=

{<*g*, *g*>: any farmer *a* who owns a donkey *b* is s.t. he owns and beats a donkey *b'*}

As the simplification in (90) shows, the formula in (89) delivers the weak reading because the donkey-owning farmers do not have to beat *all* the donkeys they own – they only have to beat *some* of their donkeys.

The strong reading of (86) is represented in (91) and simplified in (92)[43].

{<*g*, *g*>: {*a*: there is a *b* s.t. *a*∈*I*(*farmer*), *b*∈*I*(*donkey*), <*a*, *b*>∈*I*(*own*)} ⊆
{*a*: there is a *b* s.t. *a*∈*I*(*farmer*), *b*∈*I*(*donkey*), <*a*, *b*>∈ (*I*(*own*)∩*I*(*beat*))}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆
{*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈ (*I*(*own*)∩*I*(*beat*))}} =

{<*g*, *g*>: any farmer *a* who owns a donkey *b* is such that he owns and beats a donkey *b'*}.

[43] In more detail, the simplification proceeds as follows:

‖**every**$^{str}_{x}$(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*), *beat*(*x*, *y*))‖ =

{<*g*, *g*>: λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*))^{g} ⊆ λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*) → *beat*(*x*, *y*))^{g}} =

{<*g*, *g*>: {*h*(*x*): *h*∈ ([*x*]; *farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*))^{g}} ⊆
{*h*(*x*): *h*∈ ([*x*]; (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*) → *beat*(*x*, *y*)))^{g}}} =

{<*g*, *g*>: {*h*(*x*): *g*[*x*, *y*]*h*, *h*(*x*)∈*I*(*farmer*), *h*(*y*)∈*I*(*donkey*), <*h*(*x*), *h*(*y*)>∈*I*(*own*)} ⊆
{*h*(*x*): *h*∈ ([*x*]; ~(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*); ~*beat*(*x*, *y*)))^{g}}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆
{*h*(*x*): there is a *k* s.t. *g*[*x*]*k* and ‖~(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*); ~*beat*(*x*, *y*))‖<*k*, *h*> = T}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆
{*h*(*x*): *g*[*x*]*h* and there is no *l* s.t. ‖*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*); ~*beat*(*x*, *y*)‖<*h*, *l*> = T}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆
{*h*(*x*): *g*[*x*]*h* and there is no *l* s.t. *h*[*y*]*l*, *l*(*x*)∈*I*(*farmer*), *l*(*y*)∈*I*(*donkey*), <*l*(*x*), *l*(*y*)>∈*I*(*own*), <*l*(*x*), *l*(*y*)>∉*I*(*beat*)}} =

{<*g*, *g*>: {*a*: *a*∈*I*(*farmer*) and there is a *b* s.t. *b*∈*I*(*donkey*) and <*a*, *b*>∈*I*(*own*)} ⊆
{*h*(*x*): *g*[*x*]*h* and for any *l*, if *h*[*y*]*l*, *l*(*x*)∈*I*(*farmer*), *l*(*y*)∈*I*(*donkey*), <*l*(*x*), *l*(*y*)>∈*I*(*own*), then <*l*(*x*), *l*(*y*)>∈*I*(*beat*)}} =

91. **every**$^{str}_x$(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*), *beat*(*x*, *y*))

92. ‖**every**$^{str}_x$(*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*), *beat*(*x*, *y*))‖ =

   {<*g*, *g*>: λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*))^g ⊆

   λ*x*. (*farmer*(*x*); [*y*]; *donkey*(*y*); *own*(*x*, *y*) → *beat*(*x*, *y*))^g} =

   {<*g*, *g*>: {*a*: *a*∈**I**(*farmer*) and there is a *b* s.t. *b*∈**I**(*donkey*) and <*a*, *b*>∈**I**(*own*)} ⊆

   {*a*: any *b* s.t. *a*∈**I**(*farmer*), *b*∈**I**(*donkey*), <*a*, *b*>∈**I**(*own*) is s.t. <*a*, *b*>∈**I**(*beat*)}} =

   {<*g*, *g*>: any farmer *a* who owns a donkey *b* beats any donkey *b′* that he owns}

As the simplification in (92) shows, the formula in (91) delivers the strong reading because the donkey-owning farmers have to beat *all* the donkeys they own.

5.3. Solving Proportions

Selective generalized quantification also solves the proportion problem. Consider again sentence (1), repeated in (93) below. The most salient reading of this sentence seems to be the strong one, represented in (94), just as the most salient reading of the structurally similar sentence in (95) is the weak one, represented in (96) below.

If the reader's intuitions about the 'strength' of (93) are not very sharp, s/he should consider sentence (97) instead (example (49) in Heim 1990: 162), whose most salient reading is indeed the strong one.

93. Most^x house-elves who fall in love with a^y witch buy her$_y$ an^z alligator purse.

94. **most**$^{str}_x$(*house_elf*(*x*); [*y*]; *witch*(*y*); *fall_in_love*(*x*, *y*),

   [*z*]; *alligator_purse*(*z*); *buy*(*x*, *y*, *z*))

95. Most^x drivers who have a^y dime will put it$_y$ in the meter.

96. **most**$^{wk}_x$(*driver*(*x*); [*y*]; *dime*(*y*); *have*(*x*, *y*), *put_in_meter*(*x*, *y*))

{<*g*, *g*>: {*a*: *a*∈**I**(*farmer*) and there is a *b* s.t. *b*∈**I**(*donkey*) and <*a*, *b*>∈**I**(*own*)} ⊆
{*h*(*x*): *g*[*x*]*h* and for any *b*, if *h*(*x*)∈**I**(*farmer*), *b*∈**I**(*donkey*) and <*h*(*x*), *b*>∈**I**(*own*), then <*h*(*x*), *b*>∈**I**(*beat*)}}
=

{<*g*, *g*>: {*a*: *a*∈**I**(*farmer*) and there is a *b* s.t. *b*∈**I**(*donkey*) and <*a*, *b*>∈**I**(*own*)} ⊆
   {*a*: any *b* s.t. *a*∈**I**(*farmer*), *b*∈**I**(*donkey*) and <*a*, *b*>∈**I**(*own*) is s.t. <*a*, *b*>∈**I**(*beat*)}}} =

{<*g*, *g*>: any farmer *a* who owns a donkey *b* beats any donkey *b′* that he owns}.

97. Most^x people that owned a^y slave also owned his$_y$ offspring.

    (Heim 1990: 162, (49))

The formula in (94) is true iff more than half of the house-elves who fall in love with a witch are such that they buy *any* witch that they fall in love with (*strong* reading) some alligator purse or other. This formula is false in the 'Dobby as Don Juan' scenario above, in agreement with our intuitions about the corresponding English sentence in (93).

The formula in (96) makes similarly correct predictions about the truth-conditions of the English sentence in (95): both of them are true in a scenario in which there are ten drivers, each of them has ten dimes in his/her pocket and nine of them put exactly one dime in their respective meters. Out of the one hundred possible pairs of drivers and dimes they have, only nine pairs (far less than half) satisfy the nuclear scope of the quantification, but this is irrelevant as long as a majority of drivers (and not of pairs) satisfies it.

6. Limitations of DPL+GQ: Mixed Weak & Strong Donkey Sentences

However, the dynamic notion of selective generalized quantification introduced in the previous section does not offer a completely general account of the weak/strong donkey ambiguity: it fails for more complex weak & strong donkey sentences much as the unselective notion failed for the simplest ones.

Consider again the *dime* example from Pelletier & Schubert (1989), repeated in (98) below. Unselective generalized quantification fails to assign the correct weak interpretation to this example because it cannot distinguish between the various discourse referents (dref's) introduced in the restrictor of the generalized quantifier: x (the persons) should be quantified over universally, while y (their dimes) should be quantified over existentially.

Selective generalized quantification provides a solution to this problem because it can distinguish between x, which is the dref contributed by the generalized determiner, and y, which is the dref contributed by the indefinite in the restrictor of the determiner.

98. Every^x person who has a^y dime will put it$_y$ in the meter.

Thus, selective generalized quantification can only distinguish between the 'main' quantified-over dref and the other dref's introduced in the restrictor – it cannot further distinguish between the latter ones, which are collectively interpreted as either weak or strong. Since the decision about the 'strength' of their interpretation is not made on an individual basis, selective generalized quantification as defined in (82) above fails to account for any examples in which two indefinites in the restrictor of a generalized quantifier are not interpreted as both weak or both strong.

Sentences (4) and (5) below are such counter-examples.

99. Every^x person who buys a^y book on amazon.com and has a^z credit card uses it$_z$ to pay for it$_y$.
100. Every^x man who wants to impress a^y woman and who has an^z Arabian horse teaches her$_y$ how to ride it$_z$.

The most salient interpretation of (4) is strong with respect to *a^y book* and weak with respect to *a^z credit card*, i.e. for *every* book bought on amazon.com by any person that is a credit-card owner, the person uses *some* credit card or other to pay for the book. In particular, note that the credit card might vary from book to book, i.e. the strong indefinite *a^y book* seems to be able to 'take scope' over the weak indefinite *a^z credit card*: I can use my Mastercard to buy set theory books and my Visa to buy fantasy novels. This means that, despite the fact that it receives a weak reading, the indefinite *a$^{u'}$ credit card* can introduce a possibly non-singleton set of credit cards.

Similarly, in the case of (5), the indefinite *a^y woman* is interpreted as strong and the indefinite *an^z Arabian horse* as weak; and yet again, the strong indefinite seems to 'take scope' over the weak one: the horse used in the pedagogic activity might vary from female student to female student.

Finally, note that we can easily construct examples of this kind if we are willing to countenance other anaphoric expressions besides pronouns. Sentence (4) for example

does not sound clumsy anymore if we replace one of the non-animate pronouns with a definite description – as shown in (101) below[44].

101. Every^x person who buys a^y book on <u>amazon.com</u> and has a^z credit card uses the$_z$ card to pay for it$_y$.

I will not attempt to explicitly extend the DPL-style selective quantification in a way that can discriminate between the dref's introduced by indefinites in the restrictor. The basic idea would be to introduce additional lexical entries for generalized determiners which would bind universally or existentially the indefinites in their restrictor, e.g. the determiner *most* would have a 'single quantifier' entry of the form **every**$_x$, two 'double quantifier' entries of the form **most**$_x\forall_y$ and **most**$_x\exists_y$, four 'triple quantifier' entries of the form **most**$_x\forall_y\forall_z$, **most**$_x\forall_y\exists_z$, **most**$_x\exists_y\forall_z$, **most**$_x\exists_y\exists_z$ etc.[45]

Note that interpreting English sentences in terms of such determiners is not compositional, e.g. to interpret (5), we need a 'triple quantifier' of the form **every**$_x\forall_y\exists_z$, which requires us to look inside the second relative clause, identify the indefinite *an^z Arabian horse* and assign it a weak interpretation.

The situation is in fact even more complicated and non-compositional: as already indicated, the indefinites in the restrictor can enter pseudo-scopal relations since the value of the weak indefinite can vary with the value of the strong indefinite, e.g. the same 'triple quantifier' **every**$_x\forall_y\exists_z$ has a choice of scoping $\forall_y$ over $\exists_z$ or the other way around, i.e. **every**$_x\exists_z\forall_y$ [46].

I take these relations to be *pseudo*-scopal because the two donkey indefinites in both (4) and (5) are 'trapped' in a coordination island and none of them can scope out of their

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[44] I substitute a definite description for the pronoun that enters the anaphoric dependency receiving a weak reading; substituting a definite description for the strong pronoun might bring in the additional complexity that the strong reading is in fact due to the use of the (maximal) definite description (see for example the D-/E-type analyses in Neale 1990, Lappin & Francez 1994 and Krifka 1996b).

[45] See for example Heim (1990): 163-164 for sample lexical entries for the 'double' quantifiers formulated both in terms of quantification over individuals and in terms of quantification over minimal situations.

[46] Note that the latter entry **every**$_x\exists_z\forall_y$ is not identical with the other 'triple quantifier' **every**$_x\exists_y\forall_z$.

VP- or CP-conjunct to take scope over the other[47]. Note that the impossibility of scoping out of a coordination structure is not dependent on any particular scoping mechanism; to see this, consider the two sentences in (102) and (103) below showing that a quantifier like *every* cannot scope out of VP- or CP-coordination structures.

102. #Every person who buys every^x *Harry Potter* book on *amazon.com* and gives it$_x$ to a friend must be a *Harry Potter* addict.

103. #Every boy who wanted to impress every^x girl in his class and who planned to buy her$_x$ a fancy Christmas gift asked his best friend for advice.

Quite a few accounts of weak and strong readings – including the dynamic account in van den Berg (1994, 1996) and the hybrid dynamic & D-/E-type approach in Chierchia (1992, 1995) – fail to analyze such conjunction-based, mixed weak & strong donkey sentences: the main difficulty for them is that they cannot allow for the weak 'strong' indefinite to be a (possibly) *non-singleton* set and to co-vary with the value of the strong indefinite despite the fact that the strong donkey indefinite cannot scope over the weak donkey indefinite.

It will be the main goal of chapter **5** to provide an analysis of the weak/strong donkey ambiguity which (i) is completely general in the sense that it is able to discriminate between the indefinites in the restrictor of a dynamic generalized determiner, (ii) does not postulate an ambiguity in the generalized determiner but only in the interpretation of the indefinites and (iii) is compositional.

This completes the review of the DPL (and therefore of classical DRT / FCS) and of the two most straightforward extensions of DPL with generalized quantification. The next two chapters (i.e. chapters **3** and **4**) are dedicated to the reformulation of DPL and of its extensions with generalized quantification in type logic, following Muskens (1995b, 1996).

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[47] Incidentally, note that that any such scope-taking has to ensure that the indefinites still have narrow scope with respect to the quantifiers *every^x person* and *every^x man*.

Chapter **3** reformulates DPL. The goal is to define an interpretation procedure for English sentences that is both dynamic and compositional at the sub-sentential / sub-clausal level.

Chapter **4** will extend the type-logical formulation of DPL with the notions of unselective and selective quantification defined for DPL in sections **4** and **5** above.

Chapter 3. Compositional DRT

1. Introduction

The main goal of this chapter is to reformulate DPL (i.e. DPL without unselective and selective generalized quantification.) in type logic so that we will be able to define an interpretation procedure for English sentences that is both dynamic and compositional at the sub-sentential / sub-clausal level.

Section **2** provides the basics of the dynamic type-logical system, i.e. it defines the underlying dynamic logic, which I will label Dynamic Ty2[1]. I follow the Compositional DRT (CDRT) and the Logic of Change in Muskens (1996) as closely as possible, the single most obvious exception being that I model discourse referents (dref's) as functions – as Muskens (1995b) does.

The choice to model dref's as functions is motivated by the fact that the resulting system can be more easily compared with situation-based D-/E-type approaches (in which pronouns are basically functions from minimal situations to individuals) and by the fact that the system can be more easily generalized to account for:

- multiple donkey sentences involving both weak and strong donkey anaphora, e.g. *Every person who buys a book on amazon.com* (strong) *and has a credit card* (weak) *uses it* (the credit card) *to pay for it* (the book);
- plural anaphora, e.g. *Some / Three men came in. They sat down*;
- modal anaphora and modal subordination, e.g. *A wolf might enter the cabin. It would attack John first* (example based on Roberts (1989))

Section **3** shows how to translate the DPL system into Dynamic Ty2.

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[1] Dynamic Ty2 is basically the Logic of Change in Muskens (1991, 1995b, 1996). I label it "Dynamic Ty2" to make more transparent the fact that it actually is a generalization of Dynamic Predicate Logic (Groenendijk & Stokhof 1991).

Building on the translation of DPL into type logic, the following two sections introduce compositionality at the sub-sentential level: section **4** describes a rough-and-ready syntax for the English fragment we will be concerned with, while section **5** defines its semantics (i.e. a type-driven translation procedure). The resulting version of CDRT is the basis for all the formal systems introduced throughout the remainder of the dissertation.

Section **6** provides the CDRT analyses of a fairly wide range of examples – for example, we show how CDRT derives the two possible quantifier scopings of the '*every man loves a woman*' kind of examples.

The differences between the material in this chapter and Muskens (1995b, 1996) are for the most part presentational. The main four differences are:

- the fact that section **2** provides a complete, detailed definition of the underlying Dynamic Ty2 logic;
- the fact that Dynamic Ty2 allows static objects of *arbitrary* types as dref values;
- the different analysis of proper names I end up adopting;
- the novel dynamic analysis of *ditransitive* verbs and the *scoping properties* of their Dative and Accusative objects in section **6**.

Building on the foundations layed out in this chapter, the next chapter will add to the previous literature in a more substantial way by reformulating the DPL-style definitions of *unselective* and *selective* generalized quantification in type logic and, thus, extending CDRT to CDRT+GQ in a way that enables it to account for the weak / strong donkey ambiguity and the proportion problem.

The chapter concludes with a summary of the main results (section **7**).

2. Dynamic Ty2

I have already indicated in the previous chapter that Compositional DRT (CDRT) combines Montague semantics and DRT in a formalism based on ordinary type logic. As Muskens (1991) puts it:

"[The unification is] based on two assumptions and one technical insight. The first assumption is that meaning is compositional. The meaning of words (roughly) are the smallest building blocks of meaning and meanings may combine into larger and larger structures by the rule that the meaning of a complex expression is given by the meaning of its parts.
The second assumption is that meaning is computational. Texts effect change, in particular, texts effect changes in context. The meaning of a sentence or text can be viewed as a relation between context states, much in the way that the meaning of a computer program can be viewed as a relation between program states.
[…] The technical insight […] is that virtually all programming concepts to be found in the usual imperative computer languages are available in classical type theory. We can do any amount of programming in type theory. This suggests that type theory is an adequate tool for studying how languages can program context change. Since there is also some evidence that type theory is also a good vehicle for modelling how the meaning of a complex expression depends on the meaning of its parts, we may hope that it is adequate for a combined theory: a compositional theory of the computational aspects of natural language meaning."(Muskens (1991): 3-4[2])

2.1. Preliminaries

The logic that underlies the entire present investigation is Ty2 (Gallin 1975; see also Janssen 1986 and Carpenter 1998). The set of basic types is $\{t, e, s\}$. Type t is the type of truth values; the logic is bivalent and total: the domain of type t (D_t) is $\{T, F\}$. Type e is the type of individuals; I assume (for the time being) that D_e contains only atomic entities, i.e. there are no pluralities. The domain of type s (D_s) models DPL's variable assignments; several axioms will be needed to ensure that the entities of type s do actually behave as DPL variable assignments.

Dref's are modeled as functions that take 'assignments' as arguments (i.e. entities of type s) and return a static object as value, e.g. an individual (type e). A dref for individuals will therefore be of type se. This is not as different from the DPL way of modeling dref's as it might seem: DPL models dref's as variables and a variable x is basically an instruction to look in the current info state, i.e. the current variable assignment g, and retrieve whatever individual the current info state associates with x – that individual is, of course, $g(x)$.

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[2] Page references are to the online version.

Therefore, instead of working directly with variables, we can work with their 'type-lifted' versions, i.e. instead of x, we can take a dref to be a function of the form $\lambda g.\ g(x)$, which is basically the (set-theoretic) x^{th} projection function, which projects the sequence g onto the coordinate x.

This is precisely what Dynamic Ty2 does: instead of modeling discourse referents as atomic entities (variables) and info states as functions taking dref's as arguments (i.e. variable assignments), we model info states as atomic entities (of type s)[3] and dref's as functions taking info states as arguments. Thus, dref's are similar to Montague's individual concepts: they do not refer directly, but only as a function of the current discourse context.

2.2.   Definitions and Abbreviations

Let us turn now to the definition of Dynamic Ty2. For the most part, the definitions are the usual Ty2 ones. I will state what makes this logic a *Dynamic* Ty2 in plain words before or after each definition. The reader should feel free to just glance at these observations and move on to section **3,** which shows how to translate DPL into Dynamic Ty2 and, by doing this, indirectly provides the empirical motivation for Dynamic Ty2.

The definition of types in (1) below isolates a subset of types as the types of dref's: these are functions from 'assignments' (type s) to static objects of arbitrary type. This seems to be more than sufficient for all the analyses in the present work[4]. We restrict our dref's to functions from 'variable assignments' to *static* objects of arbitrary types because, if we allow for arbitrary dref types, e.g. $s(st)$, we might run into counterparts of Russell's paradox – see Muskens (1995b): 179-180, fn. 10.

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[3] We can define a notion of 'info state' g that is closer to the DPL variable assignments, e.g. for any 'assignment' i of type s, let g^i be $\lambda v_\tau.\ vi$, where $\tau \in \mathbf{DrefTyp}$. If we consider only dref's for individuals, i.e. $\tau := se$, g^i is a function of type $(se)e$, that assigns values to dref's much like a DPL variable assignment assigns values to variables.

[4] But see Stone & Hardt (1999) for an account of strict/sloppy readings that employs 'dynamic' drefs, i.e. dref's of type $s(s(\ldots))$. These are just the *pointers* introduced in Janssen (1986).

In fact, throughout this work, I will be much more restrictive and use dref's that have *basic* static types (other than *t*) as codomains. The fact that we do not require dref's for higher-order static objects in our analyses can be to a certain extent taken to support the *empirical* hypothesis that natural languages strongly prefer to use discourse referents for entities of a basic static type[5].

The *logic*, however, should allow for dref's that have any arbitrary static type as their codomain, given that the logic should provide a framework within which any plausible analysis can be formulated (including analyses involving dref's for higher-order static objects) and compared with alternative accounts.

1. **Dynamic Ty2 –** the set of dref types **DRefTyp** and the set of types **Typ**.

   a. The set of basic static types **BasSTyp**: {*t*, *e*} (truth-values and individuals).

   b. The set of static types **STyp**: the smallest set including **BasSTyp** and s.t., if $\sigma, \tau \in$ **STyp**, then $(\sigma\tau) \in$ **STyp**.

   c. The set of dref types **DRefTyp**: the smallest set s.t., if $\sigma \in$ **STyp**, then $(s\sigma) \in$ **DRefTyp**.

   d. The set of basic types **BasTyp**: **BasSTyp**$\cup\{s\}$ ('variable assignments').

   e. The set of types **Typ**: the smallest set including **BasTyp** and s.t., if $\sigma, \tau \in$ **Typ**, then $(\sigma\tau) \in$ **Typ**.

The definition in (2) below provides some typical examples of expressions of various types and introduces several notational conventions that will improve the readability of the subsequent analyses.

2. **Dynamic Ty2 – basic expressions.**

   For any type $\tau \in$ **Typ**, there is a denumerable set of τ-constants **Con**$_\tau$ and a denumerably infinite set of τ-variables **Var**$_\tau = \{v_{\tau,0}, v_{\tau,1}, \ldots\}$, e.g.

   Con$_e$ = {*john*, *mary*, *dobby*, ..., *a*, *a'*, ..., *b*, *b'*, ..., a_0, a_1, a_2, ...}

   Con$_{et}$ = {*donkey*, *farmer*, *house_elf*, *witch*, ..., *leave*, *drunk*, *walk*, ...}

   Con$_{e(et)}$ = {*fall_in_love*, *own*, *beat*, *have*, ...}

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[5] See also the related "No higher-order variables" hypothesis in Landman (2006).

$\mathbf{Con}_s = \{h, h', \ldots, i, i', \ldots, j, j', \ldots, k, k', \ldots, h_0, h_1, \ldots, i_0, i_1, \ldots\}$

$\mathbf{Con}_{se} = \{u, u', u'', \ldots, u_0, u_1, u_2, \ldots\}$

Notational conventions:

$x, x', \ldots, y, y', \ldots, z, z', \ldots, x_0, x_1, \ldots$ are variables of type e;

$h, h', h'', \ldots, i, i', i'', \ldots, j, j', j'', \ldots$ are variables of type s;

$f, f', f'', \ldots f_0, f_1, f_2, \ldots$ are variables over terms of type τ,

for any $\tau \in \mathbf{STyp}$;

$v, v', v'', \ldots, v_0, v_1, v_2, \ldots$ are variables over terms of type τ,

for any $\tau \in \mathbf{Typ.}$

The definition in (3) introduces the term $i[\delta]j$ of type t that is meant to model the DPL random variable assignment. Intuitively, the formula $i[\delta]j$ requires the info states i and j to differ at most with respect to the value they assign to def δ. Unlike Muskens (1995b, 1996), I introduce this as a basic formula of the language and not as an abbreviation, because the set **DRefTyp** of dref types is infinite and the abbreviation would require an infinite conjunction of formulas (as indicated in (4d) below).

I also introduce identity as a basic operator of the language; although it can be defined when the logic is interpreted relative to standard frames (as in (4a) below), I want to allow for the possibility of interpreting it relative to generalized (Henkin) frames, in which case identity is not definable anymore, just as it is not in many-sorted first-order logic.

Finally, note that proper names with capitals, e.g. *John*, are dref's for individuals (type *se*) and they are constant functions, a.k.a. *specific* dref's (see Muskens (1996)). They are defined in terms of the corresponding constant of type *e*, e.g. *john*.

3. **Dynamic Ty2 – terms.**

For any type $\tau \in \mathbf{Typ}$, the set of τ-terms $\mathbf{Term}_\tau$ is the smallest set s.t.:

$\mathbf{Con}_\tau \cup \mathbf{Var}_\tau \subseteq \mathbf{Term}_\tau$;

$\alpha(\beta) \in \mathbf{Term}_\tau$ if $\alpha \in \mathbf{Term}_{\sigma\tau}$ and $\beta \in \mathbf{Term}_\sigma$ for any $\sigma \in \mathbf{Typ}$;

$(\lambda v.\ \alpha) \in \mathbf{Term}_\tau$ if $\tau = (\sigma\rho)$, $v \in \mathbf{Var}_\sigma$ and $\alpha \in \mathbf{Term}_\rho$ for any $\sigma, \rho \in \mathbf{Typ}$;

$(\alpha = \beta) \in \mathbf{Term}_\tau$ if $\tau = t$ and $\alpha, \beta \in \mathbf{Term}_\sigma$ for any $\sigma \in \mathbf{Typ}$;

$(i[\delta]j) \in \mathbf{Term}_\tau$ if $\tau = t$ and $i, i' \in \mathbf{Var}_s$ and $\delta \in \mathbf{Term}_\sigma$, for any $\sigma \in \mathbf{DRefTyp}$.

Abbreviations (the subscripts on terms indicate their type):

$John_{se} := \lambda i_s.\ john_e,\ Mary_{se} := \lambda i_s.\ mary_e\ \dots;$

$\mathbf{T} := \lambda f_t.\ f = \lambda f_t.\ f; \qquad\qquad \mathbf{F} := \lambda f_t.\ f_t = \lambda f_t.\ \mathbf{T};$

$\neg := \lambda f_t.\ f_t = \mathbf{F}; \qquad\qquad \wedge := \lambda f_t f'_t.\ (\lambda f''_{tt}.\ f''(f)=f') = \lambda f''_{tt}.\ f''(\mathbf{T}))$ [6];

$\rightarrow := \lambda f_t f'_t.\ (f \wedge f') = f; \qquad \vee := \lambda f_t f'_t.\ \neg f \rightarrow f';$

$\forall v(\phi) := \lambda v.\ \phi = \lambda v.\ \mathbf{T}; \qquad \exists v(\phi) := \lambda v.\ \neg\phi \neq \lambda v.\ \mathbf{T}.$

Definition (4) introduces four axioms that Dynamic Ty2 models have to satisfy. These axioms make sure that the entities of type s actually behave as variable assignments intuitively do [7].

First, **Axiom1** employs a non-logical constant **udref** to identify *unspecific* dref's, i.e. the dref's that are supposed to behave as the DPL variables, e.g. u_0, u_1 etc. The constant function *John* (*John* $:= \lambda i.\ john_e$ – see (3) above) for example is a *specific* dref: although it is of type *se*, i.e. the type of dref's for individuals, it does not behave as a DPL variable – its value does not vary from 'assignment' to 'assignment'; if anything, specific dref's are the counterpart of DPL constants, not variables.

Axiom2 makes sure that all the unspecific dref names actually name different functions: if two distinct names denoted the same function, we would accidentally update *both* whenever we would update one of them.

Axiom3 ensures that, just like DPL variable assignments, two 'assignments' (i.e. two entities of type *s*) are different only if they assign different values to some dref δ. If they assign the same values to all dref's, the 'assignments' are identical.

Finally, **Axiom4** ensures that we have enough 'assignments': for any given 'assignment' i, any unspecific dref v and any possible dref value (i.e. static object) f of the

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[6] Equivalently, $\wedge := \lambda f_t f'_t.\ \forall f''_{t(tt)}(f''(f, f') = f''(\mathbf{T}, \mathbf{T}))$ or $\wedge := \lambda f_t f'_t.\ \forall f''_{tt}(f = (f''(f) = f''(f')))$.

[7] To get a better grasp of the axioms, the reader might find it instructive to construct a model for them, i.e. to construct the domain D_s given the domain D_e and the set of **udref** names.

appropriate type, there is an 'assignment' j that differs from i at most with respect to the value it assigns to v and which in fact assigns f as the value of v.

4. **Dynamic Ty2 – frames, models, assignments, interpretation and truth.**

   a. A *standard frame F* for Dynamic Ty2 is a set $D = \{D_\tau: \tau \in \textbf{Typ}\}$ s.t. D_e, D_t and D_s are pairwise disjoint sets and $D_{\sigma\tau} = \{f\!: f\text{ is a total function from } D_\sigma \text{ to } D_\tau\}$, for any $\sigma, \tau \in \textbf{Typ}$.

   b. A *model M* for Dynamic Ty2 is a pair $<F^M, \|\cdot\|^M>$ s.t.:

   - F^M is a standard frame for Dynamic Ty2;

   - $\|\cdot\|^M$ assigns an object $\|\alpha\|^M \in D^M{}_\tau$ to each $\alpha \in \textbf{Con}_\tau$ for any $\tau \in \textbf{Typ}$,

     i.e. $\|\cdot\|^M$ respects typing;

   - M satisfies the following axioms[8]:

   Axiom1 ("Unspecific dref's"): **udref**(δ),

   for any unspecific dref name δ of any type $(s\tau) \in \textbf{DRefTyp}$,

        e.g. u_0, u_1, … but not *John*, *Mary*, …;

   udref is a non-logical constant[9] intuitively identifying the 'variable' dref's,

        i.e. the non-constant functions of type $s\tau$ (for any $\tau \in \textbf{STyp}$)

        intended to model DPL-like variables.

   Axiom2 ("Dref's have unique dref names"): **udref**(δ) $\wedge$ **udref**(δ') $\rightarrow$ $\delta \neq \delta'$,

   for any two distinct dref names δ and δ' of type τ,

   for any type $\tau \in \textbf{DRefTyp}$,

        i.e. we make sure that we do not accidentally update a dref δ' when we update δ.

   Axiom3 ("Identity of 'assignments'"): $\forall i_s j_s(i[\,]j \rightarrow i=j)$.

   Axiom4 ("Enough 'assignments'"): $\forall i_s \forall v_{s\tau} \forall f_\tau(\textbf{udref}(v) \rightarrow \exists j_s(i[v]j \wedge vj=f))$,

   for any type $\tau \in \textbf{STyp}$.

---

[8] The axioms / axiom schemes are based on Muskens (1995b, 1996).

[9] In fact, **udref** stands for an infinite family of non-logical constants of type (τt) for any $\tau \in \textbf{DRefTyp}$. Alternatively, we can assume a polymorphic type logic with infinite sum types, in which **udref** is a polymorphic function. For a discussion of sum types, see for example Carpenter (1998): 69 et seqq.

c. An *M-assignment* θ is a function that assigns to each variable $v \in \mathbf{Var}_\tau$ an element $\theta(v) \in D^M{}_\tau$ for any $\tau \in \mathbf{Typ}$. Given an *M*-assignment θ, if $v \in \mathbf{Var}_\tau$ and $d \in D^M{}_\tau$, then $\theta^{v/d}$ is the *M*-assignment identical with θ except that it assigns d to v.

d. The interpretation function $\|\cdot\|^{M,\theta}$ is defined as follows:

$$\|\alpha\|^{M,\theta} = \|\alpha\|^M \quad \text{if} \quad \alpha \in \mathbf{Con}_\tau \text{ for any } \tau \in \mathbf{Typ};$$

$$\|\alpha\|^{M,\theta} = \theta(\alpha) \quad \text{if} \quad \alpha \in \mathbf{Var}_\tau \text{ for any } \tau \in \mathbf{Typ};$$

$$\|\alpha(\beta)\|^{M,\theta} = \|\alpha\|^{M,\theta}(\|\beta\|^{M,\theta});$$

$$\|\lambda v.\,\alpha\|^{M,\theta} = \langle \|\alpha\|^{M,\theta^{v/d}} : d \in D^M{}_\sigma \rangle \quad \text{if} \quad v \in \mathbf{Var}_\sigma;$$

$$\|\alpha{=}\beta\|^{M,\theta} = \mathbf{T} \quad \text{if} \quad \|\alpha\|^{M,\theta} = \|\beta\|^{M,\theta}$$
$$\qquad\qquad = \mathbf{F} \quad \text{otherwise.}$$

$$\|i[\delta]j\|^{M,\theta} = \mathbf{T} \quad \text{if} \quad \delta \in \mathbf{Term}_\sigma,\ \sigma \in \mathbf{DRefTyp} \text{ and}$$

$$\|\forall v_\sigma(\mathbf{udref}(v) \wedge v \neq \delta \to vi{=}vj)\|^{M,\theta} = \mathbf{T} \text{ and}$$

$$\|\forall v_\tau(\mathbf{udref}(v) \to vi{=}vj\|^{M,\theta} = \mathbf{T} \text{ for all } \tau \neq \sigma,\ \tau \in \mathbf{DRefTyp}$$
$$\qquad\qquad = \mathbf{F} \quad \text{otherwise.}$$

e. Truth: A formula $\phi \in \mathbf{Term}_t$ is *true* in *M* relative to θ iff $\|\phi\|^{M,\theta} = \mathbf{T}$.

A formula $\phi \in \mathbf{Term}_t$ is *true* in *M* iff it is true in *M* relative to any assignment θ.

3. Translating DPL into Dynamic Ty2

In this section, we will see how to encode DPL (and therefore classical DRT / FCS) in Dynamic Ty2. We do this by providing a list of abbreviations that follows closely the definition of DPL in the previous chapter: the *definiendum* has the form of a DPL expression, while the *definiens* is a term of Dynamic Ty2. As soon as the abbreviations are in place, we will see how they are employed by analyzing the examples we have previously used as empirical motivation for DPL.

3.1. Definitions and Abbreviations

Definition (5) below corresponds to the DPL definition in the preceding chapter. Note that '∧' is the Dynamic Ty2 conjunction, i.e. the official, type-logical conjunction, and '¬' is the Dynamic Ty2 negation, i.e. the official, type-logical negation. In contrast, dynamic conjunction ';' and dynamic negation '~' are simply abbreviations.

Note also that the DPL notion of random assignment [x] has as its direct correspondent the random assignment [u] of Dynamic Ty2.

The DPL distinction between conditions and DRS's is formulated in terms of types. Conditions are terms of type st, i.e. they denote sets of 'assignments'; intuitively, conditions denote the set of 'assignments' that satisfy them. DRS's are terms of type $s(st)$, i.e. binary relations between 'assignments'; intuitively, a DRS D is satisfied by a pair of two 'assignments' i and j, i.e. $Dij=\mathbf{T}$[10], iff the output 'assignment' j is the result of non-deterministically updating the input 'assignment' i with D.

5. **DPL in Dynamic Ty2** (subscripts on terms represent their types)**.**

   a. Atomic conditions – type st:

   $R\{u_1, \ldots, u_n\} := \lambda i_s.\ R(u_1 i, \ldots, u_n i)$,

   for any non-logical constant R of type $e^n t$,

   where $e^n t$ is defined as follows: $e^0 t := t$ and $e^{m+1}t := e(e^m t)$[11]

   $u_1 = u_2 := \lambda i_s.\ u_1 i = u_2 i$

   b. Atomic DRS's (DRS's containing exactly one atomic condition) – type $s(st)$ (corresponding to DPL atomic formulas):

   $[R\{u_1, \ldots, u_n\}] := \lambda i_s j_s.\ i=j \wedge R\{u_1, \ldots, u_n\}j$

   $[u_1 = u_2] := \lambda i_s j_s.\ i=j \wedge u_1 j = u_2 j$

   c. Condition-level connectives (negation), i.e. non-atomic conditions:

   $\sim D := \lambda i_s.\ \neg \exists k_s (Dik)$ [12],   where D is a DRS (term of type $s(st)$)

   i.e. $\sim D := \lambda i_s.\ i \notin \mathbf{Dom}(D)$,   where $\mathbf{Dom}(D) := \{i_s: \exists j_s(Dij)\}$

   d. Tests (generalizing 'atomic' DRS's):

---

[10] Recall that T and F are the model-theoretic objects intuitively modeling 'True' and 'False', while their bolded counterparts $\mathbf{T}$ and $\mathbf{F}$ are the Dynamic Ty2 constants whose semantic values are T and F respectively.

[11] The definition of $e^n t$ is due to Muskens (1996): 157-158.

[12] Strictly speaking, the Dynamic Ty2 translation of DPL negation is defined as $\mathbf{TR}(\sim\phi) := [\sim\mathbf{TR}(\phi)]$, i.e. $\mathbf{TR}(\sim\phi) := [\lambda i_s.\ \neg\exists k_s(\mathbf{TR}(\phi)ik)]$. $\mathbf{TR}$ is the translation function from DPL to Dynamic Ty2 which is recursively defined in the expected way, e.g. for DPL atomic formulas, we have that $\mathbf{TR}(R(x_1, \ldots, x_n)) := [R\{u_1, \ldots, u_n\}]$ and $\mathbf{TR}(x_1 = x_2) := u_1 = u_2$.

$$[C_1, \ldots, C_m] := \lambda i_s j_s. \ i{=}j \wedge C_1 j \wedge \ldots \wedge C_m j \ [13],$$

where $C_1, \ldots, C_m$ are conditions (atomic or not) of type *st*.

e. DRS-level connectives (dynamic conjunction):

$$D_1; D_2 := \lambda i_s j_s. \ \exists h_s(D_1 ih \wedge D_2 hj), \quad \text{where } D_1 \text{ and } D_2 \text{ are DRS's (type } s(st))$$

f. Quantifiers (random assignment of value to a dref):

$$[u] := \lambda i_s j_s. \ i[u]j$$

g. Truth: A DRS D (type $s(st)$) is true with respect to an input info state i_s iff $\exists j_s(Dij)$, i.e. $i \in \mathbf{Dom}(D)$ [14].

The abbreviations introduced in definition (6) below correspond to the DPL abbreviations defined in the previous chapter. '∃' and '∀' are the official type-logical existential and universal quantifiers, while '∀' and '∃' are the abbreviations corresponding to the dynamic (DPL-style) existential and universal quantifiers. I use '→' and '∨' both for the official Dynamic Ty2 and for the dynamic DPL-style implication and, respectively, disjunction; they can be easily disambiguated in context.

6. **a. Additional abbreviations – condition-level connectives (closure, disjunction, implication)**:

$$!D := \sim[\sim D] \ [15],$$

i.e. $!D := \lambda i_s. \ \exists k_s(Dik)$ or simply: $!D := \mathbf{Dom}(D)$

$$D_1 \vee D_2 := \sim([\sim D_1]; [\sim D_2]),$$

i.e. $D_1 \vee D_2 := \sim[\sim D_1, \sim D_2]$

i.e. $D_1 \vee D_2 := \lambda i_s. \ \exists k_s(D_1 ik \vee D_2 ik);$

---

[13] Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction as follows:

$[C_1, \ldots, C_m] := \lambda i_s j_s. \ ([C_1]; \ldots; [C_m])ij$, where $[C] := \lambda i_s j_s. \ i{=}j \wedge Cj$.

[14] Or, equivalently, $i \in !D$ – see the abbreviation of '!' in (6) below.

[15] Strictly speaking, DPL anaphoric closure is translated in Dynamic Ty2 as $\mathbf{TR}(!\phi) := [\sim\mathbf{TR}(\sim\phi)]$, i.e. $\mathbf{TR}(!\phi) := [\sim[\sim\mathbf{TR}(\phi)]] = [\sim[\lambda j_s. \ \neg\exists l_s(\mathbf{TR}(\phi)jl)]] = [\lambda i_s. \ \neg\exists k_s([\lambda j_s. \ \neg\exists l_s(\mathbf{TR}(\phi)jl)]ik)]$, i.e. $\mathbf{TR}(!\phi) := [\lambda i_s. \ \neg\exists k_s(i{=}k \wedge \neg\exists l_s(\mathbf{TR}(\phi)kl))] = [\lambda i_s. \ \neg(\neg\exists l_s(\mathbf{TR}(\phi)il))]$, i.e. $\mathbf{TR}(!\phi) := [\lambda i_s. \ \exists l_s(\mathbf{TR}(\phi)il)] = \mathbf{Dom}(\mathbf{TR}(\phi))$.

$\mathbf{TR}$ is the translation function from DPL to Dynamic Ty2 – see fn. **12** above.

equivalently: $D_1 \vee D_2 := \mathbf{Dom}(D_1) \cup \mathbf{Dom}(D_2)$ [16]

$D_1 \rightarrow D_2 := \sim(D_1; [\sim D_2])$,

i.e. $D_1 \rightarrow D_2 := \lambda i_s. \forall h_s(D_1 ih \rightarrow \exists k_s(D_2 hk))$ [17],

i.e. $D_1 \rightarrow D_2 := \lambda i_s. (D_1)i \subseteq \mathbf{Dom}(D_2)$,   where $(D)i := \{j_s: Dij\}$

b. Additional abbreviations – DRS-level quantifiers (multiple random assignment, existential quantification):

$[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$

$\exists u(D) := [u]; D$

c. Additional abbreviations – condition-level quantifiers (universal quantification):

$\forall u(D) := \sim([u]; [\sim D])$,

i.e. $\sim[u \mid \sim D]$ or $[u] \rightarrow D$ or equivalently $\sim\exists u([\sim D])$,

i.e. $\forall u(D) := \lambda i_s. \forall h_s(i[u]h \rightarrow \exists k_s(Dhk))$,

i.e. $\forall u(D) := \lambda i_s. ([u])i \subseteq \mathbf{Dom}(D)$

d. Additional abbreviations – DRS's (a.k.a. linearized 'boxes'):

$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda i_s j_s. ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])ij$,

where $C_1, \ldots, C_m$ are conditions (atomic or not),

i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda i_s j_s. i[u_1, \ldots, u_n]j \wedge C_1 j \wedge \ldots \wedge C_m j$.

3.2. Cross-sentential Anaphora

Going through the examples that motivated DPL and classical DRT / FCS in the first place will help us get familiar with Dynamic Ty2 translations and, at the same time, provide the empirical motivation for various features of the formal system. Consider again the mini-discourse in (7-8) below, containing two instances of cross-sentential anaphora.

---

[16] $D_1 \vee D_2 := \sim([\sim D_1]; [\sim D_2]) = \lambda i. \neg\exists k(([\sim D_1]; [\sim D_2])ik) = \lambda i. \neg\exists kl([\sim D_1]il \wedge [\sim D_2]lk) = \lambda i. \neg\exists kl(i=l \wedge \neg\exists h(D_1 ih) \wedge l=k \wedge\neg\exists h'(D_2 lh')) = \lambda i. \neg(\neg\exists h(D_1 ih) \wedge \neg\exists h'(D_2 ih')) = \lambda i. \exists h(D_1 ih) \vee \exists h'(D_2 ih') = \lambda i. \exists k(D_1 ik \vee D_2 ik)$.

[17] $D_1 \rightarrow D_2 := \sim(D_1; [\sim D_2]) = \lambda i. \neg\exists k((D_1; [\sim D_2])ik) = \lambda i. \neg\exists kl(D_1 il \wedge [\sim D_2]lk) = \lambda i. \neg\exists kl(D_1 il \wedge l=k \wedge\neg\exists h(D_2 lh)) = \lambda i. \neg\exists k(D_1 ik \wedge \neg\exists h(D_2 kh)) = \lambda i. \forall k(D_1 ik \rightarrow \exists h(D_2 kh))$.

7. A^{u_1} house-elf fell in love with a^{u_2} witch.

8. He$_{u_1}$ bought her$_{u_2}$ an^{u_3} alligator purse.

I provide its DRT-style representation in DPL and its DRT-style representation in Dynamic Ty2 in (9) and (10) below. The formula in (11) is the 'unpacked' type-logical term of type $s(st)$ translating the discourse in (7-8). Finally, the formula in (12) provides the truth-conditions associated with the Dynamic Ty2 term in (11), derived on the basis of the definition of truth for DRS's in (5g) and the "Enough States" axiom (**Axiom4** in (4b) above).

Note that the formula in (12) capturing the truth-conditions of discourse (7-8) contains a vacuous λ-abstraction over input 'assignments', which is intuitively correct given that the discourse does not contain any item whose reference is dependent on the input context, as for example a deictically used pronoun would be.

9. [*x*, *y* | *house_elf*(*x*), *witch*(*y*), *fall_in_love*(*x*, *y*)]; [*z* | *alligator_purse*(*z*), *buy*(*x*, *y*, *z*)]

10. [u_1, u_2 | *house_elf*{u_1}, *witch*{u_2}, *fall_in_love*{u_1, u_2}];
    [u_3 | *alligator_purse*{u_3}, *buy*{u_1, u_2, u_3}]

11. $\lambda i_s j_s$. $i[u_1, u_2, u_3]j \wedge$ *house_elf*($u_1 j$) $\wedge$ *witch*($u_2 j$) $\wedge$ *fall_in_love*($u_1 j$, $u_2 j$) $\wedge$
    alligator_purse($u_3 j$) $\wedge$ *buy*($u_1 j$, $u_2 j$, $u_3 j$)

12. λi_s. $\exists x_e \exists y_e \exists z_e$(*house_elf*(*x*) $\wedge$ *witch*(*y*) $\wedge$ *fall_in_love*(*x*, *y*) $\wedge$
    alligator_purse(*z*) $\wedge$ *buy*(*x*, *y*, *z*))

3.3. Relative-Clause Donkey Sentences

Let us turn now to the relative-clause donkey example in (13) below. The formula in (14) is its DPL translation (abbreviated DRT-style), while the corresponding Dynamic Ty2 formula is provided in (15). Note the double square brackets on the left- and right-hand side of the Ty2 representation: the external square brackets are due to the fact that dynamic implication '→' is a condition-level connective (see definition (6a) above), so we need the extra square brackets to obtain a test, i.e. a DRS (which is a term of type $s(st)$), out of a condition of type st.

13. Every^{u_1} house-elf who falls in love with a^{u_2} witch buys her$_{u_2}$ an^{u_3} alligator purse.

14. $[x, y \mid house\_elf(x), witch(y), fall\_in\_love(x, y)]$
$\rightarrow [z \mid alligator\_purse(z), buy(x, y, z)]$

15. $[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$
$\rightarrow [u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

The DRT-style representation in (15) above is 'unpacked' as the type-logical term in (16) below, whose truth-conditions are given in (17); note again the vacuous λ-abstraction over 'assignments', followed by a static first-order formula that captures the intuitively correct truth-conditions for sentence (13) above.

16. $\lambda i_s j_s. \; i=j \wedge \forall h_s(i[u_1, u_2]h \wedge house\_elf(u_1h) \wedge witch(u_2h) \wedge fall\_in\_love(u_1h, u_2h)$
$\rightarrow \exists k_s(h[u_3]k \wedge alligator\_purse(u_3k) \wedge buy(u_1k, u_2k, u_3k)))$

17. $\lambda i_s. \; \forall x_e \forall y_e(house\_elf(x) \wedge witch(y) \wedge fall\_in\_love(x, y)$
$\rightarrow \exists z_e(alligator\_purse(z) \wedge buy(x, y, z)))$

3.4. Conditional Donkey Sentences

The conditional donkey sentence repeated in (18) below receives the same Dynamic Ty2 translation and the same truth-conditions as the relative-clause donkey sentence in (13) above (see (15), (16) and (17)). Thus, just as DPL, the Dynamic Ty2 translations capture the intuitive correspondence between the generalized determiner *every* and bare conditional structures.

18. If a^{u_1} house-elf falls in love with a^{u_2} witch, he$_{u_1}$ buys her$_{u_2}$ an^{u_3} alligator purse.

Finally, we turn to the intuitively equivalent negative donkey sentences we have analyzed in DPL in the previous chapter – repeated in (19), (20) and (21) below.

19. No^x house-elf who falls in love with a^y witch buys her$_y$ an^z alligator purse.

20. If a^x house-elf falls in love with a^y witch, he$_x$ never buys her$_y$ an^z alligator purse.

21. If a^x house-elf falls in love with a^y witch, he$_x$ doesn't buy her$_y$ an^z alligator purse.

Just as in the case of the DPL, we can translate sentence (19) in Dynamic Ty2 in two different ways, i.e. we can translate the determiner *no* either by means of a combination of negation and existential quantification or by means of a combination of negation and universal quantification. But this is not a problem for Dynamic Ty2 just as it wasn't for DPL: as expected, the DPL partial duality between existential and universal quantification is inherited in Dynamic Ty2, so the two translations, i.e. the two *st* terms in (22) below, are identical.

The terms in (22) are of type *st* because both dynamic negation '~' and universal quantification $\forall$ are condition-level connectives. The corresponding tests – which are DRS's, i.e. terms of type $s(st)$ – are obviously identical if the conditions they are based on are identical.

22. $\sim\exists u(D; D') = \forall u([D \to [\sim D']])$ [18]

And, given the identity in (22), we can translate sentence (19) either way[19], as shown in (23) and (25) below. Furthermore, these equivalent translations are also equivalent to the DRT-style formulas in (24) and (26).

Note that the universal quantification over pairs of house-elves and witches is exhibited in the clearest way by (26), since any dref introduced in the antecedent of a conditional ends up being quantified over universally[20].

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[18] $\forall u(D \to [\sim D']) = \lambda i_s.\ \forall h_s([u]ih \to \exists k_s((D \to [\sim D'])hk))$

$= \lambda i_s.\ \forall h_s([u]ih \to \exists k_s(h=k \wedge \forall h'_s(Dhh' \to \exists k'_s([\sim D']h'k')))) = \lambda i_s.\ \forall h_s([u]ih \to \forall h'_s(Dhh' \to \exists k'_s(h'=k' \wedge (\sim D')k')))$

$= \lambda i_s.\ \forall h_s([u]ih \to \forall h'_s(Dhh' \to (\sim D')h')) = \lambda i_s.\ \forall h_s([u]ih \to \forall h'_s(Dhh' \to \neg\exists k_s(Dh'k))$

$= \lambda i_s.\ \forall h_s([u]ih \to \forall h'_s(Dhh' \to \neg\exists k_s(Dh'k)) = \lambda i_s.\ \neg\exists h_s([u]ih \wedge \neg\forall h'_s(Dhh' \to \neg\exists k_s(Dh'k))$

$= \lambda i_s.\ \neg\exists h_s([u]ih \wedge \exists h'_s\neg(Dhh' \to \neg\exists k_s(Dh'k)) = \lambda i_s.\ \neg\exists h_s([u]ih \wedge \exists h'_s(Dhh' \wedge \exists k_s(Dh'k))$

$= \lambda i_s.\ \neg\exists h_s\exists h'_s\exists k_s([u]ih \wedge Dhh' \wedge Dh'k) = \lambda i_s.\ \neg\exists k_s(([u]; D; D')ik) = \sim\exists u(D; D')$

[19] I assume that terms that are equivalent to (Dynamic Ty2 translations of DPL) translations of English sentences are also acceptable as translations.

[20] It is easily checked that the following identities hold:

$\forall u([D \to D']) = [u] \to [D \to D'] = ([u]; D) \to D' = \exists u(D) \to D'.$

23. $[\sim\exists u_1([house\_elf\{u_1\}]; \exists u_2([witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]);$

$\exists u_3([alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]))]$

24. $[\sim[u_1, u_2, u_3 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\},$

$alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

25. $[\forall u_1([house\_elf\{u_1\}]; \exists u_2([witch\{u_2\}, fall\_in\_love\{u_1, u_2\}])$

$\rightarrow [\sim\exists u_3([alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}])])]$

26. $[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$

$\rightarrow [\sim[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]]$

Note also that the formula in (26) is in fact the compositional translation of the negative conditional sentences in (20) and (21) above.

The Dynamic Ty2 truth-conditions for all three sentences are most easily derived from formula (24) – and they are provided in (27) below. Just as before, we have vacuous λ-abstraction over 'assignments', followed by a static first-order formula that captures the intuitively correct truth-conditions for the three English sentences under consideration.

27. $\lambda i_s. \neg\exists x_e\exists y_e\exists z_e(house\_elf(x) \wedge witch(y) \wedge fall\_in\_love(x, y) \wedge$

$alligator\_purse(z) \wedge buy(x, y, z))$

Thus, we see that Dynamic Ty2 can capture everything that DPL (hence classical DRT / FCS) does – including compositionality down to sentence- / clause-level. However, with Dynamic Ty2, we have all the ingredients to go compositional at the sub-sentential / sub-clausal level, which is what sections **4** and **5** below endeavor to do. I conclude this section with a brief discussion of the Dynamic Ty2 analysis of proper names.

Intermezzo: Proper Names in Dynamic Ty2

The main choice for the analysis of proper names in Dynamic Ty2 is between: (i) a *pronoun-like* analysis, whereby a proper name is basically interpreted as a deictically used pronoun, whose referent is specified by the input discourse context, and (ii) an *indefinite-like* analysis, whereby a proper name introduces a new individual-level dref

whose referent is constrained to be the individual (rigidly) designated by the proper name.

Following Muskens (1996), I have introduced specific dref's corresponding to proper names, i.e. constant functions from 'assignments' to individuals, e.g. *John* := $\lambda i_s.$ *john*$_e$. However, unlike Muskens – who chooses the pronoun-like analysis of proper names –, I will not interpret proper names as denoting such specific dref's, but I will instead let proper names introduce an unspecific dref and an identity condition between the unspecific dref and the specific dref that is the Dynamic Ty2 correspondent of the proper name. For example, the proper name *John* is represented as shown in (28) below.

28. *John*^{u} ⤳ [u | u=*John*],   i.e. $\lambda i_s j_s.$ $i[u]j \wedge uj$=*Johnj*,   i.e. $\lambda i_s j_s.$ $i[u]j \wedge uj$=*john*

As (28) shows, the newly introduced unspecific dref is constrained to have the value *john*$_e$ in the output info state *j*. This interpretation of proper names is in fact equivalent to the *external anchoring* of proper names in Kamp & Reyle (1993): 248 – and it is similar to the interpretation of proper names in Kamp (1981). Moreover, pronouns anaphoric to proper names are taken to be anaphoric to the unspecific dref introduced by the proper name, as exemplified by (29) below.

29. … *John*^{u} … *he*$_u$ …

As Muskens (1996): 151-153 observes, this kind of representation seems needlessly complex: why not simply take the proper name to be anaphoric to its corresponding specific dref? This would basically be equivalent to using the proper name as a deictic anaphor, interpreted directly relative to the input context (a.k.a. info state or 'assignment')[21]. Moreover, a pronoun anaphoric to a proper name would be anaphoric to the corresponding specific dref, as shown in (30) below. From this perspective, the use of a pronoun anaphoric to a proper name and the use of the proper name itself are not really different.

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[21] Which is basically what the causal chain theory of proper names proposes – see Kripke (1972) and Kaplan (1977/1989a, 1989b) among others.

30. … *John$_{John}$ … he$_{John}$ …*

This conflation of proper names and pronouns requires additional justification, as the two are different in at least two respects. First, a proper name is completely felicitous in a discourse initial position, while a felicitous pronoun requires a suitable context (linguistic or non-linguistic) to have previously been set up – as shown in (31) and (32) below.

31. Dobby entered The Three Broomsticks.
32. ??He$_{Dobby}$ entered The Three Broomsticks.

Second, when the proper name has been (recently) mentioned, using an anaphoric pronoun is felicitous, while using the proper name again is usually not, as shown in (33) and (34) below.

33. Dobby entered the Three Broomsticks. He$_{Dobby}$ ordered a butterbeer.
34. Dobby entered the Three Broomsticks. ??Dobby ordered a butterbeer.

These two observations seem to argue for the indefinite-like and against the pronoun-like analysis of proper names.

However, the pronoun-like analysis of proper names, i.e. representing proper names as deictic pronouns together with the assumption that proper names are by default salient in the input context, is supported by the interaction between anaphora to proper names and negation. Generally, an indefinite introduced in the scope of negation is not anaphorically accessible to a subsequent pronoun, as shown in (35) below. In contrast, a proper name is anaphorically accessible when it occurs in the scope of negation, as (36) shows.

35. Hermione didn't see a^u / any^u house-elf in the Three Broomsticks.
     #He$_u$ was on vacation in the Bahamas.
36. Hermione didn't see Dobby in the Three Broomsticks.
     He$_{Dobby}$ was on vacation in the Bahamas.

The fact that dynamic negation is defined as a condition in (5c) above, i.e. as *externally static*, correctly predicts the infelicity of anaphora in (35): the pronoun, despite

being co-indexed with the indefinite, ends up being interpreted as a deictic pronoun, picking up whatever the input context associates with the dref *u*.

And this is the reason for the infelicity of the discourse in (35): the co-indexation of the indefinite and the pronoun formally encodes that the pronoun should be 'bound' by the indefinite, i.e., as far as the speaker is concerned, the indefinite and the pronoun should be co-referent / anaphorically connected. However, the pronoun is actually 'unbound', i.e. independent of the individual non-deterministically made salient by the indefinite, since the pronoun ends up referring to some (arbitrary) individual that is already salient in the input discourse context – and this happens despite the fact that the speaker intended the pronoun to refer to the individual made salient by the indefinite.

Therefore, the hypothesis that proper names are by default salient in the input context (which underlies the representation of proper names as deictically used pronouns of some sort) correctly predicts that the anaphoric pronoun in (36) is felicitously used – while the indefinite-like analysis of proper names, according to which they introduce an unspecific dref to which subsequent pronouns are anaphoric to, makes incorrect predictions: we would expect anaphora to proper names introduced under negation to be infelicitous just as the corresponding anaphora to indefinites.

The very simple (and simplified[22]) data presented above does not completely support either the indefinite-like or the pronoun-like analysis of proper names – and it is not the goal of the present investigation to settle these difficult matters[23]. I will henceforth use the indefinite-like analysis only because it is more easily made compatible with the independently motivated formal developments in the following chapters – and the above discussion was only meant to lend some plausibility to this choice[24].

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[22] There are many other factors that can influence the accessibility of referents in discourse and that should be taken into account, e.g. information structure, epistemic specificity in the sense of Farkas (2002) etc.

[23] For a recent in-depth discussion of the linguistic and philosophical issues raised by the interpretation of proper names, see Cumming (2006).

[24] The development I have in mind is the van den Berg-style analysis of dynamic generalized quantifiers (see chapter **6** below), which requires the introduction of a dummy/'exception'/'undefined' individual # (# is

I will account for the felicitous anaphora in (36) above, which is problematic for the indefinite-like analysis of proper names, by assuming that pronouns can be indexed not only with unspecific dref's, but also with specific drefs like *Dobby* or *John*. That is, in addition to the anaphoric pattern in (29) above, I will also allow for the kind of connection between a pronoun and a proper name schematically represented in (37) below.

37. … *John^u* … *he$_{John}$* …

Strictly speaking, the pronoun is not co-referring with the proper name, i.e. the pronoun *he$_{John}$* is different from the co-indexed pronoun *he$_u$* as far as their context-change potentials go. However, the truth-conditional import of the two pronouns is the same in most cases; an exception is, of course, discourse (36) above, where only the pronoun *he$_{John}$* can account both for the felicity of the pseudo anaphoric connection and for the truth-conditions of the discourse. Sentence (36) (repeated in (38) below with the intended indexation) is analyzed as shown in (39). The reader can easily check that the representation in (39) delivers the intuitively correct truth-conditions.

38. Hermione^{u_1} didn't see Dobby^{u_2} in the Three Broomsticks.

   He$_{Dobby}$ was on vacation in the Bahamas.

39. [u_1 | u_1=*Hermione*, ~[u_2 | u_2=*Dobby*, *see_in_TB*{u_1, u_2}]];

   [*on_vacation_in_Bahamas*{*Dobby*}]

---

a designated element of type e; van den Berg's symbol is in fact ⋆). In certain contexts, we will need some 'assignments' i to assign this dummy individual to individual dref's, e.g. for some dref u, we will have ui=#.

We will ultimately have to enforce a similar requirement with respect to proper names: the mention of a proper name, e.g. *John*, will be taken to simultaneously update some 'assignments' with the actual value of the proper name, e.g. *john$_e$*, and other assignments with the dummy value #. Interpreting proper names directly in terms of specific drefs, i.e. in terms of constant functions, e.g. *John* := λi_s. *john$_e$*, does not allow for the option of introducing a dref whose values are either the individual (rigidly) designated by the proper name or the dummy individual.

4. Syntax of a Fragment of English

Now that we have translated DPL in type logic, we can go compositional at the sub-sentential/sub-clausal level. For this purpose, I will define a basic transformational syntax for a small fragment of English in the tradition of Chomsky (1981). The definition is mostly based on Muskens (1996): 159-163 and Muskens (2005).

4.1. Indexation

"The most important requirement that we impose is that the syntactic component of the grammar assigns indices to all names, pronouns and determiners" (Muskens 1996: 159). Unlike Muskens (1996), I will let indices be specific and unspecific dref's (recall that they are all constants of type *se*), e.g. u, u', u_1, *Dobby* etc. Just as before, the antecedents are indexed with superscripts and dependent elements with subscripts, following the convention in Barwise (1987).

I will also allow variables that have the appropriate dref type as indices on traces of movement, e.g. v_{se}, v'_{se}, $v_{0,se}$, $v_{1,se}$ etc. – but such indices appear only on traces, *because they are needed only on traces*. As Muskens (1996): 169 puts it: "In Montague's PTQ (Montague 197[4]) the Quantifying-in rules served two purposes: (a) to obtain scope ambiguities between noun phrases and other scope bearing elements, such as noun phrases, negations and intensional contexts, and (b) to bind pronouns appearing in the expression that the noun phrase took scope over. In the present set-up the mechanism of discourse referents takes over the second task".

The fact that we use distinct indices for the two purposes enables us to keep track of when our indexation makes an essentially dynamic contribution to the semantics and when it is an artifact of the particular scoping mechanism and the particular syntax/semantics interface we employ. In this way, it will be fairly straightforward for the reader to reformulate the analyses we develop in her/his favorite syntactic formalism.

Thus, the choice of a particular (version of a particular) syntactic formalism is largely orthogonal to the matters addressed in the present work and is motivated only by presentational considerations: whichever syntactic formalism the reader favors, it is a

reasonable expectation that s/he will have at least a nodding acquaintance with the Y-model (a.k.a. the T-model) of GB syntax. The syntax-semantics interface defined in this section (which is no more than a proof of concept) is merely meant to give the reader a basic idea of how to design a proper 'interface' between the semantics proposed here and her/his favorite syntactic formalism.

4.2. Phrase Structure and Lexical Insertion Rules

The Y-model of syntax has four components: D-structure (DS), S-Structure (SS), Logical Form (LF) and Phonological Form (PF). We will be interested in the first three, in particular in the level of LF, which provides the input to the semantic interpretation procedure.

The DS component consists of all the trees that can be generated by the phrase structure rules PS1-PS12 and the lexical insertion rules LI1-LI11 in (40) below. We could in fact do away with rule PS1 (the necessary recursion is already built into PS2), but I will keep it as a reminder that sequencing two sentences in discourse occurs at a supra-sentential, textual level.

40. **Phrase structure rules and lexical insertion rules** [25]**.**

(**PS 1**) Txt $\rightarrow$ (Txt) CP	(**PS 5**) VP $\rightarrow$ DP V'	(**PS 9**) V_{di}' $\rightarrow$ V_{di} DP
(**PS 2**) CP $\rightarrow$ (CP) IP	(**PS 6**) V' $\rightarrow$ V_{in}	(**PS 10**) DP $\rightarrow$ D NP
(**PS 3**) CP $\rightarrow$ C IP	(**PS 7**) V' $\rightarrow$ V_{tr} DP	(**PS 11**) NP $\rightarrow$ N (CP)
(**PS 4**) IP $\rightarrow$ I VP	(**PS 8**) V' $\rightarrow$ V_{di}' DP	(**PS 12**) X $\rightarrow$ X^{+} Conj X

(**LI 1**) D $\rightarrow$ *a^u, every^u, most^u, few^u, no^u, some^u, any^u, a$^{u'}$, every$^{u'}$, …*	(**LI 5**) N $\rightarrow$ *farmer, house-elf, donkey, …*	(**LI 9**) I $\rightarrow$ *Ø, doesn't, don't, -ed, -s, didn't, …*
(**LI 2**) DP $\rightarrow$ *he_u, she_u, it_u, $he_{u'}$, ..., he_{John}, she_{Mary}, …, t_v, $t_{v'}$, …*	(**LI 6**) V_{tr} $\rightarrow$ *own, beat, …*	(**LI 10**) C $\rightarrow$ *if*
(**LI 3**) DP $\rightarrow$ *John^u, Mary^u, John$^{u'}$, ...*	(**LI 7**) V_{in} $\rightarrow$ *sleep, walk, …*	(**LI 11**) Conj $\rightarrow$ *and, or*
(**LI 4**) DP $\rightarrow$ *who, whom, which*	(**LI 8**) V_{di} $\rightarrow$ *buy, give, …*	

[25] I am temporarily overloading the symbol '$\rightarrow$', which (as it is customary in the literature) is used to define the production rules of our grammar.

Subjects are assumed to be VP-internal and this is where they remain by default even at LF (they are raised out of VP only at PF). In this way, we can interpret sentential negation as having scope over quantifiers in subject position. Similarly, V-heads move to the inflectional I-head only at PF.

4.3.   Relativization and Quantifier Raising

DS and SS are connected via the obligatory movement rule of Relativization (REL). A tree Θ' *follows by REL* from a tree Θ iff Θ' is the result of replacing some sub-tree of Θ of the form $[_{CP} [_{IP} X [_{DP} wh] Y]]$, where X and Y are (possibly empty) strings and *wh* is either *who, whom* or *which*, by a tree $[_{CP} [_{DP} wh]^{v} [_{CP} [_{IP} X t_v Y]]]$, where v is a fresh variable index (not occurring in Θ as a superscript). REL is basically CP adjunction.

41. *Relativization (REL)*: $[_{CP} [_{IP} X [_{DP} wh] Y]] \rightarrow [_{CP} [_{DP} wh]^{v} [_{CP} [_{IP} X t_v Y]]]$

For example, the DP a^u *girl who* $every^{u'}$ *boy adores* has the syntactic representation in (42) below, obtained by an application of REL:

42. $[_{DP} a^u [_{NP} [_N girl] [_{CP} [_{DP} who]^{v} [_{CP} [_{IP} [_I \text{-}s] [_{VP} [_{DP} every^{u'} boy] [_{V'} [_{Vtr} adore] t_v]]]]]]]]$

Formally, SS is the smallest set of trees that includes DS and is closed under REL; thus, DS$\subseteq$SS.
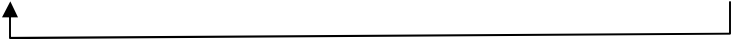
Finally, we turn to the definition of LF, the syntactic component that is the input to our semantics. This is the level where quantifier scope ambiguities are resolved. We define an optional rule of Quantifier Raising (QR) (May 1977) which adjoins DP's to IP's or DP's to VP's (we need VP-adjunction for ditransitive verbs among other things) and which is basically the Quantifying-In rule of Montague (1974).

A tree Θ' *follows by QR* from a tree Θ iff: **(a)** Θ' is the result of replacing some sub-tree Σ of Θ of the form $[_{IP} X [_{DP} Z] Y]$ by a tree $[_{IP} [_{DP} Z]^{v} [_{IP} X t_v Y]]$, where v is a fresh variable index (not occurring in Θ as a superscript); or **(b)** Θ' is the result of replacing some sub-tree Σ of Θ of the form $[_{VP} X [_{DP} Z] Y]$ by a tree $[_{VP} [_{DP} Z]^{v} [_{VP} X t_v Y]]$, where v is a fresh variable index (not occurring in Θ as a superscript). The conditions on the QR

rule are that *Z* is not a pronoun or a *wh*-word and that [$_{DP}$ *Z*] is not a proper sub-tree of a DP sub-tree [$_{DP}$ *W*] of Σ [26].

43. *Quantifier Raising (QR):* **a.** [$_{IP}$ *X* [$_{DP}$ *Z*] *Y*] → [$_{IP}$ [$_{DP}$ *Z*]^v [$_{IP}$ *X* t_v *Y*]]

                      b. [$_{VP}$ *X* [$_{DP}$ *Z*] *Y*] → [$_{VP}$ [$_{DP}$ *Z*]^v [$_{VP}$ *X* t_v *Y*]]

For example, the reverse scope of *every^u house-elf adores a$^{u'}$ witch* can be obtained by QR to IP, as shown in (44) below (of course, it could also be obtained by QR to VP).

44. [$_{IP}$ [$_{DP}$ *a$^{u'}$ witch*]^v [$_{IP}$ [$_{I}$ *-s*] [$_{VP}$ [$_{DP}$ *every^u house-elf*] [$_{V'}$ [$_{Vtr}$ *adore*] t_v]]]]

LF is defined as the smallest set of trees that includes SS and is closed under QR; thus, SS⊆LF.

5. Type-driven Translation

In a Fregean / Montagovian framework, the compositional aspect of the interpretation is largely determined by the types for the 'saturated' expressions, i.e. names and sentences. Let's abbreviate them as **e** and **t**. An extensional static logic without pluralities (i.e. classical higher-order logic) is the simplest: **e** is *e* (atomic entities) and **t** is *t* (truth-values). The English verb *sleep*, for example, is represented by a term *sleep* of type (**et**), i.e. (*et*), and the generalized quantifier (GQ) *every man* by a term of type ((**et**)**t**), i.e. ((*et*)*t*).

This setup can be complicated in various ways[27]. In particular, Dynamic Ty2 complicates it by adding another basic type *s* whose elements model DPL variable assignments, i.e. (simplified versions of) dynamic info states. A sentence is interpreted as

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[26] For example, if the DP sub-tree [$_{DP}$ *W*] of Σ contains a relative clause which in its turn contains [$_{DP}$ *Z*], we do not want to QR [$_{DP}$ *Z*] all the way out of the relative clause.

[27] See for example Lewis (1972) and Creswell (1973), which use the same technique to introduce intensionality, i.e., in their case, **t** := *st* and *s* is the sort of indices of evaluation (however one wants to think of them, e.g. as worlds, <world, time> pairs etc.; see Muskens 1995a for a set of axioms that make the atomic objects of type *s* behave as <world, time> pairs).

a relation between an input and an output 'assignment', i.e. $\mathbf{t} := (s(st))$, and a name denotes an individual dref, i.e. $\mathbf{e} := (se)$ [28].

The English verb *sleep* is still translated by a term of type (**et**), but now this means that it takes a dref u of type **e** and it relates two info states i and i' of type s if and only if $i=i'$ and the entity denoted by u in info state i, i.e. ui, has the *sleep* property of type (*et*), i.e. the static 'sleep'-property.

5.1. Translating Basic Expressions

Table (45) below provides examples of basic meanings for the lexical items in (40) above: the first column contains the lexical item, the second column its Dynamic Ty2 translation and the third column its type, assuming the above two abbreviations, i.e. $\mathbf{t} := (s(st))$ and $\mathbf{e} := (se)$. Note that the abbreviated types have exactly the form we would expect them to have in Montague semantics (e.g. the translation of the intransitive verb *sleep* is of type **et**, the translation of the pronoun *he* is of type (**et**)**t**, the translations of the indefinite article *a* and of the determiner *every* are of type (**et**)((**et**)**t**) etc.). The list of basic meanings constitutes rule **TR0** of our type-driven translation procedure for the English fragment.

Transitive verbs like *own* are assumed to take a generalized quantifier (GQ) as their direct object (type (**et**)**t**), which captures the fact that the default quantifier scoping is subject over object. The reverse scope is obtained by QR.

Ditransitive verbs like *buy* are assumed to take two GQ's as objects; the default relative scope of the two (encoded in the lexical entry) is their left-to-right surface order, i.e. the first of them (e.g. the Dative GQ) takes scope over the second (e.g. the Accusative GQ). Arguably, this is the correct prediction, since the most salient quantifier scoping in

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[28] Despite appearances, relativizing the interpretation of names to 'assignments' is not different from the Montagovian interpretation of names (or the Tarskian interpretation of individual constants in first-order logic): just as a name like 'John' is assigned the same individual, namely $john_e$, relative to any variable assignment g in a static Montagovian system, CDRT interprets proper names in terms of constant functions of type se, e.g. the semantic value of the name 'John' is given in terms of the constant function $John_{se}$ that maps each 'assignment' i_s to the individual $john_e$, i.e. $John_{se} := \lambda i_s.\, john_e$ (see also the discussion in section **0** above).

the sentence *Dobby bought every witch an alligator purse* follows the left-to-right linear order: the Dative GQ takes scope over the Accusative GQ, so that the purses co-vary with the witches. The reverse scope has to be obtained by QR (to VP or IP).

Note that the Dative GQ takes scope over the Accusative GQ despite their relative syntactic position: given the phrase structure rules **PS8** and **PS9** in (40) above, the Dative GQ is actually c-commanded by the Accusative GQ. The fact that a quantifier can take scope over another without c-commanding it syntactically is one of the advantages of working with a dynamic system, in which *quantifier scope is encoded in the order in which the updates are sequenced*.

Thus, in a dynamic framework, *syntactic structure affects quantifier scope only to the extent to which syntactic relations (e.g. c-command) are ultimately reflected in update sequencing*. The lexical entry for ditransitive verbs in (45) below 'neutralizes' syntactic c-command: it sequences the updates contributed by the two GQ objects according to their linear order and not according to their syntactic structure.

Defaulting to linear order (as opposed to syntactic c-command) has welcome empirical consequences in the case at hand: besides the fact that we capture the correlation between linear order and quantifier scope, we can also account for the fact that the Dative GQ is able to bind pronouns within the Accusative GQ without c-commanding them, as for example in *Dobby gave every^u witch her$_u$ broom*.

It is in fact not unexpected that a dynamic system can account for pronoun binding without c-command given that donkey anaphora is a paradigmatic example of such binding without c-command. The point made by the present analysis of ditransitive verbs is that the dynamic account of donkey sentences can successfully generalize beyond the phenomena that provided the initial empirical motivation.

Pronouns of the form *he$_u$* and traces of the form *t$_v$* are interpreted as in Montague (1974), i.e. as the GQ-lift of their index, which, for pronouns, is a dref (i.e. a constant of type **e** := *se*) and, for traces, is an actual variable (again of type **e** := *se*). We will see in chapter **5** that this kind of 'lifted' interpretation for pronouns (coupled with the type-raised interpretation of transitive and ditransitive verbs) is not necessarily a 'worst case'

generalization, but it is actually motivated by the *distributive* interpretation of singular number morphology occurring on donkey pronouns.

Proper names are basically analyzed as indefinites – see the discussion at the very end of section **3** above, in particular (28). The only difference is that they are now translated as the corresponding GQ-lift.

Indefinites have the type of (dynamic) generalized determiners, as needed for the definition of the compositional interpretation procedure, but their crucial dynamic contribution is the introduction of a new dref, which has to satisfy the restrictor property and the nuclear scope property *in this order*. The DPL-style abbreviation explicitly exhibits the existential quantification built into the indefinite.

The universal quantifier *every* also has the type of generalized determiners and it is interpreted as expected: the DPL-style abbreviation speaks for itself. Note the square brackets surrounding the formula – they are due to the fact that, unlike the indefinite determiner *a*, the universal determiner *every* contributes a test – it is internally dynamic but *externally static*, just as classical DRT / FCS and DPL would have it.

The negative quantifier *no* also contributes a test; I provide its two equivalent translations, one of them based on negation and existential quantification, the other based on negation and universal quantification.

The *wh*-words that enter the construction of relative clauses are analyzed as identity functions over the property contributed by the relative clause. This property will then be 'sequenced' with the property contributed by the preceding common noun to yield a 'conjoined' property that is a suitable argument for a generalized determiner. The order in which the common noun and the relative clause are sequenced follows the linear surface order. The rule that achieves this dynamic 'conjunction' / 'sequencing' of properties generalizes both the static Predicate Modification rule in Heim & Kratzer (1998) and the dynamic Sequencing rule in Muskens (1996) – see (48) below.

45. **TR 0: Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se \;\; \mathbf{t} := s(st)$
$[sleep]_{V_{in}}$	⤳ $\lambda v_{\mathbf{e}}.\ [sleep_{et}\{v\}]$	**et**
$[own]_{V_{tr}}$	⤳ $\lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [own_{e(et)}\{v, v'\}])$	**((et)t)(et)**
$[buy]_{V_{di}}$	⤳ $\lambda Q'_{(\mathbf{et})\mathbf{t}}.\lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v''_{\mathbf{e}}.\ [buy_{e(e(et))}\{v, v', v''\}]))$	**(ett)((ett)(et))**
$[house\text{-}elf]_{N}$	⤳ $\lambda v_{\mathbf{e}}.\ [house\_elf_{et}\{v\}]$	**et**
$[he_u]_{DP}$	⤳ $\lambda P_{\mathbf{et}}.\ P(u_{\mathbf{e}})$	**(et)t**
$[t_v]_{DP}$	⤳ $\lambda P_{\mathbf{et}}.\ P(v_{\mathbf{e}})$	**(et)t**
$[he_{Dobby}]_{DP}$	⤳ $\lambda P_{\mathbf{et}}.\ P(Dobby_{\mathbf{e}})$	**(et)t**
$[Dobby^u]_{DP}$	⤳ $\lambda P_{\mathbf{et}}.\ [u \mid u{=}Dobby];\ P(u)$	**(et)t**
$[a^u]_{D}$	⤳ $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [u];\ P'(u);\ P(u),$ i.e. $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ \exists u(P'(u);\ P(u))$	**(et)((et)t)**
$[every^u]_{D}$	⤳ $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [([u];\ P'(u)) \rightarrow P(u)],$ i.e. $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [\forall u(P'(u) \rightarrow P(u))]$	**(et)((et)t)**
$[no^u]_{D}$	⤳ $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [{\sim}([u];\ P'(u);\ P(u))],$ i.e. $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [{\sim}\exists u(P'(u);\ P(u))]$ ⤳ $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [([u];\ P'(u)) \rightarrow [{\sim}P(u)]],$ i.e. $\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.\ [\forall u(P'(u) \rightarrow [{\sim}P(u)])]$	**(et)((et)t)**
$[who]_{DP}$	⤳ $\lambda P_{\mathbf{et}}.\ P$	**(et)(et)**
$[\emptyset]_{I}\ /\ [\text{-}ed]_{I}\ /\ [\text{-}s]_{I}$	⤳ $\lambda D_{\mathbf{t}}.\ D$	**tt**
$[doesn't]_{I}\ /\ [didn't]_{I}$	⤳ $\lambda D_{\mathbf{t}}.\ [{\sim}D]$	**tt**
$[if]_{C}$	⤳ $\lambda D'_{\mathbf{t}}.\lambda D_{\mathbf{t}}.\ [D' \rightarrow D]$	**t(tt)**
$[and]_{Conj}$	⤳ $\lambda D'_{\mathbf{t}}.\lambda D_{\mathbf{t}}.\ D';\ D$	**t(tt)**
$[or]_{Conj}$	⤳ $\lambda D'_{\mathbf{t}}.\lambda D_{\mathbf{t}}.\ [D' \vee D]$	**t(tt)**

Non-negative inflectional heads are interpreted as identity functions over DRS meanings (type **t** := *s*(*st*)). Negative inflectional heads are interpreted as expected: their value is a test, containing a condition that negates the argument DRS.

The conditional *if* is a binary DRS connective: it takes two DRS's as arguments and it returns a test containing a dynamic implication condition that relates the two argument DRS's.

The coordinating elements *and* and *or* will be discussed in more detail in section **5.1** of the following chapter (chapter **4**); I provide here the entries for the simplest case, namely coordination of two sentences (i.e. DRS's).

5.2.   Translating Complex Expressions

Based on **TR0**, we can obtain the translation of more complex LF structures by specifying how the translation of a mother node depends on the translations of its daughters. I provide five such rules, the last of which (**TR5:** *Coordination* – see (50) below) will be generalized in the following chapter.

The first rule covers non-branching nodes: the mother inherits the translation of the daughter.

46. **TR 1 – Non-branching Nodes (NN).**

    If A ⤳ α and A is the only daughter of B,

    then B ⤳ α.

The second rule is functional application: the translation of the mother is the result of applying the translation of one daughter to the translation of the other.

47. **TR 2 – Functional Application (FA).**

    If A ⤳ α and B ⤳ β and A and B are the only daughters of C,

    then C ⤳ α(β), provided that this is a well-formed term.

The third rule is a generalized sequencing (i.e. a generalized dynamic conjunction) rule. For one thing, it translates the meaning of complex texts (Txt) that are formed out of

a text (Txt) and a sentence (CP) – see **PS1** in (40) above. In this sense, it is a generalization of the Sequencing rule in Muskens (1996). But it also handles predicate modification in general, e.g. it translates the meaning of an NP that is formed out of a common noun N and a relative clause CP – see **PS11** in (40) above. In this sense, it is a generalization of the Predicate Modification rule in Heim & Kratzer (1998).

48. **TR 3 – Generalized Sequencing (GSeq) (i.e. Sequencing + Predicate Modification).**

> If A $\rightsquigarrow$ α, B $\rightsquigarrow$ β, A and B are the only daughters of C in that order (i.e.
>
> C → A B) and α and β are of the same type τ of the form **t** or (σ**t**) for
>
> some σ∈**Typ**,
>
> then C $\rightsquigarrow$ α; β if τ=**t** or C $\rightsquigarrow$ λv_σ. α(v); β(v), if τ=(σ**t**),
>
> provided that this is a well-formed term.

The fourth rule handles Quantifying-In, both for quantifiers and for relativizers (i.e. *wh*-words).

49. **TR 4 – Quantifying-In (QIn).**

> If DP^v $\rightsquigarrow$ α, B $\rightsquigarrow$ β and DP^v and B are daughters of C,
>
> then C $\rightsquigarrow$ α(λv. β), provided that this is a well-formed term.

The final rule handles binary coordinations (it will be generalized to an arbitrary finite number of coordinated elements in the next chapter).

50. **TR 5 – Coordination (Co).**

> If A$_1$ $\rightsquigarrow$ α$_1$, Conj $\rightsquigarrow$ β, A$_2$ $\rightsquigarrow$ α$_2$ and A$_1$, Conj and A$_2$ are the only
>
> daughters of A in that order (i.e. A → A$_1$ Conj A$_2$),
>
> then A $\rightsquigarrow$ β(α$_1$)(α$_2$),
>
> provided this a well-formed term and has the same type as α$_1$ and α$_2$.

The translation procedure, i.e. the relation 'tree Θ *translates as* term α', is formally defined as the smallest relation $\rightsquigarrow$ between trees and Dynamic Ty2 terms that is conform

to **TR0**-**TR5** and is closed under the reduction of the type-logical terms, e.g. if tree Θ *translates as* term α and term β follows from α by λ-conversion, then Θ *translates as* β.

6. *Anaphora and Quantification in Compositional DRT (CDRT)*

We are now ready to go through some examples. This section will show how CDRT can account for bound variable anaphora (**6.1**), quantifier scope ambiguities (**6.2**) and quantifier scope with ditransitive verbs (**6.3**). Finally, we will see how to analyze in CDRT the three paradigm examples that motivate dynamic semantics: cross-sentential anaphora (**6.4**), relative-clause donkey sentences (**6.5**) and, finally, conditional donkey sentences (**6.6**).

6.1. **Bound Variable Anaphora**

First, we can capture bound anaphora in CDRT without using the syntactic rule QR (Quantifier Raising, see (43) above) and the corresponding semantic rule QIn (Quantifying-In, see (49) above): we simply need the pronoun to be co-indexed with the antecedent, as shown in (51) below.

51. Every^{u_1} house-elf hates himself$_{u_1}$.

Co-indexation is enough for binding because binding in CDRT (just like in DPL) is actually taken care of by the *explicit (and, in this case, unselective) quantification over 'assignments'* built into the meaning of quantifiers. In classical static logic, the quantification over assignments is only implicit (and selective, but this is not directly relevant for the matter at hand): the paradigm example is λ-abstraction, which selectively quantifies over assignments that differ at most with respect to the variable that is abstracted over. Therefore, if we want to obtain bound variable anaphora in a static system, co-indexation, i.e. using the same variable, is not enough: we also need to create a suitable λ-abstraction configuration that will ensure the semantic co-variation via selective quantification over assignments.

Sentence (51) receives the Dynamic Ty2 representation in (52) below – or, equivalently, the one in (53). The formulas deliver the intuitively correct truth-conditions, as shown in (54).

52. $[[u_1 \mid house\_elf\{u_1\}] \rightarrow [hate\{u_1, u_1\}]]$

53. $[\forall u_1([house\_elf\{u_1\}] \rightarrow [hate\{u_1, u_1\}])]$

54. $\lambda i_s. \forall x_e(house\_elf(x) \rightarrow hate(x, x))$

Most importantly, CDRT associates the correct Dynamic Ty2 translation with sentence (51) in a compositional way, as shown by the LF in (55) below, with the nodes in the tree decorated with their corresponding translations. I do not explicitly show what rules of type-driven translation are applied at various points in the calculation – the reader will have no difficulty identifying them. Note only that, by the NN rule for non-branching nodes (see (46) above), the translation of the topmost node Txt is the same as the translation of the IP node dominated by it.

55. Every ^{u_1} house-elf hates himself $_{u_1}$.

```
                          Txt
                           |
                           CP
                           |
                           IP
          [[u₁ | house_elf{u₁}] → [hate{u₁, u₁}]]
                      /              \
                  [-s]_I              VP
                  λD_t. D     [[u₁ | house_elf{u₁}] → [hate{u₁, u₁}]]
                               /                       \
                             DP                         V'
          λP_et.[[u₁ | house_elf{u₁}] → P(u₁)]     λv_e.[hate{v, u₁}]
              /            \                         /            \
      [every^u₁]_D         NP                  [hate]_Vtr      [himself u₁]_DP
  λP'_et.λP_et.[([u₁]; P'(u₁)) → P(u₁)]   λv_e.[house_elf_et{v}]   λQ_(et)t.λv_e.Q(λv'_e.[hate_e(et){v, v'}])   λP_et.P(u₁)
                                    |
                              [house-elf]_N
                           λv_e.[house_elf_et{v}]
```
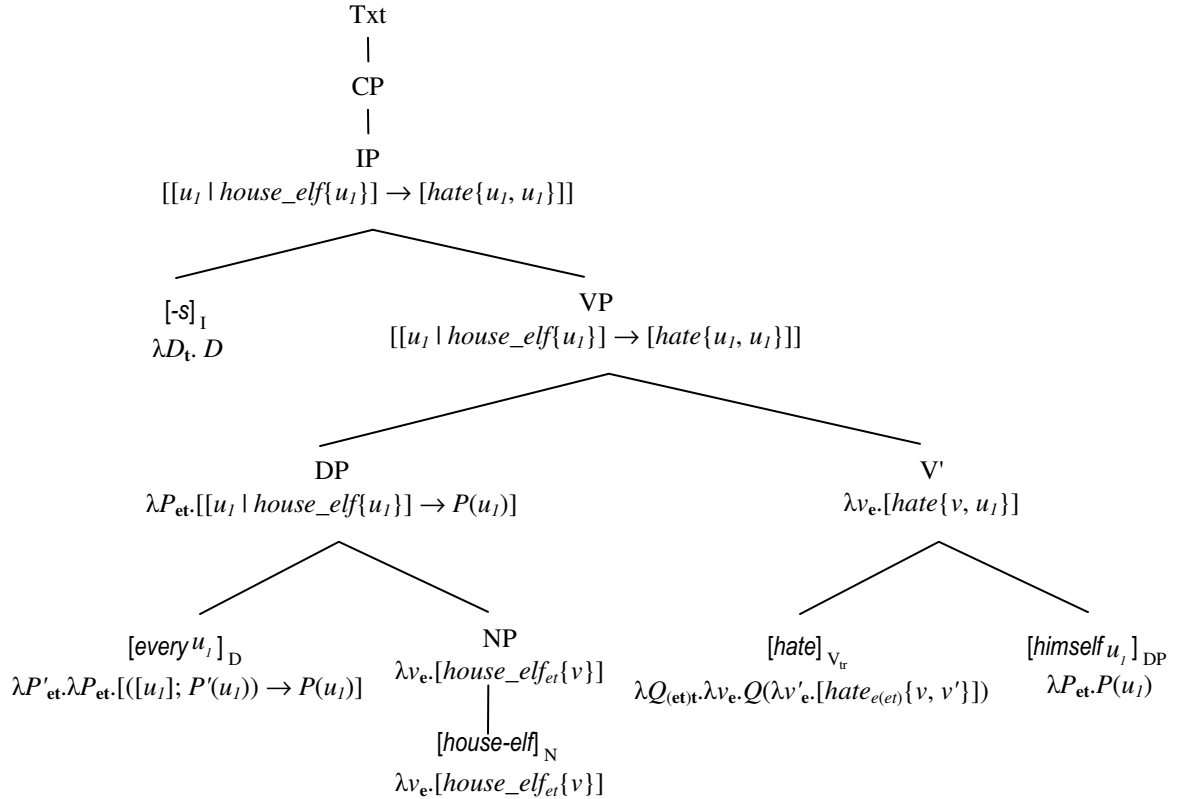
Thus, we see that CDRT can compositionally account for bound anaphora in English without QR or QIn: co-indexation is enough for binding, since the basic meaning of the determiner *every* universally quantifies over assignments. This universal quantification can be ultimately traced back to dynamic negation – see the discussion of DPL universal quantification and dynamic implication in the previous chapter.

6.2. Quantifier Scope Ambiguities

We turn now to an application of QR and QIn. Consider the sentence in (56) below, which is ambiguous between two quantifier scopings: the surface-based scope *every*^{u_1}>>*a*^{u_2} and the reverse scope *a*^{u_2}>>*every*^{u_1}. The reverse scope is obtained by an application of QR. The two LF's yield the translations in (57) and (59) below, which capture the intuitively correct truth-conditions for the two readings, as shown in (58) and (60).

56. Every^{u_1} house-elf adores a^{u_2} witch.

57. ***every*^{u_1}>>*a*^{u_2}** : $[[u_1 \mid house\_elf\{u_1\}] \rightarrow [u_2 \mid witch\{u_2\}, adore\{u_1, u_2\}]]$

58. ***every*^{u_1}>>*a*^{u_2}** : $\lambda i_s. \ \forall x_e(house\_elf(x) \rightarrow \exists y_e(witch(y) \wedge adore(x, y)))$

59. ***a*^{u_2}>>*every*^{u_1}** : $[u_2 \mid witch\{u_2\}, [u_1 \mid house\_elf\{u_1\}] \rightarrow [adore\{u_1, u_2\}]]$

60. ***a*^{u_2}>>*every*^{u_1}** : $\lambda i_s. \ \exists y_e(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y)))$

The two LFs are provided in (61) and (62) below.

61. **every**^{u_1}**>>a**^{u_2} : Every ^{u_1} house-elf adores a ^{u_2} witch.

Txt
|
CP
|
IP
$[[u_1 \mid house\_elf\{u_1\}] \rightarrow [u_2 \mid witch\{u_2\}, adore\{u_1, u_2\}]]$

[-s]$_I$
$\lambda D_t.\ D$

VP
$[[u_1 \mid house\_elf\{u_1\}] \rightarrow [u_2 \mid witch\{u_2\}, adore\{u_1, u_2\}]]$

DP
$\lambda P_{et}.[[u_1 \mid house\_elf\{u_1\}] \rightarrow P(u_1)]$

V'
$\lambda v_e.[u_2 \mid witch\{u_2\}, adore\{v, u_2\}]$

every ^{u_1} house-elf

[adore]$_{V_{tr}}$
$\lambda Q_{(et)t}.\lambda v_e.Q(\lambda v'_e.[adore_{e(et)}\{v, v'\}])$

DP
$\lambda P_{et}.[u_2 \mid witch\{u_2\}];\ P(u_2)$
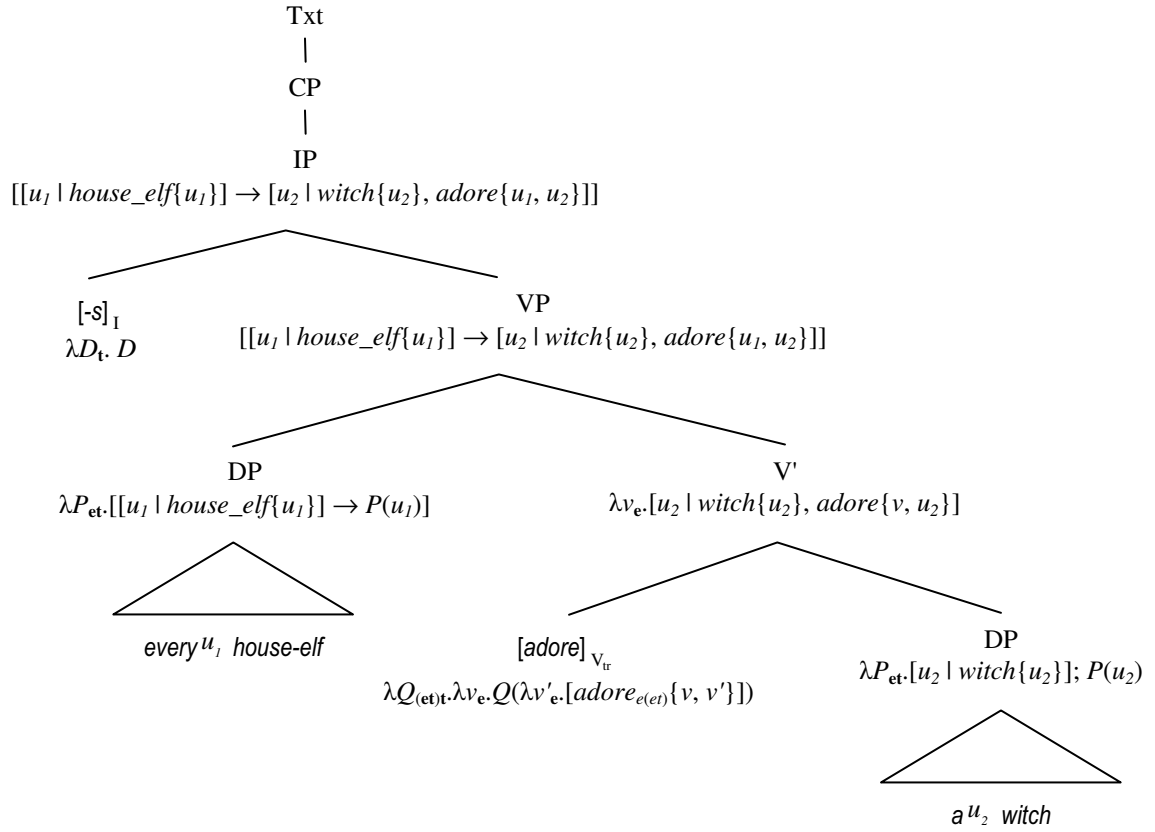
a ^{u_2} witch

The reverse scope is obtained by applying the QR rule to the indefinite DP a^{u_2} *witch*, as shown in (62) below. Note that the application of the QR rule yields the reverse scope not because it places the indefinite DP in a c-commanding position, *but because it reverses the order of updates*. From this perspective, having a syntactic level for quantifier scoping that encodes dominance in addition to linear precedence relations seems like overkill (see also the discussion of the ditransitive sentence in (63) below.

62. (a^{u_2} **>>every**^{u_1}) Every^{u_1} house-elf adores a^{u_2} witch.

Txt
|
CP
|
IP
$[u_2 \mid witch\{u_2\}, [u_1 \mid house\_elf\{u_1\}] \rightarrow [adore\{u_1, u_2\}]]$

DP$^{v''}$
$\lambda P_{et}.[u_2 \mid witch\{u_2\}]; P(u_2)$

IP
$[[u_1 \mid house\_elf\{u_1\}] \rightarrow [adore\{u_1, v''\}]]$

a^{u_2} witch

$[\text{-}s]_I$
$\lambda D_t. D$

VP
$[[u_1 \mid house\_elf\{u_1\}] \rightarrow [adore\{u_1, v''\}]]$

DP
$\lambda P_{et}.[[u_1 \mid house\_elf\{u_1\}] \rightarrow P(u_1)]$

V'
$\lambda v_e.[adore\{v, v''\}]$

every^{u_1} house-elf

$[adore]_{V_{tr}}$
$\lambda Q_{(et)t}.\lambda v_e.Q(\lambda v'_e.[adore_{e(et)}\{v, v'\}])$

$[t_{v'}]_{DP}$
$\lambda P_{et}.P(v''_e)$
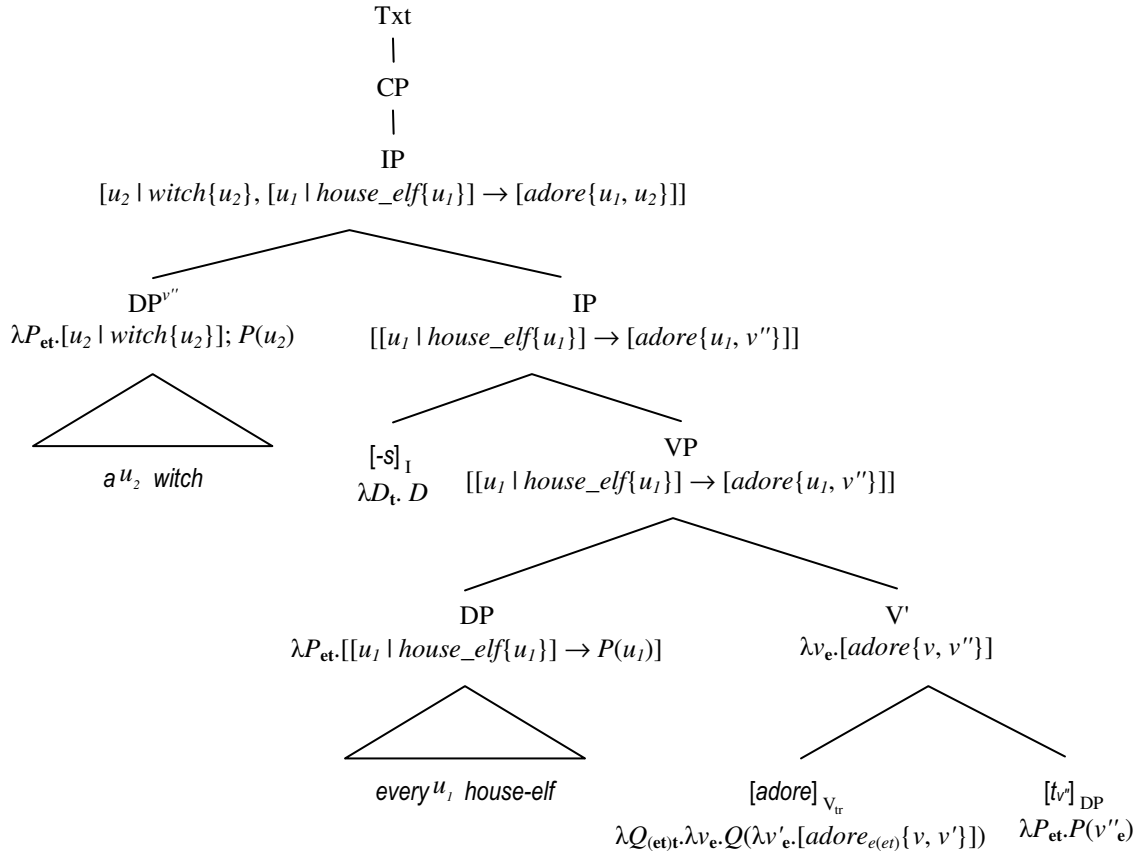
6.3. Quantifier Scope with Ditransitive Verbs

Given that donkey sentences with ditransitive verbs will feature quite prominently throughout the remainder of the dissertation, I will show in detail how sentences with ditransitive verbs are analyzed in CDRT. Consider the sentence in (63) below, in which the Dative GQ both takes scope over and binds into the Accusative GQ – without c-commanding it.

63. Dobby^{u_3} gave every^{u_1} witch her$_{u_1}$ alligator purse.

This example simultaneously exhibits two of the most interesting aspects of CDRT:

- we can have binding of pronouns without c-command and without QR, i.e. without the covert syntactic manipulations associated with the level of LF – see also (51) and (73) above;

- a quantifier can have wide scope over another without c-commanding it as long as the update it contributes is sequenced before the update of the other quantifier: the *lexical entry* for ditransitive verbs specifies that the Dative GQ update is sequenced / takes scope over the Accusative GQ update – and this is enough to nullify the fact that, *syntactically*, the former does not take scope over the latter.

Both features of CDRT point to the fact that the syntactic level of LF provides a needlessly rich, i.e. complex, input to the semantic interpretation procedure. In particular, the dominance relations that the LF level encodes are not (always) relevant for interpretation; the only two semantically relevant features of the LF level are: (i) the co-indexation of the referring expressions and (ii) the linear precedence (i.e. sequencing) of the updates[29].

Following the simplified LF for possessive DP's proposed in Heim & Kratzer (1998)[30], I analyze *her alligator purse* as the DP in (64) below[31].

64. [DP a^{u_2} [NP [N *alligator purse*] [PP *of her*$_{u_1}$]]]]

---

[29] I will not further pursue this perspective on meaning composition in the present work. Note however that coupling this perspective on meaning composition with the *plural* info states we will introduce in chapter **6** below (*plural* in the sense that the dynamic info states are sets of 'assignments' and not single 'assignments') promises to provide a novel and intuitively appealing analysis of cataphora on the one hand and the non-standard ('choice-function') scopal behavior of indefinites on the other hand (see for example Chierchia 1995: Chapter 3 for cataphora and Chierchia 2001 and references therein for 'choice-function' indefinites).

See also the online update of Bittner (2006), where said properties of CDRT (i.e. the fact that the only two semantically relevant features of the LF level are indexation and sequencing of updates) take center stage.

[30] Although the LF in (64) is similar to the one in Heim & Kratzer (1998), the analysis is different: while Heim & Kratzer (1998) take possessives to be covertly definite DP's (and adopt a Fregean analysis of definite descriptions), I analyze them here as covertly *in*definite DP's. The indefinite analysis of possessive DP's is empirically supported by the interpretation of possessives in predicative positions, e.g. *John is her / Mary's brother*, which are not associated with uniqueness implications – I am grateful to Magdalena Schwager (p.c.) for bringing this to my attention.

[31] I assume that the following phrase structure and lexical insertion rules are added to the syntax of our English fragment: (**PS 13**) NP → N (PP); (**PS 14**) PP → P DP); (**LI 12**) P → *of*.

The meaning of the preposition *of* is given in (65) below – it has the same structure as the lexical entry of a transitive verb like *own*.

65. $[of]_P \rightsquigarrow \lambda Q_{ett}. \lambda v_e. Q(\lambda v'_e. [of_{e(et)}\{v, v'\}])$

We compositionally derive the following translation for the DP in (64) (the subscript on the symbol $\rightsquigarrow$ indicates the rule applied in translating the mother node):

66. a. $[_{PP} \text{ of her}_{u_1}] \rightsquigarrow_{FA} \lambda v_e. [of\{v, u_1\}]$

b. $[_{NP} [_N \text{ alligator purse}] [_{PP} \text{ of her}_{u_1}]] \rightsquigarrow_{GSeq} \lambda v_e. [alligator\_purse\{v\}, of\{v, u_1\}]$

c. $(64) \rightsquigarrow_{FA} \lambda P_{et}. [u_2 \mid alligator\_purse\{u_2\}, of\{u_2, u_1\}]; P(u_2)$

The syntactic structure of the V' is provided in linearized form in (67) below and compositionally translated in (68). The Dative GQ *every^{u_1} witch* takes scope over the Accusative GQ and also binds the pronoun *her$_{u_1}$* contained in it.

67. $[_{V'} [_{Vdi'} \text{ give every}^{u_1} \text{ witch}] [_{DP} a^{u_2} \text{ alligator purse of her}_{u_1}]]$

68. $[_{Vdi'} \text{ give every}^{u_1} \text{ witch}] \rightsquigarrow_{FA}$

$\qquad \lambda Q_{(et)t}. \lambda v_e. [[u_1 \mid witch\{u_1\}] \rightarrow Q(\lambda v''_e. [give\{v, u_1, v''\}])]$

$\quad (67) \rightsquigarrow_{FA} \lambda v_e. [[u_1 \mid witch\{u_1\}]$

$\qquad\qquad\qquad\qquad \rightarrow [u_2 \mid alligator\_purse\{u_2\}, of\{u_2, u_1\}, give\{v, u_1, u_2\}]]$

Thus, sentence (63) is translated as shown in (69) below and it receives the intuitively correct truth-conditions (for its most salient reading), as (70) below shows.

69. $[u_3 \mid u_3=Dobby,$

$\qquad [u_1 \mid witch\{u_1\}] \rightarrow [u_2 \mid alligator\_purse\{u_2\}, of\{u_2, u_1\}, give\{u_3, u_1, u_2\}]]$

70. $\lambda i_s. \exists z_e(z=dobby \wedge$

$\qquad \forall x_e(witch(x) \rightarrow \exists y_e(alligator\_purse(y) \wedge of(y, x) \wedge give(z, x, y))))$, i.e.

$\quad \lambda i_s. \forall x_e(witch(x) \rightarrow \exists y_e(alligator\_purse(y) \wedge of(y, x) \wedge give(dobby, x, y)))$

6.4.　Cross-sentential Anaphora

We can also compositionally assign the intuitively correct interpretation to the three paradigm examples we have used in the previous chapter to motivate dynamic semantics. The examples are repeated in (71-72), (73) and (74) below; their LF's have two distinctive features: on the one hand, they put to use the previously otiose CP and Txt categories; on the other hand, they contain an application of the REL & QIn rules.

The analysis of donkey sentences exhibits a crucial property of CDRT we have already hinted at, namely the fact that, as long as these sentences receive the intuitively correct co-indexation, we can get the semantics of pronoun binding without c-command at the level of the LF.

71. A^{u_1} house-elf fell in love with a^{u_2} witch.

72. He$_{u_1}$ bought her$_{u_2}$ an^{u_3} alligator purse.

Txt
[u_1, u_2, u_3 | house_elf{u_1}, witch{u_2}, fall_in_love{u_1, u_2}, alligator_purse{u_3}, buy{u_1, u_2, u_3}]

Txt

CP

IP
[u_1, u_2 | house_elf{u_1}, witch{u_2}, fall_in_love{u_1, u_2}]

CP

IP
[u_3 | alligator_purse{u_3}, buy{u_1, u_2, u_3}]

[-ed]$_I$
$\lambda D_t. D$

VP
[u_1, u_2 | house_elf{u_1}, witch{u_2}, fall_in_love{u_1, u_2}]

VP
[u_3 | alligator_purse{u_3}, buy{u_1, u_2, u_3}]

[-ed]$_I$
$\lambda D_t. D$

[a^{u_1} house-elf]$_{DP}$
$\lambda P_{et}.[u_1$ | house_elf{u_1}]; $P(u_1)$

V'
$\lambda v_e.[u_2$ | witch{u_2}, f.i.l{v, u_2}]

[he$_{u_1}$]$_{DP}$
$\lambda P_{et}.P(u_1)$

V'
$\lambda v_e.[u_3$ | a.p{u_3}, buy{v, u_2, u_3}]

[fall in love]$_{V_{tr}}$
$\lambda Q_{(et)t}.\lambda v_e.Q(\lambda v'_e.[fall\_in\_love_{e(et)}\{v, v'\}])$

[a^{u_2} witch]$_{DP}$
$\lambda P_{et}.[u_2$ | witch{u_2}]; $P(u_2)$

[an^{u_3} alligator purse]$_{DP}$
$\lambda P_{et}.[u_3$ | a.purse{u_3}]; $P(u_3)$

V$_{di}$'
$\lambda Q_{(et)t}. \lambda v_e. Q(\lambda v''_e. [buy_{e(e(et))}\{v, u_2, v''\}])$

[buy]$_{V_{di}}$
$\lambda Q'_{(et)t}. \lambda Q_{(et)t}. \lambda v_e. Q'(\lambda v'_e. Q(\lambda v''_e. [buy_{e(e(et))}\{v, v', v''\}]))$

[her$_{u_2}$]$_{DP}$
$\lambda P_{et}.P(u_2)$

6.5.   Relative-clause Donkey Sentences

73. Every^{u_1} house-elf who falls in love with a^{u_2} witch buys her$_{u_2}$ an^{u_3} alligator purse.

Txt
|
CP
|
IP
$[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}] \rightarrow [u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

$[\text{-s}]_\text{I}$
$\lambda D_\textbf{t}. D$

VP
$[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}] \rightarrow [u_3 \mid a.p\{u_3\}, buy\{u_1, u_2, u_3\}]]$

V'
$\lambda v_\textbf{e}.[u_3 \mid a.p\{u_3\}, buy\{v, u_2, u_3\}]$

DP
$\lambda P_\textbf{et}.[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}] \rightarrow P(u_1)]$

buy her$_{u_2}$ *an*^{u_3} *alligator purse*

$[\textit{every}^{u_1}]_\text{D}$
$\lambda P'_\textbf{et}.\lambda P_\textbf{et}.[([u_1]; P'(u_1)) \rightarrow P(u_1)]$

NP
$\lambda v_\textbf{e}.[u_2 \mid house\_elf\{v\}, witch\{u_2\}, fall\_in\_love\{v, u_2\}]$

$[\textit{house-elf}]_\text{N}$
$\lambda v_\textbf{e}.[house\_elf_{et}\{v\}]$

CP
$\lambda v''_\textbf{e}.[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[\textit{who}]_{\text{DP}}{}^{v''}$
$\lambda P_\textbf{et}.P$

CP
|
IP
$[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[\text{-s}]_\text{I}$
$\lambda D_\textbf{t}. D$

VP
$[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[t_{v''}]_\text{DP}$
$\lambda P_\textbf{et}.P(v''_\textbf{e})$

V'
$\lambda v_\textbf{e}.[u_2 \mid witch\{u_2\}, fall\_in\_love\{v, u_2\}]$

$[\textit{fall in love}]_{V_\text{tr}}$
$\lambda Q_\textbf{(et)t}.\lambda v_\textbf{e}.Q(\lambda v'_\textbf{e}.[fall\_in\_love_{e(et)}\{v, v'\}])$

$[a^{u_2} \textit{ witch}]_\text{DP}$
$\lambda P_\textbf{et}.[u_2 \mid witch\{u_2\}]; P(u_2)$

6.6. Conditional Donkey Sentences

74. If a^{u_1} house-elf falls in love with a^{u_2} witch, he$_{u_1}$ buys her$_{u_2}$ an^{u_3} alligator purse.

Txt
|
CP
$[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}] \rightarrow [u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

CP
$\lambda D_t.[[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}] \rightarrow D]$

IP
$[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]$

$[if]_C$
$\lambda D'_t. \lambda D_t. [D' \rightarrow D]$

he$_{u_1}$ buys her$_{u_2}$ an^{u_3} alligator purse

IP
$[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$

a^{u_1} house-elf falls in love with a^{u_2} witch

I leave it to the reader to show that sentences (75) and (76) below – which involve coordination structures – can be compositionally assigned the intuitively correct interpretation in CDRT.

75. If a^{u_1} house-elf falls in love with a^{u_2} witch and she$_{u_2}$ likes fancy handbags, he$_{u_1}$ buys her$_{u_2}$ an^{u_3} alligator purse.

76. If a^{u_1} farmer owns a^{u_2} donkey, he$_{u_1}$ beats it$_{u_2}$ or he$_{u_1}$ feeds it$_{u_2}$ poorly.

7. Summary

The goal of this chapter and of the previous one was to situate the present research within the general enterprise of compositional dynamic semantics, in particular:

- to provide the basic framework that I will build on throughout the present work;
- to fix notation;

- to briefly recapitulate the basic empirical generalizations that motivate the dynamic approach to semantics and the basic kinds of semantic analyses that this approach makes possible.

The main achievement is the introduction of the basic compositional dynamic system couched in type logic in sections **2** and **5** above (i.e. the introduction of Dynamic Ty2 and CDRT).

The differences between the material introduced in this and the previous chapter and the cited sources are for the most part presentational. The six novel things are:

- the DPL-style definition of unselective generalized quantification that incorporates generalized quantifier *conservativity* (chapter **2**);
- the introduction of the mixed weak & strong donkey sentences, i.e. relative-clause donkey sentences with two donkey indefinites that receive different readings – one strong, the other weak –, e.g. *Every person who buys a book on amazon.com* (strong) *and has a credit card* (weak) *uses it* (the credit card) *to pay for it* (the book); this kind of sentence cannot be accounted for in DRT / FCS / DPL or CDRT for that matter, even if they are extended with *selective* generalized quantification. Mixed weak & strong donkey sentences will provide one of the primary motivations for the subsequent revisions and generalizations of CDRT (see chapter **5**);
- the complete definition of the underlying Dynamic Ty2 logic (chapter **3**);
- the fact that Dynamic Ty2 allows static objects of *arbitrary* types as dref values (chapter **3**);
- the indefinite-like analysis of proper names adopted in the present version of CDRT (chapter **3**);
- the novel dynamic analysis of ditransitive verbs and of the scoping properties of their Dative and Accusative objects (chapter **3**).

Building on the foundations layed out in this chapter, the next chapter will add to the previous literature in a more substantial way by reformulating the DPL-style definitions of *unselective* and *selective* generalized quantification in type logic and, thus,

extending CDRT to CDRT+GQ in a way that enables it to account for the weak / strong donkey ambiguity and the proportion problem.

Chapter 4. Compositional DRT with Generalized Quantification

1. Introduction

This chapter is the first one that adds to the previous literature in a more substantial way. Sections **2** and **3** are the central ones: they reformulate in type logic the DPL-style definitions of *unselective* and *selective* generalized quantification introduced in chapter **2**.

Section **4** then extends CDRT (introduced in the previous chapter) with these two notions of dynamic generalized quantification. The resulting system, which I will label CDRT+GQ, provides a *fully compositional* account of the proportion problem and of the weak / strong donkey ambiguity.

Section **5** introduces the analysis of the interaction between anaphora and generalized coordination in Muskens (1996): 176-182. I show that this analysis successfully generalizes to account for donkey sentences that contain a DP conjunction, e.g. *Every boy who has a dog and every girl who has a cat must feed it* (example (38) in Chierchia 1995: 77).

DP-conjunction donkey sentences of this kind are crucial for the argument that Chierchia (1995) develops in favor of an approach to natural language interpretation that builds (part of) the dynamics into the semantic value of natural language expressions and against approaches that build the dynamics of the interpretation into syntactic operations at the level of Logical Form (LF).

In a nutshell, the argument is that the same donkey pronoun is semantically bound by two distinct donkey indefinites, which can be naturally accounted for in a dynamic type-logical system with generalized conjunction (generalized to arbitrary types in the sense of Partee & Rooth 1983 among others). However, this kind of 'double binding' presents difficulties for approaches that require a particular syntactic configuration at the level of LF for the donkey pronouns to be semantically bound – because the same pronoun cannot enter two such distinct syntactic configurations.

Thus, by extension, such examples provide an argument for CDRT, which, just as DPL, aims to capture the dynamics of interpretation in terms of semantic values (i.e. meanings) and not syntactic representations.

The second reason for examining DP-conjunction donkey sentences is that this kind of examples will appear in the following chapter and they will help us distinguish between various (dynamic and static) accounts of the weak / strong donkey ambiguity.

Finally, section **6** shows that CDRT+GQ inherits the problems of DPL+GQ, i.e. it is not compositional enough. Just as in the case of DPL+GQ (see chapter **2**), the argument relies on mixed weak & strong donkey sentences, i.e. relative-clause donkey sentences with multiple indefinites in the restrictor of the donkey quantification that receive different readings.

In particular, I will show that determining which indefinite receives a weak reading and which one receives a strong reading cannot be compositionally implemented if we account for the weak / strong donkey ambiguity in terms of an ambiguity in the dynamic generalized determiner. This section paves the way for chapter **5**, which provides such a compositional account formulated in a version of CDRT with *plural* information states, i.e. info states which are modeled as *sets* of 'assignments' (type *st*) and not as single 'assignments' (type *s*).

The chapter ends with a summary of the main results (section **7**).

2. *Translating Unselective Quantification into Dynamic Ty2*

Consider again the definition of DPL-style unselective generalized quantification introduced in chapter **2**.

1. $\|\mathbf{det}(\phi, \psi)\| = \{<g, h>: g=h$ and $\mathbf{DET}((\phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

    where $\mathbf{DET}$ is the corresponding static determiner

    and $(\phi)^g := \{h: \|\phi\|<g, h> = \mathrm{T}\}$

    and $\mathbf{Dom}(\|\phi\|) := \{g:$ there is an h s.t. $\|\phi\|<g, h> = \mathrm{T}\}$.

2. $\mathbf{det}_x(\phi, \psi) := \mathbf{det}([x]; \phi, \psi)$

Based on the definition schemes in (1) and (2), we were able to derive meanings for the natural language determiners *every* and *no* that were equivalent to the meanings assigned to them in DPL – as shown in (3) through (6) below.

3. $\|\mathbf{every}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{EVERY}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

   i.e. $\|\mathbf{every}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } ([x]; \phi)^g \subseteq \mathbf{Dom}(\|\psi\|)\}$

4. $\|\mathbf{no}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } \mathbf{NO}(([x]; \phi)^g, \mathbf{Dom}(\|\psi\|))\}$,

   i.e. $\|\mathbf{no}_x(\phi, \psi)\| = \{<g, h>: g=h \text{ and } ([x]; \phi)^g \cap \mathbf{Dom}(\|\psi\|)=\emptyset\}$

5. $\forall x(\phi \rightarrow \psi) \Leftrightarrow \exists x(\phi) \rightarrow \psi \Leftrightarrow ([x]; \phi) \rightarrow \psi \Leftrightarrow \mathbf{every}_x(\phi, \psi)$

6. $\sim\exists x(\phi; \psi) \Leftrightarrow \forall x(\phi \rightarrow \sim\psi) \Leftrightarrow \sim([x]; \phi; \psi) \Leftrightarrow \mathbf{no}_x(\phi, \psi)$

It is straightforward to provide the corresponding definitions in Dynamic Ty2. Given that the above DPL formulas are tests, they will be translated as conditions, i.e. as terms of type *st*.

7. $\mathbf{det}(D, D') := \lambda i_s. \mathbf{DET}(Di, \mathbf{Dom}(D'))$,

   where **DET** is the corresponding static determiner

   and $Di = \{j_s: Dij\}$

   and $\mathbf{Dom}(D') := \{i_s: \exists j_s(Dij)\}$.

8. $\mathbf{det}_u(D, D') := \mathbf{det}([u]; D, D')$

Moreover, the meanings for *every* and *no* that we can derive based on the definition schemes in (7) and (8) above are equivalent to the CDRT meanings for *every* and *no* that we have provided in chapter **3**: the reader can easily check that the equalities in (11) and (12) below are true in Dynamic Ty2.

9. $\mathbf{every}_u(D, D') = \lambda i_s. \mathbf{EVERY}(([u]; D)i, \mathbf{Dom}(D'))$,

   i.e. $\mathbf{every}_u(D, D') = \lambda i_s. ([u]; D)i \subseteq \mathbf{Dom}(D')$.

10. $\mathbf{no}_u(D, D') = \lambda i_s. \mathbf{no}(([u]; D)i, \mathbf{Dom}(D'))$,

    i.e. $\mathbf{no}_u(D, D') = \lambda i_s. ([u]; D)i \cap \mathbf{Dom}(D')=\emptyset$.

11. $\forall u(D \rightarrow D') = \exists u(D) \rightarrow D' = ([u]; D) \rightarrow D' = \mathbf{every}_u(D, D')$

12. $\sim\exists u(D; D') = \forall u(D \rightarrow \sim D') = \sim([u]; D; D') = \mathbf{no}_u(D, D')$

2.1.   Limitations of Unselectivity: Proportions

Just like its DPL counterpart, the CDRT definition of unselective generalized quantification in (7-8) above enables us to derive meanings for *most* and *few* that capture the anaphoric connections in donkey sentences based on them, but are unable to provide intuitively correct truth-conditions – they too have a proportion problem.

This is explicitly shown by the truth-conditions in (16) below, which are assigned to sentence (14) by the definition of unselective generalized quantification in (7-8). Note in particular that we end up quantifying over pairs of house-elves and witches; thus, the formula in (16) is true in the 'Dobby as Don Juan' scenario mentioned in chapter **2**, in contrast to the English sentence in (14), which is intuitively false.

13. $\mathbf{most}_u(D, D') = \lambda i_s.\ \mathbf{MOST}(([u]; D)i, \mathbf{Dom}(D'))$,

   i.e. $\mathbf{most}_u(D, D') = \lambda i_s.\ |([u]; D)i \cap \mathbf{Dom}(D')| > |([u]; D)i \backslash \mathbf{Dom}(D')|$ [1],

   i.e. $\mathbf{most}_u(D, D') = \lambda i_s.\ |([u]; D; [!D'])i| > |([u]; D; [\sim D'])i|$.

14. Most^{u_1} house-elves who fall in love with a^{u_2} witch buy her$_{u_2}$ an^{u_3} alligator purse.

15. $[\mathbf{most}_{u_1}([u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$,

   $[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}])]$

16. $\lambda i_s.\ |([u_1, u_2 \mid h.elf\{u_1\}, witch\{u_2\}, f.i.l\{u_1, u_2\}]$;

   $[!([u_3 \mid a.p\{u_3\}, buy\{u_1, u_2, u_3\}])])i| >$

   $|([u_1, u_2 \mid h.elf\{u_1\}, witch\{u_2\}, f.i.l\{u_1, u_2\}]$;

   $[\sim([u_3 \mid a.p\{u_3\}, buy\{u_1, u_2, u_3\}])])i|$,

   i.e., by Axioms **3** and **4** ("Identity of 'assignments'" and "Enough 'assignments'"),

   $\lambda i_s.\ |\{<x_e, y_e>: h.elf(x) \wedge witch(y) \wedge f.i.l(x, y) \wedge \exists z_e(a.p(z) \wedge buy(x, y, z))\}| >$

   $|\{<x_e, y_e>: h.elf(x) \wedge witch(y) \wedge f.i.l(x, y) \wedge \neg \exists z_e(a.p(z) \wedge buy(x, y, z))\}|$

---

[1] '\' symbolizes set-theoretic difference.

2.2.   Limitations of Unselectivity: Weak / Strong Ambiguities

Moreover, the meaning of *every*, repeated in (17) below, is able to derive only the strong reading of donkey sentences, just like its DPL equivalent. To see this, consider again the example in (18) below (from Pelletier & Schubert 1989): this sentence is assigned intuitively *incorrect* truth-conditions because the formula in (20) below is true iff the dime-owners put *all* their dimes in the meter.

17. **every**$_u$(D, D′) = λi_s. **EVERY**(([u]; D)i, **Dom**(D′)),

    i.e. **every**$_u$(D, D′) = λi_s. ([u]; D)i ⊆ **Dom**(D′),

    i.e. **every**$_u$(D, D′) = λi_s. ([u]; D)i ⊆ ([u]; D)i∩**Dom**(D′),

    i.e. **every**$_u$(D, D′) = λi_s. ([u]; D)i ⊆ ([u]; D; !D′)i,

18. Every^{u_1} person who has a^{u_2} dime will put it$_{u_2}$ in the meter.

19. [**every**$_{u_1}$ ([u₂ | person{u₁}, dime{u₂}, have{u₁, u₂}], [put_in_meter{u₁, u₂}])]

20. λi_s. ([u₁, u₂ | person{u₁}, dime{u₂}, have{u₁, u₂}])i ⊆

    ([u₁, u₂ | person{u₁}, dime{u₂}, have{u₁, u₂}]; [!([put_in_meter{u₁, u₂}])])i,

    i.e., by Axioms **3** and **4** ("Identity of 'assignments'" and "Enough 'assignments'"),

    λi_s. {<x_e, y_e>: person(x) ∧ dime(y) ∧ have(x, y)} ⊆

    {<x_e, y_e>: person(x) ∧ dime(y) ∧ have(x, y) ∧ put_in_meter(x, y)}, i.e.

    λi_s. ∀x_e∀y_e(person(x) ∧ dime(y) ∧ have(x, y) → put_in_meter(x, y))

2.3.   Conservativity and Unselective Quantification

Finally, the observation we have made about DPL-style *conservative* unselective quantification extends to its CDRT translation [2]: assuming that the static determiner **DET** is conservative, we have that **DET**((D)i, **Dom**(D′)) holds iff **DET**((D)i, (D)i∩**Dom**(D′)) holds. Moreover, the latter formula is equivalent to **DET**((D)i, (D; !D′)i), which perspicuously encodes the intuition that a dynamic generalized determiner relates two sets of info states, the first of which is the set of output states compatible with the

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[2] The observation was in fact used in deriving the final form of the *every* definition in (17) above.

restrictor, i.e. $(D)i$, while the second one is the set of output states compatible with the restrictor that, in addition, *can* be further updated by the nuclear scope, i.e. $(D; !D')i$.

The conservative definitions of unselective generalized quantification based on the non-conservative ones in (7) and (8) above are provided in (21) and (22) below.

21. **Built-in 'unselective' dynamic conservativity:**

$$\mathbf{det}(D, D') := \lambda i_s.\ \mathbf{DET}(Di, (D; [!D'])i)$$

22. **Unselective generalized quantification with built-in dynamic conservativity:**

$$\mathbf{det}_u(D, D') := \lambda i_s.\ \mathbf{DET}(([u]; D)i, ([u]; D; [!D'])i)$$

Given that the definition of conservative unselective quantification in (22) provides access to the dref u in both the restrictor and the nuclear scope of the quantification, this definition provides the basic format for the CDRT definition of selective generalized quantification, to which we now turn.

3. Translating Selective Quantification into Dynamic Ty2

The syntax for selective generalized quantification is the same as the one used in the previous section, i.e. I will continue to use abbreviations of the form $\mathbf{det}_u(D, D')$, where u is the 'bound' dref [3], D is the restrictor and D' is the nuclear scope of the quantification.

The *selective* determiner $\mathbf{det}_u$ relates two sets of individuals (type e) and not two sets of 'assignments' (type s), as the unselective determiner $\mathbf{det}$ defined in the previous section does. The fact that $\mathbf{det}_u$ relates sets of individuals will solve the proportion problem. As far as the weak / strong donkey ambiguity is concerned, I will analyze it just as in chapter **2**, i.e. as an ambiguity in the generalized determiner, which can have a *weak* basic meaning $\mathbf{det}^{wk}_u(D, D')$ or a *strong* one $\mathbf{det}^{str}_u(D, D')$. Both basic meanings are defined in terms of the static determiner $\mathbf{DET}$ and both of them are conditions, i.e. terms of type st, as shown in (23) below.

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[3] Recall that u is a *constant* of type $\mathbf{e} := se$, so it cannot possibly be bound in the official type logical language (which is Dynamic Ty2) – hence the scare quotes on 'bound'.

23. $\mathbf{det}^{wk}_u(D, D') := \lambda i_s.\ \mathbf{DET}(u[Di],\ u[(D; D')i])$

$\mathbf{det}^{str}_u(D, D') := \lambda i_s.\ \mathbf{DET}(u[Di],\ u[([D \to D'])i]),$

where $Di := \{j_s: Dij\}$

and $u[Di] := \{u_{se}j_s: ([u]; D)ij\},$

i.e. $u[Di]$ is the image of the set of 'assignments' $([u]; D)i$

under the function u_{se}.

As already indicated, the generalized quantification defined in (23) is selective because the static determiner **DET** relates sets of *individuals*, e.g. $u[Di] := \{x_e: \exists j_s(([u]; D)ij \wedge x=uj)\}$, and not sets of info states (as it does in the unselective definitions in (21) and (22) above).

The difference between the *weak* and the *strong* lexical entry for the selective generalized determiners is localized in the nuclear scope of the quantification:

- the weak, 'existential' reading is obtained by simply sequencing (i.e. conjoining) the restrictor DRS D and the nuclear scope DRS D';
- the strong, 'universal' reading is obtained by means of the universal quantification built into the definition of dynamic implication that relates the restrictor DRS D and the nuclear scope DRS D'.

Given Axiom **3** ("Identity of 'assignments'") and Axiom **4** ("Enough 'assignments'"), the weak and strong selective determiners in (23) above can be alternatively defined in terms of generalized quantification over info states – we just need to make judicious use of the anaphoric closure operator '!', as shown in (24) below [4].

24. $\mathbf{det}^{wk}_u(D, D') := \lambda i_s.\ \mathbf{DET}(([u \mid !D])i,\ ([u \mid !(D; D')])i)$

$\mathbf{det}^{str}_u(D, D') := \lambda i_s.\ \mathbf{DET}(([u \mid !D])i,\ ([u \mid !([D \to D'])])i)$ [5],

where $Di := \{j_s: Dij\}$.

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[4] Note the formal similarities between the type-logical definition schemes in (24) and their DPL-style counterparts introduced in chapter **2**.

[5] Given that $!([D \to D']) = D \to D'$, the strong determiner can be more simply defined as $\mathbf{det}^{str}_u(D, D') := \lambda i_s.\ \mathbf{DET}(([u \mid !D])i,\ ([u \mid D \to D'])i).$

3.1. Accounting for Weak / Strong Ambiguities

It is obvious that the predictions made by the definition schemes in (23) and (24) above are identical to their DPL-style counterparts, so I will only briefly go through several examples. Consider the usual donkey example in (25) below.

25. Every^{u_1} farmer who owns a^{u_2} donkey beats it$_{u_2}$.

The weak and strong meanings for the English determiner *every* are provided in (26) below and simplified in (27).

26. **every**$^{wk}_u(D, D') := \lambda i_s.$ **EVERY**$(u[Di], \; u[(D; D')i])$

    every$^{str}_u(D, D') := \lambda i_s.$ **EVERY**$(u[Di], \; u[([D \to D'])i])$

27. **every**$^{wk}_u(D, D') := \lambda i_s. \; u[Di] \subseteq u[(D; D')i]$

    every$^{str}_u(D, D') := \lambda i_s. \; u[Di] \subseteq u[([D \to D'])i]$

The weak reading of sentence (25) is represented in Dynamic Ty2 as shown in (28) below. The formula in (29) in the scope of the vacuous λ-abstraction over 'assignments' shows that the Dynamic Ty2 representation derives the intuitively correct *weak* truth-conditions.

28. [**every**$^{wk}_{u_1}$ ([u_2 | *farmer*{u_1}, *donkey*{u_2}, *own*{u_1, u_2}], [*beat*{u_1, u_2}])]

29. $\lambda i_s. \; u_1[([u_2 \,|\, farmer\{u_1\}, donkey\{u_2\}, own\{u_1, u_2\}])i] \subseteq$

    $u_1[([u_2 \,|\, farmer\{u_1\}, donkey\{u_2\}, own\{u_1, u_2\}, beat\{u_1, u_2\}])i]$, i.e.

  $\lambda i_s. \; \{x_e: farmer(x) \wedge \exists y_e(donkey(y) \wedge own(x, y))\} \subseteq$

    $\{x_e: farmer(x) \wedge \exists z_e(donkey(z) \wedge own(x, z) \wedge beat(x, z))\}$, i.e.

  $\lambda i_s. \forall x_e(farmer(x) \wedge \exists y_e(donkey(y) \wedge own(x, y))$

    $\to \exists z_e(donkey(z) \wedge own(x, z) \wedge beat(x, z)))$

The strong reading of sentence (25) is represented in Dynamic Ty2 as shown in (30) below. The formula in (31) in the scope of the vacuous λ-abstraction over 'assignments' shows that the representation derives the intuitively correct *strong* truth-conditions.

30. [**every**$^{str}_{u_1}$ ([u_2 | *farmer*{u_1}, *donkey*{u_2}, *own*{u_1, u_2}], [*beat*{u_1, u_2}])]

31. $\lambda i_s.\ u_1[([u_2 \mid farmer\{u_1\},\ donkey\{u_2\},\ own\{u_1,\ u_2\}])i] \subseteq$

      $u_1[(([u_2 \mid farmer\{u_1\},\ donkey\{u_2\},\ own\{u_1,\ u_2\} \rightarrow [beat\{u_1,\ u_2\}]])i]$, i.e.

  $\lambda i_s.\ \{x_e: farmer(x) \wedge \exists y_e(donkey(y) \wedge own(x,\ y))\} \subseteq$

      $\{x_e: \forall z_e(farmer(x) \wedge donkey(z) \wedge own(x,\ z) \rightarrow beat(x,\ z))\}$, i.e.

  $\lambda i_s.\forall x_e(farmer(x) \wedge \exists y_e(donkey(y) \wedge own(x,\ y))$

      $\rightarrow \forall z_e(farmer(x) \wedge donkey(z) \wedge own(x,\ z) \rightarrow beat(x,\ z)))$, i.e.

  $\lambda i_s.\forall x_e\forall z_e(farmer(x) \wedge donkey(z) \wedge own(x,\ z) \rightarrow beat(x,\ z))$

3.2. Solving Proportions

The fact that the proportion problem is solved is shown by the intuitively correct truth-conditions in (35) and (38) below, which are assigned to the sentences in (33) and (36) respectively.

32. $\mathbf{most}^{wk}_{u}(D,\ D') := \lambda i_s.\ \mathbf{MOST}(u[Di],\ u[(D;\ D')i])$,

      i.e. $\mathbf{most}^{wk}_{u}(D,\ D') := \lambda i_s.\ |u[Di]\cap u[(D;\ D')i]| > |u[Di]\backslash u[(D;\ D')i]|$

  $\mathbf{most}^{str}_{u}(D,\ D') := \lambda i_s.\ \mathbf{MOST}(u[Di],\ u[([D \rightarrow D'])i])$,

      i.e. $\mathbf{most}^{str}_{u}(D,\ D') := \lambda i_s.\ |u[Di]\cap u[([D \rightarrow D'])i]| > |u[Di]\backslash u[([D \rightarrow D'])i]|$

33. Most^{u_1} house-elves who fall in love with a^{u_2} witch buy her$_{u_2}$ an^{u_3} alligator purse.

34. $[\mathbf{most}^{str}_{u_1}([u_2 \mid house\_elf\{u_1\},\ witch\{u_2\},\ fall\_in\_love\{u_1,\ u_2\}]$,

      $[u_3 \mid alligator\_purse\{u_3\},\ buy\{u_1,\ u_2,\ u_3\}])]$

35. $\lambda i_s.\ |\{x_e: h.elf(x) \wedge \exists y_e(witch(y) \wedge f.i.l(x,\ y)) \wedge$

      $\forall y'_e(witch(y') \wedge f.i.l(x,\ y') \rightarrow \exists z_e(a.p(z) \wedge buy(x,\ y',\ z)))\}| >$

    $|\{x_e: h.elf(x) \wedge \exists y'_e(witch(y') \wedge f.i.l(x,\ y') \wedge \neg\exists z_e(a.p(z) \wedge buy(x,\ y',\ z)))\}|$

36. Most^{u_1} drivers who have a^{u_2} dime will put it$_{u_2}$ in the meter.

37. $[\mathbf{most}^{wk}_{u_1}([u_2 \mid driver\{u_1\},\ dime\{u_2\},\ have\{u_1,\ u_2\}],\ [put\_in\_meter\{u_1,\ u_2\}])]$

38. $\lambda i_s.\ |\{x_e: driver(x) \wedge \exists y_e(dime(y) \wedge have(x,\ y) \wedge put\_in\_meter(x,\ y))\}| >$

    $|\{x_e: driver(x) \wedge \exists y_e(dime(y) \wedge have(x,\ y)) \wedge$

      $\forall y'_e(dime(y') \wedge have(x,\ y') \rightarrow \neg put\_in\_meter(x,\ y'))\}|$

Everything is now in place to introduce CDRT+GQ, i.e. the extension of CDRT with the notions of unselective and selective dynamic generalized quantification we have just defined in Dynamic Ty2.

4. Extending CDRT with Generalized Quantification (CDRT+GQ)

The syntax of the English fragment is the same as the one defined for CDRT in the previous chapter. As far as the semantics CDRT+GQ is concerned, we only need:

- to replace the CDRT meanings for generalized determiners with the newly defined selective generalized determiners;
- to replace the CDRT meaning for dynamic implication (i.e. for bare conditional structures) with the newly defined unselective generalized determiners; thus, CDRT+GQ will introduce a generalized definition of dynamic implication that also subsumes adverbs of quantification [6].

The CDRT+GQ meanings have the same types as the corresponding CDRT meanings, i.e. (**et**)((**et**)**t**) for determiners and **t**(**tt**) for dynamic implication+adverbs of quantification.

As expected, the meaning of the indefinite determiner *a* remains the same as in CDRT: redefining it in terms of selective generalized quantification would make it a *test*, which is empirically inadequate given that singular indefinites support cross-sentential anaphora, e.g. *A^u house-elf left the Three Broomsticks. He$_u$ was drunk.*

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[6] Of course, assigning an *unselective* meaning to conditionals fails to account for the fact that they also exhibit weak / strong donkey ambiguities; see section **6** below for more discussion.

39. **TR 0 (only the revised entries are listed): Basic Meanings (TN).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := s(st)$
$[det^{\mathbf{wk},u}]_D$ / $[det^{\mathbf{str},u}]_D$	$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{det}^{wk/str}{}_u(P'(u), P(u))],$ where:	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
e.g. *every*$^{\mathbf{str},u}$, *no*$^{\mathbf{wk},u}$, *most*$^{\mathbf{str},u}$... (but not *a*^{u})	$\mathbf{det}^{wk}{}_u(P'(u), P(u)) := \lambda i_s.\ \mathbf{DET}(u[P'(u)i],\ u[(P'(u); P(u))i])$ $\mathbf{det}^{str}{}_u(P'(u), P(u)) := \lambda i_s.\ \mathbf{DET}(u[P'(u)i],\ u[((P'(u) \rightarrow P(u))]i]),$ where $P(u)i := \{j_s: P(u)ij\}$ and $u[P(u)i] := \{uj: ([u]; P(u))ij\}$ and $\mathbf{DET}$ is the corresponding static determiner.	
$[if\ (+adv.\ of\ quant.)]_C$	$\rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{det}(D', D)],$ where: $\mathbf{det}(D, D') := \lambda i_s.\ \mathbf{DET}(Di, (D; [!D'])i),$ where $Di := \{j_s: Dij\}$ and $\mathbf{DET}$ is the corresponding static determiner.	$\mathbf{t}(\mathbf{tt})$
$[if\ ]_C$ (i.e. bare *if*)	$\rightsquigarrow \lambda D'_{\mathbf{t}}.\ \lambda D_{\mathbf{t}}.\ [\mathbf{every}(D', D)]$	$\mathbf{t}(\mathbf{tt})$

4.1. Proportions and Weak / Strong Ambiguities in CDRT+GQ

It is easily seen that, based on these lexical entries, we can compositionally derive the correct interpretation for the proportion examples and for the examples ambiguous between a weak and a strong reading. I will therefore treat only one example in detail – the reader will have no difficulties constructing and translating the LF's for the others.

40. Most $^{\textbf{str},\,u_1}$ house-elves who fall in love with a^{u_2} witch buy her$_{u_2}$ an^{u_3} alligator purse.

Txt
|
CP
|
IP

$[\textbf{most}^{str}\,u_1\,([u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}],\ [u_3 \mid alligator\_purse\,\{u_3\}, buy\{u_1, u_2, u_3\}])]$

$[\text{-}\varnothing]_I$
$\lambda D_t.\, D$

VP
$[\textbf{most}^{str}\,u_1\,([u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}],\ [u_3 \mid a.p\{u_3\}, buy\{u_1, u_2, u_3\}])]$

V'
$\lambda v_e.[u_3 \mid a.p\{u_3\}, buy\{v, u_2, u_3\}]$

DP
$\lambda P_{et}.[\textbf{most}^{str}\,u_1\,([u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}],\ P(u_1))]$

buy her$_{u_2}$ an^{u_3} alligator purse

$[most\,u_1]_D$
$\lambda P'_{et}.\lambda P_{et}.[\textbf{most}^{str}\,u_1\,(P'(u_1), P(u_1))]$

NP
$\lambda v_e.[u_2 \mid house\_elf\{v\}, witch\{u_2\}, fall\_in\_love\{v, u_2\}]$

$[house\text{-}elf]_N$
$\lambda v_e.[house\_elf_{et}\{v\}]$

CP
$\lambda v''_e.[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[who]_{DP}{}^{v''}$
$\lambda P_{et}.P$

CP
|
IP
$[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[\text{-}\varnothing]_I$
$\lambda D_t.\, D$

VP
$[u_2 \mid witch\{u_2\}, fall\_in\_love\{v'', u_2\}]$

$[t_{v''}]_{DP}$
$\lambda P_{et}.P(v''_e)$

V'
$\lambda v_e.[u_2 \mid witch\{u_2\}, fall\_in\_love\{v, u_2\}]$

$[fall\ in\ love]_{V_{tr}}$
$\lambda Q_{(et)t}.\lambda v_e.Q(\lambda v'_e.[fall\_in\_love_{e(et)}\{v, v'\}])$

$[a^{u_2}\ witch]_{DP}$
$\lambda P_{et}.[u_2 \mid witch\{u_2\}]; P(u_2)$

5. *Anaphora and Generalized Coordination in CDRT+GQ*

This section prepares the ground for the analysis of all the conjunction-based donkey sentences in the chapters to come. Most of the material is based on Muskens (1996): 176 et seqq and Partee & Rooth (1983). There are two main novelties:

- I provide direct dynamic counterparts of the definitions of conjoinable types and generalized conjunction and disjunction in Partee & Rooth (1983);
- I show that CDRT (and its extension CDRT+GQ) can account for the DP-conjunction donkey example in (41) below, from Chierchia (1995): 77, (38).

41. Every ^{u_1} boy who has a ^{u_2} dog and every ^{u_3} girl who has a ^{u_2} cat must feed it $_{u_2}$.

This is one of the central examples used in Chierchia (1995) to argue for an approach to natural language that builds (part of) the dynamics into the semantic value of natural language expressions as opposed to syntactic operations on the LF of sentences/discourses. Therefore, *mutatis mutandis*, his argument that discourse dynamics should be captured semantically and not syntactically also supports the architecture of CDRT+GQ.

5.1. Generalized Dynamic Conjunction and Disjunction

First, we need to define in Dynamic Ty2 a notion of generalized dynamic conjunction and disjunction. Following Partee & Rooth (1983), I define the set of dynamically conjoinable types as shown in (42) below.

42. **Dynamically Conjoinable Types (DCTyp).**

The set of dynamically conjoinable types **DCTyp** is the smallest subset of **Typ** s.t. $\mathbf{t} \in \mathbf{DCTyp}$ (where $\mathbf{t} := s(st)$) and, if $\tau \in \mathbf{DCTyp}$, then $(\sigma\tau) \in \mathbf{DCTyp}$ for any $\sigma \in \mathbf{Typ}$.

We can now define generalized (pointwise) dynamic conjunction and disjunction as shown in (43) below.

43. **Generalized Pointwise Dynamic Conjunction ⊓ and Disjunction ⊔.**

For any two terms α and β of type τ, for any $\tau \in$ **DCTyp**:

$$\alpha \sqcap \beta := (\alpha; \beta) \text{ if } \tau = \mathbf{t} \qquad \text{and} \qquad \alpha \sqcap \beta := \lambda v_\sigma.\ \alpha(v) \sqcap \beta(v) \text{ if } \tau = (\sigma\rho);$$

$$\alpha \sqcup \beta := [\alpha \lor \beta] \text{ if } \tau = \mathbf{t} \qquad \text{and} \qquad \alpha \sqcup \beta := \lambda v_\sigma.\ \alpha(v) \sqcup \beta(v) \text{ if } \tau = (\sigma\rho).$$

Abbreviation. $\alpha_1 \sqcap \alpha_2 \sqcap \ldots \sqcap \alpha_n := (\ldots(\alpha_1 \sqcap \alpha_2) \sqcap \ldots \sqcap \alpha_n);$

$$\alpha_1 \sqcup \alpha_2 \sqcup \ldots \sqcup \alpha_n := (\ldots(\alpha_1 \sqcup \alpha_2) \sqcup \ldots \sqcup \alpha_n).$$

Note that the translation rule GSeq (Generalized Sequencing) we have introduced in chapter **3** above is simply a restricted form of generalized dynamic conjunction ⊓.

We can now define the basic meanings for *and* and *or* by means of the schemata in Table (44) below.

44. **TR 0 (only the revised entries are listed): Basic Meanings (TN).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := s(st)$
[*and*] $_{\text{Conj}}$	⤳ $\lambda v_1.\ \ldots\ \lambda v_n.\ v_1 \sqcap \ldots \sqcap v_n$	$\tau(\ldots(\tau\tau)\ldots)$
[*or*] $_{\text{Conj}}$	⤳ $\lambda v_1.\ \ldots\ \lambda v_n.\ v_1 \sqcup \ldots \sqcup v_n$	$\tau(\ldots(\tau\tau)\ldots)$

5.2. Revising the Coordination Rule: Generalized Coordination

We generalize our translation rule for coordinated constructions as shown in (45) below, i.e. the translation of a coordinated construction is obtained by applying the translation of the coordinating element to the translations of the coordinated expressions.

45. **TR 5 (revised) – Generalized Coordination (GCo).**

If $A_1 \rightsquigarrow \alpha_1, \ldots, A_n \rightsquigarrow \alpha_n$, Conj $\rightsquigarrow \beta$, $A_{n+1} \rightsquigarrow \alpha_{n+1}$ and $A_1, \ldots, A_n$, Conj

and A_{n+1} are the only daughters of A in that order

(i.e. $A \rightarrow A_1 \ldots A_n$ Conj A_{n+1}),

then $A \rightsquigarrow \beta(\alpha_1)\ldots(\alpha_n)(\alpha_{n+1})$,

provided this a well-formed term and has the same type as $\alpha_1, \ldots, \alpha_n, \alpha_{n+1}$.

5.3. Catching and Eating a Fish in CDRT+GQ

We can now go through some examples. First, consider the sentences in (46) and (47) below, from Partee & Rooth (1983): 338, (12) and (13)[7].

46. John caught and ate a^{u_1} fish.

47. John hugged and kissed three^{u_1} women.

As Partee & Rooth (1983): 338 observe, under the most salient reading of sentence (46), John catches and eats the same fish; similarly for (47), John hugs and kisses the same three women. Unfortunately, we can obtain this reading in CDRT+GQ (or CDRT) only by quantifying-in the direct object indefinite *a^{u_1} fish* – that is, CDRT+GQ predicts that the default reading (the one without quantifying-in) should be one in which the fish that John catches and the fish that John eats are possibly different.

This is a consequence of the fact that, following Montague (1974), transitive verbs are interpreted as taking a GQ as direct object (a term of type (**et**)**t**) and not an individual dref (a term of type **e**). However, this is not an empirically unmotivated feature of the system: it correctly predicts that the preferred relative scope of the subject and direct object is the one in which the subject scopes over the object [8].

In sum, given our current setup, there is no analysis that would make the correct predictions both with respect to the preferred quantifier scoping of transitive verbs and with respect to the preferred reading of transitive verb conjunctions. I will therefore leave the system as it is and leave this matter for future research[9].

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[7] Page references are to Partee & Portner (2002).

[8] Moreover, this kind of representation receives independent empirical support from the interpretation of singular number morphology on donkey indefinites and pronouns as semantic distributivity – but this topic falls beyond the scope of the current investigation.

[9] As far as I can see, we can provide a novel solution to this problem if we leave the lexical entries for transitive verbs as they are now (i.e. as in Montague 1974) and employ cataphora (see e.g. Chierchia 1995: Chapter 3 for a discussion of what cataphora is) to obtain the desired reading for the transitive verb conjunction examples; structured cataphora (where 'structured' is to be understood in the sense of chapter **5** below) could also be used as the mechanism in terms of which reverse quantifier scope and Bach-Peters sentences are analyzed – but the exploration of these suggestions must be left for another occasion.

The two possible LF's of sentence (46) are schematically represented in (48) and (51) below, together with their respective translations.

48. John^{u_2} [$_{Vtr}$ caught and ate] a^{u_1} fish.

49. $[u_2 \mid u_2=John]$; $[u_1 \mid fish\{u_1\}, catch\{u_2, u_1\}]$; $[u_1 \mid fish\{u_1\}, eat\{u_2, u_1\}]$

50. $\lambda i_s. \exists x_e(fish(x) \wedge catch(john, x)) \wedge \exists y_e(fish(y) \wedge eat(john, y))$

51. [a^{u_1} fish]$^{v''}$ [John^{u_2} [$_{Vtr}$ caught and ate] $t_{v''}$].

52. $[u_1 \mid fish\{u_1\}]$; $[u_2 \mid u_2=John]$; $[catch\{u_2, u_1\}, eat\{u_2, u_1\}]$,

   i.e. $[u_1, u_2 \mid fish\{u_1\}, u_2=John, catch\{u_2, u_1\}, eat\{u_2, u_1\}]$

53. $\lambda i_s. \exists x_e(fish(x) \wedge catch(john, x) \wedge eat(john, x))$

As already indicated, the LF in (48) with the direct object *in situ* yields the 'possibly distinct fish' interpretation, while the LF in (51) with the QR-ed direct object yields the 'same fish' interpretation.

5.4. Coordination and Discourse Referent Reassignment

The 'possibly distinct fish' representation in (49) above and its interpretation are unlike anything in classical DRT / FCS, where reintroducing a dref, e.g. dref u_1 in (49), is either banned or, if it is allowed, it is *not* interpreted as *reassigning* a value to that dref – the output info state assigns the same value to the dref as the input info state. In contrast, CDRT+GQ allows dref reintroduction and interprets it as reassignment of value to the dref.

Thus, it would seem that the classical DRT / FCS design choice is empirically better than the CDRT+GQ one: the representation in (49) yields the 'same fish' interpretation in a DRT / FCS-like system. However, in such a system, we cannot easily obtain a representation of the 'distinct fish' interpretation, which *is* an intuitively available reading of sentence (46) (although dispreferred)[10]: we would have to postulate a mechanism whereby the indefinite object *a fish* occurs twice in the LF of sentence (46) and

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[10] Moreover, as Partee & Rooth (1983): 338 observe, the 'distinct fish' representation is the preferred one for conjunctions of intensional transitive verbs, e.g. *John needed and bought a new coat*.

contributes distinct dref's – i.e. we would have to syntactically simulate the semantic Montagovian analysis in (49) above[11].

Moreover, the necessary syntactic operations on LF's become increasingly stipulative as soon as we turn to more complex examples like the coordination donkey sentence in (41) above (from Chierchia 1995), which, as we will presently see, receives a straightforward reassignment-based analysis in CDRT+GQ.

I conclude that the reassignment-based architecture of CDRT+GQ is a desirable one and, in some form or other, it is a necessary component of any account of the interaction between anaphora and generalized coordination (exhibited by sentence (41) above, for example).

That being said, we have to admit that the particular implementation of dref reassignment in CDRT+GQ is not the empirically optimal one: reassignment in CDRT+GQ (just as in DPL) is destructive – the previous value of the dref is completely lost and cannot be later accessed in discourse.

And destructive reassignment has unwelcome empirical consequences. Consider for example the DP conjunction *Mary and Helen* in discourse (54-55) below.

54. Mary^{u_1} and Helen^{u_2} (each) bought an^{u_3} alligator purse.

55. They$_{u_3}$ were (both) bright red.

56. $[u_1, u_3 \mid u_1 = Mary, alligator\_purse\{u_3\}, buy\{u_1, u_3\}]$;
    $[u_2, u_3 \mid u_2 = Helen, alligator\_purse\{u_3\}, buy\{u_2, u_3\}]$;
    $[bright\_red\{u_3\}]$

As indicated by the parenthesized floating quantifier, the most salient reading of sentence (54) is the one in which Mary and Helen buy a purse each. However, if we analyze this sentence as shown in (56) above, we will be able to retrieve in sentence (55)

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[11] I leave for future research the dynamic reformulation of the analysis of intensional verbs in Zimmermann (1993) and its comparison with the Montagovian counterpart.

only the purse mentioned last, i.e. Helen's purse: the destructive CDRT+GQ reassignment renders Mary's purse inaccessible for subsequent anaphora.

Thus, irrespective of how we decide to analyze the plural anaphor *they* in (55), we need to somehow preserve the values that are currently overwritten by dref reintroduction.

Summarizing, we face the following problem. On the one hand, we need to provide an account of the interaction between anaphora and generalized coordination exhibited by sentence (41) and, for that, we need to allow for dref reintroduction – or, more exactly, *index* reusability – so that both donkey indefinites a^{u_2} *dog* and a^{u_2} *cat* can be anaphorically associated with the donkey pronoun it_{u_2}. On the other hand, the only way to capture index reusability in CDRT+GQ is as dref reintroduction, i.e. as destructive random (re)assignment.

However, index reusability does not have to be interpreted as destructive reassignment: we could in principle associate a new value with a previously used index while, at the same time, saving the old value for later retrieval by associating it with another index. This idea can be implemented in various ways, e.g. by taking information states to be *referent systems* (see e.g. Vermeulen 1993 and Groenendijk, Stokhof & Veltman 1996) or *stacks* (see e.g. Dekker 1994, van Eijck 2001, Nouwen 2003 or Bittner 2006) – and not DPL-style, total 'variable assignments'.

Such information states, however, are formally more complex than our current ones and their empirical superiority and intuitive appeal are largely orthogonal to the matters with which the present dissertation is concerned – so I will continue to employ total 'variable assignments' and the current notion of (destructive) random assignment for the remainder of this work. Extending CDRT+GQ and the novel dynamic system of chapters **5**, **6** and **7** below with referent systems or stacks is left for future research.

5.5. Anaphora across VP- and DP-Conjunctions

Let us turn now to sentences involving both anaphora and generalized conjunction, as it is this kind of examples that really bring out the benefits of having a dynamic type-

logical system. Consider sentences (57), (60) and (63) below (from Muskens 1996: 177-180, (52), (54) and (58)).

As shown below, the V'-conjunction example in (57) and the DP-conjunction examples in (60) and (63) are compositionally interpreted in CDRT+GQ and they are assigned the intuitively correct truth-conditions.

57. A^{u_1} cat [$_{V'}$[$_V$caught a^{u_2} fish] and [$_V$ate it$_{u_2}$]].

58. [$u_1, u_2 \mid cat\{u_1\}, fish\{u_2\}, catch\{u_1, u_2\}, eat\{u_1, u_2\}$]

59. $\lambda i_s. \exists x_e \exists y_e (cat(x) \wedge fish(y) \wedge catch(x, y) \wedge eat(x, y))$

60. John^{u_4} has [$_{DP}$[$_{DP}$a^{u_1} cat which caught a^{u_2} fish] and [$_{DP}$a^{u_3} cat which ate it$_{u_2}$]].

61. [$u_4 \mid u_4=John$]; [$u_1, u_2 \mid cat\{u_1\}, have\{u_4, u_1\}, fish\{u_2\}, catch\{u_1, u_2\}$];

     [$u_3 \mid cat\{u_3\}, have\{u_4, u_3\} eat\{u_3, u_2\}$]

62. $\lambda i_s. \exists x_e \exists y_e \exists z_e (cat(x) \wedge have(john, x) \wedge fish(y) \wedge catch(x, y) \wedge$

     $cat(z) \wedge have(john, z) \wedge eat(z, y))$

63. John^{u_3} admires [$_{DP}$[$_{DP}$a^{u_1} girl] and [$_{DP}$a^{u_2} boy who loves her$_{u_1}$]].

64. [$u_3 \mid u_3=John$]; [$u_1 \mid girl\{u_1\}, admire\{u_3, u_1\}$];

     [$u_2 \mid boy\{u_2\}, admire\{u_3, u_2\}, love\{u_2, u_1\}$]

65. $\lambda i_s. \exists x_e \exists y_e (girl(x) \wedge admire(john, x) \wedge boy(y) \wedge admire(john, y) \wedge \quad love(y, x))$

Moreover, given that CDRT+GQ interprets all generalized quantifiers as conditions / tests, the anaphoric connections in the structurally identical examples in (66), (67) and (68) below are correctly predicted to be infelicitous.

66. #A^{u_1} cat [$_{V'}$[$_V$caught no^{u_2} fish] and [$_V$ate it$_{u_2}$]].

67. #John^{u_4} has [$_{DP}$[$_{DP}$a^{u_1} cat which caught no^{u_2} fish] and [$_{DP}$a^{u_3} cat which ate

     it$_{u_2}$]].

68. #John^{u_3} admires [$_{DP}$[$_{DP}$no^{u_1} girl] and [$_{DP}$a^{u_2} boy who loves her$_{u_1}$]].

5.6. DP-Conjunction Donkey Sentences

Finally, the donkey sentence with DP-conjunction from Chierchia (1995) is compositionally interpreted as shown in (70) below; the truth-conditions – provided in

(71) – that are derived on the basis of the translation assign strong readings to the donkey indefinites, which is intuitively correct[12].

69. [[Every$^{\mathbf{str},u_1}$ boy who has a^{u_2} dog] and [every$^{\mathbf{str},u_3}$ girl who has a^{u_2} cat]] must feed it$_{u_2}$.

70. *every$^{\mathbf{str},u_1}$ boy who has a^{u_2} dog* ⤳

$\lambda P_{\mathbf{et}}.$ [**every**$^{str}{}_{u_1}$ ([u_2 | *boy*{u_1}, *dog*{u_2}, *have*{u_1, u_2}], $P(u_1)$))]

every$^{\mathbf{str},u_3}$ girl who has a^{u_2} cat ⤳

$\lambda P_{\mathbf{et}}.$ [**every**$^{str}{}_{u_3}$ ([u_2 | *girl*{u_3}, *cat*{u_2}, *have*{u_3, u_2}], $P(u_3)$))]

every$^{\mathbf{str},u_1}$ boy who has a^{u_2} dog and every$^{\mathbf{str},u_3}$ girl who has a^{u_2} cat ⤳

$\lambda P_{\mathbf{et}}.$ [**every**$^{str}{}_{u_1}$ ([u_2 | *boy*{u_1}, *dog*{u_2}, *have*{u_1, u_2}], $P(u_1)$),

every$^{str}{}_{u_3}$ ([u_2 | *girl*{u_3}, *cat*{u_2}, *have*{u_3, u_2}], $P(u_3)$))]

must feed it$_{u_2}$ ⤳ $\lambda v_{\mathbf{e}}.$ [*must_feed*{v, u_2}]

every$^{\mathbf{str},u_1}$ boy who has a^{u_2} dog and every$^{\mathbf{str},u_3}$ girl who has a^{u_2} cat must feed it$_{u_2}$

⤳ [**every**$^{str}{}_{u_1}$ ([u_2 | *boy*{u_1}, *dog*{u_2}, *have*{u_1, u_2}], [*must_feed*{u_1, u_2}]),

every$^{str}{}_{u_3}$ ([u_2 | *girl*{u_3}, *cat*{u_2}, *have*{u_3, u_2}], [*must_feed*{u_3, u_2}])]

71. $\lambda i_s.$ $\forall x_e \forall y_e (boy(x) \wedge dog(y) \wedge have(x, y) \rightarrow must\_feed(x, y)) \wedge$

$\forall x'_e \forall y'_e (girl(x') \wedge cat(y') \wedge have(x', y') \rightarrow must\_feed(x', y'))$

As Chierchia (1995): 96 observes, structurally similar sentences like (72) below are infelicitous.

72. ??[[Every$^{\mathbf{str},u_1}$ boy who has a^{u_2} dog] and [a^{u_3} girl]] must feed it$_{u_2}$.

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[12] In contrast, the corresponding translation in Chierchia (1995): 96, (76b) delivers the *weak* truth-conditions (i.e. the donkey indefinites are assigned the weak readings), which are arguably incorrect for the most salient reading of this type of example.

In all fairness, it should be noted that Chierchia (1995): 96 aims to interpret the slightly different example: *Every boy that has a dog and every girl that has a cat will beat it* (see Chierchia (1995): 96, (76a)). Therefore, his implicit claim might be that this particular example is preferably interpreted by accommodating an 'anger management' kind of scenario wherein the children are advised to beat their pets rather than each other – which would favor the weak reading of the sentence.

73. [**every**^{str}$_{u_1}$ ([u_2 | boy{u_1}, dog{u_2}, have{u_1, u_2}], [*must_feed*{u_1, u_2}])];

    [u_3 | girl{u_3}, *must_feed*{u_3, u_2}]

I suggest (following Chierchia 1995: 96) that their infelicity should be explained just as the infelicity of examples (66), (67) and (68) above: given that CDRT+GQ interprets generalized quantifiers as conditions / tests, the anaphoric connection between the pronoun *it*$_{u_2}$ and the indefinite *a*^{u_2} *dog* cannot be successfully established in the second conjunct of the translation in (73) above. That is, the occurrence of the dref u_2 in the second condition *must_feed*{u_3, u_2} is 'unbound', i.e. deictically used, despite the fact that the pronoun *it*$_{u_2}$ is co-indexed with a preceding indefinite, which is meant to encode that all occurences of the dref u_2 should be 'bound' (anaphorically used).

Alternatively, the infelicity of sentences like (72) above can be attributed to the fact that they fail to establish a discourse-level parallelism between the two DP-conjuncts relative to the anaphor in the VP. Besides accounting for the infelicity of (72), this hypothesis also provides an explanation for the particular indexing exhibited by the felicitous example in (69) above: the indefinites *a*^{u_2} *dog* and *a*^{u_2} *cat* receive the same index as a consequence of the fact that the two DP-conjuncts (or the two DRS's we obtain by combining the DP semantic values with the semantic value of the VP) are related by a *Parallel* discourse relation[13],[14].

This completes our analysis of the interaction between anaphora and generalized conjunction in CDRT+GQ – and, at the same time, the exposition of the basic framework for the present investigation.

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[13] For theories of parallelism in discourse, see Hobbs (1990) and Kehler (1995, 2002) among others. For similar observations with respect to disjunctive structures, see Stone (1992).

[14] Yet another way of thinking about examples like (69), suggested to me by Matthew Stone (p.c.), is to take the pronoun *it* in the VP to refer to the union of the referents contributed by the donkey indefinites in the two DP-conjuncts (see Stone 1992 for disjunction-based examples that seem to require this kind of analysis). This interpretation of the doubly-anteceded pronoun in the VP might emerge as a consequence of the parallelism between the two DP-conjuncts. Just like the parallelism-based explanation suggested in the main text, this hypothesis would also explain why the two indefinites receive the same index: co-indexation would be a necessary prerequisite for the union operation. The infelicity of examples like (72) above would presumably be explained just as suggested in the main text, i.e. as a failure to infer a discourse relation of parallelism – or any other discourse relation that would establish the (local) coherence of the discourse.

6. Limitations of CDRT+GQ: Mixed Weak & Strong Donkey Sentences

This section shows that CDRT+GQ, just as DPL+GQ, cannot give a compositional account of mixed weak & strong donkey sentences, i.e. relative-clause donkey sentences with multiple indefinites in the restrictor of the donkey quantification that receive different readings. In particular, we will see that determining which indefinite receives a weak reading and which one receives a strong reading cannot be compositionally implemented if we account for the weak / strong donkey ambiguity in terms of an ambiguity in the dynamic generalized determiner.

Consider again the examples with two donkey indefinites in (74) and (75) below.

74. Every ^{u_1} person who buys a ^{u_2} book on amazon.com and has a ^{u_3} credit card uses it $_{u_3}$ to pay for it $_{u_2}$.

75. Every ^{u_1} man who wants to impress a ^{u_2} woman and who has an ^{u_3} Arabian horse teaches her $_{u_2}$ how to ride it $_{u_3}$.

The most salient reading of (74) is one that is *strong* with respect to *a ^{u_2} book* and *weak* with respect to *a ^{u_3} credit card*, i.e. every person uses *some* credit card or other to pay for *any* book bought on amazon.com. Similarly, in (75) every man teaches *any* woman he wants to impress to ride *some* Arabian horse of his.

The problem with the weak and strong CDRT+GQ meanings for determiners is that they do not distinguish between the indefinites in the restrictor: all of them receive either a weak or a strong reading. The obvious fix is to make generalized determiners even more ambiguous, i.e. to redefine them as determiners binding a sequence of dref's and specifying for each dref that is different from the 'primary' one, i.e. the one that encodes the selective generalized quantification, whether it receives a weak or a strong reading.

For example, a determiner of the form $\mathbf{det}_u^{\mathbf{wk}:u',\mathbf{str}:u''}(D, D')$ quantifies over three drefs u, u' and u''; the 'primary' dref is u and the dref's u' and u'' are introduced by donkey

indefinites in the restrictor of the quantification and receive a weak and a strong reading respectively.

Such determiners could be defined by combining the weak and strong determiner meanings that we have introduced in CDRT+GQ; the definition would have the form shown in (76) below, where the DRS's D_3 and D_4 are subparts of DRS D_1. More precisely, D_3 is the subpart of D_1 relevant for the interpretation of dref u' associated with a strong donkey reading, while D_4 is the subpart of D_1 relevant for the interpretation of u'', which is associated with a weak donkey reading.

76. $\mathbf{det}_u^{\mathbf{str}:u',\mathbf{wk}:u''}(D_1, D_2) := \lambda i_s.\ \mathbf{DET}(u[D_1 i],\ u[([D_3 \rightarrow (D_4; D_2)])i])$

Sentence (74) above, for example, would be represented in CDRT+GQ as shown in (77) below.

77. $\mathbf{every}_{u_1}^{\mathbf{str}:\,u_2\,,\mathbf{wk}:\,u_3}([u_2, u_3|\ pers\{u_1\}, bk\{u_2\}, buy\{u_1, u_2\}, c.card\{u_3\}, hv\{u_1, u_3\}],$

$$[use\_to\_pay\{u_1, u_3, u_2\}]) :=$$

$$\lambda i_s.\ u_1[([u_2, u_3|\ person\{u_1\}, book\{u_2\}, buy\{u_1, u_2\}, c.card\{u_3\}, have\{u_1, u_3\}])i] \subseteq$$

$$u_1[(([u_2\ |\ person\{u_1\}, book\{u_2\}, buy\{u_1, u_2\}] \rightarrow$$

$$[u_3|\ c.card\{u_3\}, have\{u_1, u_3\}, use\_to\_pay\{u_1, u_3, u_2\}]])i])$$

There is another possible lexical entry for the determiner in (76) above, namely the entry where the two indefinites stand in the other possible relative scope, as shown in (78) below.

78. $\mathbf{det}_u^{\mathbf{str}:u',\mathbf{wk}:u''}(D_1, D_2) := \lambda i_s.\ \mathbf{DET}(u[D_1 i],\ u[(D_4; [D_3 \rightarrow D_2])i])$

For example, this meaning assigns sentence (74) a reading in which each person uses the same credit card to pay for all the books s/he buys, as shown in (79) below.

79. $\mathbf{every}_{u_1}^{\mathbf{str}:\,u_2\,,\mathbf{wk}:\,u_3}([u_2, u_3|\ pers\{u_1\}, bk\{u_2\}, buy\{u_1, u_2\}, c.card\{u_3\}, hv\{u_1, u_3\}],$

$$[use\_to\_pay\{u_1, u_3, u_2\}]),\ \text{i.e.}$$

$$\lambda i_s.\ u_1[([u_2, u_3|\ person\{u_1\}, book\{u_2\}, buy\{u_1, u_2\}, c.card\{u_3\}, have\{u_1, u_3\}])i] \subseteq$$

$$u_1[([u_3|\ person\{u_1\}, c.card\{u_3\}, have\{u_1, u_3\}];$$

$$[[u_2\ |\ book\{u_2\}, buy\{u_1, u_2\}] \rightarrow [use\_to\_pay\{u_1, u_3, u_2\}]])i])$$

It is not clear that this kind of pseudo wide-scope reading for the weak indefinite is a *separate reading* for sentence (74) as opposed to merely being a special case of the (more general and weaker) reading in which the credit card can vary from book to book [15].

For concreteness, imagine that there are two kinds of Amazon credit cards, one for the Christmas shopping period and one for the Easter shopping period and whenever a person uses the appropriate Amazon credit card at the appropriate time to buy a book on amazon.com, the person gets a discount. In this context, the sentence in (80) below is intuitively interpreted as assigning pseudo wide-scope to the weak indefinite (i.e. pseudo wide-scope relative to the strong indefinite).

80. Last Christmas, every^{u_1} person who bought a^{u_2} book on amazon.com and had an^{u_3} Amazon credit card used it$_{u_3}$ to pay for it$_{u_2}$ and got a discount.

However, the fact that the credit card does not vary from book to book in the most salient reading of sentence (80) cannot be taken as an argument for a distinct, pseudo wide-scope reading for this sentence: the lack of co-variation is a direct consequence of the way we have set up the context relative to which sentence (80) is interpreted – and, in the given situation, the indefinite *an^{u_3} Amazon credit card* is contextually restricted in such a way that it is a *singleton* indefinite[16].

Thus, it seems that the English sentence in (74) does not have two distinct readings. But as far as CDRT+GQ is concerned, we *can* in fact assign a distinct, pseudo wide-scope representation to sentence (74) (given the superscripted index notation that we have just introduced to capture the intuitively available reading of sentence) and we *should* distinguish this representation encoding an intuitively unavailable reading from the other, pseudo narrow-scope representation (see (77) above), which encodes the intuitively correct interpretation of sentence (74).

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[15] I call such readings *pseudo* wide-scope or *pseudo* narrow-scope because, as we have already noticed in chapter **2**, the donkey indefinites in sentences (74) and (75) are trapped in their respective VP-/CP-conjuncts and cannot take (syntactic) scope one relative to the other.

[16] See Schwarzschild (2002) for more discussion of singleton indefinites.

Therefore, besides having to specify which indefinite receives which reading (weak or strong), the CDRT+GQ lexical entry for a generalized determiner has to further specify the relative scope of the donkey indefinites. Thus, if we abbreviate that α has scope over β as $\alpha \gg \beta$, the lexical entries in (76) and (78) above are in fact the ones in (81) below.

81. $\mathbf{det}_u^{\mathbf{str}:u' \gg \mathbf{wk}:u''}(D_1, D_2) := \lambda i_s.\ \mathbf{DET}(u[D_1 i],\ u[([D_3 \rightarrow (D_4; D_2)])i])$

   $\mathbf{det}_u^{\mathbf{str}:u' \ll \mathbf{wk}:u''}(D_1, D_2) := \lambda i_s.\ \mathbf{DET}(u[D_1 i],\ u[(D_4; [D_3 \rightarrow D_2])i])$,

   where D_3 is the subpart of D_1 constraining dref u'

   and D_4 is the subpart of D_1 constraining dref u''.

To summarize, the CDRT+GQ strategy of analyzing mixed weak & strong 'indefinites' by locating the weak / strong ambiguity at the level of generalized determiners is undesirable for at least three reasons:

- first, it greatly increases the number of lexical entries for each determiner: we do not only have to specify for each indefinite in the restrictor of the determiner whether it receives a strong or a weak reading, but we also need to specify their relative scope;

- second, the interpretation procedure is not compositional – and this happens precisely because we pack in the lexical entry of the determiner many features that should in fact be encoded in the LF of its restrictor, i.e. what indefinites it contains, what reading they receive and what their relative scope is;

- third, the large number of lexical entries leads to (rampant) over-generation; for example, we have noticed that sentence (74) intuitively has a single reading[17], while CDRT+GQ assigns it several other readings that are intuitively unavailable.

Thus, if we want to give a precise definition of the CDRT+GQ interpretation procedure for sentence (74) for example, we have to either reject the Montagovian notion of compositionality or define fairly wild syntactic operations at the LF level, e.g.

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[17] This is not to say that other readings, e.g. a '**strong**:u_2, **strong**:u_3' reading, are not available for other examples.

identifying the substrees in the restrictor that correspond to the indefinites, duplicating them in the nuclear scope, moving them around to obtain various relative scopes, relating the subtrees via dynamic implication or dynamic conjunction depending on whether they have a strong or a weak reading etc. And, even if this daunting task were accomplished in a relatively plausible way, we would still face the ensuing over-generation problem.

I take the above reasoning to establish that CDRT+GQ (and similar systems) cannot account for mixed weak & strong donkey sentences containing VP- or CP-conjunctions like the ones in (74) and (75) above. It is the goal of the following chapter (chapter **5**) to offer a compositional account of the mixed weak & strong donkey sentences.

In particular, I will show that modifying CDRT+GQ so that information states are modeled as *sets* of 'assignments' (type *st*) and not as single 'assignments' (type *s*)[18], together with the hypothesis that any indefinite is ambiguous[19] between a weak and a strong reading, enables us to assign a unique meaning to each generalized determiner and to provide a *fully compositional* and intuitively correct interpretation for a wide range of donkey sentences, including the mixed weak & strong examples in (74) and (75) above.

CDRT+GQ faces the same basic kind of problems with respect to conditionals that exhibit asymmetric readings, i.e. weak / strong ambiguities. Recall Kadmon's generalization: a multi-case conditional with two indefinites in the antecedent generally allows for three interpretations, one where the QAdverb (which is a covert *always* or *usually* in the case of bare conditionals) quantifies over pairs, one where it quantifies over instances of the first indefinite and one where it quantifies over instances of the second. For example, consider sentences (82), (83) and (84) below.

82. If a^u village is inhabited by a$^{u'}$ painter, it$_u$ is usually pretty.

    (Kadmon 1987)

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[18] The idea of extending DPL by using sets of variable assignments for info states is due to van den Berg (1994, 1996a), which proposes this to account for a different kind of phenomena, namely discourses involving *plural cross-sentential* anaphora of the form *Every^u man saw a$^{u'}$ woman. They$_u$ greeted them$_{u'}$*.

[19] Or, to put in (possibly) more appealing terms: each indefinite is *underspecified* with respect to its 'strength' (it can be either weak or strong) and its 'strength' needs to be specified in each particular donkey sentence; for a discussion of the various factors that influence this 'strength' specification, see chapter **5**.

83. If a^u drummer lives in an$^{u'}$ apartment complex, it$_{u'}$ is usually half empty.

(Bäuerle & Egli 1985, apud Heim 1990: 151, (29))

84. If a^u woman owns a$^{u'}$ cat, she$_u$ usually talks to it$_{u'}$.

(Heim 1990: 175, (91))

The most salient reading of (82) is an asymmetric one in which we quantify over villages u inhabited by a painter; thus, that conditional is translated in CDRT+GQ by means of the selective determiner **most**$^{wk}_u$ or **most**$^{str}_u$.

The most salient reading of (83) is an asymmetric one in which we quantify over apartment complexes u' inhabited by a drummer; hence, the conditional is translated in CDRT+GQ by means of the selective determiner **most**$^{wk}_{u'}$ or **most**$^{str}_{u'}$.

Finally, the most salient reading of (84) is one where we quantify over woman-cat pairs; therefore, the conditional is translated in CDRT+GQ by means of the unselective determiner **most**.

Various factors influence what is the most salient reading of a donkey conditional: Bäuerle & Egli (1985) notice that it depends on which indefinites from the antecedent are anaphorically picked up in the consequent. Rooth (1985) and Kadmon (1987) (see also Heim 1990 and Chierchia 1995 among others) observe that the focus-background structure of the sentence also determines which indefinites receive which reading, the generalization being that the non-focused indefinite in the antecedent is the one that is bound by the *if*+QAdverb quantification. As Heim (1990): 152 observes, the sentence in (85) below receives precisely the most salient interpretation of (83) above, i.e. an 'apartment complex' asymmetric reading, while sentence (86), with a different focus-background, receives a 'drummer' asymmetric reading.

85. Do you think there are vacancies in this apartment complex? – Well, I heard that Fulano lives there, and if a DRUMMER lives in an apartment complex, it is usually half empty.

86. Drummers mostly live in crowded dormitories. But if a drummer lives in an APARTMENT COMPLEX, it is usually half empty.

Accounting for how these factors determine what reading is the most salient, i.e. which one of the determiners **most**$^{wk/str}_u$, **most**$^{wk/str}_{u'}$ and **most** should be selected in the translation, is clearly beyond the scope of CDRT+GQ. What I want to point out is simply that the translation of a conditional by means of a selective determiner like **most**$^{wk/str}_u$ or **most**$^{wk/str}_{u'}$ exhibits the same kind of non-compositionality as the translation of relative-clause donkey sentences with mixed readings: one of the indefinites in the antecedent of the conditional is somehow supposed to 'fuse' with the QAdverb and be interpreted as a selective generalized determiner.

There are even more complex examples with three indefinites – like the one in (87) below. Its most salient reading seems to be one in which we quantify over most woman-man pairs that have some son or other (i.e. the indefinite $a^{u'}$ *son* receives a weak reading).

87. If a^u woman has a$^{u'}$ son with a$^{u''}$ man, she$_u$ usually keeps in touch with him$_{u''}$.

   (I. Heim, apud Chierchia 1995: 67, (14b))

Because of their different, multi-sentential syntactic structure and because of their somewhat different behavior with respect to the weak / strong donkey ambiguity, I will generally avoid conditional structures (with or without QAdverbs) and use mainly mixed weak & strong *relative-clause* donkey sentences to motivate the novel dynamic system I will introduce in chapter **5** and the analysis of the weak / strong ambiguity I will propose there.

Let's turn now to another welcome consequence of the fact that CDRT (and its extension CDRT+GQ) unifies Montague semantics and dynamic semantics, namely the account of the interaction between donkey anaphora and generalized conjunction (generalized to arbitrary types in the sense of Partee & Rooth 1983 among others).

7. Summary

The overarching goal of this chapter and of the previous two was to incrementally describe a compositional dynamic system formulated in many-sorted type logic with selective dynamic generalized quantification and generalized conjunction, which I have labeled CDRT+GQ. Three phenomena provide the primary motivation for CDRT+GQ:

- the proportion problem;
- the weak / strong donkey ambiguity;
- the interaction between donkey anaphora, dynamic quantification and generalized dynamic conjunction.

Besides providing a streamlined notation integrating various DRT, DPL and CDRT notational conventions, CDRT+GQ contributes several new notions and analyses:

- it shows how to define in Dynamic Ty2 the DPL-style conservative definition of unselective dynamic generalized quantification in chapter **2**;
- it integrates CDRT (Muskens 1996) and the DPL-style notion of selective generalized quantification (Bäuerle & Egli 1985, Root 1986, Rooth 1987, van Eijck & de Vries 1992, Chierchia 1992, 1995);
- it provides the dynamic counterparts of the definitions of conjoinable types and generalized conjunction and disjunction in Partee & Rooth (1983)**;**
- it accounts for the DP-conjunction donkey example *Every^{u_1} boy who has a^{u_2} dog and every^{u_3} girl who has a^{u_2} cat must feed it$_{u_2}$* from Chierchia (1995).

Finally, section **6** showed that CDRT+GQ has the same problems as DPL+GQ when confronted with relative-clause donkey sentences with mixed weak & strong readings, e.g. *Every person who buys a book on amazon.com and has a credit card uses it / the card to pay for it*. Such examples are difficult to analyze in CDRT+GQ and, if they can be analyzed at all, the account is stipulative, non-compositional and over-generates fairly wildly.

The last three chapters in general and the introduction of CDRT+GQ in particular pave the way for chapter **5**, which shows that a minimal modification of CDRT+GQ, i.e. the introduction of *plural* info states (type *st*) as opposed to 'singular' ones (type *s*), enables us to give a compositional analysis of a wide range of singular donkey sentences, including mixed weak & strong relative-clause donkey sentences.

Chapter 5. Structured Nominal Reference: Donkey Anaphora

1. Introduction

This chapter incrementally introduces a new dynamic system that extends CDRT+GQ and within which we can give a compositional account of the multiple donkey sentences in (1) and (2) below. This pair of sentences shows that the analysis of singular donkey anaphora requires a notion of plural discourse reference, i.e. reference to a quantificational dependency between sets of objects (atomic individuals, possible worlds etc.), which is established and subsequently referred to in discourse.

1. Every^{u_1} person who buys a^{u_2} book on amazon.com and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$ [1].

2. Every^{u_1} boy who bought a^{u_2} Christmas gift for a^{u_3} girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

Both examples contain multiple instances of singular donkey anaphora that are semantically correlated: (1) shows that singular donkey anaphora can refer to (possibly non-singleton) *sets*, while (2) shows that singular donkey anaphora can refer to a *dependency* between such sets.

Sentence (1) is a *mixed weak & strong* donkey sentence[2]: it is interpreted as asserting that, for *every* book (strong) that any credit-card owner buys on amazon.com,

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[1] Some speakers find the variants in (i) below intuitively more compelling:

(i) Every person who buys a computer / TV and has a credit card uses it to pay for it.

[2] To my knowledge, the existence of mixed reading relative-clause donkey sentences was observed for the first time by van der Does (1993). His example is provided in (i) below – and it is accompanied by the observation that "clear intuitions are absent, but a combined reading in which a whip is used to lash all horses seems available" (van der Does 1993: 18). The intuitions seem much clearer with respect to example (1) above; moreover, it is crucial for our purposes that the weak reading of *a credit card* in (1) does not require the set of credit cards to be a singleton set (that is, some people might use different credit cards to buy different (kinds of) books).

(i) Every farmer who has a horse and a whip in his barn uses it to lash him. (van der Does 1993: 18, (26))

there is *some* credit card (weak) that s/he uses to pay for the book. Note in particular that the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels – which means that even the *weak* indefinite *a^{u_3} credit card* can introduce a (possibly) *non-singleton set*.

For each buyer, the two sets of objects, i.e. all the books purchased on amazon.com and some of the credit cards that the buyer has, are *correlated* and the *dependency* between these sets is specified in the nuclear scope of the quantification: each book is correlated with the credit card that was used to pay for it. The translation of sentence (1) in classical (static) first-order logic is provided in (3) below.

3. $\forall x(person(x) \wedge \exists y(book(y) \wedge buy\_on\_amazon(x, y)) \wedge \exists z(c.card(z) \wedge have(x, z))$

   $\rightarrow \forall y'(book(y') \wedge buy\_on\_amazon(x, y')$

   $\rightarrow \exists z'(c.card(z') \wedge have(x, z') \wedge use\_to\_pay(x, z', y'))))$

The challenge posed by this sentence is to *compositionally* derive its interpretation while allowing for: (**i**) the fact that the two donkey indefinites in the restrictor of the quantification receive two distinct readings (strong and weak respectively) and (**ii**) the fact that the value of the weak indefinite *a^z credit card* co-varies with / is dependent on the value of the strong indefinite *a^y book* although the strong indefinite cannot syntactically scope over the weak one, since both DP's are trapped in their respective conjuncts.

The *dependency* between the two sets of objects is the most transparent in sentence (2). Both instances of donkey anaphora are strong: we are considering *every* Christmas gift and *every* girl. The restrictor introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. The nuclear scope of the donkey quantification retrieves not only the two sets of objects, but also the *structure* associated with them, i.e. the dependency between them: each gift was wrapped by the

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The existence of mixed reading conditional donkey sentences has been observed at least since Dekker (1993); his example is provided in (ii) below.

(ii) If a man has a dime in his pocket, he throws it in the parking meter. (Dekker 1993: 183, (25)).

deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to *structure* in addition to donkey anaphora to *values*.

Importantly, the structure associated with the two sets, i.e. the dependency between gifts and girls, is *semantically* encoded and not pragmatically inferred: the correlation between the two sets is not left vague / underspecified and subsequently made precise based on various extra-linguistic factors. To see this, consider the following situation. John buys two gifts, one for Mary and the other for Helen. The two girls are deskmates (note that the *deskmate* relation is symmetric). Intuitively, sentence (2) is true if John asked Mary to wrap Helen's gift and Helen to wrap Mary's gift and it is false if John asked each girl to wrap her own gift (i.e. if John asked Mary to wrap the gift bought for her and, similarly, he asked Helen to wrap the gift bought for her). But if the relation between gifts and girls were vague / underspecified, we would predict that sentence (2) should be true even in the second (somewhat odd) situation[3,4].

In sum, we need a *semantic* framework which can account for reference to *non-singleton structured sets*, where the quantificational structure associated with the sets is introduced in a (syntactically) non-local manner – for example, in (1), across a coordination island – and subsequently accessed in a non-local manner – for example, in (2), from outside the relative clause that introduces the structured dependency.

The chapter is structured as follows. Section **2** provides a brief outline of the proposed account. Section **3** introduces an extension of CDRT+GQ with plural info

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[3] Note the similarity between example (2) (which crucially involves the symmetric relation *deskmate*) and the 'indistinguishable participants' examples involving symmetric relations due to Hans Kamp, Jan van Eijck and Irene Heim (see Heim 1990: 147, fn. 6):

(i) If a man shares an apartment with another man, he shares the housework with him. (Heim 1990: 147, (22))

(ii) If a bishop meets a bishop, he blesses him. (Heim 1990: 148, (23)).

[4] The donkey sentence in (2) does not pose problems for CDRT+GQ (or indeed DRT / FCS / DPL) – at least to the extent to which CDRT+GQ can provide a suitable analysis of possessive definite descriptions like *her deskmate*. However, as the remainder of this section will show, the donkey sentence in (2) is an important companion to the mixed reading donkey sentence in (1); it is only together that these two sentences provide an argument for extending CDRT+GQ with plural information states (i.e. the main technical innovation of this chapter) as opposed to a more conservative extension of CDRT+GQ with dref's for sets.

states, which I dub Plural CDRT (PCDRT)[5]. Section **4** shows in detail how PCDRT can be used to compositionally interpret a variety of donkey sentences, including mixed weak & strong relative-clause donkey sentences.

Section **6** compares PCDRT with alternative approaches to donkey anaphora and evaluates how they fare with respect to the proportion problem, the weak/strong donkey ambiguity and mixed reading relative-clause donkey sentences. The appendix contains a summary of the PCDRT system and some of the more technical results about its formal properties.

2. Outline of the Proposed Account

The first issue that we need to address is the weak / strong donkey ambiguity. I will attribute this ambiguity to the donkey indefinites – and not to any other element involved in the donkey anaphora structure, e.g. the generalized determiner, as CDRT+GQ (following Rooth 1987, Heim 1990, Kanazawa 1994a) would have it.

The two basic meanings for the donkey indefinites have the format in (4) below, where the **max** operator taking scope over both the restrictor and the nuclear scope properties delivers the strong (maximal) donkey reading. The **max** operator ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref u the maximal set of individuals satisfying both the restrictor dynamic property P' and the nuclear scope dynamic property P.

4. **weak indefinites:** $a^{\text{wk}:u} \rightsquigarrow \lambda P'_{\text{et}}. \lambda P_{\text{et}}. [u]; P'(u); P(u)$

    strong indefinites: $a^{\text{str}:u} \rightsquigarrow \lambda P'_{\text{et}}. \lambda P_{\text{et}}. \textbf{max}^u(P'(u); P(u))$

Attributing the weak / strong ambiguity to the donkey indefinites enables us to give a *compositional* account of the mixed weak & strong donkey sentence in (1) above because we *locally* decide for each indefinite article whether it receives a weak or a

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[5] One possible mnemonic for PCDRT is Politically Correct DRT. The author vigorously denies responsibility for any entailments, presuppositions, implicatures or implications of any other kind associated with the use of this mnemonic in any discourse and / or utterance context whatsoever.

strong reading. Moreover, selective generalized determiners like *every*, *no* etc. have the kind of dynamic meaning that we would expect them to have based on their static Montague-style meanings: they are associated with / 'bind' only one dref (their own) and do not need to encode which readings the donkey indefinites in their restrictor have and that is the relative (pseudo-)scope of these indefinites.

Furthermore, this analysis of the weak / strong donkey ambiguity is couched within a framework that enables us to account for the fact that donkey anaphora involves reference to (possibly non-singleton) *structured sets* of individuals. The main innovation (relative to CDRT+GQ) is to minimally complicate the notion of info state: instead of using singular info states consisting of a single 'assignment' i, j, … (type s), I follow the proposal in van den Berg (1994, 1996a) and use *plural* info states I, J, …, consisting of sets of 'assignments' (type st). I will call the resulting system Plural CDRT (PCDRT).

In PCDRT, individual dref's have the same type as in CDRT+GQ, i.e. type se. A dref u (of type se) stores a set of individuals uI with respect to such a plural info state I: as shown in (5) below, the set of individuals uI is the image of the set of 'assignments' I under the function u.

5. **Abbreviation:** $uI := u_{se}[I_{st}] = \{u_{se}i_s: i_s \in I_{st}\} = \{x_e: \exists i_s \in I(ui=x)\}$

Storing a set of individuals by means of a plural info state and not by means of a dref for sets (its type would be $s(et)$) enables us to access in discourse not only the set of individuals, but also the structure associated with it by the plural info state: for example, two drefs u and u' store two sets of individuals relative to a plural info state I, i.e. $uI = \{ui: i \in I\}$ and $u'I = \{u'i: i \in I\}$; but the info state I also stores the dependency (i.e. the binary relation) between the two dref's, which is the set of pairs of individuals $\{<ui, u'i>: i \in I\}$[6].

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[6] In DRT / FCS / DPL terminology, we can think of the sets of individuals as being contributed by sets of variable assignments (or sets of embedding functions) G, G' etc. A set of variable assignments introduces both *sets* of individuals, e.g. a variable x is associated with the set of individuals $\{g(x): g \in G\}$, and a *relation* between them, e.g. two variables x and y determine the binary relation $\{<g(x), g(y)>: g \in G\}$ between the two sets of individuals associated with x and y, i.e. between $\{g(x): g \in G\}$ and $\{g(y): g \in G\}$.

6. Info State I

	...	u		u'		...
i_1	...	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	...
i_2	...	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	...
i_3	...	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	...
...	...	...		...		...

Values – sets: $\{x_1, x_2, x_3, \ldots\}, \{y_1, y_2, y_3, \ldots\}$ **Structure – relations:** $\{\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \langle x_3, y_3\rangle, \ldots\}$

As (6) above shows, plural info states encode discourse reference to both values and structure. The values are the sets of objects that are stored in the *columns* of the matrix, e.g. a dref u for individuals stores a set of individuals relative to a plural info state, since u is assigned an individual by each assignment (i.e. row). The structure is *distributively* encoded in the *rows* of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment.

Thus, plural info states enable us to capture the structured dependencies between the multiple donkey anaphoric connections in (1) and (2) above. Let us start with the PCDRT analysis of sentence (2): by the time we are done processing the restrictor of the donkey quantification, we will be in an info state I which can be represented as the matrix in (7) below. Note that the strong donkey indefinites introduce both *values*, i.e. the set of gifts $u_2I = \{a_1, a_2, \ldots\}$ and the set of girls $u_3I = \{b_1, b_2, \ldots\}$, and *structure*, i.e. for each 'assignment' $i \in I$, the gift u_2i was bought for girl u_3i.

7. Every^{u_1} boy who bought a$^{\textbf{str:}u_2}$ Christmas gift for a$^{\textbf{str:}u_3}$ girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

Info state I	...	u_2 (**all gifts**)	u_3 (**all girls**)	...
i_1	...	a_1 $(=u_2i_1)$	b_1 $(=u_3i_1)$	...
i_2	...	a_2 $(=u_2i_2)$	b_2 $(=u_3i_2)$	...
i_3	...	a_3 $(=u_2i_3)$	b_3 $(=u_3i_3)$	...
...	...	...	...	...

gift a_1 was bought for girl b_1

When we process the nuclear scope of the donkey quantification, we are anaphoric to both values and structure: we require each 'assignment' $i \in I$ to be such that the deskmate of girl u_3i was asked to wrap gift u_2i. Thus, the nuclear scope of the donkey quantification elaborates on the structured dependency between the set of gifts u_2I and the set of girls u_3I introduced in the restrictor of the donkey quantification.

The interpretation of sentence (1) is different in two important respects: (**i**) the indefinite a^{u_3} *credit card* receives a weak reading and (**ii**) the structural dependency between books and credit cards remains implicit in the restrictor and is explicitly established only in the nuclear scope. That is, by the time we are done processing the restrictor of the donkey quantification in (1), we will be in an info state I like the one in (8) below. We introduce the *maximal* set of books for u_2 (the strong indefinite), we non-deterministically introduce *some* set of credit cards for u_3 (the weak indefinite) and we non-deterministically introduce *some structure* correlating the values of u_2 and u_3.

8. Every^{u_1} person who buys a$^{\textbf{str:}u_2}$ book on <u>amazon.com</u> and has a$^{\textbf{wk:}u_3}$ credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

Info state I	...	u_2 (**all books**)	u_3 (**some credit cards**)	...
i_1	...	$a_1 (=u_2i_1)$	$b_1 (=u_3i_1)$	...

<p style="text-align:center">book a_1 is somehow correlated with card b_1</p>

i_2	...	$a_2 (=u_2i_2)$	$b_2 (=u_3i_2)$	...
i_3	...	$a_3 (=u_2i_3)$	$b_3 (=u_3i_3)$	...
...	...	...	...	...

The nuclear scope is again anaphoric to both values and structure; in particular, we test that the non-deterministically introduced value for u_3 and the non-deterministically introduced structure associating u_3 and u_2 satisfy the nuclear scope condition, i.e., for each 'assignment' $i \in I$, the credit card u_3i is used to pay for the book u_2i. Yet again, the nuclear scope elaborates on the unspecified dependency between u_3 and u_2 introduced in the restrictor of the donkey quantification. Crucially, the credit cards co-vary with / are dependent on the books and introducing such a dependency does not require the strong indefinite a^{u_2} *book* to scope over the weak indefinite a^{u_3} *credit card* – which cannot happen because the two DP's are trapped in their respective conjuncts.

As the semi-formal paraphrases above indicate, PCDRT follows CDRT+GQ and interprets a sentence as a DRS, i.e. as a relation between an input and an output info state. The only difference is that the PCDRT info states are plural, hence the type of a DRS is $(st)((st)t)$, i.e. a relation between an input info state I_{st} and an output info state J_{st}. The example in (1) provides the empirical motivation for modeling DRS's as *relations* between plural info states (of type $(st)((st)t)$), i.e. as *non-deterministically* updating a plural info state. We need the non-determinism to introduce both (**i**) the value of the weak indefinite a^{u_3} *credit card* and (**ii**) the dependency between the weak indefinite a^{u_3} *credit card* and the strong indefinite a^{u_2} *book*: both the plural value of dref u_3 and the dependency relative to the dref u_2 are non-deterministically introduced in the restrictor and elaborated upon in the nuclear scope.

The structural non-determinism, i.e. the fact that the dynamics of structural dependencies is essentially the same as the dynamics of values, is a core design feature of PCDRT, which sets it apart from many previous dynamic systems with plural info states (including van den Berg 1996a, Krifka 1996b and Nouwen 2003).

One final observation before turning to the formal development of the account sketched in this section. The hypothesis that singular indefinite articles are ambiguous is not entirely desirable: for one thing, the two readings of the indefinite are always morphologically identical in English; moreover, I do not know of any natural language that would systematically reflect the difference between these two readings in the surface form of the indefinites. Thus, an analysis that would avoid the ambiguity and would derive the two distinct readings solely on the basis of independently motivated semantic and pragmatic factors would be preferable.

However, the proposed analysis of the weak / strong ambiguity gets fairly close to achieving this goal: the only difference between a weak and a strong indefinite article is the presence vs. absence of a maximization operator. We can therefore think of the singular indefinite article as *underspecified* with respect to the presence vs. absence of this maximization operator: the decision to introduce it or not is made online depending on the discourse and utterance context of a particular donkey sentence – much like aspectual coercion[7] or the selection of a particular type for the denotation of an expression[8] are context-driven online processes.

3. CDRT+GQ with Plural Information States: Plural CDRT (PCDRT)

This section incrementally develops Plural CDRT (PCDRT), i.e. the promised extension of CDRT+GQ with plural info states. Section **3.1** gives the new definition of atomic conditions, section **3.2** the definition of new dref introduction, section **3.3** defines

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[7] For example, the iterative interpretation of *John sent a letter to the company for years* or of *The light is flashing*.

[8] For example, when proper names are conjoined with generalized quantifiers.

negation, section **3.4** introduces maximization and, finally, section **3.5** defines selective and unselective generalized quantification in PCDRT. I provide the empirical and theoretical motivation for the formal innovations as I introduce them.

3.1. Atomic Conditions

No changes need to be made to our underlying logic Dynamic Ty2, i.e. our 'low-level programming language': we will be working with the same bivalent total logic with no non-atomic individuals. And the changes to our DRT-style abbreviation system, i.e. our 'high-level programming language', are minimal: we introduce plural info states I, J, K, … of type st and we consequently reset the type **t** of (saturated) sentences to $(st)((st)t)$: **t** is still the type of a binary relation between info states, it's just that the info states themselves are plural[9].

9. Plural info states (type st): H_{st}, I_{st}, J_{st}, K_{st}, H'_{st}, I'_{st}, J'_{st}, K'_{st}, …;

'Saturated' expressions in PCDRT:

- sentences (DRSs) – relations between plural info states: $\mathbf{t} := (st)((st)t)$;

- names (individual dref's) – the same as in CDRT+GQ: $\mathbf{e} := se$.

Just as in CDRT+GQ, the atomic conditions are sets of info states. However, given that we are now working with plural info states, their type will be $(st)t$. Moreover, the atomic conditions will be *unselectively distributive*, where 'unselective' is used in the sense of Lewis (1975), i.e. the atomic conditions are *distributive* over the *plural info states* they accept: they accept a set of 'assignments' iff they accept, in a pointwise manner, every single 'assignment' in the set.

This is implemented by means of universal quantification over the set of assignments in a plural info state I_{st}, as shown in (10) below. The requirement of non-emptiness $I \neq \emptyset$ rules out the 'degenerate' case in which the universal quantification $\forall i_s \in I(\ldots)$ is vacuously satisfied.

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[9] Incidentally, note that **t** is the type of generalized determiners over entities of type s, parallel to static (extensional) determiners of type $(et)((et)t)$.

10. **Atomic conditions** – type $(st)t$.

$$R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I(R(u_1 i, \ldots, u_n i)),$$

for any non-logical constant R of type $e^n t$,

where $e^n t$ is defined as follows: $e^0 t := t$ and $e^{m+1}t := e(e^m t)$.

$$u_1{=}u_2 := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I(u_1 i{=}u_2 i)$$

As already suggested, the requirement enforced by an atomic condition can be intuitively depicted by means of a matrix, as shown in (11) below.

11.Info state I	…	u_1	…	u_n	…
i	…	$a_1\ (=u_1 i)$	…	$a_n\ (=u_n i)$	…

$$R(u_1 i, ..., u_n i),\ \textbf{i.e. } R(a_1, ..., a_n)$$

i'	…	$a_1'\ (=u_1 i')$	…	$a_n'\ (=u_n i')$	…
i''	…	$a_1''\ (=u_1 i'')$	…	$a_n''\ (=u_n i'')$	…
…	…	…	…	…	…

The unselectively distributive structure of the atomic conditions endows the set of plural information states characterized by them with a lattice-theoretic ideal structure.

12. $\Im$ is an *ideal* with respect to the partial order induced by set inclusion $\subseteq$ on the power set of the domain of 'assignments' $\wp(D_s^M)$ (i.e. $<\wp(D_s^M), \subseteq>$) iff:

 a. $\Im \subseteq \wp(D_s^M)$;

 b. for any I_{st} and J_{st}, if $I{\in}\Im$ and $J{\subseteq}I$, then $J{\in}\Im$ (closure under subsets);

 c. for any I_{st} and J_{st}, if $I{\in}\Im$ and $J{\in}\Im$, then $(I{\cup}J){\in}\Im$ (closure under finite unions).

 $\Im$ is a *complete ideal* iff (**a**) and (**b**) are as above and, instead of (**c**), we require closure under arbitrary unions.

A complete ideal $\Im$ has a supremum, namely $\cup\Im$. Given the requirement of closure under subsets and closure under arbitrary unions, a complete ideal $\Im$ is a complete Boolean algebra, as stated in (13) below.

13. $\Im = \wp(\cup\Im),$ for any complete ideal $\Im$ (in the atomic lattice $\wp(D_s^M)$).

We introduced the notation in (14) below to handle the non-emptiness requirement in the definition of atomic conditions.

14. Let $\wp^+(D_s^M)$ be the power set of the domain of 'assignments' without the empty set Ø. A *(complete) ideal without a bottom element* is defined just as in (12) above, except that, instead of (12a), we require inclusion in $\wp^+(D_s^M)$ and, instead of (12b), we require closure under non-empty subsets.

Since we are concerned here only with complete ideals without a bottom element, I will henceforth use "c-ideal" instead of the longer "complete ideal without a bottom element". The most important fact is that, for any c-ideal $\Im$, we have that $\Im = \wp^+(\cup\Im)$, i.e. c-ideals are complete Boolean algebras without a bottom element.

The definition of atomic conditions in (10) above ensures that they always denote c-ideals (in the atomic lattice $\wp(D_s^M)$). We can in fact characterize them in terms of the supremum of their denotation.

15. **Atomic Conditions as C-Ideals.**

For any non-logical constant R of type $e^n t$ and sequence of unspecific[10] dref's $\langle u_1, \ldots, u_n \rangle$, let $\mathbb{I}(R, \langle u_1, \ldots, u_n \rangle) := \lambda i_s. R(u_1 i, \ldots, u_n i)$, abbreviated $\mathbb{I}^R$ whenever the sequence of dref's can be recovered from context. Then, $R\{u_1, \ldots, u_n\} = \wp^+(\mathbb{I}^R)$ [11].

The fact that atomic conditions denote c-ideals will be useful in showing that PCDRT has a range of desirable properties and it will guide several design choices we have to make on the way.

3.2. New Discourse Referents

We turn now to defining the introduction of new dref's in PCDRT. I will consider only two candidate definitions, both given in (16) below, and I will argue that the first

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[10] For the notion of unspecific dref, see definition **4** in section **2.2** of chapter **3** above.

[11] Convention: $\wp^+(\emptyset_{st}) = \emptyset_{(st)t}$.

one, namely (16a), is the empirically and theoretically better choice. Both definitions relate two plural info states I_{st} and J_{st} in terms of the pointwise relation $i_s[u]j_s$.

16. **Introducing new dref's in PCDRT – two candidate definitions**:

    a. $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

    b. $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et} \neq \emptyset (J = \bigcup_{i_s \in I} \{j_s : i[u]j \wedge uj \in X\})$,

        equivalently: $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et} \neq \emptyset (J = \{j_s : \exists i_s \in I(i[u]j \wedge uj \in X)\})$.

Definition (16a) is the more general and logically weaker one: it simply requires any 'assignment' i in the input info state I to have a successor 'assignment' j in the output state J and, similarly, any 'assignment' j in the output info state J should have an ancestor 'assignment' i in the input state I. In this way, we will necessarily preserve all the discourse information[12] in the input state I when we non-deterministically update it and obtain the output state J.

Definition (16b) has an extra-requirement over and above definition (16a): we need to *uniformly* reassign the value of the dref u for all the 'assignments' i_s in the input info state I_{st}, i.e. there is some random set X_{et} of new values for u and each input 'assignment' i is updated (relative to u) with each and every single value in X. The effect of definition (16b) is shown in (17) below: the input state I_{st} contains two 'assignments' i and i' and the set X_{et} of new values for u contains two individuals a and b.

17. **$I\{u\}J$, where $X_{et}=\{a, b\}$**

$I\{u\}J$	Output state J_{st}	…	u	…
Input state I_{st} …	i_a	…	$a\ (=u(i_a))$	…
i …	i_b	…	$b\ (=u(i_b))$	…
i' …	i'_a	…	$a\ (=u(i'_a))$	…
	i'_b	…	$b\ (=u(i'_b))$	…

---

[12] Recall that, in PCDRT, the preserved discourse information consists of: (i) the previously established *values* for all the dref's other than u and (ii) the previously established *structured dependencies* between the dref's other than u.

The choice between the two definitions in (16)[13,14] boils down to how we want to handle the new component of our information states, i.e. the *structure* associated with the values of the dref's. The singular info states of CDRT+GQ encode only *values* – and we non-deterministically assign new values to a particular dref. Thus, for each particular info state, the value of the dref is *determined*, but throughout the entire discourse context, i.e. throughout the space of all possible output info states for the random assignment [*u*], the value of the dref is *not determined*: for every possible value that the dref *u* can take, there will be some output info state that assigns that value to *u*.

The plural info states of PCDRT encode *values* and, in addition, *structure*, i.e. they encode dependencies between the values of the dref's in a pointwise manner ('assignment' by 'assignment'). Our first definition *I*[*u*]*J* treats the structural component in parallel to the value component of the info state: we non-deterministically introduce both new *values* for *u* and new *structure*, as the values for *u* in the output state can be stored in a particular configuration of pointwise associations with the other dref's.

Thus, in each info state, the value and the structure of dref *u* are *determined*, but throughout the entire discourse context, i.e. throughout the space of all possible plural output states, the value and the structure of dref *u* are *not determined*: for every possible non-empty set of values, for every possible structure (i.e. pointwise distribution) of that set, there is some plural output state that assigns to *u* that particular value with that particular associated structure.

The second definition *I*{*u*}*J* does not treat the two components of a plural info state, i.e. value and structure, in a parallel way: we are still non-deterministic with respect to the value, but we are *deterministic* with respect to the structure – for any set of

[13] Both definitions appear in van den Berg's work: an equivalent of (16a) is used in van den Berg (1994): 15, fn 12 and in van den Berg (1996b): 18, (49), while van den Berg (1996a): 134-135, (2.7) & (2.8) uses a version of (16b). The two definitions I consider differ from van den Berg's definitions in several respects: first, (16a) and (16b) are formulated in type logic, unlike van den Berg's, which are formulated in DPL terms; second, the definitions of random assignment in van den Berg are more complex because he works with a three-valued logic and also countenances a dummy / 'undefined' individual ⋆. To my knowledge, there is no comparison of the two alternative definitions in van den Berg's work.

[14] Nouwen (2003) follows van den Berg (1996a) and assumes the definition of {*u*} in (16b); the alternative option is not mentioned.

individuals that is randomly assigned as a value, there is only one possible structure (i.e. pointwise distribution) of that set throughtout the output discourse context (i.e. throughout the space of output info states).

This choice seems to be preferable if we want to make the system computationally more efficient because it would significantly cut down the number of possible output info states for any given instance of new dref introduction (a.k.a. plural random assignment). Moreover, a more constrained system (presumably) runs a lower risk of over-generation. Finally, the structure we choose for every random value is the least 'biased' one: we introduce the entire set assigned to u with respect to each input 'assignment' i, so there is no 'biased' correspondence / dependency between the values of some other dref u' and the values newly assigned to u. That is, although the update is structurally deterministic, it always associates *the least possible amount of structural information* with each new value.

Despite the fact that the second definition $\{u\}$ is more constrained (hence, *ceteris paribus*, more desirable), I will provide three reasons, one empirical and two theoretical, for preferring the first definition, namely $[u]$. The first, empirical reason is provided by our mixed weak & strong donkey sentences, repeated below for convenience.

18. Every^{u_1} person who buys a^{u_2} book on amazon.com and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

19. Every^{u_1} man who wants to impress a^{u_2} woman and who has an^{u_3} Arabian horse teaches her$_{u_2}$ how to ride it$_{u_3}$.

Recall that, intuitively, we want to allow for credit cards that vary from book to book and also for Arabian horses that vary from woman to woman. Consider now the definition in (16a), i.e. $[u]$, and its effect on the interpretation of the quantification in (19) (the same reasoning applies to (18)). By the time we process the second conjunct in the restrictor, i.e. *who has an^{u_3} Arabian horse*, we have already processed the first one *who wants to impress a^{u_2} woman* and, therefore, the dref u_2 has already been introduced and was assigned appropriate womanly values. Now we introduce u_3 by means of the update

[u_3] and we *non-deterministically* assign it a set of equine values and *non-deterministically* associate a structure with this set of values, i.e. we non-deterministically associate each u_3-horse with some u_2-woman.

The nuclear scope subsequently filters the non-deterministically assigned values and structure: we require the u_3-horses to stand in the 'u_2 *rides* u_3' relation to the u_2-set of women and this requirement has to be satisfied in a pointwise manner, i.e. relative to each individual 'assignment' in the plural info state.

In contrast, the definition of random assignment in (16b), i.e. $\{u_3\}$, requires us to introduce the *same* set of horses with respect to each and every u_2-woman. This yields intuitively incorrect, overly strong truth-conditions since, for sentence (19) to be intuitively true, we do not have to require each and every woman to ride the same horse or the same set of horses as the other women.

Thus, the structural non-determinism built into the definition of random assignment in (16a) allows us to introduce a value and a structure for u_3 that can verify sentence (19) without imposing overly strong truth-conditions.

The second, theoretical reason in favor of $I[u]J$ and against $I\{u\}J$ is that $I[u]J$ preserves the formally desirable properties of the pointwise relation $i[u]j$, while $I\{u\}J$ doesn't. More exactly, $I[u]J$ is an equivalence relation[15], just as $i[u]j$, while the relation $I\{u\}J$ is neither reflexive nor symmetric (as the reader can easily check).

The third and final reason in favor of $I[u]J$ and against $I\{u\}J$ is that the relation $[u]$, but not the relation $\{u\}$, preserves the c-ideal structure that the atomic conditions have[16]

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[15] The reflexivity, symmetry and transitivity of the relation $I[u]J$ follow from the reflexivity, symmetry and transitivity of $i[u]j$ in a straightforward way.

[16] A relation $\mathbb{R}$ between plural info states (of type $\mathbf{t} := (st)((st)t)$) preserves c-ideals under images iff if $\mathfrak{I}$ is a c-ideal, then $\mathfrak{I}'=\{J_{st}: \exists I_{st}(\mathbb{R}IJ \wedge I\in\mathfrak{I})\}$ is a c-ideal. A relation $\mathbb{R}$ between plural info states preserves c-ideals under pre-images iff if $\mathfrak{I}'$ is a c-ideal, then $\mathfrak{I}=\{I_{st}: \exists J_{st}(\mathbb{R}IJ \wedge J\in\mathfrak{I}')\}$ is a c-ideal. The relation $[u]$ preserves c-ideals under both images and pre-images.

(again, the reader can easily verify this statement). I conclude that the relation $I[u]J$ is the empirically most adequate and theoretically most natural generalization of $i[u]j$ [17].

20. **Introducing new dref's in PCDRT:**

$$[u] := \lambda I_{st}.\lambda J_{st}. \ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$$

Introducing new dref's by means of $[u]$ has an immediate benefit. We now have a clear understanding of the denotation of a DRS D containing only atomic conditions or of arbitrary dynamic conjunctions of such DRS's. The relevant definitions are provided in (21) below.

21. **Atomic DRS's (DRS's containing only one atomic condition)** – type $(st)((st)t)$.

$$[R\{u_1, \ldots, u_n\}] := \lambda I_{st}.\lambda J_{st}. \ I{=}J \wedge R\{u_1, \ldots, u_n\}J$$

$$[u_1{=}u_2] := \lambda I_{st}.\lambda J_{st}. \ I{=}J \wedge (u_1{=}u_2)J$$

DRS-level connectives (dynamic conjunction):

$$D_1; D_2 := \lambda I_{st}.\lambda J_{st}. \ \exists H_{st}(D_1IH \wedge D_2HJ),$$

where D_1 and D_2 are DRSs (type $(st)((st)t)$)

Tests (generalizing atomic DRS's):

$$[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. \ I{=}J \wedge C_1J \wedge \ldots \wedge C_mJ \ [18],$$

where $C_1, \ldots, C_m$ are conditions (atomic or not) of type $(st)t$.

We know that the domain and the range of any atomic DRS are c-ideals. We also know that the domain and the range of an arbitrary dynamic conjunction of atomic DRSs

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[17] We can in fact define $\{u\}$ in terms of $[u]$ and the closure condition **enough_assignments** defined in (i) below. The name of the condition indicates the formal similarity between this PCDRT condition and Axiom **4** ("Enough 'assignments'") of Dynamic Ty2, repeated in (ii) below. The definition $\{u\}$ in terms of $[u]$ is provided in (iii).

(i) **enough_assignments**$\{u\} := \lambda I_{st}. \ \forall x_e \in uI\forall i_s \in I(\exists i'_s \in I(i[u]i' \wedge ui'{=}x))$

(ii) **Axiom4**: $\forall i_s\forall v_{s\tau}\forall f_\tau(\textbf{udref}(v) \rightarrow \exists j_s(i[v]j \wedge vj{=}f))$, for any type $\tau \in$ **STyp**.

(iii) $\{u\} := \lambda I_{st}J_{st}. \ I[u]J \wedge \textbf{enough\_assignments}\{u\}J$,
    i.e. $[u \mid \textbf{enough\_assignments}\{u\}]$ in DRT-style abbreviation.

[18] Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction as follows:

$[C_1, \ldots, C_m] := \lambda I_{st}J_{st}. \ ([C_1]; \ldots; [C_m])IJ$, where $[C] := \lambda I_{st}J_{st}. \ I{=}J \wedge CJ$.

are c-ideals because the intersection of a set of c-ideals is a c-ideal (assuming that the intersection is non-empty). This is summarized in (22) below.

22. **Dom**($[C]$) = **Ran**($[C]$) = C = $\wp^+(\cup C)$, for any condition C that is a c-ideal.

   Dom($[C_1, \ldots, C_m]$) = **Ran**($[C_1, \ldots, C_m]$) = $C_1 \cap \ldots \cap C_m$

$$= \wp^+((\cup C_1) \cap \ldots \cap (\cup C_m)),$$

   for any conditions $C_1, \ldots, C_m$ that are c-ideals.

These results are generalized to DRS's in which new dref's are introduced: they are defined in (23) below and the general form of their denotation is provided in (24).

23. **Multiple random assignment**.

$$[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$$

   DRS's with new dref's – type $(st)((st)t)$.

$$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])IJ,$$

   where $C_1, \ldots, C_m$ are conditions,

   i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. I[u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J$.

24. **DRS's in terms of C-Ideals over Relations.**

   Given a DRS D of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, we have that:

$$\mathbf{Ran}(D) = C_1 \cap \ldots \cap C_m = \wp^+((\cup C_1) \cap \ldots \cap (\cup C_m));$$

$$\mathbf{Dom}(D) = \wp^+(\{i_s: \exists j_s(i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m))\}).$$

   Note that, since $i[u_1, \ldots, u_n]j$ is reflexive, $\mathbf{Ran}(D) \subseteq \mathbf{Dom}(D)$.

   Let $\mathbb{R}^D := \{<i_s, j_s>: i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)\}$. Then:

$$D = \{<I_{st}, J_{st}>: \exists \mathbb{R}_{s(st)}(I=\mathbf{Dom}(\mathbb{R}) \wedge J=\mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \in \wp^+(\mathbb{R}^D))\},$$

   i.e. $D = \{<I_{st}, J_{st}>: \exists \mathbb{R}_{s(st)} \neq \emptyset(I=\mathbf{Dom}(\mathbb{R}) \wedge J=\mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^D)\}$.

   That is:

$$\mathbb{R}^D := \lambda i_s.\lambda j_s. i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)$$

$$D := \lambda I_{st}.\lambda J_{st}. \exists \mathbb{R}_{s(st)} \in \wp^+(\mathbb{R}^D)(I=\mathbf{Dom}(\mathbb{R}) \wedge J=\mathbf{Ran}(\mathbb{R})).$$

The properties of DRS denotations identified in (22) and (24) above will prove useful when we decide how to define negation in PCDRT. Two final observations before we address negation. First, just as in CDRT+GQ, the existential force of the random

assignment [*u*] (see (20) above) is an automatic consequence of the way it is defined when coupled with the PCDRT definition of *truth* for DRS's, provided in (25) below.

25. **Truth:** A DRS *D* (type $(st)((st)t)$) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$, i.e. iff $I \in \mathbf{Dom}(D)$, where $\mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}$.

Second, note that we can already translate discourse (7-8) below in PCDRT (assuming that all the indefinites are weak). Given the definition of truth for DRS's in (25) above, the translation in (10) below derives the intuitively correct truth-conditions, as shown in (29).

26. A $^{\mathbf{wk}:u_1}$ house-elf fell in love with a $^{\mathbf{wk}:u_2}$ witch.

27. He $_{u_1}$ bought her $_{u_2}$ an $^{\mathbf{wk}:u_3}$ alligator purse.

28. $[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$;
    $[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]$

29. $\lambda I_{st}. \exists J_{st}(([u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}];$
    $[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}])IJ)$, i.e.
    $\lambda I_{st}. I \neq \emptyset \land \exists x_e \exists y_e \exists z_e(house\_elf(x) \land witch(y) \land fall\_in\_love(x, y) \land$
    $alligator\_purse(z) \land buy(x, y, z))$

3.3. Negation

Let us turn now to the definition of negation in PCDRT. The fact that plural info states encode both *values* and *structure* makes the issue non-trivial. A first attempt would be to simply import the CDRT+GQ definition, which is basically the DRT / FCS / DPL one, as shown in (30) below [19].

30. **Negation – first attempt**:
    $\sim D := \lambda I_{st}. I \neq \emptyset \land \neg \exists K_{st}(DIK)$,   where *D* is a DRS (type $(st)((st)t)$),
    i.e. $\sim D := \lambda I_{st}. I \neq \emptyset \land I \notin \mathbf{Dom}(D)$,   where $\mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}$.

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[19] Factoring out various complications, i.e. the fact that van den Berg's Dynamic Plural Logic is intended to handle anaphora to dref's introduced within the scope of negation and the fact that it is a partial logic, the DPL-style definition in (30) is the one used in van den Berg's Dynamic Plural Logic – see van den Berg (1994): 10, (27), van den Berg (1996a): 136, (6), van den Berg (1996b): 18, Definition D, (e).

However, given the PCDRT definition of atomic conditions, the definition in (30) yields incorrect truth-conditions for the negation example in (31) below.

31. Every^{u_1} farmer who owns a$^{\textbf{str:}u_2}$ donkey doesn't feed it$_{u_2}$ properly. [20]

Consider example (31) more closely: intuitively, the indefinite $a^{\textbf{str:}u_2}$ *donkey* is strong (hence the notation $a^{\textbf{str:}u_2}$) and the interpretation of (31) is that no donkey-owning farmer feeds *any* of his donkeys properly. Thus, by the time we process the restrictor of the quantification in (31), we have a plural information state I of the form shown in (32) below in which, for a given donkey-owning farmer a, every 'assignment' $i \in I$ stores some donkey d_1, d_2 etc. that a owns.

32. Info state I	…	u_1 (**one farmer**)		u_2 (**all donkeys**)	…
i_1	…	$a\ (=ui_1)$	$\xrightarrow{\ a\ owns\ d_1\ }$	$d_1\ (=u'i_1)$	…
i_2	…	$a\ (=ui_2)$	$\xrightarrow{\ a\ owns\ d_2\ }$	$d_2\ (=u'i_2)$	…
i_3	…	$a\ (=ui_3)$	$\xrightarrow{\ a\ owns\ d_3\ }$	$d_3\ (=u'i_3)$	…
…	…	…		…	…

Now, we reach the nuclear scope condition in (33) below, interpreted according to the definition of negation in (30) above.

33. $(\sim[\textit{feed\_proper}\{u_1, u_2\}])I \ = \ I \neq \emptyset \wedge \exists i_s \in I(\neg \textit{feed\_proper}(u_1i, u_2i))$

The truth-conditions derived by (33) are too weak: they only require farmer a to feed *some* donkey he owns poorly and they allow for cases in which he feeds properly all his other donkeys – while intuitively we should require him to feed *all* his donkeys poorly. We see that the DPL-style definition of negation in conjunction with the PCDRT definition of atomic conditions, which is unselectively distributive, yields overly weak

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[20] See also the example in (i) below from van der Does (1993): 18, (27c).

(i) A$^{\textbf{wk/str:}u}$ boy who had an$^{\textbf{str:}u'}$ apple in his rucksack didn't give it$_{u'}$ to his sister.

truth-conditions. I will therefore give a stronger definition for negation, provided in (34) below.

34. **Negation in PCDRT.**

$$\sim D := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall H_{st}(H{\neq}\emptyset \wedge H{\subseteq}I \rightarrow \neg\exists K_{st}(DHK)),$$

where D is a DRS (type $(st)((st)t)$),

i.e. $\sim D := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall H_{st}{\neq}\emptyset(H{\subseteq}I \rightarrow H{\notin}\mathbf{Dom}(D)).$

The PCDRT definition of negation in (34) requires that:

I is not in $\mathbf{Dom}(D)$ – just as the DPL-style definition (30);

no singleton subset of I is in $\mathbf{Dom}(D)$ – which enables us to account for the donkey sentence in (31) above, since the nuclear scope condition ($\sim[feed\_proper\{u_1, u_2\}])I$ is 'unpacked' as $I{\neq}\emptyset \wedge \forall i_s{\in}I(\neg feed\_proper(u_1 i, u_2 i))$, which yields the intuitively correct, strong truth-conditions;

all the other non-empty subsets of I are not in $\mathbf{Dom}(D)$.

The third and final requirement ensures that the denotation of a negative condition preserves the c-ideal structure of the negated DRS. For example, if the negated DRS D is of the form given in (23) above, its domain $\mathbf{Dom}(D)$ is a c-ideal and, if $\mathbf{Dom}(D)$ is a c-ideal, $\sim D$ is the maximal c-ideal disjoint from $\mathbf{Dom}(D)$. This is stated in (35) below.

35. If $\mathbf{Dom}(D)$ is a c-ideal, $\sim D$ is the unique maximal c-ideal disjoint from $\mathbf{Dom}(D)$[21].

That is, $\sim D = \wp^+(D_s^M\backslash\cup\mathbf{Dom}(D))$ if $\mathbf{Dom}(D) = \wp^+(\cup\mathbf{Dom}(D))$.

---

[21] **$\sim D$ is a c-ideal if Dom(D) is a c-ideal.**

Proof: **(i)** $\sim D{\subseteq}\wp^+(D_s^M)$; **(ii)** for any I_{st} and J_{st}, if $I{\in}\sim D$ and $J{\subseteq}I$ and $J{\neq}\emptyset$, then $J{\in}\sim D$ (this follows directly from definition (34)); **(iii)** if $\Upsilon{\subseteq}\sim D$, then $\cup\Upsilon{\in}\sim D$. (Proof: suppose (iii) doesn't hold, i.e. $\Upsilon{\subseteq}\sim D$ and $\cup\Upsilon{\notin}\sim D$. Then, there is an H s.t. $H{\neq}\emptyset$ and $H{\subseteq}\cup\Upsilon$ and $H{\in}\mathbf{Dom}(D)$. Since $H{\subseteq}\cup\Upsilon$ and $H{\neq}\emptyset$, there must be at least one $I{\in}\Upsilon$ s.t. $H{\cap}I{\neq}\emptyset$. Let $I'=H{\cap}I$. Since $I'{\subseteq}H$ and $H{\in}\mathbf{Dom}(D)$ and $\mathbf{Dom}(D)$ is a c-ideal, we have that $I'{\in}\mathbf{Dom}(D)$. But $I'{\subseteq}I$ and $I{\in}\Upsilon{\subseteq}\sim D$, so, by definition (34), $I'{\notin}\mathbf{Dom}(D)$. Contradiction. □). □

$\sim D$ is maximal.

Proof: Suppose $\sim D$ is not maximal. Then, there is a c-ideal $\Im$ s.t. $\Im{\cap}\mathbf{Dom}(D)=\emptyset$ and $\sim D{\subset}\Im$. Then, there is some $I{\in}\Im$ s.t. $I{\notin}\sim D$; hence, there is an H s.t. $H{\neq}\emptyset$ and $H{\subseteq}I$ and $H{\in}\mathbf{Dom}(D)$. Since $\Im$ is a c-ideal, $I{\in}\Im$ and $H{\subseteq}I$, we have that $H{\in}\Im$. Hence, $\Im{\cap}\mathbf{Dom}(D){\neq}\emptyset$. Contradiction. □.

$\sim D$ is unique.

In sum, given the properties of the denotations of DRS's in PCDRT, the dynamic negation defined in (34) above is as well-behaved as possible[22].

We can now represent the discourse in (36-37) below. The representation, provided in (38), derives the intuitively correct truth-conditions, given in (39): there is a house-elf that fell in love with some witch and that bought her no alligator purse.

36. A $^{\mathbf{wk}:u_1}$ house-elf fell in love with a $^{\mathbf{wk}:u_2}$ witch.

37. (Surprisingly) He $_{u_1}$ didn't buy her $_{u_2}$ an $^{\mathbf{wk}:u_3}$ alligator purse.

38. $[u_1, u_2 \mid house\_elf\{u_1\}, witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]$;

 $[\sim[u_3 \mid alligator\_purse\{u_3\}, buy\{u_1, u_2, u_3\}]]$

39. $\lambda I_{st}.\ I \neq \emptyset \land \exists x_e \exists y_e(house\_elf(x) \land witch(y) \land fall\_in\_love(x, y) \land$

$$\neg \exists z_e(alligator\_purse(z) \land buy(x, y, z)))$$

3.4. Maximization

Now that the core part of PCDRT is in place, we can turn to the maximization operator, which is the essential ingredient in the analysis of the weak / strong donkey ambiguity. The definition of the **max** operator is provided in (40) below; **max** is an operator over DRS's: its argument is a DRS, i.e. a term of type $\mathbf{t} := (st)((st)t)$, and its value is another DRS, i.e. another term of type $\mathbf{t}$. Note that we actually define a family of maximization operators, each one specified for the dref u over which we maximize.

Proof: Suppose $\sim D$ is not unique. Then, there is a maximal c-ideal $\Im$ s.t. $\Im \cap \mathbf{Dom}(D) = \emptyset$ and $\sim D \neq \Im$. Since both $\sim D$ and $\Im$ are maximal, there is some $I \in \Im$ s.t. $I \notin \sim D$ and some $J \in \sim D$ s.t. $I \notin \Im$. The reasoning is now similar to the maximality proof: since $I \notin \sim D$, there must be an H s.t. $H \neq \emptyset$ and $H \subseteq I$ and $H \in \mathbf{Dom}(D)$). Since $\Im$ is a c-ideal, $I \in \Im$ and $H \subseteq I$, we have that $H \in \Im$. Hence, $\Im \cap \mathbf{Dom}(D) \neq \emptyset$. Contradiction. $\square$.

[22] For completeness, I provide the definitions of anaphoric closure, disjunction and implication in PCDRT.

(i) **Anaphoric closure:** $!D := \lambda I_{st}.\ \exists K_{st}(DIK)$,   i.e. $!D := \mathbf{Dom}(D)$

(ii) **Disjunction:** $D_1 \lor D_2 := \lambda I_{st}.\ \exists K_{st}(D_1IK \lor D_2IK)$,   i.e. $D_1 \lor D_2 := \mathbf{Dom}(D_1) \cup \mathbf{Dom}(D_2)$

(iii) **Implication:** $D_1 \to D_2 := \lambda I_{st}.\ \forall H_{st}(D_1IH \to \exists K_{st}(D_2HK))$,

 i.e. $D_1 \to D_2 := \lambda I_{st}.\ D_1I \subseteq \mathbf{Dom}(D_2)$,   where $DI := \{J_{st}: DIJ\}$,

 i.e. $D_1 \to D_2 := (\wp^+(D_s^M) \backslash \mathbf{Dom}(D_1)) \cup \{I \in \mathbf{Dom}(D_1): D_1I \subseteq \mathbf{Dom}(D_2)\}$.

40. $\mathbf{max}^u(D) := \lambda I_{st}J_{st}. \exists H_{st}(I[u]H \wedge DHJ) \wedge \forall K_{st}(\exists H'_{st}(I[u]H' \wedge DH'K) \rightarrow uK \subseteq uJ)$,

where D is a DRS, i.e. a term of type $\mathbf{t} := (st)((st)t)$,

i.e. $\mathbf{max}^u(D) := \lambda I_{st}J_{st}. ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ)$.

The first conjunct in (40) introduces u as a new dref (i.e. $I[u]H$) and makes sure (by DHJ) that each individual in uJ 'satisfies' D, i.e. we store *only* individuals that 'satisfy' D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e. any other set of individuals that 'satisfies' D) is included in uJ, i.e. we store *all* the individuals that satisfy D.

Note that, because of its maximality requirement, the **max** operator does not preserve the c-ideal structure of the range of the DRS over which it scopes. To see this, consider the second, shorter formulation of the definition in (40). This formulation explicitly shows that the relation between info states denoted by the maximized DRS $\mathbf{max}^u(D)$ is always a subset of the relation denoted by $[u]; D$, i.e. we 'strengthen' the DRS $[u]; D$ by ruling out the output info states J that assign to u strict subsets of maximal set that is assigned to u throughout $\mathbf{Ran}([u]; D)$ [23].

The DRS $\mathbf{max}^u(D)$ can be thought of as dynamic λ-abstraction over individuals: the 'abstracted variable' is the individual dref u, the 'scope' is the DRS D and the result of the 'abstraction' is a set of individuals uJ (where J is the output info state) containing *all* and *only* the individuals that 'satisfy' D. Thus, *maximization* together with *plural info states* and the *unselective distributivity* built into the definition of atomic conditions enables us to 'dynamize' λ-abstraction: (**i**) the maximization operator stores the λ-abstracted set in a dref, so that we can access it in discourse; (**ii**) unselective distributivity enables us to λ-abstract one value at a time; (**iii**) finally, plural info states enable us to store the dependency structure associated with each λ-abstracted value.

The empirical motivation for the *selectivity* of the $\mathbf{max}^u$ operator (as definition (40) shows, $\mathbf{max}^u$ *selectively* maximizes over the dref u) is provided by the mixed weak &

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[23] The update $\mathbf{max}^u(D)$ fails if such a supremum set does not exist, i.e. $\mathbf{max}^u(D)$ fails for an input info state I if the family of sets $\{uJ: ([u]; D)IJ\}$ does not have a supremum.

strong donkey sentences: we do not want to indiscriminately maximize over all donkey indefinites, but only over those that receive a strong reading. So, the selective $\mathbf{max}^u$ operator enables us to define the strong meaning for donkey indefinites in such a way that it is minimally different from the weak meaning. Both basic meanings are provided in (41) below[24].

41. **weak indefinites:** $a^{\mathsf{wk}:u} \rightsquigarrow \lambda P'_{\mathbf{et}}. \lambda P_{\mathbf{et}}. [u]; P'(u); P(u)$

    strong indefinites: $a^{\mathsf{str}:u} \rightsquigarrow \lambda P'_{\mathbf{et}}. \lambda P_{\mathbf{et}}. \mathbf{max}^u(P'(u); P(u))$,

        where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$.

Note that it is the compositional system that makes sure we have the correct 'configuration' within the scope of $\mathbf{max}^u$, i.e. that the DRS $P'(u)$ over which we maximize has the dref u in the appropriate (argument) places. This is very much like the technique employed in static semantics: it is the compositional system that ensures that the λ-abstraction over the variable x takes scope over a formula that has x in the appropriate 'slots'.

The definition of $\mathbf{max}^u$ and the way it is used in the analysis of strong donkey anaphora will become clearer if we look at an example. Consider (42) below and assume that it is uttered in a context in which there is some unique salient boy with apples in his rucksack. For example, twenty children (ten brother-sister pairs) travel by bus and the bus passes an apple orchard; as the story goes, the girls are overwhelmed with desire for the fruit, but none of them gets it because no one on the bus has any apples – except for one boy, but he doesn't care about anyone's plea, not even his sister's. In this context, we can felicitously utter that the other boys would have given an apple to their sisters if they had one, but:

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[24] Note the similarity between the PCDRT representation of weak indefinites and the representation of indefinites in CDRT+GQ.

42. The$_{u_1}$ (one) boy who had an$^{\textbf{str:}u_2}$ apple in his rucksack didn't give it$_{u_2}$ to his sister [25].

In this context, (42) is interpreted as: the boy who had (some) apples in his rucksack didn't give any to his sister. I will assume that the definite article the$_{u_1}$ functions as an anaphor, i.e. it simply tests that some contextually salient dref u_1 satisfies both its restrictor and its nuclear scope, as shown in (43) below. For simplicity, the restrictor $P'(u)$ in (43) is not represented as a presupposition, but as part of the assertion[26].

43. **Definite articles as anaphors:**

$the_u \rightsquigarrow \lambda P'_{\textbf{et}}. \lambda P_{\textbf{et}}. [\textbf{unique}\{u\}]; P'(u); P(u),$

where $\textbf{e} := se$ and $\textbf{t} := (st)((st)t)$.

44. $\textbf{unique}\{u\} := \lambda I_{st}. I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(ui = ui'),$

i.e. $\textbf{unique}\{u\} := \lambda I_{st}. |uI| = 1,$

where $|uI|$ is the cardinality of the set uI.

I take the definite article to contribute an atomic condition $\textbf{unique}\{u\}$, defined in (44), which encodes a *weak* form of uniqueness: it requires that the dref anaphorically retrieved by the definite article has a unique value with respect to the current plural info state, i.e. it requires the set uI stored by the dref to be a singleton. This kind of uniqueness is *weak* because it is relativized to the current info state (i.e. it is salience-dependent uniqueness); I take *strong* uniqueness to be uniqueness relative to the entire model. As we will see in (51) below, strong uniqueness can be obtained by combining weak uniqueness, i.e. the condition $\textbf{unique}\{u\}$, and the $\textbf{max}^u$ operator.

Sentence (42) is represented as shown in (45) below.

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[25] I ignore throughout most of this chapter the uniqueness implications sometimes associated with donkey anaphora, e.g., in example (42), the intuition that the apple is unique. For more discussion about the uniqueness effects associated with singular anaphora in quantificational subordination and donkey anaphora, see sections **6.1** and **6.2** of chapter 6 below.

I am indebted to Roger Schwarzschild (p.c.) for suggesting the sentence in (i) below as an alternative example that does not exhibit uniqueness effects.

(i) A / The boy who had a^u pen in his backpack didn't give it$_u$ to his sister.

[26] See Muskens (1995b): 165 for a similar lexical entry in a CDRT kind of system.

45. [**unique**$\{u_1\}$, *boy*$\{u_1\}$]; **max**^{u_2} ([*apple*$\{u_2\}$, *have_in_rucksack*$\{u_1, u_2\}$]);

   [~[*give_to_sister*$\{u_1, u_2\}$]]]

By the end of the **max**^{u_2} update, we are in a plural information state *I* like the one in (46) below. The dref u_1 stores the same boy *b* throughout the info state *I* (due to **unique**$\{u\}$) and the dref u_2 stores all the apples a_1, a_2, a_3 etc. that boy *b* has in his rucksack (due to **max**^{u_2}).

46. Info state *I*	…	u_1 (**the boy**)		u_2 (**all apples**)	…
i_1	…	b $(=u_1i_1)$	$\xrightarrow{\text{b has } a_1}$	a_1 $(=u_2i_1)$	…
i_2	…	b $(=u_1i_2)$	$\xrightarrow{\text{b has } a_2}$	a_2 $(=u_2i_2)$	…
i_3	…	b $(=u_1i_3)$	$\xrightarrow{\text{b has } a_3}$	a_3 $(=u_2i_3)$	…
…	…	…		…	…

Given the PCDRT definition of negation, the translation in (45) derives the intuitively correct truth-conditions: the formula in (47) below is true iff there is exactly one contextually salient boy that has some apples and gives none of them to his sister.

47. $\lambda I.\ \exists J([$**unique**$\{u_1\}$, *boy*$\{u_1\}$]; **max**^{u_2} ([*apple*$\{u_2\}$, *have_in_rucksack*$\{u_1, u_2\}$]);

   [~[*give_to_sister*$\{u_1, u_2\}$]]])$IJ)$ =

   $\lambda I.\ \exists x_e(u_1I=\{x\} \wedge boy(x) \wedge \exists Y_{et}\neq\emptyset(\forall y_e(apple(y) \wedge h.i.r(x, y) \leftrightarrow y\in Y) \wedge$

   $\forall y_e\in Y(\neg g.t.s(x, y))))$

This example makes clear that the **max**^{u} operator defined in (40) is *selective* in exactly the sense in which the dynamic quantification in CDRT+GQ is selective: the set of output states that are in the range of a **max**^{u} DRS is determined based on the set of *individuals* that such an output state stores with respect to the dref *u*. However, in view of the fact that donkey conditionals seem to exhibit *unselectively* strong readings, e.g. the conditional in (48) below, I will define an unselective form of maximization – as shown in (49).

48. If a $^{\text{str:}u_1}$ house-elf borrows a $^{\text{str:}u_2}$ broom from a $^{\text{str:}u_3}$ witch, he$_{u_1}$ (always) gives it$_{u_2}$ back to her$_{u_3}$ the next day.

49. **unselective maximization:**

$$\mathbf{max}(D) := \lambda I_{st} J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \to K{\subseteq}J)\ ^{27}$$

The unselective **max** operator in (49) retrieves the supremum in an inclusion partial order over sets of info states and not over sets of individuals (i.e. it is unselective in the sense of Lewis 1975). This operator will be used to defined unselective generalized quantification in PCDRT.

I conclude the section with two observations about selective maximization, one empirical and the other theoretical. First, note that selective maximization seems to be independently motivated by the Russellian uses of definite descriptions in natural language, i.e. the definite descriptions that intuitively require strong uniqueness (uniqueness relative to the entire model). The definite DP in (50) below exemplifies the Russellian kind of definite descriptions, i.e. definite descriptions that are non-anaphoric and that require existence and strong uniqueness.

50. Hagrid fell in love with the^u tallest witch in the world.

In PCDRT, we can analyze Russellian definite descriptions by suitably combining weak uniqueness, i.e. the condition **unique**$\{u\}$, and the **max**^{u} operator. In fact, PCDRT can analyze definite articles in any of the four ways listed in (51) below; deciding which one (if any) is the right meaning falls outside the scope of the current investigation.

51. **The definite article – possible meanings in PCDRT.**

    a. anaphoric and weakly unique:

$$the_u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ P'(u);\ P(u),$$

        where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$

        and $\mathbf{unique}\{u\} := \lambda I_{st}.\ \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui')$.

    b. anaphoric, no uniqueness:

---

[27] Note that, for any I_{st}, the set $\{J_{st}:\ \mathbf{max}(D)IJ\}$ is either empty or a singleton set.

$$the_u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ P'(u);\ P(u)$$

c. existence and strong uniqueness, non-anaphoric (the Russellian analysis):

$$the^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u));\ [\mathbf{unique}\{u\}];\ P(u)$$

d. existence and maximality (no uniqueness), non-anaphoric:

$$the^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ \mathbf{max}^u(P'(u));\ P(u)\ ^{28}$$

I conclude this section with the examination of DRS's in which one $\mathbf{max}^u$ operator is embedded within the scope of another, as schematically shown in (52) below.

52. $\mathbf{max}^u(D;\ \mathbf{max}^{u'}(D'))$

Such structures occur fairly frequently in the PCDRT translations of natural language discourses and they are difficult to grasp at an intuitive level. To simplify derivations and make translations more transparent, I show that the values assigned to multiply embedded $\mathbf{max}^u$ operators are often reducible to non-embedded ones.

The main result is stated in the corollary in (53) below – see section **0** of the Appendix to this chapter for its proof.

53. **Simplifying 'max-under-max' representations (corollary):**

$$\mathbf{max}^u(D;\ \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D;\ [u'];\ D');\ \mathbf{max}^{u'}(D'),$$

if the following three conditions obtain:

a. u is not reintroduced in D';

b. $\mathbf{Dom}([u'];\ D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

If $C_1, \ldots, C_m$ are c-ideals, condition (53b) follows from (53c) [29].

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[28] Note that this meaning is different from the strong meaning of the indefinite article with respect to the scope of the $\mathbf{max}^u$ operator: in the case of the definite, this operator has scope only over the restrictor DRS, i.e. $\mathbf{max}^u(P'(u))$, while in the case of the indefinite, it has scope over both the restrictor and nuclear scope DRS's, i.e. $\mathbf{max}^u(P'(u);\ P(u))$.

[29] **If $C_1, \ldots, C_m$ are c-ideals, condition (53b) follows from (53c).**

Proof: In general, we have that $\mathbf{Dom}(\mathbf{max}^{u'}(D')) \subseteq \mathbf{Dom}([u'];\ D')$, so we only have to prove that $\mathbf{Dom}([u'];\ D') \subseteq \mathbf{Dom}(\mathbf{max}^{u'}(D'))$. But an info state $I \in \mathbf{Dom}([u'];\ D')$ fails to be in $\mathbf{Dom}(\mathbf{max}^{u'}(D'))$ iff the family of sets $\{u'J: ([u'];\ D')IJ\}$ does not have a supremum. And the existence of the supremum follows by an application

Let us reanalyze the example in (42) above, repeated in (54), in terms of the Russellian analysis of definite descriptions, i.e. letting the definite article *the*^{u_1} contribute existence and uniqueness as in (51c) above: *the*^{u} ⤳ $\lambda P'_{et}.\ \lambda P_{et}.$ **max**$^u(P'(u))$; [**unique**{u}]; $P(u)$. The example is translated as shown in (55).

54. The^{u_1} (one) boy who had an$^{\textbf{str:}u_2}$ apple in his rucksack didn't give it$_{u_2}$ to his sister.

55. **max**^{u_1} ([*boy*{u_1}]; **max**^{u_2} ([*apple*{u_2}, *have_in_rucksack*{u_1, u_2}]));

   [**unique**{u_1}]; [~[*give_to_sister*{u_1, u_2}]]

The representation in (55) gives us the opportunity to apply the corollary in (53) above. Conditions (53a) and (53c) are clearly satisfied; checking that condition (53b) holds is also straightforward: given that both conditions *apple*{u_2} and *have_in_rucksack*{u_1, u_2} are c-ideals, (53b) follows from (53c).

Thus, the translation in (55) is equivalent to the one in (56) below. The truth-conditions, provided in (57), are the intuitively correct ones (assuming that the definite article should indeed receive the Russellian analysis): sentence (54) is true iff there is a unique boy with some apples in his rucksack such that he didn't give any of his apples to his sister.

56. **max**^{u_1} ([*boy*{u_1}]; [u_2 | *apple*{u_2}, *have_in_rucksack*{u_1, u_2}]);

   max^{u_2} ([*apple*{u_2}, *have_in_rucksack*{u_1, u_2}]);

   [**unique**{u_1}]; [~[*give_to_sister*{u_1, u_2}]]

57. λI_{st}. $I \neq \emptyset \land \exists x_e(\forall z_e(boy(z) \land \exists y_e(apple(y) \land have\_in\_rucksack(z, y)) \leftrightarrow z=x) \land$

   $\exists Y_{et} \neq \emptyset(\forall y_e(apple(y) \land have\_in\_rucksack(x, y) \leftrightarrow y \in Y) \land$

   $\forall y_e \in Y(\neg give\_to\_sister(x, y))))$

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of the result stated in (24) above: just take the image of the info state I under the relation $\mathbb{R}^{[u'];\ D'} = \{<i, j>$: $i[u', u_1, \ldots, u_n]j \land j \in (\cup C_1) \cap \ldots \cap (\cup C_m)\}$, i.e. $\mathbb{J} = \{j$: $\exists i \in I(\mathbb{R}^{[u'];\ D'}ij)\}$ and note that $u'\mathbb{J}$ is the supremum of $\{u'J$: ([u']; $D')IJ\}$. □

To see more clearly that the truth-conditions enforced by PCDRT formulas are of the form "there are *some values* and there is *some structure* associated with those values such that…", I will rewrite the formula in (57) as shown in (58) below, i.e. by using of a relation $R_{e(et)}$ between individuals which encodes the *structure* associated with the values in question (i.e. with the unique boy and his apples). $\mathbf{Dom}(R)$ and $\mathbf{Ran}(R)$ are defined as usual, i.e. $\mathbf{Dom}(R) := \{x_e: \exists y_e(Rxy)\}$ and $\mathbf{Ran}(R) := \{y_e: \exists x_e(Rxy)\}$

58. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists R_{e(et)}(\mathbf{Dom}(R){\neq}\emptyset \wedge \mathbf{Ran}(R){\neq}\emptyset \wedge |\mathbf{Dom}(R)|{=}1 \wedge$

$\mathbf{Dom}(R) = \{x_e: boy(x) \wedge \exists y_e(apple(y) \wedge have\_in\_rucksack(x, y))\} \wedge$

$\mathbf{Ran}(R) = \{y_e: apple(y) \wedge \exists x_e{\in}\mathbf{Dom}(R)(have\_in\_rucksack(x, y))\} \wedge$

$\forall x_e \forall y_e(Rxy \rightarrow have\_in\_rucksack(x, y)) \wedge$

$\forall x_e \forall y_e(Rxy \rightarrow \neg give\_to\_sister(x, y)))$

3.5. Generalized Quantification

The only thing left to define in PCDRT is generalized quantification. We start with selective generalized quantification.

Selective generalized determiners are relations between two dynamic properties P'_{et} (the restrictor) and P_{et} (the nuclear scope), i.e. their denotations are of the expected type $(\mathbf{et})((\mathbf{et})\mathbf{t})$. The PCDRT definition of selective generalized determiners has to be formulated in such a way that:

- on the one hand, we capture the fact that anaphors in the nuclear scope can have antecedents in the restrictor;
- on the other hand, we avoid the proportion problem and, at the same time, allow for the weak / strong donkey ambiguity.

To avoid the proportion problem, a selective generalized determiner has to relate sets of individuals and not sets of 'assignments'. Thus, the main problem in a dynamic system is to find an appropriate way to extract the two sets of individuals, i.e. the restrictor set and the nuclear scope set, based on the restrictor and the nuclear scope dynamic properties.

The proposed ways to solve this problem fall into two broad categories. The first category of solutions is the one exemplified by CDRT+GQ (following DRT / FCS / DPL): we employ a dynamic framework based on singular info states and we analyze generalized quantification as internally dynamic and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property, while the nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope dynamic property, so that the anaphoric connections between them are captured.

The second category of solutions employs a dynamic framework based on plural information states and it analyzes generalized quantification as both internally and externally dynamic. The main reference for this kind of solution is van den Berg (1994, 1996a) (but see also Krifka (1996b) and Nouwen (2003) among others). The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property and, then, the nuclear scope set of individuals is the maximal subset of the restrictor set of individuals that satisfies the nuclear scope dynamic property. The restrictor and the nuclear scope sets are stored in the output plural info state and are available for anaphoric retrieval, e.g. *Every^u man saw a$^{u'}$ woman / two$^{u'}$ women. They$_u$ greeted them$_{u'}$.*

Given that the notion of a dref being a *subset* of another required for van den Berg's definition of quantification involves non-trivial complexities[30] that are largely orthogonal to the donkey issues we are interested in, I will analyze selective generalized quantification following the format of the CDRT+GQ definition[31].

However, since PCDRT is a system based on *plural* info states, the definition of selective generalized determiners I will provide is novel. This definition is intermediate between the above two strategies of defining selective dynamic quantification and, as

[30] E.g., it requires the introduction of a dummy / 'undefined' / exception individual # – see chapter **6**. For the corresponding notion of dummy / 'undefined' / exception possible world, see the analysis of structured discourse reference to propositions in chapter **7**.

[31] But see chapter **6** for a van den Berg-style definition of generalized quantification in PCDRT which is used in the analysis of quantificational subordination.

such, it is useful in exhibiting the commonalities and differences between them in a formally explicit way.

The generalized quantifiers we will be considering throughout the present investigation are domain-level and discourse-level distributive in the sense that they relate two sets of *atomic* individuals (i.e. domain-level distributivity) and these sets of atomic individuals are required to satisfy the restrictor and nuclear scope dynamic properties *one individual at a time* (i.e. discourse-level distributivity). We enforce the first kind of distributivity (i.e. domain-level) by restricting our domain of individuals D_e to atomic individuals (there are no non-atomic individuals in the sense of Link 1983). We enforce the second kind of distributivity by making use of the dynamic condition **unique**$\{u\}$, which was introduced in the previous chapter for the analysis of definite descriptions. The definition of selective quantification is provided in (59) below.

59. **Selective Generalized Determiners** ($\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$).

$\qquad$ **det**$^u \rightsquigarrow \lambda P'_{\mathbf{et}}.\ \lambda P_{\mathbf{et}}.\ [\mathbf{det}_u(P'(u),\ P(u))]$,

$\qquad\qquad$ where $\mathbf{det}_u(D,\ D') := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[DI],\ u[(D;\ D')I])$

$\qquad\qquad$ and $u[DI] := \cup\{uJ:\ ([u \mid \mathbf{unique}\{u\}];\ D)IJ\}$

$\qquad\qquad$ and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui')$

$\qquad\qquad$ and **DET** is the corresponding static determiner.

Intuitively, the definition $u[DI] := \cup\{uJ:\ ([u \mid \mathbf{unique}\{u\}];\ D)IJ\}$ above instructs us to do the following 'operations' to an input matrix *I*: add a column *u* to matrix *I*, fill it up with only one individual *x* and then check that the resulting matrix satisfies *D* (the resulting matrix satisfies *D* iff it can be updated with *D*, i.e. iff it has at least one output state *J* relative to *D*). If this matrix satisfies *D*, then *x* is in the set $u[DI]$, otherwise not.

The definition of generalized quantification in (59) is *selective* because the static determiner **DET** relates sets of individuals. The individuals in these sets are obtained in basically the same way as they are obtained in the case of the CDRT+GQ weak generalized determiners. The restrictor set contains all the individuals which 'satisfy' the restrictor DRS *D* when plugged in one atomic individual at a time, i.e. $[u \mid \mathbf{unique}\{u\}]$. The nuclear scope contains all the individuals which satisfy both the restrictor DRS *D*

and the nuclear scope DRS D' when plugged in one atomic individual at a time. Clearly, this definition of selective quantification enables us to avoid the proportion problem just as the corresponding CDRT+GQ definition does.

The definition of unselective generalized quantification is provided in (60) below.

60. **Unselective generalized determiners (in terms of unselective maximization):**

$$\textbf{det} \leadsto \lambda D'_\textbf{t}. \lambda D_\textbf{t}. [\textbf{det}(D', D)],$$

where $\textbf{det}(D, D') := \lambda I_{st}. I{\neq}\emptyset \wedge \textbf{DET}(\textbf{max}[DI], \ \textbf{max}[(D; [!D'])I])$

and $\textbf{max}[DI] := \cup\{J_{st}: \textbf{max}(D)IJ\}$ and $!D' := \textbf{Dom}(D')$

and $\textbf{DET}$ is the corresponding static determiner.

This definition is unselective because the static determiner $\textbf{DET}$ relates sets of 'assignments', i.e. sets of cases in the terminology of Lewis (1975). The info states in these sets are obtained much as they are obtained in the case of CDRT+GQ unselective determiners: the restrictor is the set of all the 'assignments' / cases that 'satisfy' the restrictor DRS D relative to input info state I and the nuclear scope is the set of all the 'assignments' / cases that 'satisfy' both the restrictor DRS D and the nuclear scope DRS D'.

The unselective $\textbf{max}$ operator functions as a dynamic λ-abstraction over 'assignments', i.e. over the cases of Lewis (1975) – much like the condition $\textbf{unique}\{u\}$ together with the union of sets of individuals in the definition of selective generalized quantification in (59) above functions as dynamic λ-abstraction over individuals[32].

4. Solutions to Donkey Problems

In this section, we see in detail how the PCDRT system introduced in the preceding section can be used to compositionally interpret a variety of donkey sentences, including mixed weak & strong relative-clause donkey sentences.

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[32] Recall that, since $\textbf{max}(D) := \lambda I_{st}J_{st}. DIJ$ & $\forall K_{st}(DIK \rightarrow K{\subseteq}J)$, the set $\{J_{st}: \textbf{max}(D)IJ\}$ is either empty or a singleton set, so the union over unselectively maximized info states is basically vacuous and needed here only for technical reasons: we want to access the maximal plural info state and not the singleton set whose only member is the maximal plural info state.

As we have already observed in chapter **3**, the compositional aspect of the interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences, which we abbreviate as **e** and **t**. An extensional static logic without pluralities (i.e. the static component of our Dynamic Ty2) identifies **e** and e (atomic entities) and also **t** and t (truth-values). CDRT+GQ complicates this setup by interpreting a sentence as a relation between an input and an output 'assignment', hence $\mathbf{t} := (s(st))$, and a name as an individual dref, i.e. as a function from 'assignments' to individuals, hence $\mathbf{e} := (se)$.

In PCDRT, names are interpreted just as in CDRT+GQ, but sentences are interpreted as relations between plural info states, i.e. as relations between an input *set* of 'assignments' and an output *set* of 'assignments', hence $\mathbf{t} := (st)((st)t)$. Everything else in our definition of type-driven translation remains the same. In particular, the only translation rule we need to change is **TR0**, i.e. the translation rule for the basic meanings – and even here, the modifications are minimal, as the table in (45) below shows.

61. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[sleep]_{V_{in}}$	$\rightsquigarrow \lambda v_\mathbf{e}.\ [sleep_{et}\{v\}]$	**et**
$[own]_{V_{tr}}$	$\rightsquigarrow \lambda Q_\mathbf{(et)t}.\ \lambda v_\mathbf{e}.\ Q(\lambda v'_\mathbf{e}.\ [own_{e(et)}\{v, v'\}])$	**((et)t)(et)**
$[buy]_{V_{di}}$	$\rightsquigarrow \lambda Q'_\mathbf{(et)t}.\ \lambda Q_\mathbf{(et)t}.\ \lambda v_\mathbf{e}.\ Q'(\lambda v'_\mathbf{e}.\ Q(\lambda v''_\mathbf{e}.\ [buy_{e(e(et))}\{v, v', v''\}]))$	**(ett)((ett)(et))**
$[house\text{-}elf]_N$	$\rightsquigarrow \lambda v_\mathbf{e}.\ [house\_elf_{et}\{v\}]$	**et**
$[he_u]_{DP}$	$\rightsquigarrow \lambda P_\mathbf{et}.\ P(u_\mathbf{e})$	**(et)t**
$[the_u]_D$	$\rightsquigarrow \lambda P'_\mathbf{et}.\ \lambda P_\mathbf{et}.\ [\mathbf{unique}\{u\}];\ P'(u);\ P(u),$ 　　　where $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui'),$ 　　　i.e. anaphoric and 'weakly' unique. $\rightsquigarrow \lambda P'_\mathbf{et}.\ \lambda P_\mathbf{et}.\ P'(u);\ P(u),$ 　　　i.e. anaphoric.	**(et)((et)t)**
$[t_v]_{DP}$	$\rightsquigarrow \lambda P_\mathbf{et}.\ P(v_\mathbf{e})$	**(et)t**
$[he_{Dobby}]_{DP}$	$\rightsquigarrow \lambda P_\mathbf{et}.\ P(Dobby_\mathbf{e})$	**(et)t**

61. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[Dobby^u]_{DP}$	$\rightsquigarrow \lambda P_{et}.\ [u \mid u{=}Dobby];\ P(u)$	**(et)t**
$[who]_{DP}$	$\rightsquigarrow \lambda P_{et}.\ P$	**(et)(et)**
$[\emptyset]_I\ /\ [\text{-}ed]_I\ /\ [\text{-}s]_I$	$\rightsquigarrow \lambda D_t.\ D$	**tt**
$[doesn't]_I\ /\ [didn't]_I$	$\rightsquigarrow \lambda D_t.\ [{\sim}D]$	**tt**

$[a^{wk:u}]_D$ $\rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ [u];\ P'(u);\ P(u),$
  i.e. $\lambda P'_{et}.\ \lambda P_{et}.\ \exists u(P'(u);\ P(u)),$
  where $\exists u(D) := [u];\ D$ — **(et)((et)t)**

$[a^{str:u}]_D$ $\rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ \mathbf{max}^u(P'(u);\ P(u)),$ where — **(et)((et)t)**
  $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ),$
  i.e. $\lambda P'_{et}.\ \lambda P_{et}.\ \exists^{\mathbf{m}}u(P'(u);\ P(u)),$
  where $\exists^{\mathbf{m}}u(D) := \mathbf{max}^u(D)$

$[the^u]_D$ $\rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ \mathbf{max}^u(P'(u));\ [\mathbf{unique}\{u\}];\ P(u),$ — **(et)((et)t)**
  where $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$ and
  $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ),$
  i.e. $\lambda P'_{et}.\ \lambda P_{et}.\ \exists^{\mathbf{m}}u(P'(u));\ [\mathbf{unique}\{u\}];\ P(u),$
  i.e. existence and uniqueness – the Russellian analysis

  $\rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ \mathbf{max}^u(P'(u));\ P(u),$ where
  $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ),$
  i.e. $\lambda P'_{et}.\ \lambda P_{et}.\ \exists^{\mathbf{m}}u(P'(u));\ P(u),$
  i.e. existence and maximality

$[det^u]_D$ $\rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ [\mathbf{det}_u(P'(u),\ P(u))],$ where: — **(et)((et)t)**

e.g. *every^u, no^u,* $\quad \mathbf{det}_u(D_1, D_2) := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[D_1I],\ u[(D_1;\ D_2)I]),$
  most^u...
(but not $a^{wk:u}$, $a^{str:u}$, $\quad$ where $u[DI] := \cup\{uJ: ([u \mid \mathbf{unique}\{u\}];\ D)IJ\}$
the_u or *the^u*) $\quad$ and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$
  $\quad$ and **DET** is the corresponding static determiner

$[if\ (+adv.\ of\ quant.)]_C$ $\rightsquigarrow \lambda D'_t.\ \lambda D_t.\ [\mathbf{det}(D',\ D)],$ where: — **t(tt)**

  $\mathbf{det}(D_1, D_2) := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(\mathbf{max}[D_1I],\ \mathbf{max}[(D_1;\ [!D_2])I]),$

  $\quad$ where $\mathbf{max}[DI] := \cup\{J_{st}:\ \mathbf{max}(D)IJ\}$
  $\quad$ and $\mathbf{max}(D) := \lambda I_{st}J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K{\subseteq}J)$
  $\quad$ and $!D := \mathbf{Dom}(D)$
  $\quad$ and **DET** is the corresponding static determiner

$[if]_C$ (i.e. bare *if*) $\rightsquigarrow \lambda D'_t.\ \lambda D_t.\ [\mathbf{every}(D',\ D)]$ — **t(tt)**

61. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $e := se$ $t := (st)((st)t)$
[*and*] $_{\text{Conj}}$	$\leadsto \lambda v_1. \ldots \lambda v_n. v_1 \sqcap \ldots \sqcap v_n$	$\tau(\ldots(\tau\tau)\ldots)$
[*or*] $_{\text{Conj}}$	$\leadsto \lambda v_1. \ldots \lambda v_n. v_1 \sqcup \ldots \sqcup v_n$	$\tau(\ldots(\tau\tau)\ldots)$

The definition of dynamically conjoinable types (**DCTyp**) is the same as in CDRT+GQ modulo the fact that we reset **t** to $(st)((st)t)$, as shown in (62) below.

62. **PCDRT Dynamically Conjoinable Types (DCTyp).**

The set of PCDRT dynamically conjoinable types **DCTyp** is the smallest subset of **Typ** s.t. $\mathbf{t} \in \mathbf{DCTyp}$ ($\mathbf{t} := (st)((st)t)$) and, if $\tau \in \mathbf{DCTyp}$, then $(\sigma\tau) \in \mathbf{DCTyp}$ for any $\sigma \in \mathbf{Typ}$.

We define generalized (pointwise) dynamic conjunction and disjunction as shown in (43) below (the same as the CDRT+GQ definition) – and thereby complete the definition of *and* and *or* in table (45) above.

63. **Generalized Pointwise Dynamic Conjunction $\sqcap$ and Disjunction $\sqcup$.**

For any two terms α and β of type τ, for any $\tau \in \mathbf{DCTyp}$:

$\alpha \sqcap \beta := (\alpha; \beta)$ if $\tau = \mathbf{t}$ and $\alpha \sqcap \beta := \lambda v_\sigma. \alpha(v) \sqcap \beta(v)$ if $\tau = (\sigma\rho)$;

$\alpha \sqcup \beta := [\alpha \vee \beta]$ if $\tau = \mathbf{t}$ and $\alpha \sqcup \beta := \lambda v_\sigma. \alpha(v) \sqcup \beta(v)$ if $\tau = (\sigma\rho)$.

Abbreviation. $\alpha_1 \sqcap \alpha_2 \sqcap \ldots \sqcap \alpha_n := (\ldots(\alpha_1 \sqcap \alpha_2) \sqcap \ldots \sqcap \alpha_n)$

$\alpha_1 \sqcup \alpha_2 \sqcup \ldots \sqcup \alpha_n := (\ldots(\alpha_1 \sqcup \alpha_2) \sqcup \ldots \sqcup \alpha_n)$.

We are now ready to analyze the donkey examples we have introduced in the preceding sections and chapters.

4.1. Bound Variable Anaphora

First, we show that PCDRT preserves the compositional CDRT+GQ account of the basic kinds of examples. Let's start with the bound anaphora example in (51) below.

64. Every^{u_1} house-elf hates himself$_{u_1}$.

Just as CDRT+GQ, PCDRT can compositionally account for bound anaphora without Quantifier Raising and Quantifying-In: co-indexation is enough for binding because the meaning of the determiner *every* dynamically conjoins the restrictor and the nuclear scope DRS's to determine the set of individuals in its nuclear scope – thus, *every* quantifies over 'assignments' in a selective way.

Sentence (55) is compositionally translated as shown in (65) below. The PCDRT representation derives the intuitively correct truth-conditions, provided in (66).

65. Every^{u_1} house-elf hates himself$_{u_1}$.
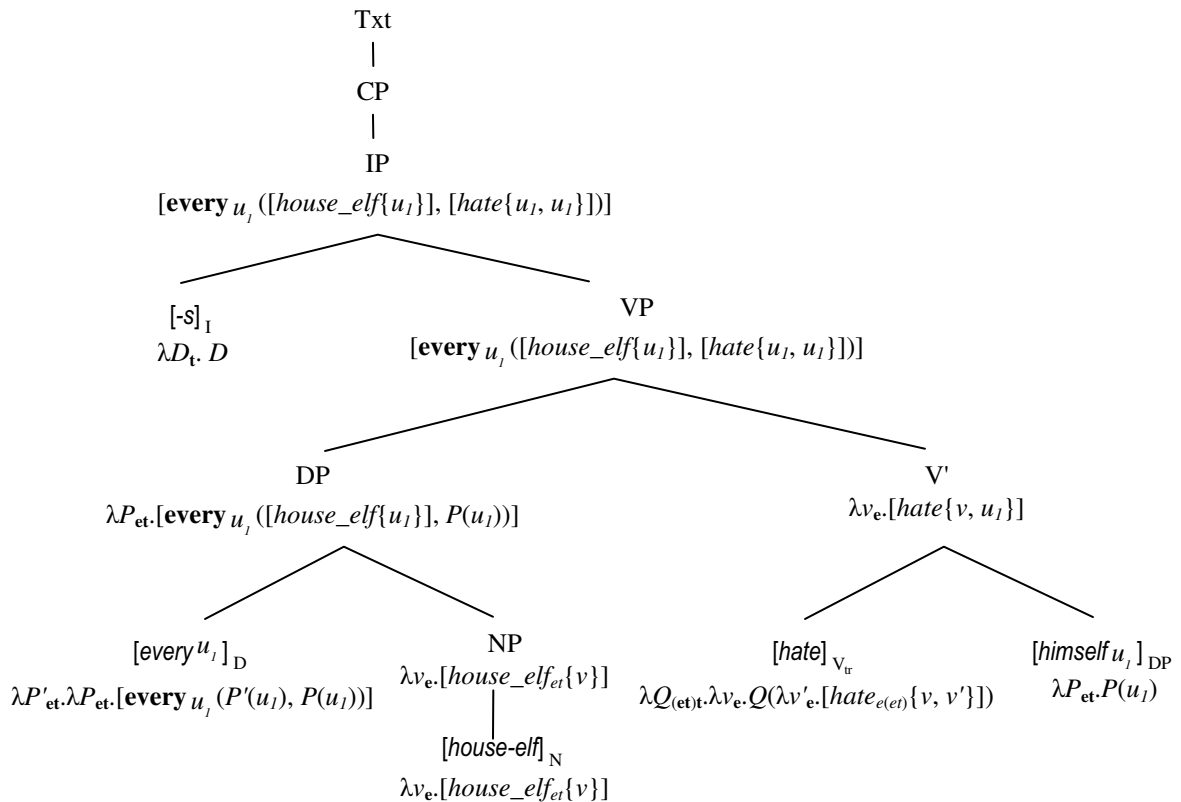
66. $\lambda I_{st}.\ I \neq \emptyset \land \forall x_e(house\_elf(x) \rightarrow hate(x, x))$

4.2. Quantifier Scope Ambiguities

Let us turn now to the example in (67) below exhibiting quantifier scope ambiguities over and above the lexical ambiguity of the indefinite. We start with the

weak reading of the indefinite – and we assign intuitively correct truth-conditions to both LFs, as shown in (69) and (71) below.

67. Every ^{u_1} house-elf adores a $^{\textbf{wk:}u_2}$ witch.

68. **every**^{u_1}**>>a**$^{\textbf{wk:}u_2}$: [**every**$_{u_1}$([*house_elf*{u_1}], [u_2 | *witch*{u_2}, *adore*{u_1, u_2}])]

69. **every**^{u_1}**>>a**$^{\textbf{wk:}u_2}$: λI_{st}. $I{\neq}\emptyset \wedge$

$$\forall x_e(house\_elf(x) \rightarrow \exists Y_{et}{\neq}\emptyset(\forall y_e{\in} Y(witch(y) \wedge adore(x, y)))) \text{ [33]}$$

70. **a**$^{\textbf{wk:}u_2}$**>>every**^{u_1}: [u_2 | *witch*{u_2}, **every**$_{u_1}$([*house_elf*{u_1}], [*adore*{u_1, u_2}])]

71. **a**$^{\textbf{wk:}u_2}$**>>every**^{u_1}: λI_{st}. $I{\neq}\emptyset \wedge$

$$\exists Y_{et}{\neq}\emptyset(\forall y_e{\in} Y(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y)))) \text{ [34]}$$

Take the update in (70), for instance. Intuitively, this update instructs us to do the following 'operations' on an input matrix I: fill column u_2 only with witches; then, check that each way of filling column u_1 with a single elf x is a way of filling column u_1 with the elf x such that x adores every (corresponding) u_2-witch.

The LF's for the two readings are provided in (72) and (73) below.

---

[33] I use quantification over sets $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in} Y(witch(y) \wedge adore(x, y)))$ in (69) only to make more explicit the relation between truth-conditions and *plural* info states in PCDRT (which plural info states store possibly non-singleton sets of individuals). In this particular case, quantification over sets is clearly not essential since $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in} Y(witch(y) \wedge adore(x, y)))$ is equivalent to the first order formula $\exists y_e(witch(y) \wedge adore(x, y))$.

[34] Just as before (see fn. **33** above), note that quantification over sets is not essential: the formula $\exists Y_{et}{\neq}\emptyset(\forall y_e{\in} Y(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y))))$ is equivalent to the first-order formula $\exists y_e(witch(y) \wedge \forall x_e(house\_elf(x) \rightarrow adore(x, y)))$.

72. **every**^{u_1} **>>a** $^{\textbf{wk:}u_2}$: Every ^{u_1} house-elf adores a $^{\textbf{wk:}u_2}$ witch.

Txt
|
CP
|
IP

[**every** $_{u_1}$ ([*house_elf*{u_1}], [u_2 | *witch*{u_2}, *adore*{u_1, u_2}])]

[-*s*] $_\text{I}$
$\lambda D_\textbf{t}.\, D$

VP

[**every** $_{u_1}$ ([*house_elf*{u_1}], [u_2 | *witch*{u_2}, *adore*{u_1, u_2}])]

DP

$\lambda P_\textbf{et}.[\textbf{every}\,_{u_1}\,([\textit{house\_elf}\{u_1\}],\ P(u_1))]$

V'

$\lambda v_\textbf{e}.[u_2\ |\ \textit{witch}\{u_2\},\ \textit{adore}\{v, u_2\}]$

every ^{u_1} *house-elf*

[*adore*] $_{\text{V}_\text{tr}}$

$\lambda Q_\textbf{(et)t}.\lambda v_\textbf{e}.Q(\lambda v'_\textbf{e}.[\textit{adore}_{e(et)}\{v, v'\}])$

DP

$\lambda P_\textbf{et}.[u_2\ |\ \textit{witch}\{u_2\}];\ P(u_2)$

a ^{u_2} *witch*

73. $a^{\textbf{wk:}u_2} >> \textbf{every}^{u_1}$: Every^{u_1} house-elf adores a$^{\textbf{wk:}u_2}$ witch.

Txt
|
CP
|
IP
$[u_2 \mid witch\{u_2\}, \textbf{every}_{u_1}([house\_elf\{u_1\}], [adore\{u_1, u_2\}])]$

DP$^{v''}$
$\lambda P_{\textbf{et}}.[u_2 \mid witch\{u_2\}]; P(u_2)$

IP
$[\textbf{every}_{u_1}([house\_elf\{u_1\}], [adore\{u_1, v''\}])]$

a^{u_2} witch

$[\textit{-s}]_{\textbf{I}}$
$\lambda D_{\textbf{t}}. D$

VP
$[\textbf{every}_{u_1}([house\_elf\{u_1\}], [adore\{u_1, v''\}])]$

DP
$\lambda P_{\textbf{et}}.[\textbf{every}_{u_1}([house\_elf\{u_1\}], P(u_1))]$

V'
$\lambda v_{\textbf{e}}.[adore\{v, v''\}]$

every^{u_1} house-elf

$[adore]_{\textbf{V}_{\textbf{tr}}}$
$\lambda Q_{(\textbf{et})\textbf{t}}.\lambda v_{\textbf{e}}.Q(\lambda v'_{\textbf{e}}.[adore_{e(et)}\{v, v'\}])$

$[t_{v'}]_{\textbf{DP}}$
$\lambda P_{\textbf{et}}.P(v''_{\textbf{e}})$
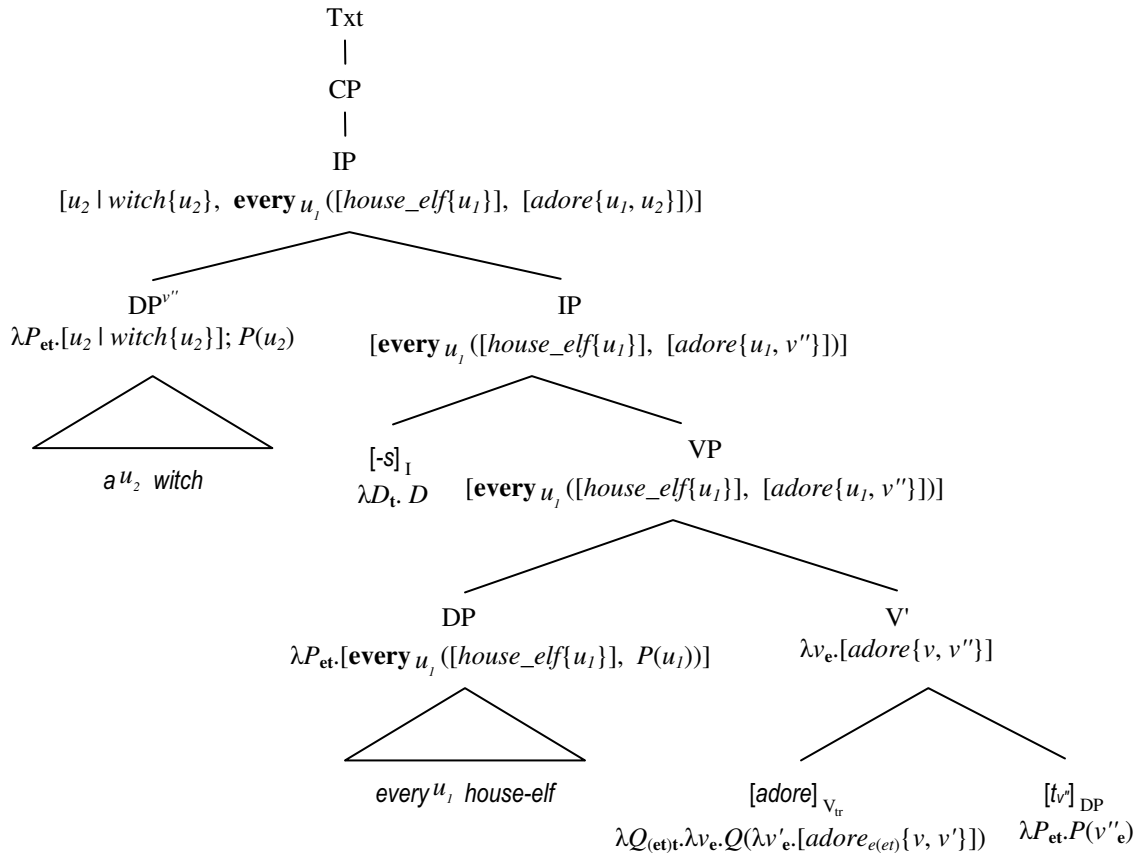
If the indefinite is strong – as shown in (74) below –, we have two more LF's with the same structure as (72) and (73) above. Yet again, we assign intuitively correct truth-conditions to both LF's, as shown in (76) and (78) below.

74. Every^{u_1} house-elf adores a$^{\textbf{str:}u_2}$ witch.

75. $\textbf{every}^{u_1} >> a^{\textbf{str:}u_2}$: $[\textbf{every}_{u_1}([h.elf\{u_1\}], \textbf{max}^{u_2}([witch\{u_2\}, adore\{u_1, u_2\}]))]$

76. $\textbf{every}^{u_1} >> a^{\textbf{str:}u_2}$: $\lambda I_{st}. I \neq \emptyset \wedge$

$\forall x_e(h.elf(x) \rightarrow \exists Y_{et} \neq \emptyset(\forall y_e(witch(y) \wedge adore(x, y) \leftrightarrow y \in Y)))$ [35]

77. $a^{\textbf{str:}u_2} >> \textbf{every}^{u_1}$: $\textbf{max}^{u_2}([witch\{u_2\}, \textbf{every}_{u_1}([h.elf\{u_1\}], [adore\{u_1, u_2\}])])$

---

[35] Yet again, we can do away with quantification over sets since, for our purposes (i.e. the interpretation of (74)), we can substitute $\exists y_e(Fy)$ for $\exists Y_{et} \neq \emptyset(\forall y_e(Fy \leftrightarrow y \in Y))$ in both (76) and (78) – where F stands for the predicate that is appropriate in each of the two cases.

78. **a** **str:**u_2 **>>every** u_1 : $\lambda I_{st}.\ I \neq \emptyset\ \wedge$

$$\exists Y_{et} \neq \emptyset (\forall y_e(witch(y) \wedge \forall x_e(h.elf(x) \rightarrow adore(x,\ y)) \leftrightarrow y \in Y))$$

4.3. Weak / Strong Ambiguities

Consider first the strong donkey example in (79) below, which is most readily understood as a generalization about the habits of house-elves that are in love – this being the reason for the fact that the donkey indefinite receives a strong reading. The LF of the sentence and the main steps of its compositional translation are provided in (79).

Intuitively, the translation in (79), i.e. the update associated with the Txt / CP / IP node, instructs us to check that, for any given matrix *I*, each way of pairing up a witch-loving elf with each of the witches he loves is a way of pairing up a witch-loving elf with each of the witches he loves and with some purse he bought her.

The translation derives the intuitively correct truth-conditions, given in (80).

79. Every^{u_1} house-elf who falls in love with a$^{\textbf{str:}u_2}$ witch buys her$_{u_2}$ an$^{\textbf{wk:}u_3}$ alligator purse. [36]

Txt
|
CP
|
IP

[**every**$_{u_1}$ ([*house_elf*{u_1}]; **max**^{u_2} ([*witch*{u_2}, *fall_in_love*{u_1, u_2}]), [u_3 | *a.purse*{u_3}, *buy*{u_1, u_2, u_3}])]

[-s]$_I$
$\lambda D_t. D$

VP

[**every**$_{u_1}$ ([*house_elf*{u_1}]; **max**^{u_2} ([*witch*{u_2}, *f.i.l*{u_1, u_2}]), [u_3 | *a.p*{u_3}, *buy*{u_1, u_2, u_3}])]

V'
$\lambda v_e.[u_3 | a.p\{u_3\}, buy\{v, u_2, u_3\}]$

DP
$\lambda P_{et}.$[**every**$_{u_1}$ ([*house_elf*{u_1}]; **max**^{u_2} ([*witch*{u_2}, *f.i.l*{u_1, u_2}]), $P(u_1)$)]

buy her$_{u_2}$ *an* $^{\textbf{wk:}u_3}$ *alligator purse*

[*every*^{u_1}]$_D$
$\lambda P'_{et}.\lambda P_{et}.$[**every**$_{u_1}$ ($P'(u_1), P(u_1)$)]

NP
$\lambda v_e.[house\_elf\{v\}];$ **max**^{u_2} ([*witch*{u_2}, *fall_in_love*{v, u_2}])

[*house-elf*]$_N$
$\lambda v_e.[house\_elf_{et}\{v\}]$

CP
$\lambda v''_e.$**max**^{u_2} ([*witch*{u_2}, *fall_in_love*{v'', u_2}])

[*who*]$_{DP}$$^{v''}$
$\lambda P_{et}.P$

CP
|
IP

max^{u_2} ([*witch*{u_2}, *fall_in_love*{v'', u_2}])

[-s]$_I$
$\lambda D_t. D$

VP
max^{u_2} ([*witch*{u_2}, *fall_in_love*{v'', u_2}])

[$t_{v''}$]$_{DP}$
$\lambda P_{et}.P(v''_e)$

V'
$\lambda v_e.$**max**^{u_2} ([*witch*{u_2}, *fall_in_love*{v, u_2}])

[*fall in love*]$_{V_{tr}}$
$\lambda Q_{(et)t}.\lambda v_e.Q(\lambda v'_e.[fall\_in\_love_{e(et)}\{v, v'\}])$

[*a*$^{\textbf{str:}u_2}$ *witch*]$_{DP}$
$\lambda P_{et}.$**max**^{u_2} ([*witch*{u_2}]; $P(u_2)$)

---

[36] For more discussion of the particular interpretation of *who* in (79), see section **5** of chapter **3**.

80. $\lambda I_{st}.$ $I \neq \emptyset \wedge \forall x_e \forall y_e (house\_elf(x) \wedge witch(y) \wedge adore(x, y)$

$$\rightarrow \exists z_e (alligator\_purse(z) \wedge buy(x, y, z)))$$

The analysis of the classical weak donkey sentence in (81) below proceeds as expected – see the PCDRT translation in (82) and the truth-conditions in (83).

Intuitively, the update in (82) instructs us to check the following, for any given matrix I: for each person x, if you can form a matrix based on I which stores x in column u_1 and which stores some non-empty set of dimes that x has in column u_2, then you should be able to form a (possibly different) matrix based on I which stores x in column u_1 and some non-empty set of dimes that x has and puts in the meter in column u_2.

81. Every ^{u_1} person who has a $^{\mathbf{wk:}u_2}$ dime will put it $_{u_2}$ in the meter.

82. [**every** $_{u_1}$ ([u_2 | $person\{u_1\}$, $dime\{u_2\}$, $have\{u_1, u_2\}$], [$put\_in\_meter\{u_1, u_2\}$])]

83. $\lambda I_{st}.$ $I \neq \emptyset \wedge \forall x_e (person(x) \wedge \exists y_e (dime(y) \wedge have(x, y))$

$$\rightarrow \exists z_e (dime(z) \wedge have(x, z) \wedge put\_in\_meter(x, z)))$$

4.4. Proportions

The proportion problem is solved because we work with a *selective* form of generalized quantification – as exemplified by the analysis of sentence (84) below. This sentence is most readily understood as a generalization about the behavior of most house-elves that are in love with a witch: *every* such witch ends up getting an alligator purse from the house-elf that is in love with her. Thus, the donkey indefinite *a witch* receives a strong reading. The PCDRT translation derives the intuitively correct truth-conditions (identical to the CDRT+GQ truth-conditions), provided in (85) below; note in particular that the PCDRT representation is false in the "Dobby as Don Juan" scenario, as desired.

84. Most^{u_1} house-elves who fall in love with a$^{\mathbf{str:}u_2}$ witch buy her$_{u_2}$ an$^{\mathbf{wk:}u_3}$ alligator purse.

Txt
|
CP
|
IP

$[\mathbf{most}_{u_1} ([house\_elf\{u_1\}]; \mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{u_1, u_2\}]),\ [u_3 \mid a.purse\{u_3\}, buy\{u_1, u_2, u_3\}])]$

$[-\emptyset]_{\mathrm{I}}$
$\lambda D_{\mathbf{t}}.\ D$

VP
$[\mathbf{most}_{u_1} ([house\_elf\{u_1\}]; \mathbf{max}^{u_2} ([witch\{u_2\}, f.i.l\{u_1, u_2\}]),\ [u_3 \mid a.p\{u_3\}, buy\{u_1, u_2, u_3\}])]$

V'
$\lambda v_{\mathbf{e}}.[u_3 \mid a.p\{u_3\}, buy\{v, u_2, u_3\}]$

DP
$\lambda P_{\mathbf{et}}.[\mathbf{most}_{u_1} ([house\_elf\{u_1\}]; \mathbf{max}^{u_2} ([witch\{u_2\}, f.i.l\{u_1, u_2\}]),\ P(u_1))]$

buy her$_{u_2}$ *an*^{u_3} *alligator purse*

$[most^{u_1}]_{\mathrm{D}}$
$\lambda P'_{\mathbf{et}}.\lambda P_{\mathbf{et}}.[\mathbf{most}_{u_1} (P'(u_1), P(u_1))]$

NP
$\lambda v_{\mathbf{e}}.[house\_elf\{v\}]; \mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{v, u_2\}])$

$[house\text{-}elf]_{\mathrm{N}}$
$\lambda v_{\mathbf{e}}.[house\_elf_{et}\{v\}]$

CP
$\lambda v''_{\mathbf{e}}.\mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{v'', u_2\}])$

$[who]_{\mathrm{DP}}{}^{v''}$
$\lambda P_{\mathbf{et}}.P$

CP
|
IP

$\mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{v'', u_2\}])$

$[-\emptyset]_{\mathrm{I}}$
$\lambda D_{\mathbf{t}}.\ D$

VP
$\mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{v'', u_2\}])$

$[t_{v''}]_{\mathrm{DP}}$
$\lambda P_{\mathbf{et}}.P(v''_{\mathbf{e}})$

V'
$\lambda v_{\mathbf{e}}.\mathbf{max}^{u_2} ([witch\{u_2\}, fall\_in\_love\{v, u_2\}])$

$[fall\ in\ love]_{\mathrm{V_{tr}}}$
$\lambda Q_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}.Q(\lambda v'_{\mathbf{e}}.[fall\_in\_love_{e(et)}\{v, v'\}])$

$[a^{\mathbf{str:}u_2}\ witch]_{\mathrm{DP}}$
$\lambda P_{\mathbf{et}}.\mathbf{max}^{u_2} ([witch\{u_2\}]; P(u_2))$

85. $\lambda I_{st}.\ I \neq \emptyset \wedge \mid\{x_e: h.elf(x) \wedge \exists y_e(witch(y) \wedge f.i.l(x, y)) \wedge$

$\forall y_e(witch(y) \wedge f.i.l(x, y) \rightarrow \exists z_e(a.p(z) \wedge buy(x, y, z)))\}\mid >$

$$\mathsf{|}\{x_e: house\_elf(x) \wedge \exists y_e(witch(y) \wedge fall\_in\_lovel(x, y) \wedge$$
$$\neg\exists z_e(a.purse(z) \wedge buy(x, y, z)))\}\mathsf{|}$$

4.5. Mixed Weak & Strong Sentences

The PCDRT definition of selective generalized quantification enables us to assign intuitively correct interpretations to our mixed weak & strong donkey sentences, repeated in (86) and (87) below.

86. Every ^{u_1} person who buys a $^{\mathbf{str:}u_2}$ book on amazon.com and has a $^{\mathbf{wk:}u_3}$ credit card uses it $_{u_3}$ to pay for it $_{u_2}$.

87. Every ^{u_1} man who wants to impress a $^{\mathbf{str:}u_2}$ woman and who has an $^{\mathbf{wk:}u_3}$ Arabian horse teaches her $_{u_2}$ how to ride it $_{u_3}$.

Given that their PCDRT analyses are basically identical, I will analyze only sentence (86). Its PCDRT translation – obtained compositionally in much the same way as the translations for the donkey sentences we have just examined – is provided in (88).

88. **[every** $_{u_1}$ ([*pers*{u_1}]; **max** ^{u_2} ([*bk*{u_2}, *buy*{u_1, u_2}]); [u_3 | *c.card*{u_3}, *hv*{u_1, u_3}],

$$[use\_to\_pay\{u_1, u_2, u_3\}])]$$

The PCDRT translation in (88) derives the intuitively correct truth-conditions, provided in (89) below.

89. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(person(x) \wedge \exists R_{e(et)}{\neq}\emptyset(\forall y_e(book(y) \wedge buy(x, y) \leftrightarrow y{\in}\mathbf{Dom}(R)) \wedge$
$$\forall z_e{\in}\mathbf{Ran}(R)(c.card(z) \wedge have(x, z)))$$
$$\rightarrow \exists R_{e(et)}{\neq}\emptyset(\forall y_e(book(y) \wedge buy(x, y) \leftrightarrow y{\in}\mathbf{Dom}(R)) \wedge$$
$$\forall z_e{\in}\mathbf{Ran}(R)(c.card(z) \wedge have(x, z))$$
$$\forall y_e\forall z_e(Ryz \rightarrow use\_to\_pay(x, y, z)))),\ \text{i.e.}$$
$$\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(person(x) \wedge \exists y_e(bk(y) \wedge buy(x, y)) \wedge \exists z_e(c.card(z) \wedge hv(x, z))$$
$$\rightarrow \forall y_e(bk(y) \wedge buy(x, y) \rightarrow \exists z_e(c.card(z) \wedge hv(x, z) \wedge u.t.p(x, y, z))))$$

4.6.   Donkey Anaphora to Structure

Let us turn to an example that involves *structured* donkey anaphora, i.e. the nuclear scope anaphorically retrieves not only the values of the donkey indefinites, but also the relational structure associated with those values.

Consider (90) below: as we have already noticed, both indefinites, i.e. $a^{\mathbf{str}:u_2}$ *Christmas gift* and $a^{\mathbf{str}:u_3}$ *girl in his class*, receive a strong reading, i.e. for each u_1-boy, we consider the set of *all* gifts that he bought for some girl in his class and the set of *all* girls that said u_1-boy bought a gift for. However, we need to store not only the sets, but also the correspondences between them established by the buying events, so that we can retrieve this correspondence in the nuclear scope, where we assert that, for each u_3-girl, her deskmate was asked to wrap the u_2-gift that was bought for said u_3-girl.

90. Every^{u_1} boy who bought a$^{\mathbf{str}:u_2}$ Christmas gift for a$^{\mathbf{str}:u_3}$ girl in his class asked

   her$_{u_3}$ deskmate to wrap it$_{u_2}$.

I will analyze *her$_{u_3}$ deskmate* as *the^{u_4} deskmate of her$_{u_3}$* and give a Russellian translation for the definite description, i.e. I assume it contributes existence and uniqueness. Since the uniqueness of the u_4-deskmate needs to be relativized to the u_3-girl, I will use an anaphoric uniqueness condition of the form $\mathbf{unique}_{u'}\{u\}$, as shown in (91) below.

91. *her$_{u_3}$ ^{u_4} deskmate* (i.e. *the$_{u_3}$ ^{u_4} deskmate of her$_{u_3}$*)

   $\rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u_4}\ ([deskmate\{u_4\},\ of\{u_4,\ u_3\}]);\ [\mathbf{unique}_{u_3}\{u_4\}];\ P(u),$

   where $\mathbf{unique}_{u_3}\{u_4\} := \lambda I_{st}.\ I \neq \varnothing \wedge \forall i_s \in I \forall i'_s \in I(u_3 i = u_3 i' \rightarrow u_4 i = u_4 i')$

The PCDRT translation of sentence (90) is provided in (92) below. The translation derives the intuitively correct truth-conditions, given in (93).

92. $[\textbf{every}_{u_1}([boy\{u_1\}]; \textbf{max}^{u_2}([gift\{u_2\}]; \textbf{max}^{u_3}([girl\{u_3\}, buy\{u_1, u_2, u_3\}])),$

$\qquad \textbf{max}^{u_4}([deskmate\{u_4\}, of\{u_4, u_3\}]); [\textbf{unique}_{u_3}\{u_4\}]; [a.t.w\{u_1, u_4, u_2\}])]^{37}$

93. $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e(boy(x) \wedge$

$\qquad\qquad \exists R_{e(et)} \neq \emptyset(\textbf{Dom}(R) = \{y_e: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\qquad\qquad\qquad \textbf{Ran}(R) = \{z_e: girl(z) \wedge \exists y_e \in \textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\qquad\qquad\qquad \forall y_e \forall z_e(Ryz \to buy(x, y, z)))$

$\qquad\qquad \to \exists R_{e(et)} \neq \emptyset(\textbf{Dom}(R) = \{y_e: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\qquad\qquad\qquad \textbf{Ran}(R) = \{z_e: girl(z) \wedge \exists y_e \in \textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\qquad\qquad\qquad \forall y_e \forall z_e(Ryz \to buy(x, y, z)) \wedge$

$\qquad\qquad\qquad \forall y_e \forall z_e(Ryz \to$

$\qquad\qquad\qquad \exists z'_e(\forall z''_e(d.m(z'') \wedge of(z'', z) \leftrightarrow z''=z') \wedge a.t.w(x, z', y))))),$

i.e. given the natural assumption that no boy bought the same gift for two distinct girls, so that there is only one relation R with the required properties,

$\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e \forall R_{e(et)} \neq \emptyset(boy(x) \wedge$

$\qquad\qquad \textbf{Dom}(R) = \{y_e: gift(y) \wedge \exists z_e(girl(z) \wedge buy(x, y, z))\} \wedge$

$\qquad\qquad \textbf{Ran}(R) = \{z_e: girl(z) \wedge \exists y_e \in \textbf{Dom}(R)(buy(x, y, z))\} \wedge$

$\qquad\qquad \forall y_e \forall z_e(Ryz \to buy(x, y, z))$

$\qquad \to \forall y_e \forall z_e(Ryz \to$

$\qquad\qquad\qquad \exists z'_e(\forall z''_e(d.m(z'') \wedge of(z'', z) \leftrightarrow z''=z') \wedge a.t.w(x, z', y))))$

5. Summary

The main goal of this chapter was to give a compositional account of weak / strong ambiguities that generalizes to mixed reading relative-clause donkey sentences like the one in (1) above. The main proposal is that the weak / strong donkey ambiguity is located at the level of the indefinite article, which is ambiguous (or underspecified) between a weak and a strong / maximal reading.

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[37] Intuitively, uniqueness needs to be relativized to u_3 in $\textbf{unique}_{u_3}\{u_4\}$ because, otherwise, we would require every u_4-individual to be the same, i.e. there would have to be just one deskmate over all.

The two crucial ingredients of the analysis are: (**i**) plural information states (modeled as sets of 'variable assignments', which can be represented as matrices with 'assignments' as rows) and (**ii**) a maximization operator use to specify the meaning of strong indefinite articles. The resulting system is dubbed Plural Compositional DRT (PCDRT). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

In PCDRT, sentences denote relations between an input and an output plural info state, i.e. sentences non-deterministically update a plural info state. Indefinites non-deterministically introduce both values and structure, i.e. they introduce structured sets of individuals, and pronouns are anaphoric to such structured sets. Quantification over individuals is defined in terms of matrices (i.e. plural info states) instead of single 'assignments' and the semantics of the non-quantificational part becomes rules for how to fill out a matrix.

PCDRT enables us to give a compositional account of a variety of phenomena, including mixed reading relative-clause donkey sentences, while keeping the dynamic meanings of generalized determiners, pronouns and indefinite articles very close to their static, Montagovian counterparts.

6. Comparison with Alternative Approaches

To my knowledge, the existence of mixed reading donkey sentences was observed for the first time by van der Does (1993) for relative-clause donkey sentences and by Dekker (1993) for conditional donkey sentences. Their examples are provided in (94) and (95) below.

94. Every farmer who has a horse and a whip in his barn uses it to lash him.
    (van der Does 1993: 18, (26))
95. If a man has a dime in his pocket, he throws it in the parking meter.
    (Dekker 1993: 183, (25)).

The example in (1) above (*Every*^{u_1} *person who buys a*^{u_2} *book on* amazon.com *and has a*^{u_3} *credit card uses it*$_{u_3}$ *to pay for it*$_{u_2}$) makes one additional point that is obscured by the examples in (94) and (95), namely that the weak reading of the indefinite *a credit card* in (1) is compatible with the set of credit cards being a non-singleton set – since I could use different credit cards to buy different (kinds of) books.

As already remarked in the previous chapters, weak / strong donkey ambiguities in general and mixed weak & strong relative-clause donkey sentences in particular pose problems for many influential dynamic theories of donkey sentences, including Heim (1982/1988), Kamp & Reyle (1993), Dekker (1993), Kanazawa (1994a) and Chierchia (1995). The main reason is that, in these dynamic theories, donkey indefinites do not have any quantificational force whatsoever, so all the truth-conditional effects associated with donkey anaphora have to be built into whatever element in the environment gives the quantificational force of the indefinite.

In the case of the mixed reading example in (1), this requires us to pack an entire logical form into the meaning of the generalized determiner *every*. As shown explicitly by the classical first-order translation of example (1), repeated in (96) below, the generalized determiner *every* needs to specify three things: (**i**) the fact that the indefinite *a book* is strong; (ii) the fact that the indefinite *a credit card* is weak and (**iii**) the fact that the strong indefinite *a book* can take scope over the weak indefinite *a credit card*, since I can use different cards to buy different (kinds of) books.

96. $\forall x(person(x) \wedge \exists y(book(y) \wedge buy\_on\_amazon(x, y)) \wedge \exists z(c.card(z) \wedge have(x, z))$
$$\rightarrow \forall y'(book(y') \wedge buy\_on\_amazon(x, y')$$
$$\rightarrow \exists z'(c.card(z') \wedge have(x, z') \wedge use\_to\_pay(x, z', y'))))$$

Thus, dynamic approaches of this kind are forced to give increasingly complex and stipulative meanings for selective generalized determiners. In contrast, the proposal I have pursued in this chapter is that indefinites should be endowed with a minimal quantificational force of their own: (**i**) just as in DPL, I let them contribute an existential quantification; (**ii**) what is new is that I also let them specifiy whether the existential

quantification they introduce is maximal or not, i.e. whether they introduce in discourse *some* witness set or the *maximal* witness set that satisfies the nuclear scope update[38].

The pseudo-scopal relation between the strong indefinite *a book* and the weak indefinite *a credit card* in (1) above ("pseudo" because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite) arises as a consequence of the fact that PCDRT uses plural information states, which store and pass on information about both the sets of objects and the dependencies between these objects that are introduced and elaborated upon in discourse.

Before examining alternative approaches in more detail, I want to indicate three respects in which PCDRT differs from most previous dynamic approaches (irrespective of whether or how they analyze weak / strong ambiguities).

The first difference is conceptual: PCDRT explicitly embodies the idea that reference to *structure* is as important as reference to value and that the two should be treated in parallel (see the definition of dref introduction and its justification in section **3.2** above).

Capturing reference to structure as *discourse* reference to structure, i.e. by means of plural information states rather than by means of choice and / or Skolem functions (or dref's for such functions), is preferable for the following reason: such functions can in principle be used to capture donkey anaphora to structure, but they have to have variable arity depending on how many simultaneous donkey anaphoric connections there are, i.e. the arity of the functions is determined by the discourse context. It is therefore more desirable to encode this context dependency in the database that stores discourse information, i.e. the info state, and not in the representation of a lexical item (the donkey pronoun and / or the donkey indefinite); for a related argument, see also section **7.2** in chapter **7** below.

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[38] A witness set for a static quantifier **DET**(A) (where **DET** is a static determiner and A is a set of individuals) is any set of individuals B such that $B \subseteq A$ and **DET**(A)(B). See Barwise & Cooper 1981: 103 (page references to Portner & Partee 2002).

The second difference is empirical: the motivation for *plural* information states is provided by singular and intra-sentential donkey anaphora, in contrast to the previous literature which relies on plural and cross-sentential anaphora (see van den Berg 1994, 1996a, b, Krifka 1996b, and Nouwen 2003 among others).

Importantly, donkey anaphora to structure provides a much stronger argument for the idea that plural info states are *semantically* necessary. To see this, consider anaphora to value first: a pragmatic account is plausible for cases of cross-sentential anaphora (e.g. in *A man came in. He sat down*, the pronoun *he* can be taken to refer to whatever man is pragmatically brought to salience by the use of an indefinite in the first sentence), but less plausible for cases of intra-sentential donkey anaphora (no single donkey is brought to salience in *Every farmer who owns a donkey beats it*).

Similarly, a pragmatic account of anaphora to structure is plausible for cases of cross-sentential anaphora like *Every man saw a woman. They greeted them.* This discourse asserts that every man greeted the woman / women that he saw, i.e. the greeting structure is the same as the seeing structure – but the identity of structure might be a pragmatic addition to semantic values that are *unspecified* for structure (e.g. the second sentence *They greeted them* could be interpreted cumulatively in the sense of Scha 1981). However, a pragmatic approach is much less plausible for cases of intra-sentential donkey anaphora to structure instantiated by sentence (2) above.

Third, PCDRT takes the research program in Muskens (1996) of unifying different semantic frameworks, i.e. Montague semantics and dynamic semantics, one step further: PCDRT unifies in *classical type logic* the static, compositional analysis of generalized quantification in Montague semantics and van den Berg's Dynamic Plural Logic. The unification is not a trivial task, given certain peculiarities of Dynamic Plural Logic, e.g. the fact that its underlying logic is partial and the fact that discourse-level plurality (i.e. the use of plural information states) and domain-level plurality (i.e. non-atomic individuals) are conflated[39].

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[39] For more on the distinction between discourse-level and domain-level plurality, see chapter **8** below and Brasoveanu (2006c).

One of the advantages of the resulting type-logical framework is that it can be extended in the usual way with additional sorts for eventualities, times and possible worlds, which enables PCDRT to account for *temporal* and *modal* anaphora and quantification in a way that is *parallel* to the account of individual-level anaphora and quantification. The modal extension is worked out in chapter **7** below.

The previous accounts of weak / strong donkey sentences fall (roughly) into three categories:

- accounts that locate the ambiguity at the level of the generalized determiner (e.g. the determiner *every* in the classic example *Every farmer who owns a donkey beats it*); most dynamic accounts fall into this category, including Rooth (1987), Van Eijck & de Vries (1992), Dekker (1993), Kanazawa (1994a, b), but also the D-/E-type approach in Heim (1990); these approaches will be discussed in section **6.1**;
- accounts that locate the ambiguity at the level of the donkey pronoun, e.g. the D-/E-type approaches in van der Does (1993) and Lappin & Francez (1994); these approaches will be discussed in section **6.2**;
- accounts that locate the ambiguity at the level of the indefinite article; this is the approach pursued in this chapter and in van den Berg (1994, 1996a); van den Berg's approach will be discussed in section **6.3**.

In addition, there is also the hybrid dynamic/E-type approach pursued in Chierchia (1995). This approach will be discussed in section **6.2**.

6.1. Weak / Strong Determiners

I can see two reasons for locating the weak / strong ambiguity in the donkey indefinites and not in the dynamic meaning of generalized determiners.

The first one – already presented above – has to do with the syntax/semantics aspect of the interpretation of donkey sentences, in particular, with the requirement of (strict) compositionality. If we attribute the weak / strong ambiguity to the determiner and we want to derive the intuitively correct truth-conditions for the mixed reading donkey sentence in (1), we basically need to pack an entire logical form into the meaning of the

generalized determiner *every*, which needs to non-locally / non-compositionally determine both the readings associated with different donkey indefinites and their relative (pseudo-)scope.

The other reason for locating the weak / strong ambiguity in the indefinites is concerned with the semantics/pragmatics side of the interpretation of donkey sentences, namely the *variety of factors* that influence which reading is selected in any given instance of donkey anaphora and the *defeasible character* of the generalizations correlating these factors and the resulting readings.

Some of these factors are:

- *the logical properties of the determiners* – see Kanazawa (1994a, b);
- *world-knowledge* – see the 'dime' example in Pelletier & Schubert (1989) and, also, the examples and discussion in Geurts (2002);
- the *information* (focus-topic-background) *structure* of the sentence – see Kadmon (1987), Heim (1990);
- the kind of *predicates* that are used, i.e. total vs. partial predicates – see Krifka (1996a) and references therein;
- whether the donkey indefinite is referred back to by a donkey pronoun – see Bäuerle & Egli (1985)[40].

Given the variety of factors that influence which reading is selected in any given instance of donkey anaphora and also the defeasible character of the generalizations correlating these factors and the resulting readings, I think that the most conservative hypothesis is to locate the weak / strong ambiguity at the level of the donkey indefinites themselves, i.e. to make the donkey items ambiguous between a weak and a strong meaning[41], and let more general and defeasible pragmatic mechanisms decide which meaning is selected in any particular case.

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[40] Apud Heim (1990).

[41] Ambiguous between a weak and a strong reading or, alternatively, underspecified for weak / strong readings (like quantifier scope, for example, is underspecified) or vague (like adjectives).

One of the most theoretically appealing accounts of the weak / strong donkey ambiguity is due to Kanazawa (1994a, b), which locates the ambiguity in the meaning of the generalized determiners. I will therefore dedicate the remainder of this section to making two more points that seem to favor the PCDRT indefinite-based theory – or, at least, to give it sufficient initial plausibility.

First, Kanazawa's account is ultimately pragmatic, just like the account I am suggesting. In fact, except for the fact that he chooses to make the dynamic generalized determiner – and not the indefinite – underspecified, I think that all the observations below also apply to the PCDRT account.

> "The primary assumption I make is the following: […] The grammar rules in general underspecify the interpretation of a donkey sentence.
> Thus, I assume that, for any donkey sentence, the grammar only partially characterizes its meaning, with which a range of specific interpretations are compatible. So the truth value of donkey sentences in particular situations may be left undecided by the grammar. This may not be such an outrageous idea; it may explain the lack of robust intuitions about donkey sentences.
> For the sake of concreteness, I assume that the underspecified interpretation of a donkey sentence Det N' VP assigned to by the grammar can be represented using an indeterminate dynamic generalized determiner Q which is related to the static generalized determiner Q denoted by Det and which satisfies certain natural properties. […]
> Even if its interpretation is underspecified, a sentence may be assigned a definite truth-value in special circumstances. […] It is not unreasonable to suppose that people are capable of assessing the truth value of a donkey sentence without resolving the 'vagueness' of the meaning given by the grammar when there is no need to do so. […] underspecification causes no problems for people in assigning a truth value to a donkey sentence in situations where the uniqueness condition for the donkey pronoun is met."
> (Kanazawa 1994a: 151-152)

Note in particular the situations in which the "uniqueness condition" is met are precisely the situations in which the PCDRT weak and strong meanings for the indefinite article are conflated; for more discussion about uniqueness effects in donkey sentences, see section **6.2** of chapter **6** below.

Thus, both accounts of the weak / strong donkey ambiguity defer the task of disambiguation to pragmatics – which brings me to the second, empirical point. The hypothesis that the weak / strong ambiguity (or underspecification) should be located in

the generalized determiner has more plausibility than the PCDRT hypothesis only if we observe that the logical properties of the determiners are, consistently, the main deciding disambiguation factor. This is clearly not true for the determiner *every*: its monotonicity-based bias for strong readings is easily trumped by world knowledge (as shown by the 'dime' example; see also the discussion in Kanazawa 1994a: 122-124 and Geurts 2002).

I will now point out that the monotonicity-based bias can be systematically overridden for most other determiners in a particular kind of construction that involves nuclear scope negation. This observation – together with the above list of five unrelated factors that influence the choice between weak and strong readings – provides support for the conservative hypothesis that the source of the weak / strong donkey ambiguity should be located in the donkey indefinites and not in some other element in their environment.

Donkey Readings and Nuclear Scope Negation[42]

I use "nuclear scope negation" as a cover term for negative items, e.g. sentential negation or negative verbs like *fail*, *forget* and *refuse*, that occur within the nuclear scope of a quantification and that semantically take scope over the other elements in the nuclear scope. To my knowledge, the only examples of nuclear scope negation discussed in the previous literature are the ones provided in (97), (98), (99) and (100) below[43].

97. A boy who had an^u apple in his rucksack didn't give it$_u$ to his sister.

   (van der Does 1993: 18, (27c))

98. No man who had a^u credit card failed to use it$_u$.

   (Kanazawa 1994a: 117, fn. 16)

99. Every person who had a^u dime in his pocket did not put it$_u$ into the meter.

   (Lappin & Francez 1994: 401, (22a))

100. Every person who had a^u dime in his pocket refused to put it$_u$ into the meter.

   (Lappin & Francez 1994: 401, (22a))

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[42] I am grateful to Hans Kamp (p.c.) for pointing out to me that there seems to be a systematic correlation between sentential negation and donkey readings. Most of the empirical observations in this sub-section emerged during or as a result of our conversations.

[43] Geurts (2002) also mentions the examples due to van der Does (1993) and Kanazawa (1994a), but he believes that "such examples are hard to find" (Geurts (2002): 131).

The generalization that emerges based in these examples and which trumps the monotonicity-based bias observed in Kanazawa 1994a is that nuclear scope negation generally requires the strong reading for donkey sentences; see also Lappin & Francez (1994) for observations that point towards the same generalization (p. 408 in particular) and for a critique of Kanazawa (1994a) based on sentences (99) and (100) (pp. 410-411). Sentence (97) is interpreted as asserting that there is some boy such that, for *every* apple in his rucksack, he didn't give that apple to his sister. Sentence (98) is interpreted as asserting that no man is such that, for *every* credit card of his, he failed to use that card, i.e. no man failed to use every credit card of his – or, equivalently, every man used some credit card or other.

The examples in (97) and (98) form minimal pairs with sentences (101) and (102) below, where there is no nuclear scope negation and where the most salient donkey reading is the weak one (just as Kanazawa 1994a predicts they should).

101. A boy who had an^u apple in his rucksack gave it$_u$ to his sister.
102. No man who had a^u credit card used it$_u$ (to pay the bill).

We can observe a similar contrast for non-monotone intersective determiners of the form *exactly n*, also predicted by Kanazawa (1994a) to favor the weak reading (just as the intersective but monotone determiners *a* and *no* do). The most salient reading of (103) below is the strong donkey reading: exactly two men are such that, for *every* credit card they had, they failed to use that card. The most salient reading of (104) is the weak one: exactly two men used *some* credit card they had.

103. Exactly two men who had a^u credit card failed to use it$_u$ / didn't use it$_u$ / forgot to use it$_u$.
104. Exactly two men who had a^u credit card used it$_u$.

The same applies to the *only*-based donkey examples in (105) and (106) below.

105. Only two men who had a^u credit card failed to use it$_u$ / didn't use it$_u$ / forgot to use it$_u$.
106. Only two men who had a^u credit card used it$_u$.

As the examples (99) and (100) above show, even the classical weak reading example in (107) below becomes strong under the influence of nuclear scope negation: the example in (108) below is interpreted as asserting that every man who had a quarter was such that, for *every* quarter of his, he refused to put that quarter in the meter. The pairs of *at least n-*, *at most n-* and *most*-sentences in (109)-(110), (111)-(112) and (113)-(114) below instantiate the same kind of contrast.

107. Every man who had a^u quarter put it$_u$ in the meter.

108. Every man who had a^u quarter refused to put it$_u$ in the meter / forgot to put it$_u$ in the meter.

109. At least two men who had a^u quarter put it$_u$ in the meter.

110. At least two men who had a^u quarter refused to put it$_u$ in the meter / forgot to put it$_u$ in the meter.

111. At most two men who had a^u quarter put it$_u$ in the meter.

112. At most two men who had a^u quarter refused to put it$_u$ in the meter / forgot to put it$_u$ in the meter.

113. Most men who had a^u nice suit wore it$_u$ at the town meeting.
     (based on Kanazawa 2001: 386, (17))

114. Most men who had a^u nice suit refused to wear it$_u$ at the town meeting / forgot to wear it$_u$ at the town meeting / didn't wear it$_u$ at the town meeting.

In contrast, note that negation with scope over the entire donkey quantification does not have a similar 'strengthening' effect, as the examples in (115), (116) and (117) below show. Consider (116) for example: its strong reading is that not every man who had a credit card is such that, for *every* credit card he had, he used that card to pay the bill – an assertion that borders on triviality. Intuitively, sentence (116) asserts that not every man who had a credit card used *some* credit card of his to pay the bill – or, equivalently, that there is a man who had a credit card and who didn't use any of his cards to pay, i.e. the weak donkey reading.

115. Not every man who had a^u quarter put it$_u$ in the meter.

116. Not every man who had a^u credit card used it$_u$ to pay the bill.

117. Not every person who buys a^u book on amazon.com and who has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_u$.

However, just like the other generalizations about the distribution of weak vs. strong donkey readings, the correlation between nuclear scope negation and the strong donkey reading is not without exception. A top-level negation cancels the 'strengthening' effect of the nuclear scope negation, as the examples in (118) and (119) below show.

Incidentally, note that the weak donkey sentences in (118) and (119) and the ones in (115), (116) and (117) above show that $\uparrow$MON$\downarrow$ determiners like *not every* and *not all* reliably tolerate weak readings, contra Kanazawa (1994a): 118 et seqq.

118. Not every man who had a^u credit card failed to use it$_u$.
119. Not every man who had a nice suit refused to wear it$_u$ at the town meeting / forgot to wear it$_u$ at the town meeting.

Sentences (118) and (119) indicate that, if there is any correlation between negation, the monotonicity properties of the generalized determiners and the choice between weak and strong donkey readings, this correlation cannot be locally and deterministically established by taking into account only some particular item in the context of the donkey indefinites, be it the generalized determiner or the nuclear scope negation – we need to take into account the whole quantification and, on top of that, factors of a different nature, e.g. world knowledge about how credit card owners normally behave (they don't pay with all their credit cards) or about how people normally wear their suits (not all of them at the same time, even if they are very nice).

I conclude with the example in (120) below, which provides one more exception to the correlation between nuclear scope negation and strong donkey readings. The most salient reading of (120) is that every man who placed a suitcase on the belt took back every suitcase after it was X-rayed, i.e. no man who placed a suitcase on the belt failed, for *some* such suitcase, to take it back, i.e. the weak donkey reading.

120. (At the airport "self check-in", where customers place their suitcase / suitcases on the belt to have them X-rayed:)

No man who placed a^u suitcase on the belt forgot to take it$_u$ back after it$_u$ was X-rayed / failed to take it$_u$ back after it$_u$ was X-rayed.

I leave the analysis of the above generalizations for future research – but I hope to have established that the volatile nature of the weak / strong donkey ambiguity makes the PCDRT account at least as plausible as the alternative dynamic strategy of locating the source of the ambiguity in the selective generalized determiners.

6.2. Weak / Strong Pronouns

D-/E-type accounts of donkey anaphora fall into two categories with respect to the problem posed by weak / strong ambiguities. If they address the problem (e.g. Neale 1990 and Elbourne 2005 do not), they either locate the weak / strong ambiguity in the meaning of the generalized determiner, e.g. Heim (1990), or in the meaning of the donkey pronoun, e.g. van der Does (1993) and Lappin & Francez (1994).

Given that the strategy in Heim (1990) is basically the same as the one pursued by the dynamic accounts discussed in the previous section, the resulting analysis faces the same kind of problems (*mutatis mutandis*).

In this section, I will focus on accounts that take the donkey pronoun to be the source of the weak / strong ambiguity; in particular, I will focus on the account in Lappin & Francez (1994), but the general argument also applies to van der Does (1993).

Lappin & Francez (1994) assume the ontology in Link (1983), which countenances both (atomic) individuals and individual sums thereof – or *i*-sums. Lappin & Francez (1994): 403 propose to analyze donkey pronouns as functions from individuals to *i*-sums, e.g., in the classical donkey example *Every farmer who owns a donkey beats it*, the pronoun *it* denotes a function *f* that, for every donkey-owning farmer *x*, returns some *i*-sum *f(x)* of donkeys that *x* owns, i.e. the sum of some subset of the donkeys that *x* owns.

Strong donkey readings are obtained by placing a maximality constraint on the function *f*, which requires *f* to select, for each *x* in its domain, the supremum of its possible values, i.e., in the case at hand, the maximal *i*-sum of donkeys that *x* owns. Weak

donkey readings are obtained by suspending the maximality constraint, i.e. f is a choice function from x to one of the i-sums of donkeys that x owns.

I will use DP-conjunction donkey sentences of the kind analyzed in section **5.6** of chapter **4** above to distinguish between the D-/E-type strategy of locating the weak / strong ambiguity in the meaning of the donkey pronoun and the PCDRT strategy of locating it in the meaning of the donkey indefinite.

DP-Conjunction Donkey Sentences with Mixed Readings

Consider the mixed weak & strong donkey sentences in (121) below, whose subjects is a conjunction of two DP's.

121. (Today's newspaper claims that, based on the most recent statistics:)

Every^{u_1} company who hired a^{u_2} Moldavian man, but no^{u_3} company who hired a^{u_2} Transylvanian man promoted him$_{u_2}$ within two weeks of hiring.

Intuitively, the sentence asserts that every company who hired a Moldavian promoted *every* Moldavian it hired within two weeks, while there is no company who hired some Transylvanian and promoted *some* Transylvanian it hired within two weeks – that is, the donkey anaphora to *a Moldavian man* is strong and the donkey anaphora to *a Transylvanian man* is weak.

Crucially, the very same pronon *it* is intuitively anaphoric to both indefinites. Example (121) poses a problem for approaches like Lappin & Francez (1994) and van der Does (1993), which locate the weak / strong ambiguity in the donkey pronouns, because there is only one pronoun in (121), but two distinct donkey readings.

Note that there is no immediately obvious was in which covert syntactic operations could 'reconstruct' two pronouns in the case of (121) – or in the case of the similar example in (122) below. Examples (121) and (122) do not seem to be instances of ellipsis or Right Node Raising, in which case we could have assumed that the pronoun is covertly duplicated at the level of LF. Also, covertly duplicating at LF the pronoun in (121) (or (122)) by rightward Across-the-Board (ATB) movement of the VP does not seem to be an independently motivated syntactic operation in English. And, even if rightward ATB

movement of the VP is possible, one still needs to reconstruct the VP in both places to get two pronouns and, presumably, assign the reconstructed pronouns two different indices.

Sentence (122) below makes the same point as (121) – the only difference is that, in (122), we conjoin two DP's headed by the same generalized determiner[44,45]. The sentence in (122) can be felicitously uttered in the following context: there is this Sunday fair where, among other things, people come to sell their young puppies – and they do want to get rid of all of them before they are too old. Also, the fair entrance fee is one dollar. Now, the fair rules are strict: all the puppies need to be checked for fleas at the gate and, at the same time, the one dollar bills also need to be checked for authenticity because of the many faux-monnayeurs in the area. So:

122. Everyone^{u_1} who has a^{u_2} puppy (to sell) and everyone^{u_3} who has a^{u_2} dollar (to pay the fee) brings it$_{u_2}$ to the gate to be checked.

The most salient interpretation of sentence (122) is that every potential seller brings *all* her or his puppies to the gate to be checked, while every potential buyer needs to bring only *one* of her or his dollars, i.e. anaphora to *a^{u_2} puppy* is strong, while anaphora to *a^{u_2} dollar* is weak.

Thus, I assume that, in the case of both (121) and (122) above, what one sees is what one gets: two donkey indefinites, one donkey pronoun and two donkey readings. These mixed weak & strong donkey sentences pose problems for the approach in Lappin

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[44] Note also that the intonational tune in example (122) is the same as the one associated with declarative sentences like *Every student and every professor was invited to the party*, the LF of which is not derived by ellipsis and / or Right Node Raising.

[45] Variants of the mixed reading example in (122) are given in (i) and (ii) below; note that, in all cases, the context needs to be tweaked in a way that prevents the default parallel interpretation of the two conjuncts (i.e. both donkey indefinites are strong or both are weak).

The example in example (122) is the refinement of (ii), due to Sam Cumming, following Klaus von Heusinger's and Hans Kamp's suggestions (p.c.).

(i) (There aren't that many ambulant theater troupes anymore in Romania. This is because of the following custom:) At the end of a play, every person that liked the play and has a^u dime and every person that didn't like the play and has a^u rotten tomato throws it$_u$ at the actors.

(ii) (It's market day. So:) Every farmer who owns a^u donkey and every spectator who has a^u dollar – for entry – brings it$_u$ to the saleyard.

& Francez (1994) because either the donkey pronouns *him*$_{u_2}$ in (121) and *it*$_{u_2}$ in (122) are subject to the maximality constraint and therefore can deliver only strong donkey readings or the maximality constraint is suspended and the donkey pronouns can deliver only the weak reading.

These sentences pose an even more severe problem for the hybrid approach to weak / strong ambiguities proposed in Chierchia (1995), where the weak reading is derived within a dynamic framework and the strong reading is attributed to a D-/E-type reading of the donkey pronoun. Given that Chierchia (1995) agrees with the observation that examples like (121) and (122) above involve a single pronoun (he actually uses examples of the same form to argue for a semantic as opposed to a syntactic approach to donkey anaphora), his approach is faced with the problem of deriving, by means of a single pronoun, two different donkey readings which are furthermore claimed to involve two different kinds of semantic representations for the pronoun.

One more move seems to still be open possible for the D-/E-type approach in Lappin & Francez (1994); following a suggestion from Chierchia (1995): 116-117, the donkey pronouns *him*$_{u_2}$ in (121) and *it*$_{u_2}$ in (122) could be interpreted as denoting the union of two different functions, a maximal one that is contributed by the first DP in their respective sentences and a non-maximal, choice-based one that is contributed by the second DP. Note, however, that this strategy does not work in general because the union of two functions is not necessarily a function. In particular, suppose that, in (121), the very same company *x* hired both a Moldavian man and a Transylvanian man; the first function will return the Moldavian man as value for the argument *x*, while the second function will return the Transylvanian man, so the result of their union is not function and, therefore, not a suitable kind of meaning for a donkey pronoun.

Finally, suppose that we take the function union approach one step further and assume that, when we take the union of two functions *f* and *f'*, we require the resulting function to return, for any *x* that is in the domain of both *f* and *f'*, the sum of the individuals *f(x)* and *f'(x)*. This "union & sum" strategy could yield the correct truth-conditions for example (122) where, for a person *x*, *x* brings to the gate to be checked

every individual in the *i*-sum formed out of *x*'s puppies and one of *x*'s dollar bills – but it will not yield the intuitively correct truth-conditions for (121).

Moreover, the "union & sum" strategy (and D-/E-type approaches in general) predict that the sum should be available for subsequent *singular* cross-sentential anaphora – if the function that provides the meaning of the pronoun is salient enough the first time around, it should still be salient enough immediately afterwards. However, subsequent singular anaphora to puppy-dollar sums is unacceptable, as shown in (123) below[46].

123. **a.** Everyone who has a^u puppy and everyone who has a^u dollar brings it$_u$ to the gate to be checked.

   b. #They do so because the rules of the fair require that it$_u$ (should) be checked.

PCDRT, on the other hand, can account for this kind of examples without any additional stipulations: their analysis is parallel to the CDRT+GQ analysis of the example *Every^{u_1} boy who has a^{u_2} dog and every^{u_3} girl who has a^{u_2} cat must feed it$_{u_2}$* from Chierchia (1995) (see section **5.6** of chapter **4** above). Sentences (121) and (122) receive the readings in (124) and (125) below.

124. Every^{u_1} company who hired a$^{\text{str:}u_2}$ Moldavian man, but no^{u_3} company who hired a$^{\text{wk:}u_2}$ Transylvanian man promoted him$_{u_2}$ within two weeks of hiring.

125. Everyone^{u_1} who has a$^{\text{str:}u_2}$ puppy and everyone^{u_3} who has a$^{\text{wk:}u_2}$ dollar brings it$_{u_2}$ to the gate to be checked.

I will only analyze (124), since the analysis of (125) is parallel. The PCDRT translation is given in (126) below the derived truth-conditions, which are intuitively correct, are provided in (127).

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[46] Plural anaphora is, however, possible, as shown by (i) below. But D-/E-type approaches cannot offer any explanation for this asymmetry. I believe that PCDRT can and that the explanation would be similar to the account of the infelicitous telescoping cases in section **6.3** of chapter **6** below.

(i) **a.** Everyone who has a^u puppy and everyone who has a^u dollar brings it$_u$ to the gate to be checked. **b.** They do so because the rules of the fair require that they$_u$ (should) be checked.

126. every ^{u_1} company who hired a $^{\textbf{str:}u_2}$ Moldavian man

    ↝ $\lambda P_{\textbf{et}}.\ [\textbf{every}_{u_1}\ ([company\{u_1\}];\ \textbf{max}^{u_2}\ ([mold\{u_2\},\ hire\{u_1,\ u_2\}]),\ P(u_1))]$

no ^{u_3} company who hired a $^{\textbf{wk:}u_2}$ Transylvanian man

    ↝ $\lambda P_{\textbf{et}}.\ [\textbf{no}_{u_3}\ ([company\{u_3\}];\ [u_2\ |\ trans\{u_2\},\ hire\{u_3,\ u_2\}],\ P(u_3))]$

every ^{u_1} company who hired a $^{\textbf{str:}u_2}$ M.man, but no ^{u_3} company who hired a $^{\textbf{wk:}u_2}$ T.man

    ↝ $\lambda P_{\textbf{et}}.\ [\textbf{every}_{u_1}\ ([company\{u_1\}];\ \textbf{max}^{u_2}\ ([mold\{u_2\},\ hire\{u_1,\ u_2\}]),\ P(u_1)),$

      $\textbf{no}_{u_3}\ ([company\{u_3\}];\ [u_2\ |\ trans\{u_2\},\ hire\{u_3,\ u_2\}],\ P(u_3))]$

promoted him $_{u_2}$ within two weeks of hiring ↝ $\lambda v_{\textbf{e}}.\ [promote\{v,\ u_2\}]$

every ^{u_1} company who hired a $^{\textbf{str:}u_2}$ Moldavian man, but no ^{u_3} company who hired a $^{\textbf{wk:}u_2}$

Transylvanian man promoted him $_{u_2}$ within two weeks of hiring

    ↝ $[\textbf{every}_{u_1}\ ([company\{u_1\}];\ \textbf{max}^{u_2}\ ([mold\{u_2\},\ hire\{u_1,\ u_2\}]),\ [prom\{u_1,\ u_2\}]),$

      $\textbf{no}_{u_3}\ ([company\{u_3\}];\ [u_2\ |\ trans\{u_2\},\ hire\{u_3,\ u_2\}],\ [prom\{u_3,\ u_2\}])]$

127. $\lambda I_{st}.\ I{\neq}\emptyset\ \wedge\ \forall x_e \forall y_e(company(x)\ \wedge\ mold(y)\ \wedge\ hire(x,\ y)\ \rightarrow\ promote(x,\ y))\ \wedge$

      $\forall x'_e \forall y'_e(company(x')\ \wedge\ trans(y')\ \wedge\ hire(x',\ y')\ \rightarrow\ \neg promote(x',\ y'))$

To conclude, note that the PCDRT account of mixed reading donkey sentences (including the DP-conjunction examples above) predicts that the same indefinite cannot be interpreted as strong with respect to one pronoun (or any other kind of anaphor, e.g. a definite) and weak with respect to another pronoun. This prediction seems to be borne out[47]. By the same token, the D-/E-type analysis in Lappin & Francez (1994) (the points also applies to the hybrid approach in Chierchia 1995), which locates the weak / strong ambiguity at the level of the pronoun (or anaphor, in the general case), predicts the exact opposite – and, it seems, incorrectly so. That is, according to the D-/E-type analysis, the same indefinite should be able to be interpreted as strong with respect to one pronoun and as weak with respect to another. I am not aware of any example of this form.

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[47] I am indebted to Roger Schwarzschild (p.c.) for emphasizing this point.

Unifying Dynamic Semantics and Situation Semantics

In this sub-section, I want to suggest that PCDRT effectively unifies dynamic and situation-based D-/E-type approaches of the kind proposed in Heim (1990) (among others) in a way that remains faithful to many of their respective goals and underlying intuitions.

In particular, the type s in PCDRT can be taken to be the type of partial situations as they are used in Heim (1990) – with the added advantage that PCDRT does not have the problem of indistinguishable participants (a.k.a. Kamp's 'bishop' problem) and does not need to address the issues raised by the 'formal link' condition.

Moreover, two major differences between dynamic and D-/E-type approaches to anaphora mentioned in Heim (1990): 137 are effectively invalidated by PCDRT. These differences (see the contrasting items (ii)-(iii) and (ii')-(iii') in (Heim 1990: 137) concern:

- the treatment of anaphoric pronouns: they are "plain bound variables" in dynamic approaches, while D-/E-type approaches analyze them as "semantically equivalent to (possibly complex) definite descriptions" (Heim 1990: 137);
- the treatment of quantificational determiners: they are "capable of binding multiple variables" in dynamic approaches, while they "bind just one variable each" (Heim 1990: 137) in D-/E-type approaches.

In PCDRT, anaphoric pronouns are basically analyzed as individual-level dref's, i.e. as functions from entities of type s to individuals (type e). Depending on how we prefer to intuitively think about the entities of type s, i.e. as 'variable assignments' or 'partial situations', the anaphoric pronouns are bound variables, i.e. they are the equivalent of projection functions on variable assignments (type s), or definite descriptions characterizing a unique individual in a given partial situation (again, type s).

Similarly, quantificational structures contributed by determiners or the generic operator in conditionals are analyzed as having the general form in (59) above (see section **3.5** of the present chapter), i.e. **det**$_u(D, D')$. Insofar as these quantificational structures operate over the DRS's D and D', hence over relations between info states, they

are capable of binding multiple variables, but insofar as they contribute a particular dref *u* that is crucial in relating the two updates *D* and *D'*, they bind one variable each.

Finally, it seems to me that, if situation-based D-/E-type approaches are to be extended to account for mixed weak & strong donkey sentences like (1) above, they will have to introduce mechanisms that involve quantification over sets of partial situations and, also, updates of such sets that will be very similar to the notions of plural info state, quantification and info state update in PCDRT. I leave a more thorough investigation and comparison between PCDRT and situation-based D-/E-type approaches for future research.

6.3. Weak / Strong Indefinites

I will conclude with a brief examination of the approach in van den Berg (1994, 1996a), which, just as PCDRT, locates the weak / strong donkey ambiguity in the meaning of the donkey indefinites.

The first thing we need to do is to introduce van den Berg's notion of dynamic maximization. Abstracting away from the fact that it is formulated in a three-valued logic, the definition in van den Berg (1994): 15, (45) is different from the PCDRT definition in only one respect: it is is a weaker version of the **max**^{u} operator insofar as it does not require the existence of a supremum – it simply requires an output state to non-deterministically store a (locally) maximal set[48]. A PCDRT definition that is as close as possible to the maximization operator in van den Berg (1994) is given in (128) below, where '⊂' stands for strict inclusion. This operator and the corresponding one in PCDRT stand in the relation shown in (129) below.

128. **max-wk**^{u}(D) := $\lambda I_{st}.\lambda J_{st}.$ ([u]; D)IJ ∧ ¬∃K_{st}(([u]; D)IK ∧ uJ⊂uK)

129. **max**^{u}(D) ⊆ **max-wk**^{u}(D)

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[48] For example, assume that if we update a given input info state *I* with a DRS of the form [*u*]; *D*, we get three possible output states J_1, J_2 and J_3 such that uJ_1={*a*}, uJ_2={*a*, *b*} and uJ_3={*a*, *c*}. The PCDRT supremum-based form of maximization will simply discard the input info state *I* altogether because there is no supremum in the set {uJ_1, uJ_2, uJ_3}. The weak, maxima-based form of maximization will retain the input info state *I* and the corresponding output states J_2 and J_3, but not J_1.

Van den Berg (1994, 1996a) crucially needs the weaker form of maximization **max-wk**^{u} (as opposed to the PCDRT one) to be able to account for weak / strong ambiguities. The reason for this is that he takes indefinites to be generalized quantifiers and, in his framework, generalized quantifiers are defined in terms of maximization[49]. He, therefore, uses a maximization operator to give the meaning of both weak and strong donkey indefinites[50].

In the case of the weak indefinites, however, van den Berg needs to neutralize the maximization effect (since people usually do not put *all* their dimes in the meter), so he adds an additional *singular* condition (basically the same as the **unique**$\{u\}$ condition defined in (44) above), which requires the weak indefinite dref to store a singleton set relative to a plural info state. Obviously, this can work only in tandem with weak maximization: as we saw in section **3.4** above (see definition (51) in particular), strong maximization plus a singular condition **unique**$\{u\}$ requires model-level uniqueness and yields the Russellian analysis of definite description – and not the desired weak donkey indefinites. Van den Berg's meanings for weak and strong indefinites are provided in (130) below, rendered in a compositional PCDRT format for ease of comparison.

130. **Van den Berg's weak indefinites in PCDRT format:**

$$a^{\text{wk}:u} \rightsquigarrow \lambda P_{\text{et}}.\lambda P'_{\text{et}}. \; \textbf{max-wk}^u([\textbf{unique}\{u\}]; P(u); P'(u)),$$

where $\textbf{e} := se$ and $\textbf{t} := (st)((st)t)$

and $\textbf{unique}\{u\} := \lambda I_{st}. \; I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui').$

Van den Berg's strong indefinites in PCDRT format:

$$a^{\text{str}:u} \rightsquigarrow \lambda P_{\text{et}}.\lambda P'_{\text{et}}. \; \textbf{max-wk}^u(P(u); P'(u))$$

Van den Berg's analysis can account for simple instances of weak / strong donkey ambiguities, but it does not generalize to the mixed weak & strong donkey sentences analyzed in this chapter – and repeated in (131) and (132) below for convenience. The

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[49] For a similar definition of generalized quantification in PCDRT – which, crucially, does not include indefinites – see chapter **6** below.

[50] Analyzed in terms of his "collective" and "distributive" existential quantification respectively: see van den Berg (1994): 18-19 and van den Berg (1996a): 163-164.

reason is that van den Berg's weak donkey indefinites always introduce singleton sets, while the sentences in (131) and (132) are compatible with situations in which the value of the weak indefinites ($a^{\textbf{wk:}u_3}$ *credit card* and $an^{\textbf{wk:}u_3}$ *Arabian horse* respectively) is different for different values of the strong indefinites ($a^{\textbf{str:}u_2}$ *book* and $a^{\textbf{str:}u_2}$ *woman* respectively), i.e. in situations in which the credit cards vary from book to book and the horses from woman to woman. In the case of (131), for example, Van den Berg's analysis incorrectly pairs all the u_2-books with the same u_3-credit card, as shown in (133) below.

131. Every^{u_1} person who buys a$^{\textbf{str:}u_2}$ book on <u>amazon.com</u> and has a$^{\textbf{wk:}u_3}$ credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

132. Every^{u_1} man who wants to impress a$^{\textbf{str:}u_2}$ woman and who has an$^{\textbf{wk:}u_3}$ Arabian horse teaches her$_{u_2}$ how to ride it$_{u_3}$.

133. [*person*$\{u_1\}$]; $\textbf{max-wk}^{u_2}$ ([*book*$\{u_2\}$, *buy_on_amazon*$\{u_1, u_2\}$]);

$\textbf{max-wk}^{u_3}$ ([$\textbf{unique}\{u_3\}$, *credit_card*$\{u_3\}$, *have*$\{u_1, u_3\}$])

Moreover, extracting the strong indefinite out of its VP-conjunct and scoping it over the weak one is not possible because the resulting syntactic structure violates the Coordinate Structure Constraint[51]. As far as the analysis of the weak / strong donkey ambiguity is concerned, the definition of maximization in van den Berg (1996a): 139, (3.1)[52] is the same as the definition in van den Berg (1994)[53], so the above observations apply to it too.

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[51] That the Coordinate Structure Constraint does indeed apply to this kind of examples is shown by the two sentences in (i) and (ii) below, where the *every*-quantifiers cannot scope out of their own conjuncts to bind pronouns.

(i) #Every person who buys every^{u} *Harry Potter* book on <u>amazon.com</u> and gives it$_u$ to a friend must be a *Harry Potter* addict.

(ii) #Every boy who wanted to impress every^{u} girl in his class and who planned to buy her$_u$ a fancy Christmas gift asked his best friend for advice.

[52] See also the alternative formulation in van den Berg (1996a): 141, (3.2) and Lemma (3.3) for the relation between the two.

[53] Although it is not relevant for the weak / strong ambiguity problem, it is interesting to compare the two definitions. The definition of maximization in van den Berg (1996a): 139, (3.1) is different from the definition in van den Berg (1994) in two respect. First, the way in which new dref's are introduced is

In closing, note that van den Berg's system could in principle provide an alternative analysis of mixed weak & strong donkey sentences if it is extended with a form of anaphoric / relativized uniqueness of the kind defined in (134) below. If the uniqueness condition contributed by the weak indefinite is anaphoric / relativized to the strong indefinite, the value of the weak indefinite will be able to vary with the value of the strong indefinite; we will, therefore, be able to adequately translate the quantifier restrictor of sentence (131), as shown in (135) below.

134. $\mathbf{unique}_{u'}\{u\} := \lambda I_{st}.\ I \neq \emptyset \land \forall i_s \in I \forall i'_s \in I(u'i = u'i' \to ui = ui')$

135. $[person\{u_1\}];\ \mathbf{max\text{-}wk}^{u_2}\ ([book\{u_2\},\ buy\_on\_amazon\{u_1, u_2\}]);$

$\mathbf{max\text{-}wk}^{u_3}\ ([\mathbf{unique}_{u_2}\{u_3\},\ credit\_card\{u_3\},\ have\{u_1, u_3\}])$

Such an analysis, however, is more complex than the PCDRT one: the meaning of the weak indefinites involves a maximization operator, just like the meaning of the strong indefinites, and, in addition, the weak indefinites involve a relativized uniqueness condition that effectively neutralizes their maximization operator. Moreover, the **max-**

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different from the one we have chosen in PCDRT: it is the relation $I\{u\}J$ defined in (16b) above. As argued in section **3.2**, introducing new dref's by means of $\{u\}$ makes incorrect (overly strong) predictions with respect to mixed reading donkey sentences, so this does not amend the incorrect predictions made by the notion of maximization in van den Berg (1994).

The second difference, however, provides us with an interesting notion of pseudo-selective maximization. As the definition in van den Berg (1996a): 133, (2.6) shows, he requires maximality not only with respect to output sets of individuals, but also with respect to the output sets of info states, which results in a notion of maximization that is intermediate between selective and unselective maximization. The definition of $\mathbf{max\text{-}unsel}^u$ in (i) below is the PCDRT correspondent of this notion – compare it with the PCDRT definitions of selective (strong) maximization in (ii) below and unselective (strong) maximization in (iii).

(i) $\mathbf{max\text{-}unsel}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \land \forall K_{st}(([u]; D)IK \to uK \subseteq uJ \land K \subseteq J)$

(ii) $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \land \forall K_{st}(([u]; D)IK \to uK \subseteq uJ)$

(iii) $\mathbf{max}(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \to K \subseteq J)$

However, the $\mathbf{max\text{-}unsel}^u$ operator does not add to the expressive power of the PCDRT system: as the identity in (iv) below shows, it can be defined in terms of unselective maximization, much as we were able to define the DRT/FCS/DPL version of pseudo-selective generalized quantification (repeated in (vi) below) in terms of unselective generalized quantification (repeated in (v)) (see section **2** of chapter **4**).

(iv) $\mathbf{max\text{-}unsel}^u(D) = \mathbf{max}([u]; D)$

(v) $\mathbf{det}(D, D') := \lambda i_s.\ \mathbf{DET}(Di, \mathbf{Dom}(D'))$,
    where **DET** is the corresponding static determiner, $Di = \{j_s: Dij\}$ and $\mathbf{Dom}(D') := \{i_s: \exists j_s(Dij)\}$

(vi) $\mathbf{det}_u(D, D') := \mathbf{det}([u]; D, D')$

$\mathbf{wk}^u$ operator is more complex than the PCDRT $\mathbf{max}^u$ operator and its added complexity (i.e. the fact that it is maxima-based and not supremum-based) obscures the correspondence between dynamic maximization and static λ-abstraction. Therefore, I believe that the PCDRT account is theoretically preferable.

Moreover, empirically, it is not clear how to independently motivate the fact that the run-of-the-mill indefinite $a^{\mathbf{wk}:u_3}$ *credit card* in sentence (131) above contributes an anaphoric condition $\mathbf{unique}_{u_2}\{u_3\}$, since it is not anaphorically dependent in any obvious way on the strong indefine $a^{\mathbf{str}:u_2}$ *book*.

Appendix

A1. Plural CDRT (PCDRT): The Formal System

136. **PCDRT** (subscripts on terms represent their types)**.**

    a. Atomic conditions – type $(st)t$:

$$R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(R(u_1 i, \ldots, u_n i)),$$

        for any non-logical constant R of type $e^n t$,

            where $e^n t$ is defined as follows: $e^0 t := t$ and $e^{m+1} t := e(e^m t)$.

$$u_1 = u_2 := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u_1 i = u_2 i)$$

All atomic conditions are c-ideals.

    b. Atomic DRS's (DRS's containing exactly one atomic condition) – type $(st)((st)t)$

$$[R\{u_1, \ldots, u_n\}] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge R\{u_1, \ldots, u_n\}J$$

$$[u_1 = u_2] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge (u_1 = u_2)J$$

The domain $\mathbf{Dom}(D)$ and range $\mathbf{Ran}(D)$ of an atomic DRS D are c-ideals, where $\mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}$ and $\mathbf{Ran}(D) := \{J_{st}: \exists I_{st}(DIJ)\}$.

    c. Condition-level connectives (negation, closure, disjunction, implication), i.e. non-atomic conditions:

$$\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st}(H \neq \emptyset \wedge H \subseteq I \rightarrow \neg \exists K_{st}(DHK)),$$

        where D is a DRS (type $(st)((st)t)$),

    i.e. $\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st} \neq \emptyset (H \subseteq I \rightarrow H \notin \mathbf{Dom}(D))$.

If $\mathbf{Dom}(D)$ is a c-ideal (hence $\mathbf{Dom}(D) = \wp^+(\cup\mathbf{Dom}(D)))$, $\sim D$ is the unique maximal c-ideal disjoint from $\mathbf{Dom}(D)$: $\sim D = \wp^+(D_s^M\backslash\cup\mathbf{Dom}(D))$.

$$!D := \lambda I_{st}.\ \exists K_{st}(DIK),$$

i.e. $!D := \mathbf{Dom}(D)$.

If $\mathbf{Dom}(D)$ is a c-ideal, $\sim[\sim D] = !D$.

$$D_1 \vee D_2 := \lambda I_{st}.\ \exists K_{st}(D_1IK \vee D_2IK),$$

i.e. $D_1 \vee D_2 := \mathbf{Dom}(D_1)\cup\mathbf{Dom}(D_2)$.

$$D_1 \to D_2 := \lambda I_{st}.\ \forall H_{st}(D_1IH \to \exists K_{st}(D_2HK)),$$

i.e. $D_1 \to D_2 := \lambda I_{st}.\ D_1I \subseteq \mathbf{Dom}(D_2)$,   where $DI := \{J_{st}: DIJ\}$,

i.e. $D_1 \to D_2 := (\wp^+(D_s^M)\backslash\mathbf{Dom}(D_1)) \cup \{I_{st}\in\mathbf{Dom}(D_1): D_1I \subseteq \mathbf{Dom}(D_2)\}$.

d. Tests (generalizing 'atomic' DRS's):

$$[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge C_1J \wedge \ldots \wedge C_mJ^{\,54},$$

where $C_1, \ldots, C_m$ are conditions (atomic or not) of type $(st)t$.

The domain $\mathbf{Dom}(D)$ and range $\mathbf{Ran}(D)$ of any test D is a c-ideal if all the conditions are c-ideals.

e. DRS-level connectives (dynamic conjunction):

$$D_1; D_2 := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(D_1IH \wedge D_2HJ),$$

where D_1 and D_2 are DRSs (type $(st)((st)t)$)

f. Quantifiers (random assignment of value to a dref):

$$[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s\in I(\exists j_s\in J(i[u]j)) \wedge \forall j_s\in J(\exists i_s\in I(i[u]j))$$

If a DRS D has the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, we have that:

i. $\mathbf{Ran}(D) = C_1\cap \ldots \cap C_m = \wp^+((\cup C_1)\cap \ldots \cap(\cup C_m))$;

ii. $\mathbf{Dom}(D) = \wp^+(\{i_s: \exists j_s(i[u_1, \ldots, u_n]j \wedge j\in(\cup C_1)\cap \ldots \cap(\cup C_m))\})$.

Since $i[u_1, \ldots, u_n]j$ is reflexive, $\mathbf{Ran}(D)\subseteq\mathbf{Dom}(D)$.

g. Selective maximization:

$$\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(I[u]H \wedge DHJ) \wedge \forall K_{st}(\exists H_{st}(I[u]H \wedge DHK) \to uK\subseteq uJ),$$

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[54] Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction as follows:

$[C_1, \ldots, C_m] := \lambda I_{st}J_{st}.\ ([C_1]; \ldots; [C_m])IJ$, where $[C] := \lambda I_{st}J_{st}.\ I{=}J \wedge CJ$.

where D is a DRS of type $(st)((st)t)$,

i.e. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \land \forall K_{st}(([u]; D)IK \rightarrow uK \subseteq uJ)$

The $\mathbf{max}^u$ operator does not preserve the c-ideal structure of the domain or range of the embedded DRS.

Multiply embedded $\mathbf{max}^u$ operators can be reduced as follows:

$$\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D'),$$

if: i. u is not reintroduced in D';

ii. $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

iii. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

h. Unselective maximization:

$$\mathbf{max}(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ\ \&\ \forall K_{st}(DIK \rightarrow K \subseteq J)$$

i. Selective Generalized Determiners (non-atomic conditions):

$$\mathbf{det}_u(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \land \mathbf{DET}(u[D_1 I],\ u[(D_1; D_2)I]),$$

where $u[DI] := \cup\{uJ: ([u \mid \mathbf{unique}\{u\}]; D)IJ\}$

and $\mathbf{unique}\{u\} := \lambda I_{st}.\ I \neq \emptyset \land \forall i_s \in I \forall i'_s \in I(ui = ui')$

and $\mathbf{DET}$ is the corresponding static determiner.

The lexical entries for selective generalized determiners are:

$$\mathbf{det}^u \rightsquigarrow \lambda P'_{et}.\ \lambda P_{et}.\ [\mathbf{det}_u(P'(u), P(u))], \quad \text{where } \mathbf{e} := se \text{ and } \mathbf{t} := (st)((st)t)$$

j. Unselective Generalized Determiners (non-atomic conditions):

$$\mathbf{det}(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \land \mathbf{DET}(\mathbf{max}[D_1 I],\ \mathbf{max}[(D_1; [!D_2])I]),$$

where $\mathbf{max}[DI] := \cup\{J_{st}: \mathbf{max}(D)IJ\}$

and $\mathbf{DET}$ is the corresponding static determiner.

The lexical entries for unselective generalized determiners are:

$$\mathbf{det} \rightsquigarrow \lambda D'_t.\ \lambda D_t.\ [\mathbf{det}(D', D)], \quad \text{where } \mathbf{t} := (st)((st)t).$$

k. Truth: A DRS D (type $(st)((st)t)$) is true with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$, i.e. iff $I \in \mathbf{Dom}(D)$ (or, equivalently, $I \in !D$).

We supplement the definition of basic PCDRT in (5) with the list of abbreviations in (137) below.

137. **a. Additional abbreviations – DRS-level quantifiers (multiple random assignment, existential quantification, maximal existential quantification)**:

$$[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$$

$$\exists u(D) := [u]; D$$

$$\exists^{\mathbf{m}} u(D) := \mathbf{max}^u(D)$$

b. Additional abbreviations – condition-level quantifiers (universal quantification):

$$\forall u(D) := {\sim}([u]; {\sim}D),$$

i.e. $\forall u(D) := {\sim}\exists u({\sim}D)$.

c. Additional abbreviations – DRS's (a.k.a. linearized 'boxes'):

$$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. \, ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])IJ,$$

where $C_1, \ldots, C_m$ are conditions (atomic or not),

i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}. \, I[u_1, \ldots, u_n]J \wedge C_1J \wedge \ldots \wedge C_mJ$.

d. Additional abbreviations – negation based condition-level connectives (N-closure, N-disjunction, N-implication):

N-Closure: $\mathord{\text{!}}D := {\sim}[{\sim}D]$

N-Disjunction: $D_1 \vee\kern-0.9ex\vee D_2 := {\sim}[{\sim}D_1, {\sim}D_2]$

If **Dom**(D_1) and **Dom**(D_2) are c-ideals, then $D_1 \vee\kern-0.9ex\vee D_2 = \wp^+(\cup(\mathbf{Dom}(D_1)\cup\mathbf{Dom}(D_2)))$. Therefore, if **Dom**$(D_1)$ and **Dom**(D_2) are c-ideals, we have that $D_1 \vee D_2 \subseteq D_1 \vee\kern-0.9ex\vee D_2$.

N-Implication: $D_1 \Rightarrow\kern-1.2ex\Rightarrow D_2 := {\sim}(D_1; [{\sim}D_2])$

Note that $\forall u(D) = [u] \Rightarrow\kern-1.2ex\Rightarrow D$.

If $D_1 = [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ and $C_1, \ldots, C_m$, **Dom**(D_2) are c-ideals, then $D_1 \Rightarrow\kern-1.2ex\Rightarrow D_2 = \wp^+(\{i_s: \forall j_s(i[u_1, \ldots, u_n]j \wedge j\in(\cup C_1)\cap \ldots \cap(\cup C_m) \to j\in(\cup\mathbf{Dom}(D_2)))\})$. Therefore, if $D_1 = [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ and $C_1, \ldots, C_m$, **Dom**(D_2) are c-ideals, we have that $D_1 \Rightarrow\kern-1.2ex\Rightarrow D_2 \subseteq D_1 \to D_2$.

If, in addition, we can establish that $\forall i_s j_s(i[u_1, \ldots, u_n]j \wedge j\in(\cup C_1)\cap \ldots \cap(\cup C_m) \to j\in(\cup\mathbf{Dom}(D_2)))$, then $D_1 \to D_2 = D_1 \Rightarrow\kern-1.2ex\Rightarrow D_2 = \wp^+(\mathbf{D}_s^M)$.

The definitions of the dynamic universal and existential quantifiers in (137b-c) above preserve their DPL / CDRT+GQ partial duality if we quantify over DRS's whose domains are c-ideals.

138. $\sim\exists u(D) = \forall u(\sim D)$,

        if $\mathbf{Dom}(D)$ is a c-ideal (hence $\mathbf{Dom}(D) = \mathbf{Dom}([\sim[\sim D]])$).

Just as in CDRT+GQ, the partial duality in (138) can be generalized by means of N-implication as shown in (139) below.

139. $\sim\exists u(D; D') = \forall u(D \Rightarrow [\sim D'])$,

        if: **a.** $\mathbf{Dom}(D')$ is a c-ideal (hence $\mathbf{Dom}(D) = \mathbf{Dom}([\sim[\sim D]])$);

          b. D preserves c-ideals under pre-images [55,56].

A2. Simplifying 'Max-under-Max' Representations

The general version of the theorem is stated in (140) below.

140. **Simplifying 'max-under-max' representations:**

        $\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D')$,

   if the following three conditions obtain:

        a. u is not reintroduced in D';

        b. $\forall I_{st}\forall X_{et} (\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ))$;

        c. $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$[57].

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[55] D preserves c-ideals under pre-images iff if $\Im'$ is a c-ideal, then $\Im=\{I_{st}: \exists J_{st}(DIJ \wedge J\in\Im'\}$ is a c-ideal.

[56] Proof: The reader can easily check that the following identities hold: $\forall u([D \Rightarrow [\sim D']) = \forall u([\sim(D; [\sim[\sim D']]) = \forall u([\sim(D; D')]) = \sim([u]; [\sim[\sim(D; D')]]) = \sim([u]; D; D') = \sim\exists u(D; D')$. □

[57] Proof:

$\mathbf{max}^u(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ \wedge \forall K(\exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HK) \rightarrow uK\subseteq uJ)$

We have that $\forall I_{st}\forall X_{et}(\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ))$ (condition (140b)). Hence:

$\mathbf{max}^u(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ \wedge \forall K(\exists H(([u]; D)IH \wedge ([u']; D')HK) \rightarrow uK\subseteq uJ)$

$= \exists H(([u]; D)IH \wedge \mathbf{max}^{u'}(D')HJ \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK\subseteq uJ)$.

We have that $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$ (condition (140c)). Hence:

Given that, by condition (140a), u is not reintroduced in D', the second condition (140b) can be further reduced to the condition in (141) below.

141. Given (140a), condition (140b) is equivalent to:

$$\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D')).$$

Moreover, based on the two facts in (142) below, we can further simplify condition (140c).

142. a. If D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$,

then $\forall I_{st} \forall J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$[58].

   b. If $\forall I_{st} \forall J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$,

then $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$[59].

$\mathbf{max}^u(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D)IH \wedge ([u']; D'; \mathbf{max}^{u'}(D'))HJ) \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ)$

$= \exists H(([u]; D; [u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ) \quad \wedge \quad \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ)$

$= \exists H(([u]; D; [u']; D')IH \wedge \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uJ) \wedge \mathbf{max}^{u'}(D')HJ)$

Since u is not reintroduced in D' (condition (140a)), we have that $uJ=uH$. Hence:

$\mathbf{max}^u(D; \mathbf{max}^{u'}(D'))IJ = \exists H(([u]; D; [u']; D')IH \wedge \forall K(([u]; D; [u']; D')IK \rightarrow uK \subseteq uH) \wedge \mathbf{max}^{u'}(D')HJ)$

$= \exists H(\mathbf{max}^u(D; [u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ) = (\mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D'))IJ.$ □

[58] Proof: $([u']; D') := \lambda I_{st} J_{st}. I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J$. Therefore:

$([u']; D')IJ \wedge ([u']; D')JK$ iff $I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J \wedge J[u', u_1, \ldots, u_n]K \wedge C_1 K \wedge \ldots \wedge C_m K$

iff $I[u', u_1, \ldots, u_n]J \wedge C_1 J \wedge \ldots \wedge C_m J \wedge I[u', u_1, \ldots, u_n]K \wedge C_1 K \wedge \ldots \wedge C_m K$ iff $([u']; D')IJ \wedge ([u']; D')IK$. □

[59] Proof:

Claim1: If $\forall I_{st} J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$, then $[u']; D' = [u']; D'; [u']; D'$ (note that the premise ensures that the relation denoted by $[u']; D'$ is a KD45 kind of accessibility relation).

Proof of Claim1: $([u']; D'; [u']; D')IJ$ iff $\exists H(([u']; D')IH \wedge ([u']; D')HJ)$ iff (by the premise)

$\exists H(([u']; D')IH \wedge ([u']; D')IJ)$ iff $\exists H(([u']; D')IH) \wedge ([u']; D')IJ$ iff $([u']; D')IJ$. □

$([u']; D'; \mathbf{max}^{u'}(D'))IJ$ iff $\exists H(([u']; D')IH \wedge \mathbf{max}^{u'}(D')HJ)$ iff

$\exists H(([u']; D')IH \wedge ([u']; D')HJ \wedge \forall K(([u']; D')HK) \rightarrow u'K \subseteq u'J))$ iff (by the premise)

$\exists H(([u']; D')IH \wedge ([u']; D')HJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff

$([u']; D'; [u']; D')IJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff (by **Claim1**)

$([u']; D')IJ \wedge \forall K(([u']; D')IK) \rightarrow u'K \subseteq u'J)$ iff $\mathbf{max}^{u'}(D')IJ$. □

Thus, we obtain the corollary given in the main text of the chapter, repeated in (143) below.

143. **Simplifying 'max-under-max' representations (corollary):**

$$\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D'),$$

if the following three conditions obtain:

a. u is not reintroduced in D';

b. $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

Chapter 6. Structured Nominal Reference: Quantificational Subordination

1. Introduction

The present chapter proposes an account of the contrast between the interpretations of the discourses in (1) and (2) below from Karttunen (1976).

1. **a.** Harvey courts a^u girl at every convention.

    b. She$_u$ is very pretty.

2. **a.** Harvey courts a^u girl at every convention.

    b. She$_u$ always comes to the banquet with him.

    [**c.** The$_u$ girl is usually also very pretty.]

The initial sentence (1a/2a) by itself is ambiguous between two readings (i.e. two quantifier scopings): it "can mean that, at every convention, there is some girl that Harvey courts or that there is some girl that Harvey courts at every convention. […] Harvey always courts the same girl […] [or] it may be a different girl each time" (Karttunen 1976: 377).

The contrast between the continuations in (1b) and (2b) is that the former allows only for the "same girl" reading of sentence (1a/2a), while the latter is also compatible with the "possibly different girls" reading.

Discourse (1) raises the following question: how can we capture the fact that a *singular anaphoric pronoun* in sentence (1b) can interact with and disambiguate *quantifier scopings* in sentence (1a)?

To see that it is indeed quantifier scopings that are disambiguated, substitute *exactly one^u girl* for *a^u girl* in sentence (1a); this will yield two truth-conditionally independent scopings: (**i**) *exactly one girl>>every convention*, which is true in a situation in which Harvey courts more than one girl per convention, but there is exactly one (e.g. Faye Dunaway) that he never fails to court, and (**ii**) *every convention>>exactly one girl*.

To see that number morphology on the pronoun *she* is indeed crucial, consider the discourse in (3) below, where the (preferred) relative scoping of *every convention* and *a girl* is the opposite of the one in discourse (1).

3.   **a.** Harvey courts a^u girl at every convention. **b.** They$_u$ are very pretty.

Discourse (2) raises the following questions. First, why is it that adding an adverb of quantification, i.e. *always*/usually, makes both readings of sentence (2a) available?

Moreover, on the newly available reading of sentence (2a), i.e. the *every convention>>a girl* scoping, how can we capture the intuition that the singular pronoun *she* and the adverb *always* in sentence (2b) elaborate on the quantificational dependency between conventions and girls introduced in sentence (2a), i.e. how can we capture the intuition that we seem to have simultaneous *anaphora to two quantifier domains* and to the *quantificational dependency* between them?

The phenomenon instantiated by discourses (1) and (2) is subsumed under the more general label of *quantificational subordination* (see for example Heim 1990: 139, (2)), which covers a variety of phenomena involving interactions between generalized quantifiers and morphologically singular cross-sentential anaphora.

One of the main goals of this chapter is to show that the PCDRT system introduced in chapter **5** and motivated by mixed reading (weak & strong) donkey sentences receives independent empirical justification based on the phenomenon of quantificational subordination.

To account for quantificational subordination, we will only need to modify the definition of selective generalized quantification. As already remarked in section **3.5** of chapter **5**, there are two main strategies for the definition of generalized quantification in a dynamic system; the previous chapter explored one of them, namely the one that is closer to the DRT / FCS / DPL-style definition, while this chapter explores the other,

formally more complex strategy, namely the one that is closer to van den Berg's definition of generalized quantification[1].

The chapter is structured as follows. Section **2** informally presents the PCDRT analysis of the Karttunen discourses in (1) and (2) above. Section **3** introduces and justifies the new definition of dynamic generalized quantification that enables us to account for quantificational subordination. Section **4** presents the formal PCDRT analysis of the Karttunen discourses based on the novel notion of dynamic quantification introduced in section **3**. Finally, section **6** briefly compares the PCDRT analysis of quantificational subordination with alternative accounts.

The appendix to the chapter introduces generalized selective distributivity, i.e. selective distributivity generalized to arbitrary types, and studies some of the formal properties of DRS-level selective distributivity.

The presentation of the PCDRT analysis of quantificational subordination in sections **3** and **4** repeats some of the basic notions and ideas introduced in the previous chapters. I hope that the resultant global redundancy is outweighed by the local improvement in readability.

2. *Structured Anaphora to Quantifier Domains*

This section shows semi-formally that the semantic framework proposed in the previous chapter (chapter **5**), i.e. PCDRT, enables us to account for discourses (1) and (2) above. In particular, the main proposal will be that compositionally assigning natural language expressions *finer-grained semantic values* (finer grained than the usual meanings assigned in static Montague semantics) enables us to capture the interaction between generalized quantifiers, singular pronouns and adverbs of quantification exhibited by the contrast between the interpretations of (1) and (2).

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[1] The fact that we are able to reformulate the two kidns of definitions of dynamic generalized quantification within the same type-logical system greatly facilitates their formal and empirical comparison, which (unfortunately) I must leave for a different occasion.

Just as in the previous chapter, the PCDRT semantic values are finer-grained in a very precise sense: the 'meta-types' $\mathbf{e}$ and $\mathbf{t}$ assigned to the denotations of the two kinds of 'saturated' expressions (names and sentences respectively) are assigned types that are complex than the corresponding types in static extensional Montague semantics. That is, the 'meta-type' $\mathbf{t}$ abbreviates $(st)((st)t)$, i.e. a sentence is interpreted as a DRS, and the 'meta-type' $\mathbf{e}$ abbreviates se, i.e. a name is interpreted as a dref. The denotation of a common noun like *girl* will still be of type $\mathbf{et}$ – see (4) below – and the denotation of a selective generalized determiner like every will still be of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$.

4. *girl* $\leadsto \lambda v_{\mathbf{e}}.\ [girl_{et}\{v\}]$, i.e. *girl* $\leadsto \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge girl_{et}\{v\}J$

Accounting for *cross-sentential* phenomena in semantic terms (as opposed to purely / primarily pragmatic terms) requires some preliminary justification. First, the same kind of finer-grained semantic values are independently motivated by intra-sentential phenomena, as shown by the account of mixed weak & strong donkey sentences in the previous chapter.

Second, the phenomenon instantiated by discourses (1) and (2) is as much intra-sentential as it is cross-sentential. Note that there are four separate components that come together to yield the contrast in interpretation between (1) and (2): (**i**) the generalized quantifier *every convention*, (**ii**) the indefinite *a girl*, (**iii**) the singular number morphology on the pronoun *she* and (**iv**) the adverb of quantification *always/usually*. To derive the intuitively correct interpretations for (1) and (2), we have to attend to both the cross-sentential connections *a girl–she* and *every convention–always/usually* and the intra-sentential interactions *every convention–a girl* and *always–she*.

I conclude that an account of the contrast between (1) and (2) that involves a revamping of semantic values has sufficient initial plausibility to make its pursuit worthwhile.

The PCDRT plural info states enable us to encode discourse reference to both *quantifier domains*, i.e. *values*, and *quantificational dependencies*, i.e. *structure*, as shown in the matrix in (5) below.

5. Info State I	…	u		u'		…
i_1	…	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	…
i_2	…	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	…
i_3	…	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	…
…	…	…		…		…

Quantifier domains (sets):
$\{x_1, x_2, x_3, …\}, \{y_1, y_2, y_3, …\}$

Quantifier dependencies (relations):
$\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, …\}$

Just as before, the values are the sets of objects that are stored in the *columns* of the matrix, e.g. a dref u for individuals stores a set of individuals relative to a plural info state, since u is assigned an individual by each assignment (i.e. row). The structure is *distributively* encoded in the *rows* of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment.

Thus, plural info states enable us to pass information about both quantifier domains and quantificational dependencies across sentential / clausal barriers, which is exactly what we need to account for the interpretation of discourses (1) and (2). More precisely, we need the following two ingredients.

First, we need a suitable interpretation for *selective generalized determiners*, e.g. *every* in (1a/2a), which needs to do two things: (**i**) it stores in the plural info state the *restrictor and nuclear scope sets* of individuals that are related by the generalized determiner; (**ii**) it stores in the plural info state the *quantificational dependencies* between the individuals in the restrictor and / or nuclear scope set and any other quantifiers or indefinites in the restrictor or nuclear scope of the quantification.

For example, the indefinite *a girl* in (1a/2a) is in the nuclear scope of the *every*-quantification, while in the usual donkey examples (*Every farmer who owns a^u donkey beats it$_u$*), we have an indefinite in the restrictor of the quantification.

Given that a plural info state stores (**i**) sets of individuals and (**ii**) dependencies between such sets, both of them are available for subsequent anaphoric retrieval, e.g.

always and *she* in (2b) are simultaneously anaphoric to (**i**) *every convention* and *a girl* on the one hand and (**ii**) the dependency between conventions and girls on the other hand.

The second ingredient is a suitable interpretation of *singular number morphology* on pronouns, e.g. *she* in (1b) and (2b), that can interact with quantifiers and indefinites in the previous discourse, e.g. *every convention* and *a girl* in (1a/2a), and with quantifiers in the same sentence, e.g. the adverb *always* in (2b).

In particular, I will take the singular number morphology on *she* in (1b) to require that the set of individuals stored by the current plural info state relative to u be a singleton. This set of individuals is introduced by the indefinite *a girl* in (1a) – irrespective of whether the indefinite has wide or narrow scope relative to *every convention*. This is possible because we use plural info states, by means of which we store sets of individuals and pass them across sentential boundaries – we can thus constrain their cardinality by subsequent anaphoric elements like *she*.

If the indefinite *a girl* has narrow scope relative to *every convention*, the singleton requirement contributed by *she* applies to the set of all girls that are courted by Harvey at some convention or other. Requiring this set to be a singleton boils down to removing from consideration all the plural information states that would satisfy the narrow scope *every convention>>a girl*, but not the wide scope *a convention>>every girl*.

We therefore derive the intuition that, irrespective of which quantifier scoping we assume for sentence (1a), any plural info state that we obtain after a successful update with sentence (1b) is bound to satisfy the representation in which the indefinite a^u *girl* (or a quantifier like *exactly one^u girl*) takes wide scope.

In the case of discourse (2) however, the adverb of quantification *always* in (2b) – which is anaphoric to the nuclear scope set introduced by *every convention* in (2a) – can take scope over the singular pronoun *she*. In doing so, the adverb 'breaks' the plural info state containing all the conventions into smaller sub-states, each storing a particular convention. Then, the singleton requirement contributed by singular morphology on *she$_u$* is enforced locally, relative to these sub-states, and not globally, relative to the whole

plural info state. We therefore end up requiring that the courted girl is unique *per convention* and not across the board (the latter option being instantiated by discourse (1)).

The following section will introduce, explain and motivate the new definition of selective generalized quantification in PCDRT – and the corresponding (minor) adjustments of the meanings of indefinites, pronouns and definites.

3. Redefining Generalized Quantification

We turn now to the definition of *selective* generalized quantification in PCDRT.

3.1. Four Desiderata

The definition has to satisfy four desiderata, the first three of which are about anaphoric connections that can be established *internally*, within the generalized quantification (i.e. between antecedents in the restrictor and anaphors in the nuclear scope) and the last of which is about anaphora that can be established *externally* (i.e. between antecedents introduced by or within the quantification and anaphors that are outside the quantification).

First, we want our definition to be able to account for the fact that anaphoric connections between the restrictor and the nuclear scope of the quantification can in fact be established, i.e. we want to account for donkey anaphora.

Second, we want to account for such anaphoric connections while avoiding the proportion problem which *unselective* quantification (in the sense of Lewis 1975) runs into, i.e. we need the generalized determiner to relate *sets of individuals* (i.e. sets of objects of type *e*) and not sets of 'assignments' (i.e. sets of objects of type *s*).

Sentence (6) below provides a typical instance of the proportion problem: intuitively, (6) is false in a situation in which there are ten farmers, nine have a single donkey each that they do not beat, while the tenth has twenty donkeys and he is busy beating them all. But the unselective formalization of *most*-quantification as quantification over 'assignments' incorrectly predicts that (6) is true in the above situation

because more than half of the *<farmer, donkey>* pairs (twenty out of twenty-nine) are such that the farmer beats the donkey.

6. Most farmers who own a^u donkey beat it$_u$.

The third desideratum is that the definition of selective generalized quantification should be compatible with both strong and weak donkey readings: we want to allow for the different interpretations associated with the donkey anaphora in (7) (Heim 1990) and (8) (Pelletier & Schubert 1989) below.

7. Most people that owned a^u slave also owned his$_u$ offspring.
8. Every person who has a^u dime will put it$_u$ in the meter.

Sentence (7) is interpreted as asserting that most slave-owners were such that, for *every* (strong reading) slave they owned, they also his offspring. Sentence (8) is interpreted as asserting that every dime-owner puts *some* (weak reading) dime of her/his in the meter.

We also need to allow for mixed weak & strong relative-clause sentences like the one in (9) below (i.e. the kind of sentence we have analyzed in chapter **5**). Sentence (9) is interpreted as asserting that, for any person that is a computer buyer and a credit card owner, for *every* computer s/he buys, s/he uses *some* credit card of her/his to pay for the computer.

9. Every person who buys a^u computer and has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_u$.

Thus, the first three, internal desiderata simply recapitulate the main points we have made in chapters **2** through **5** and they are only meant to ensure that the new definition of selective generalized quantification preserves all welcome the results we have previously obtained.

The fourth desideratum, however, is about the novel phenomenon of quantificational subordination we have introduced by means of the discourses in (1) and (2) above. These discourses indicate that selective generalized determiners need to make anaphoric information externally available, i.e. they need to introduce dref's for the *restrictor and nuclear scope sets of individuals* related by the generalized determiner that

can be retrieved by subsequent anaphora. Furthermore, we also need to make available for anaphoric take-up the *quantificational dependencies* between different quantifiers and/or indefinites (see the discussion of discourse (2) in the previous section).

In more detail, generalized quantification supports anaphora to two sets: (**i**) the maximal set of individuals satisfying the restrictor DRS, i.e. the *restrictor set*, and (**ii**) the maximal set of individuals satisfying the restrictor and nuclear scope DRS's, i.e. the *nuclear scope set*[2]. Note that the latter set is the nuclear scope set that emerges as a consequence of the *conservativity* of natural language quantification – and, as Chierchia (1995) and van den Berg (1996a) (among others) observe, we need to build conservativity into the definition of dynamic quantification to account for the fact that the nuclear scope DRS can contain anaphors dependent on antecedents in the restrictor[3].

The discourse in (10) below exemplifies anaphora to nuclear scope sets: sentence (10b) is interpreted as asserting that the people that went to the beach are the students that left the party after 5 am (which, in addition, formed a majority of the students at the party).

10. **a.** Most^u students left the party after 5 am.

    b. They$_u$ went directly to the beach.

The discourses in (11) and (12) below exemplify anaphora to restrictor sets. Both examples involve determiners that are right downward monotonic, which strongly favor anaphora to restrictor sets as opposed to anaphora to nuclear scope sets.

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[2] Throughout the paper, I will ignore anaphora to complement sets, i.e. sets obtained by taking the complement of the nuclear scope relative to the restrictor, e.g. *Very few students were paying attention to the lecture. They were hungover.*

[3] Thus, in a sense, Chierchia (1995) and van den Berg (1996a) suggest that the conservativity universal proposed in Barwise & Cooper (1981) should be replaced by / derived from an 'anaphoric' universal that would have the form: the meanings of natural language determiners have to be such that they allow for anaphoric connections between the restrictor and nuclear scope of the quantification (I am indebted to Roger Schwarzschild, p.c., for making this observation clearer to me).

In a dynamic system, the 'anaphoric' universal boils down to the requirement that the nuclear scope update be interpreted relative to the info state that is the output of the restrictor update. And the two strategies of defining dynamic generalized quantification explored in chapter **5** and chapter **6** respectively are two different ways of implementing this requirement (see in particular the discussion in section **3.5** of chapter **5** and section **1** of chapter **6**, i.e. the present chapter).

11. **a.** No^u student left the party later than 10 pm.

    b. They$_u$ had classes early in the morning.

12. **a.** Very few^u people with a rich uncle inherit his fortune.

    b. Most of them$_u$ don't.

Consider (11) first: any successful update with a *no^u* quantification ensures that the nuclear scope set is empty and anaphora to it is therefore infelicitous; the only anaphora possible in (11) is anaphora to the restrictor set. The same thing happens in (12) albeit for a different reason: anaphora to the restrictor set is the only possible one because anaphora to the nuclear scope set would yield a contradiction, namely: most of the people with a rich uncle that inherit his fortune don't inherit his fortune.

Thus, a selective generalized determiner will receive a translation of the form provided in (13) below, which is in the spirit – but fairly far from the letter – of van den Berg (1996a) (see his definition (4.1) on p. 149).

13. $\text{det}^{u,u' \sqsubseteq u} \rightsquigarrow \lambda P_{\textbf{et}}.\lambda P'_{\textbf{et}}.\ \textbf{max}^u(_{\langle u \rangle}(P(u)));\ \textbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')));\ [\textbf{DET}\{u,\ u'\}]$

The translation in (13) can be semi-formally paraphrased as follows.

First note that, as expected, $\text{det}^{u,u' \sqsubseteq u}$ relates a restrictor dynamic property $P_{\textbf{et}}$ and a nuclear scope dynamic property $P'_{\textbf{et}}$. When these dynamic properties are applied to individual dref's, i.e. $P(u)$ and $P'(u')$, we obtain a restrictor DRS $P(u)$ and a nuclear scope DRS $P'(u')$ of type $\textbf{t} := (st)((st)t)$.

Which brings us to the three sequenced updates in (13), namely $\textbf{max}^u(_{\langle u \rangle}(P(u)))$, $\textbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')))$ and $[\textbf{DET}\{u,\ u'\}]$. The first update is formed out of three distinct pieces, namely the restrictor DRS $P(u)$, the operator $_{\langle u \rangle}(\ldots)$ which takes scope over the restrictor DRS and, finally, the operator $\textbf{max}^u(\ldots)$ that takes scope over everything else. The second update is formed out of the same basic pieces, i.e. the restrictor DRS $P'(u')$, the operator $_{\langle u' \rangle}(\ldots)$ and the operator $\textbf{max}^{u' \sqsubseteq u}(\ldots)$. The last update is a test containing the static condition $\textbf{DET}\{u,\ u'\}$ contributed by the particular determiner under consideration and which relates two individual dref's u and u'.

These are the individual dref's introduced by the generalized determiner, more exactly by the operators $\mathbf{max}^u(\ldots)$ and $\mathbf{max}^{u'\sqsubseteq u}(\ldots)$: they introduce the dref's u and u' respectively and u stores the restrictor set of individuals, while u' stores the nuclear scope set of individuals obtained via conservativity, which is encoded by the superscripted inclusion $u'\sqsubseteq u$.

The restrictor set u is the maximal set of individuals (maximality is contributed by $\mathbf{max}^u(\ldots)$) such that, when we take each u-individual separately (distributivity is contributed by $_{\langle u\rangle}(\ldots)$), this individual satisfies the restrictor dynamic property (i.e. $P(u)$).

The nuclear scope set u' is obtained in a similar way except for the requirement that it is the maximal structured subset of the restrictor set u (i.e. $\mathbf{max}^{u'\sqsubseteq u}(\ldots)$). The notion of structured subset $u'\sqsubseteq u$ is introduced and discussed in the very next section.

We finally reach the third update, which tests that the restrictor set u and the nuclear scope set u' stand in the relation denoted by the corresponding static determiner $\mathbf{DET}$ (i.e. $\mathbf{DET}\{u, u'\}$).

As already mentioned, the three updates in (13) are sequenced, i.e. dynamically conjoined. Recall that dynamic conjunction ';' is interpreted as relation composition, as shown in (14) below.

14. $D_1; D_2 := \lambda I_{st}.\lambda J_{st}. \exists H_{st}(D_1 IH \wedge D_2 HJ)$ [4],

        where D_1 and D_2 are DRS's of type $\mathbf{t} := (st)((st)t)$.

The remainder of this section is dedicated to formally spelling out the meaning of generalized determiners in (13) above and, also, the PCDRT meanings for indefinite articles and pronouns.

We will need: (**i**) two operators over plural info states, namely a *selective maximization* operator $\mathbf{max}^u(\ldots)$ and a *selective distributivity* operator $_{\langle u\rangle}(\ldots)$, which will

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[4] Also, recall the difference between dynamic conjunction ';', which is an abbreviation, and the official, classical static conjunction '$\wedge$'.

enable us to define updates of the form $\mathbf{max}^{u}(_{\langle u \rangle}(\ldots))$ and (**ii**) a notion of *structured subset* between two sets of individuals that requires the subset to preserve the quantificational dependencies, i.e. the structure, associated with the individuals in the superset – which will enable us to define $u' \sqsubseteq u$ and, thereby, updates of the form $\mathbf{max}^{u' \sqsubseteq u}(\ldots)$.

3.2.   Structured Inclusion

Let us start with the notion of structured subset. Recall that plural info states store both values (quantifier domains) – in the columns of the matrix – and structure (quantifier dependencies) – in the rows of the matrix. We can therefore define two different notions of inclusion: one that takes into account only values, i.e. value inclusion, and one that takes into account both values and structure, i.e. structured inclusion. Let us examine them in turn.

Requiring a dref u_3 to simply be a *value* subset of another dref u_1 relative to an info state I is defined as shown in (15) below. For example, the info state I in (16) satisfies the condition $u_3 \subseteq u_1$ because $u_3 I = \{x_1, x_2, x_3\} \subseteq u_1 I = \{x_1, x_2, x_3, x_4\}$.

15. $u_3 \subseteq u_1 := \lambda I_{st}.\ u_3 I \subseteq u_1 I$

16. Info State I

	u_1	u_2	u_3
i_1	x_1	y_1	x_1
i_2	x_2	y_2	x_3
i_3	x_3	y_3	x_1
i_4	x_4	y_4	x_2

As the info state I in (16) shows, value inclusion disregards structure completely: the correlation / dependency between the u_1-individuals and the u_2-individuals, i.e. the relation $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, <x_4, y_4>\}$, is lost in going from the u_1-superset to the u_3-subset: as far as u_3 and u_2 are concerned, x_1 is still correlated with y_1, but it is now also correlated with y_3; moreover, x_2 is now correlated with y_4 and x_3 with y_2.

If we were to use the notion of value subset in (15) to define dynamic generalized quantification, we would make incorrect predictions. To see this, consider the discourse

in (17) below, where u_1 stores the set of conventions[5] and u_2 stores the set of corresponding girls. Furthermore, assume that *every^{u_1} convention* takes scope over *a^{u_2} girl* and that the correlation between u_1-conventions and courted u_2-girls is the one represented in (16) above.

    17. **a.** Harvey courts a^{u_2} girl at every^{u_1} convention.

       b. She$_{u_2}$ usually$^{u_3 \subseteq u_1}$ comes to the banquet with him.

Intuitively, the adverb *usually* is anaphoric to the set of conventions and sentence (17b) is interpreted as asserting that at most conventions, the girl courted by Harvey *at that convention* comes to the banquet with him. The dref u_3 in (16) above does store most conventions (three out of four), but it does not preserve the correlation between conventions and girls established in sentence (17a).

Note that a similarly incorrect result is achieved for donkey sentences like the one in (18) below: the restrictor of the quantification introduces a dependency between all the donkey-owning u_1-farmers and the u_2-donkeys that they own; the nuclear scope set u_3 needs to contain most u_1-farmers, but in such a way that the correlated u_2-donkeys remain the same. That is, the nuclear scope set contains a *most*-subset of donkey owning farmers that beat *their respective donkey(s)*. The info state in (16) above and the notion of value-only inclusion in (15) are yet again inadequate.

    18. Most$^{u_1, u_3 \subseteq u_1}$ farmers who own a^{u_2} donkey beat it$_{u_2}$.

Thus, to capture the intra-sentential and cross-sentential interaction between anaphora and quantification, we need a notion of *structured inclusion*, i.e. a notion of *value inclusion* that *preserves structure*. That is, the only way to go from a superset to a subset should be by *discarding rows in the matrix*: in this way, we are guaranteed that the subset will contain *only* the dependencies associated with the superset (but not necessarily *all* dependencies – see below).

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[5] Note that, in the case of a successful *every*-quantification, the restrictor and the nuclear scope sets end up being identical (both with respect to value and with respect to structure – for more details, see (65) below and its discussion), so, for simplicity, I conflate them into dref u_1.

Following van den Berg (1996a), I will introduce a dummy / exception individual #
that I will use as a tag for the rows in the matrix that should be discarded in order to
obtain a structured subset u' of a superset u – as shown by the matrix in (20) below. The
formal definition is provided in (19).

19. $u_3 \in u_1 := \lambda I_{st}. \forall i_s \in I(u_3 i = u_1 i \vee u_3 i = \#)$

20. Info State I

	u_1	u_2	u_3
i_1	x_1	y_1	x_1
i_2	x_2	y_2	x_2
i_3	x_3	y_3	#
i_4	x_4	y_4	x_4

Unlike van den Berg (1996a), I will not take the introduction of the dummy
individual # to require us to make the underlying logic partial, i.e. I will not assigned the
undefined truth-value to a lexical relation that takes the dummy individual # as an
argument, e.g. *girl*(#) or *courted_at*(#, x_1). Instead, I will take such lexical relations to
simply be false[6,7], which will allow us to keep the underlying type logic classical. The
fact that the dummy individual # always yields falsity (as opposed to always yielding
truth) is meant to ensure that we do not introduce # as the default value of a dref that
vacuously satisfies any lexical relation.

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[6] Conflating undefinedness and falsity in this way is a well-known 'technique' in the presupposition
literature: a Fregean / Strawsonian analysis of definite descriptions distinguishes between what such
descriptions contribute to the asserted content and what they contribute to the presupposed content
associated with any sentence in which they occur. In contrast, the Russellian analysis of definite
descriptions takes everything to be asserted, i.e. it conflates what is asserted and what is presupposed
according to the Fregean / Strawsonian analysis. Therefore, if the presupposed content is not true, the
Russellian will have falsity whenever the Fregean / Strawsonian will have undefinedness.

While this conflation seems to be counter-intuitive and ultimately incorrect in the case of presupposition, it
does not seem to be so in the case of structured inclusion. At this point, I cannot see any persuasive
argument (empirical or otherwise) for a formally unified treatment of structured inclusion and
presupposition (albeit van den Berg seems to occasionally suggest the contrary, see for example van den
Berg 1994: 11, fn. 9), so I will work with the simplest possible system that can model structured inclusion.

[7] We ensure that any lexical relation R of arity n (i.e. of type $e^n t$, defined recursively as in Muskens 1996:
157-158, i.e. as $e^0 t := t$ and $e^{m+1} t := e(e^m t)$) yields falsity whenever # is one of its arguments by letting
$R \subseteq (D_e^M \setminus \{\#\})^n$.

At the same time, requiring the dummy individual # to falsify any lexical relation makes it necessary for us to define lexical relations in PCDRT as shown in (22) below. That is, atomic conditions discard / ignore the dummy party of the plural info state, i.e. $I_{u_1 = \#} \cup \ldots \cup I_{u_n = \#}$, and are interpreted only relative to the non-discarded part of the plural info state, i.e. $I_{u_1 \neq \#, \ldots, u_n \neq \#}$. Note also that they are interpreted distributively relative to this non-discarded part, i.e. we universally quantify over every 'assignment' i in $I_{u_1 \neq \#, \ldots, u_n \neq \#}$.

21. $I_{u_1 \neq \#, \ldots, u_n \neq \#} := \{ i_s \in I : u_1 i \neq \# \wedge \ldots \wedge u_n i \neq \# \}$

22. $R\{u_1, \ldots, u_n\} := \lambda I_{st}. \, I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset \wedge \forall i_s \in I_{u_1 \neq \#, \ldots, u_n \neq \#} (R(u_1 i, \ldots, u_n i))$

Discarding the 'dummy' part of the info state when we evaluate the condition (as shown in (22) above) is crucial: if we were to interpret conditions relative to the entire plural info state, the condition would very often be false because the dummy individual # yields falsity – and we would not be able to allow for output info states like the one in (20) above, which we need to define dynamic quantification. Finally, the non-emptiness requirement enforced by the first conjunct in (22) rules out the degenerate cases in which a plural info state vacuously satisfies an atomic condition by being entirely 'dummy'.

Let us return to the notion of structured inclusion needed for dynamic quantification. Note that the notion of structured inclusion $\subseteq$ defined in (19) above ensures that the subset inherits *only* the superset structure – but we also need it to inherit *all* the superset structure, which we achieve by means of the definition in (23) below.

23. $u' \sqsubseteq u := \lambda I_{st}. \, (u' \subseteq u)I \wedge \forall i_s \in I(ui \in u'I_{u' \neq \#} \rightarrow ui = u'i)$

To see that we need the second conjunct in (23), consider again the donkey sentence in (7) above, i.e. *Most people that owned a^u slave also owned his$_u$ offspring*. This sentence is interpreted as talking about *every* slave owned by any given person – therefore, the nuclear scope set, which needs to be a *most*-subset of the restrictor set, needs to inherit *all* the superset structure, i.e., for any slave owner in the nuclear scope set, we need to associate with her/him *every* slave (and his offspring) that s/he owned.

3.3.   Maximization, Distributivity and Selective Quantification

We turn now to the definition of the maximization and distributivity operators $\mathbf{max}^u$ and $\mathbf{dist}_u$, which are defined in the spirit – but not the letter – of the corresponding operators in van den Berg (1996a). Selective maximization plus selective distributivity[8] enable us to dynamize λ-abstraction over both *values*, i.e. individuals, and *structure*, i.e. the quantificational dependencies associated with the individuals. We will consequently be able to extract and store the restrictor and nuclear scope structured sets needed to define dynamic generalized quantification.

To see that we need maximization over both values and structure, consider the discourse in (24) below. Sentence (24b) elaborates on the relation between students and cakes introduced by the first sentence. Note that this relation is the Cartesian product of the set of students and the set of cakes, i.e. we want to introduce the set of *all* students, the set of *all* cakes and the *maximal relation / structure* associating the two sets. That is, we want to introduce the entire set of cakes relative to each and every student. We will achieve this by means of a distributivity operator $\mathbf{dist}_u$ over students taking scope over a maximization operator $\mathbf{max}^{u'}$ operator over cakes. Note that the distributivity operator is anaphoric to the dref u introduced by a preceding maximization operator $\mathbf{max}^u$ over students, as shown in (25) below.

24. **a.** Every^u student ate from every$^{u'}$ cake. **b.** They$_u$ liked them$_{u'}$ (all) [9].
25. $\mathbf{max}^u([student\{u\}]); \mathbf{dist}_u(\mathbf{max}^{u'}([cake\{u'\}])); [eat\_from\{u, u'\}]; [like\{u, u'\}]$

Intuitively, the update in (25) instructs us to perform the following operations on a given input matrix *I*:

- $\mathbf{max}^u([student\{u\}])$: add a new column u and store all the students in it;

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[8] Both maximization and distributivity are selective in the sense that they target a particular dref u over which they maximize or distribute), i.e. exactly in the sense in which the DPL/FCS/DRT-style dynamic generalized quantification introduced in chapters **2**, **4** and **5** is selective and, by being so, solves the proportion and weak / strong ambiguity problems which mar the notion of unselective quantification introduced in Lewis (1975).

[9] Another example with a similar 'Cartesian product' interpretation is *Every guest tasted every dish at the potluck party.*

- **dist**$_u$(**max**$^{u'}$([*cake*{u'}])): look at each u-individual separately – more exactly, for each such individual x, look at that subpart of the matrix that has only x in column u; relative to each such sub-matrix, add a new column u' and store all the cakes in that column; then, take the union of all the resulting matrices: the big union matrix will associated every u-individual separately with each and every cake;

- [*eat_from*{u, u'}]; [*like*{u, u'}]: test that, for each row in the big union matrix, the u-individual stored in that row ate from the u'-individual stored in that row; finally, test that, for every rowin the big union matrix, the u-individual stored in that row liked the u'-individual stored in that row.

A different kind of example indicating that we need selective distributivity operators over and above the unselective distributivity built into the atomic conditions[10] to obtain structure maximization is provided by the donkey sentence in (26) below. Intuitively, the donkey indefinite receives a strong reading, i.e. every farmer kicked *every* donkey he saw (and not only some). In particular, if two farmers happened to see the same donkeys, each one of them kicked each one the donkeys, i.e. we need to consider each farmer in turn and introduce every seen donkey with respect to each one of them. Again, this can be achieved by means of a **dist**$_u$ operator over farmers taking scope over a **max**$^{u'}$ operator over donkeys, as shown in (27) below.

26. Every^u farmer who saw a$^{u'}$ donkey kicked it$_{u'}$.
27. **max**^{u}([*farmer*{u}]; **dist**$_u$(**max**$^{u'}$([*donkey*{u'}, *see*{u, u'}]))); [*kick*{u, u'}]

Notice that the example in (24) above indicates that we need a **dist**$_u$ operator over the nuclear scope of *every student* (since we need to introduce every cake relative to each student), while the example (26) above indicates that we need a **dist**$_u$ operator over the restrictor of *every farmer* (since we need to introduce every donkey that was seen relative to each farmer). We therefore expect our final definition of dynamic generalized determiners to contain two distributivity operators – and this is exactly how it will be.

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[10] Atomic conditions are unselectively distributive because they contain the universal quantifications over 'assignments' of the form $\forall i_s \in I(\ldots)$, i.e. they unselectively target 'assignments' (i.e. cases in the sense of Lewis 1975) and not individuals or individual dref's, as the selectively distributive operator **dist**$_u$ does.

The **max**$_u$ and **dist**$_u$ operators are defined in (28) and (31) below. Consider the definition of **max**^{u} first: the first conjunct in (28) introduces u as a new dref (i.e. $[u]$) and makes sure that each individual in uJ 'satisfies' D, i.e. we store *only* individuals that 'satisfy' D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e. any other set of individuals that 'satisfies' D) is included in uJ, i.e. we store *all* the individuals that satisfy D.

28. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK_{u\neq\#} \subseteq uJ_{u\neq\#})$

29. $\mathbf{max}^{u' \sqsubseteq u}(D) := \mathbf{max}^{u'}([u' \sqsubseteq u]; D)$

30. $I_{u=x} := \{i_s \in I\colon ui=x\}$

31. $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ \forall x_e(I_{u=x} \neq \emptyset \leftrightarrow J_{u=x} \neq \emptyset) \wedge \forall x_e(I_{u=x} \neq \emptyset \rightarrow DI_{u=x}J_{u=x})$,

        i.e. $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ uI=uJ \wedge \forall x_e \in uI(DI_{u=x}J_{u=x})$

The basic idea behind *distributively* updating an input info state I with a DRS D is that we first partition the info state I and then *separately* update each partition cell (i.e. subset of I) with D.

Moreover, the partition of the info state I is induced by the dref u as follows: consider the set of individuals $uI := \{ui\colon i \in I\}$; each individual x in the set uI generates one cell in the partition of I, namely the subset $\{i \in I\colon ui=x\}$. Clearly, the family of sets $\{\{i \in I\colon ui=x\}\colon x \in uI\}$ is a partition of the info state I.

Thus, updating an info state I with a DRS D *distributively* over a dref u means updating each cell in the u-partition of I with the DRS D and then taking the union of the resulting output info states. The first conjunct in definition (31) above, i.e. $uI=uJ$, is required to ensure that there is a bijection between the partition cells induced by the dref u over the input state I and the partition cells induced by u over the output state J; without this requirement, we could introduce arbitrary new values for p in the output state J, i.e. arbitrary new partition cells[11].

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[11] Nouwen (2003): 87 was the first to observe that the first conjunct in this definition, namely $uI=uJ$, is necessary (the original definition in van den Berg 1996a: 145, (18) lacks it).

The second conjunct, i.e. $\forall x \in uI(DI_{u=x}J_{u=x})$, is the one that actually defines the distributive update: every partition cell in the input info state I is related by the DRS D to the corresponding partition cell in the output state J. The figure in (32) below schematically represents how the input state I is u-distributively updated with the DRS D^{12}.

32. **Updating info state I with D distributively over u.**

$$Input\ state\ I \quad - update\ with\ D\ distributively\ over\ u \rightarrow \quad Output\ state\ J$$
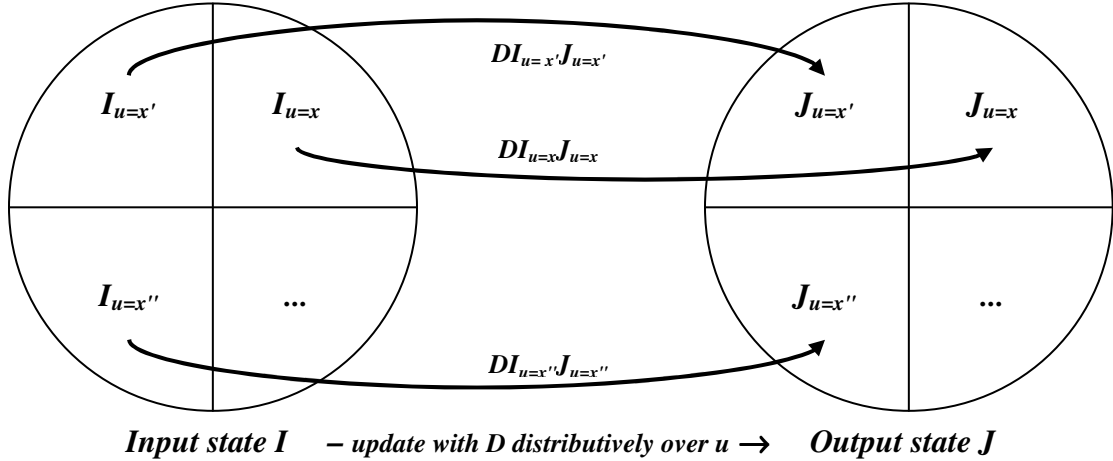
The definitions of generalized determiners and weak / strong indefinites are provided in (36), (37) and (38) below. For the justification of the account of weak / strong donkey ambiguities in terms of weak / strong indefinite articles, see chapter **5**.

33. $_u(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}=J_{u=\#} \wedge I_{u\neq\#}\neq\emptyset \wedge \mathbf{dist}_u(D)I_{u\neq\#}J_{u\neq\#}$ ^{13}

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[12] Some properties of the distributivity operator (see also the appendix of this chapter):

(i) $\mathbf{dist}_u(D; D') = \mathbf{dist}_u(D); \mathbf{dist}_u(D')$, for any D and D' s.t. $\forall <I,J> \in D(uI=uJ)$ and $\forall <I,J> \in D'(uI=uJ)$ (i.e. $\mathbf{dist}_u$ distributes over dynamic conjunction)

(ii) $\mathbf{dist}_u(\mathbf{dist}_{u'}(D)) = \mathbf{dist}_{u'}(\mathbf{dist}_u(D))$

(iii) $\mathbf{dist}_u(\mathbf{dist}_u(D)) = \mathbf{dist}_u(D)$.

[13] Some properties of the $_u(\ldots)$ operator:

(i) $_u(D; D') = {_u(D)}; {_u(D')}$, for any D and D' s.t. $\mathbf{dist}_u(D; D') = \mathbf{dist}_u(D); \mathbf{dist}_u(D')$

(ii) $_u(_u(D)) = {_u(D)}$

However, note that, in general, $_u(_{u'}(D)) \neq {_{u'}(_u(D))}$. Consider for example the info state I in (42) below: while it is true that $<I,I>$ is in the denotation of $_u(_{u'}([u \in u]))$, it is not true that $<I,I>$ is in the denotation of $_{u'}(_u([u \in u]))$. Moreover, we can easily construct an info state I' such that $<I',I'>$ is in the denotation of $_{u'}(_u([u \in u]))$, but not in the denotation of $_u(_{u'}([u \in u]))$.

34. $_{\langle u \rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}=J_{u=\#} \wedge (I_{u\neq\#}=\emptyset \rightarrow I=J) \wedge (I_{u\neq\#}\neq\emptyset \rightarrow \mathbf{dist}_u(D)I_{u\neq\#}J_{u\neq\#})$

35. $\mathbf{DET}\{u, u'\} := \lambda I_{st}.\ \mathbf{DET}(uI_{u\neq\#}, u'I_{u'\neq\#})$,

   where $\mathbf{DET}$ is a static determiner.

36. $det^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\langle u \rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}(_{\langle u \rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

37. $a^{\mathsf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ _u(P(u));\ _u(P'(u))$

38. $a^{\mathsf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_u(P(u));\ _u(P'(u)))$

Note that the **max**-based definition of selective generalized quantification correctly predicts that anaphora to restrictor and nuclear scope sets is always anaphora to *maximal sets*, i.e. E-type anaphora (recall the Evans examples: *Few^u congressmen admire Kennedy and they$_u$ are very junior* and *Harry bought some^u sheep. Bill vaccinated them$_u$*[14]; see also (10), (11) and (12) above). The maximality of anaphora to quantifier sets follows automatically as a consequence of the fact that we need maximal sets to correctly compute the meaning of dynamic generalized quantifiers. This is one of the major results in van den Berg (1996a) and PCDRT preserves it[15].

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[14] See Evans (1980): 217, (7) and (8) (page references are to Evans 1985).

[15] That the restrictor set needs to be maximal is established by *every*-quantifications: to determiner the truth of *Every man left*, we need to have access to the set of all men. That the nuclear scope set also needs to be maximal, namely the maximal subset of the restrictor set that satisfies the nuclear scope update, is established by downward monotonic quantifiers (i.e. by determiners that are downward monotonic in their right argument); for example, *Few men left* intuitively means that, among the set of men, the *maximal* set of men that left is a *few*-subset, i.e. it is less than half of the set of men. In particular, if *Few men left* is true, then *Most men left* is false – and the use of maximal nuclear scope sets correctly predicts that.

If we were to use non-maximal subsets of the restrictor set of individuals, we would be able to capture the meaning of upward monotonic quantifiers, e.g. *Most* (*some*, *two*, *at least two*, etc.) *men left* can be interpreted as: introduce the maximal set of men (i.e. the maximal restrictor set); then, introduce some subset of the restrictor set that is a *most*-subset (i.e. it is more than half of the restrictor set) and that also satisfies the nuclear scope update. If you can do this, then the quantification update is successful. Note that, in this case, the nuclear scope set is not necessarily the maximal subset of the restrictor set that satisfies the nuclear scope update. The relevant definition is given in (i) below.

(i) $det^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\langle u \rangle}(P(u)));\ [u' \mid u'\sqsubseteq u, \mathbf{DET}\{u, u'\}];\ _{\langle u' \rangle}(P'(u'))$

But this strategy will not work with downward monotonic quantifiers, e.g. *Few* (*no*, *at most two* etc.) *students left* cannot be interpreted as: introduce the maximal set of men (i.e. the maximal restrictor set); then, introduce some subset of the restrictor set that is a *few*-subset (i.e. it is less than half of the restrictor set, possibly empty) and that also satisfies the nuclear scope update (if the *few*-subset that was introduced is empty, we can assume that it vacuously satifies the nuclear scope update). We cannot do this because, even if we are successful in introducing a *few*-subset that satisfies the nuclear scope update, it can still be the

Moreover, this result is an important argument for a dynamic approach to generalized quantification in general and, in particular, for a dynamic approach to generalized quantification of the kind pursued in this chapter.

3.4.   The Dummy Individual and Distributivity Operators

We have already established that the definition of generalized determiners in (36) above requires a distributivity operator **dist**$_u$. The distributivity operator is contributed by the operators $_u(D)$ and $_{\langle u \rangle}(D)$ defined in (33) and (34) above. The question is: why do we need the additional conjuncts in the definition of these operators over and above distributivity?

To see the necessity of the first conjunct $I_{u=\#}=J_{u=\#}$ in (33) and (34), consider the simple sentence in (39) below, represented in (40) without the operator $_u(\ldots)$ and in (41) with the operator $_u(\ldots)$ [16].

39. A^u man fell in love with a$^{u'}$ woman.
40. $[u \mid man\{u\}]; [u' \mid woman\{u'\}, f\_i\_l\{u, u'\}]$
41. $[u]; {}_u([man\{u\}]; [u']; {}_u([woman\{u'\}, f\_i\_l\{u, u'\}]))$

After processing sentence (39), we want our output info state to be such that each non-dummy u-man loves some non-dummy u'-woman and each non-dummy u'-woman loves some non-dummy u-man. However, if the conjunct $I_{u=\#}=J_{u=\#}$ is lacking – as it is lacking in (40) above –, we might introduce some u'-women relative to 'assignments' that

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case that a *most*-subset, for example, also satisfies the update, i.e. a successful update with *Few men left* does not rule out the possibility that *Most men left*, which is intuitively incorrect.

For the quantification *Few men left* to rule out the possibility that a *most*-subset of the restrictor also satisfies the nuclear scope update, we need to introduce the maximal nuclear scope set, i.e. the maximal subset of the restrictor that satisfies the nuclear scope update and only afterwards test that the two maximal sets are related by the static determiner. This is a direct consequence of the proposition relating witness sets and quantifier monotonicity in Barwise & Coopeer (1981): 104 (page references to Partee & Portner 2002).

In conclusion, to correctly compute the truth-conditions of generalized quantifications, the dynamic meaning of generalized determiners have to relate two maximal sets of individuals (i.e. the restrictor set and the nuclear scope set) – and this automatically and correctly predicts that E-type (i.e. unbound, 'quantifier external') anaphora to quantificational domains is maximal.

[16] These oversimplified representations are good enough for our current purposes. For the actual PCDRT analysis of this example, see (55) and (60) in section **3.6** below.

store the dummy individual # with respect to the dref u, see for example 'assignment' i_3 in (42) below.

42. Info State I	u (men)	u' (women)
i_1	x_1	y_1
i_2	x_2	y_2
i_3	#	y_3
i_4	#	#

Given that we ignore both i_3 and i_4 in the evaluation of the lexical relation $f\_i\_l\{u, u'\}$, y_3 can be any woman whatsoever (including a woman that is not loved by any man) – which can inadvertently falsify subsequent anaphoric sentences, e.g. the follow-up *She$_{u'}$ was pretty*, which might actually be true of y_1 and y_2, but not of y_3. The discourse *Every^u man fell in love with a$^{u'}$ woman. They$_{u'}$ were pretty* provides a similar argument for the necessity of the first conjunct $I_{u=\#}=J_{u=\#}$ in (33) and (34).

The second conjunct $I_{u \neq \#} \neq \emptyset$ in the definition the operator $_u(\ldots)$ in (33) above encodes existential commitment. Note that the existential commitment associated with dref introduction is built into two distinct definitions: (**i**) the definition of lexical relations (see the conjunct $I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset$ in (22) above) and (**ii**) the definition of the operator $_u(\ldots)$ (see the conjunct $I_{u \neq \#} \neq \emptyset$ in (33)).

We need the former (i.e. the conjunct $I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset$ in the definition of lexical relations) because the pair $\langle \emptyset_{st}, \emptyset_{st} \rangle$ belongs to the denotation of $[u]$ for any dref u (since both conjuncts in the definition of $[u]$ are universal quantifications).

We need the latter (i.e. the conjunct $I_{u \neq \#} \neq \emptyset$ in the definition of the operator $_u(\ldots)$) because the definition of the **dist**$_u$ operator is a universal quantification and is therefore trivially satisfied relative to the empty input info state $\emptyset_{st}$; that is, the pair $\langle \emptyset_{st}, \emptyset_{st} \rangle$ belongs to the denotation of **dist**$_u(D)$ for any dref u and DRS D.

Thus, we capture the existential commitment associated with indefinites by using the operator $_u(\ldots)$ in their translation – see (37) and (38) above.

In contrast, there is no such existential commitment in the definition of the the operator $_{\langle u\rangle}(\ldots)$ in (34) above and, therefore, there is no such existential commitment in the definition of generalized determiners $det^{u,u'\sqsubseteq u}$ in (36). This enables us to capture the meaning of both upward and (especially) downward monotonic quantifiers by means of the same definition. The problem posed by downward monotonic quantifiers is that their nuclear scope set can or has to be empty.

For example, after a successful update with a $no^{u,u'\sqsubseteq u}$ quantification (e.g. *No man left*), the nuclear scope set is necessarily empty (recall that we use nuclear scope sets with built-in conservativity), i.e. the dref u' will always store only the dummy individual # relative to the output info state. This, in turn, entails that no lexical relation in the nuclear scope DRS that has u' as an argument can be satisfied (because the first conjunct of any such lexical relation is $I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \varnothing$ – see (22) above). Thus, we need the operator $_{\langle u\rangle}(\ldots)$ – more precisely, the second conjunct in its definition in (34) above – to resolve the conflict between the emptiness requirement enforced by a *no*-quantification and the non-emptiness requirement enforced by lexical relations.

Similarly, given that we use the same operator $_{\langle u\rangle}(\ldots)$ in the formation of restrictor sets, we predict that *John visited every Romanian colony* is true (although it might not always be felicitous) in case there are no Romanian colonies, i.e. in case the restrictor set of the *every*-quantification is empty.

Note that, despite the fact that definition (34) allows for empty restrictor and nuclear scope sets, we are still able to capture the fact that subsequent anaphora to such sets is infelicitous. This follows from: (**i**) the fact that lexical relations have a non-emptiness / existential requirement built in and (**ii**) pronouns will be defined by means of the operator $_{u}(\ldots)$ (see (44) below), which also has a non-emptiness / existential requirement built in.

Finally, note that the second conjunct the definition of $_{\langle u'\rangle}(\ldots)$ in (34) requires the identity of the input state I and the output state J. That is, the nuclear scope DRS of a successful $no^{u,u'\sqsubseteq u}$ quantification, i.e. $_{\langle u'\rangle}(P'(u'))$, will always be a test. Consequently, we correctly predict that anaphora to any indefinites in the nuclear scope of a $no^{u,u'\sqsubseteq u}$

quantification is infelicitous, e.g. *No$^{u,u'\sqsubseteq u}$ farmer owns a$^{u''}$ donkey. #It$_{u''}$ is unhappy / #They$_{u''}$ are unhappy* (or *Harry courts a girl$^{u''}$ at no$^{u,u'\sqsubseteq u}$ convention. #She$_{u''}$ is very pretty.*).

3.5.   Singular Number Morphology on Pronouns

Let us turn now to the last component needed for the account of discourses (1) and (2), namely the representation of singular pronouns. Their PCDRT translation, provided in (44) below, has the expected Montagovian form: it is the distributive type-lift of the dref u, i.e. $\lambda P_{et}.\,_u(P(u))$, with the addition of the condition **unique**$\{u\}$, which is contributed by the singular number morphology and which requires uniqueness of the non-dummy value of the dref u relative to the current plural info state – see (43) below.

43. **unique**$\{u\}$:= $\lambda I_{st}.\ I_{u\neq\#}\neq\emptyset \wedge \forall i_s, i'_s \in I_{u\neq\#}(ui=ui')$

44. *she$_u$* ⤳ $\lambda P_{et}.\ [\textbf{unique}\{u\}];\ _u(P(u))$

In contrast, plural pronouns do not require uniqueness, as shown in (45) below.

45. *they$_u$* ⤳ $\lambda P_{et}.\ _u(P(u))$

Singular and plural *anaphoric* definite descriptions – we need them to interpret the anaphoric DP *the girl* in (2c) above among others – are interpreted as shown in (46) and (47) below. They exhibit the same kind of unique/non-unique contrast as the pronouns.

46. *the_sg$_u$* ⤳ $\lambda P_{et}.\lambda P'_{et}.\ [\textbf{unique}\{u\}];\ _u(P(u));\ _u(P'(u))$

47. *the_pl$_u$* ⤳ $\lambda P_{et}.\lambda P'_{et}.\ _u(P(u));\ _u(P'(u))$

The uniqueness enforced by the condition **unique**$\{u\}$ is *weak* in the sense that it is relativized to the current plural info state. However, we can require *strong* uniqueness, i.e. uniqueness relative to the entire model, by combining the **max**^{u} operator and the condition **unique**$\{u\}$ – as shown by the Russellian, non-anaphoric meaning for definite descriptions provided in (48) below, which, as expected from a Russellian analysis,

requires both existence and strong uniqueness. This alternative meaning for definite articles is needed to interpret the non-anaphoric DP *the banquet* in (2b) above.

48. *the_sg^u* $\rightsquigarrow$ $\lambda P_{et}.\lambda P'_{et}.$ $\mathbf{max}^u(_u(P(u)))$; [$\mathbf{unique}\{u\}$]; $_u(P'(u))$ [17]

The PCDRT translation of proper names is provided in (49) below. The definitions of dynamic negation and truth are identical to the ones in chapter **5**, as shown by (50) and (51) respectively.

49. *Harvey^u* $\rightsquigarrow$ $\lambda P_{et}.$ [$u \mid u \in Harvey$]; $_u(P(u))$,

   where $Harvey := \lambda i_s.\ harvey_e$.

50. $\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st} \neq \emptyset(H \subseteq I \rightarrow \neg\exists K_{st}(DHK))$ [18]

51. A DRS D (of type $\mathbf{t} := (st)((st)t)$) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

3.6.   An example: Cross-Sentential Anaphora to Indefinites

I will conclude this section with the PCDRT analysis of the simple example in (39) above. The transitive verb *fall in love* is translated as shown in (52) below. Also, for simplicity, I will assume that both indefinites are weak and are therefore translated as

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[17] The plural counterpart of the Russellian singular definite article in (48) is provided in (i) below – the only difference is that we remove the **unique**$\{u\}$ condition from its singular counterpart, just as we did for plural pronouns and anaphoric plural definite articles in (45) and (47) above.

(i) *the_pl^u* $\rightsquigarrow$ $\lambda P_{et}.\lambda P'_{et}.$ $\mathbf{max}^u(_u(P(u)))$; $_u(P'(u))$

Note that the Russellian plural definite translation in (i) above is identical to the simplified translation of *every* in (65) below (see section **4.1**), which preserves the intuitive equivalence between *every*-DP's and (distributive uses of) plural *the*-DP's, e.g. *Every student left* and *The students left*, already observed and captured in Link (1983).

[18] This definition of negation enables us to capture the interaction between negation and intra-sentential donkey anaphora in (i), (ii) and (iii) below (as already indicated in section **3.3** of chapter **5**) and also between negation and cross-sentential anaphora in (iv).

(i) Most farmers who own a^u donkey do not beat it$_u$.

(ii) Every farmer who owns a^u donkey doesn't feed it$_u$ properly.

(iii) Most house-elves who fall in love with a^u witch do not buy her$_u$ an$^{u'}$ alligator purse.

(iv) Every^u student bought several$^{u'}$ books. But they$_u$ didn't read (any of) them$_{u'}$.

shown in (53) below. The semantic composition[19] proceeds based on the syntactic structure schematically represented in (54) and yields the representation in (55).

52. fall_in_love ⤳ $\lambda Q'_{(et)t}.\lambda v_e.\ Q'(\lambda v'_e.\ [f\_i\_l\{v, v'\}])$

53. $a^{wk:u}$ man ⤳ $\lambda P_{et}.\ [u];\ _u([man\{u\}]);\ _u(P(u))$

   $a^{wk:u'}$ woman ⤳ $\lambda P_{et}.\ [u'];\ _{u'}([woman\{u'\}]);\ _{u'}(P(u'))$

54. $a^{wk:u}$ man [fall_in_love [$a^{wk:u'}$ woman]]

55. $[u];\ _u([man\{u\}]);\ _u([u'];\ _{u'}([woman\{u'\}]);\ _{u'}([f\_i\_l\{u, u'\}]))$

To simplify the representation in (55), I will introduce the abbreviations in (56) and (57) below. The reader can easily check that the identities in (58) and (59) hold.

56. $_u(C) := \lambda I_{st}.\ I_{u\neq\#}\neq\emptyset \wedge \forall x \in uI_{u\neq\#}(CI_{u=x})$,

        where C is a condition (of type $(st)t$).

57. $_u(u_1, \ldots, u_n) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}=J_{u=\#} \wedge I_{u\neq\#}[u_1, \ldots, u_n]J_{u\neq\#}$,

        where $u\notin\{u_1, ..., u_n\}$ and $[u_1, ..., u_n] := [u_1];\ ...;\ [u_n]$ [20].

58. $_u([C_1, \ldots, C_m]) = [_u(C_1), \ldots, _u(C_m)]$

59. $_u([u_1, ..., u_n \mid C_1, \ldots, C_m]) = [_u(u_1, ..., u_n) \mid _u(C_1), \ldots, _u(C_m)]$

Based on the identities in (58) and (59) and several fairly obvious simplifications, we obtain the final PCDRT translation of sentence (39), provided in (60) below. Based on the definition of truth in (51) above, we derive the truth-conditions in (61) below, which agree with our intuitions about the truth-conditions of sentence (39).

60. $[u, _u(u') \mid man\{u\}, _u(woman\{u'\}), _u(f\_i\_l\{u, u'\})]$

61. $\lambda I_{st}.\ I\neq\emptyset \wedge \exists x_e \exists y_e(man(x) \wedge woman(y) \wedge f\_i\_l(x, y))$

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[19] That is, the type-driven translation of example (39); for the precise definition, see section **5** of chapter **3**.

[20] That is: $[u_1, ..., u_n] := \lambda I_{st}.\lambda J_{st}.\ \exists H_1...\exists H_{n-1}(I[u_1]H_1 \wedge ... \wedge H_{n-1}[u_n]J)$.

3.7.   The Dummy Info State as Default Discourse Context

In general, I take the default context of interpretation for all discourses to be the singleton info state $\{i_\#\}$, where $i_\#$ is the 'assignment' that stores the dummy individual # relative to all individual dref's. When we apply the truth-conditions in (61) above to the default input info state $\{i_\#\}$, we obtain $\exists x_e \exists y_e(man(x) \wedge woman(y) \wedge f\_i\_l(x, y))$, i.e. precisely the classical first-order truth-conditions assigned to sentence (39).

Moreover, taking $\{i_\#\}$ to be the default context of interpretation enables us to capture the infelicity of discourse-initial anaphors, e.g. #*She$_u$ is pretty*, because multiple meaning components (in particular, the condition **unique**$\{u\}$, the lexical relation **pretty**$\{u\}$ and the operator $_u(…)$) cannot be satisfied relative to the input info state $\{i_\#\}$.

Hence, the felicitous *deictic* use of a pronoun like *she$_u$* requires us to *non-linguistically* update the default input info state $\{i_\#\}$ before processing the sentence containing the pronoun; intuitively, this update is contributed by the deixis associated with the pronoun (see Heim 1982/1988: 309 et seqq for a similar assumption[21]).

4. Quantificational Subordination in PCDRT

This section presents the PCDRT analysis of the contrast in interpretation between the discourses in (1) and (2) above.

4.1.   Quantifier Scope

We start with the two possible quantifier scopings for the discourse-initial sentence (1a/2a). For simplicity, I will assume that the two scopings are due to the two different lexical entries for the ditransitive verb *court_at*, provided in (62) and (63) below. As chapter **5** showed, PCDRT is compatible with Quantifier Raising / Quantifying-In and, in

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[21] "If something has been mentioned before, there will always be a card for it in the file […] But does the file also reflect what is familiar by contextual salience? So far we have not assumed it does, but let us make the assumption now. […] An obvious implication is that files must be able to change, and in particular, must be able to have new cards added, without anything being uttered. For instance, if halfway through a conversation between A and B a dog comes running up to them and draws their attention, then that event presumably makes the file increase by a new card" (Heim 1982/1988: 309-310).

general, with any of the quantifier scoping mechanisms proposed in the literature, there is no need to use any of them for our current purposes.

Furthermore, I will assume that the syntactic structure of the sentence is the one schematically represented in (64) below.

62. *court_at¹* $\rightsquigarrow \lambda Q'_{\mathbf{(et)t}}.\lambda Q''_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}. \ Q'(\lambda v'_{\mathbf{e}}. \ Q''(\lambda v''_{\mathbf{e}}. \ [court\_at\{v, v', v''\}]))$

63. *court_at²* $\rightsquigarrow \lambda Q'_{\mathbf{(et)t}}.\lambda Q''_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}. \ Q''(\lambda v''_{\mathbf{e}}. \ Q'(\lambda v'_{\mathbf{e}}. \ [court\_at\{v, v', v''\}]))$

64. *Harvey* [[*court_at¹ᐟ²* [*a girl*]] [*every convention*]]

Thus, *court_at¹* assigns the indefinite *a girl* wide scope relative to *every convention*, while *court_at²* assigns it narrow scope.

Turning to the meaning of the quantifier *every convention*, note that we can safely identify the restrictor dref *u* and the nuclear scope dref *u'* of any *every$^{u,u'\sqsubseteq u}$*-quantification: the definition in (36) above entails that, if *J* is an arbitrary output state of a successful *every$^{u,u'\sqsubseteq u}$*-quantification, *u* and *u'* have to be identical both with respect to value and with respect to structure, i.e. we will have that $\forall j_s \in J(uj = u'j)$. We can therefore conflate the two dref's and assume that *every* contributes only one, as shown in (65) below. I will also assume that the restrictor set of the *every^{u_1}*-quantification is non-empty, so I will replace the operator $_{\langle u \rangle}(\ldots)$ with the simpler operator $_u(\ldots)$.

65. *every^{u_1}* $\rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}. \ \mathbf{max}^{u_1}(_{u_1}(P(u_1))); \ _{u_1}(P'(u_1))$

The PCDRT translations of the generalized quantifier *every^{u_1} convention* and of the indefinite *a$^{\mathbf{wk}\ u_2}$ girl* (which, for the moment, I assume to be weak) are given in (66) and (67) below, followed by the compositionally derived representations of the two quantifier scopings of sentence (1a/2a), which are provided in (68) and (69).

To make the representations simpler, I will assume that the PCDRT translation of the proper name *Harvey* is $\lambda P_{\mathbf{et}}.P(Harvey)$ instead of the one provided in (49) above. The reader can easily convince herself that this simplification does not affect the PCDRT truth-conditions for the two discourses under consideration.

66. *every*^{u_1} *convention* $\leadsto \lambda P_{et}.$ **max**^{u_1} ([*convention*{u_1}]); $_{u_1}(P(u_1))$

67. *a*$^{\textbf{wk:}\,u_2}$ *girl* $\leadsto \lambda P_{et}.$ [u_2 | *girl*{u_2}]; $_{u_2}(P(u_2))$

68. (*a*$^{\textbf{wk:}\,u_2}$ *girl*>>*every*^{u_1} *convention*)

   [u_2 | *girl*{u_2}]; $_{u_2}$(**max**^{u_1}([*convention*{u_1}])); [$_{u_2}$(*court_at*{*Harvey*, u_2, u_1})]

69. (*every*^{u_1} *convention*>>*a*$^{\textbf{wk:}\,u_2}$ *girl*)

   max^{u_1}([*convention*{u_1}]); [$_{u_1}(u_2)$ | $_{u_1}$(*girl*{u_2}), $_{u_1}$(*court_at*{*Harvey*, u_2, u_1})]

The reader can check that the (truth-conditions derived by the) representations in (68) and (69) are the intuitively correct ones. I will examine them only in informal terms.

The "wide-scope indefinite" representation in (68) updates the default input info state $\{i_\#\}$ as follows. First, we introduce some non-empty (i.e. non-dummy) set of individuals relative to the dref u_2. Then, we test that each u_2-individual is a girl. Then, relative to each u_2-individual, we introduce the non-empty set containing all and only conventions and store it relative to the dref u_1. Finally, we test that, for each u_2-girl, for each of the corresponding u_1-conventions (which, in this case, means: for every convention), Harvey courted the girl currently under consideration at the convention currently under consideration.

By the time we are done processing (68), the output info state contains a non-empty set of u_2-girls that where courted by Harvey at every convention and, relative to each u_2-girl, u_1 stores the set of all conventions.

The "narrow-scope indefinite" representation in (69) updates the default input info state $\{i_\#\}$ as follows. First, we introduce the non-empty set of individuals containing all and only conventions relative to the dref u_1. Then, for each u_1-convention, we introduce a u_2-set of individuals. Finally we test that, for each u_1-convention, each of the corresponding u_2-individuals are girls and are such that Harvey courted them at the convention currently under consideration.

By the time we are done processing (69), the output info state stores the set of all conventions under the dref u_1 and, relative to each u_1-convention, the dref u_2 stores a non-

empty set of girls (possibly different from convention to convention) that Harvey courted at that particular convention.

4.2. Quantifier scope and Singular Anaphora, Cross-Sententially

It is now easy to see how sentence (1b) – and, in particular, the singular number morphology on the pronoun she_{u_2} – forces the "indefinite wide-scope" reading for the preceding sentence (1a): the condition **unique**$\{u_2\}$ effectively *conflates* the two readings by requiring the set of u_2-girls obtained after processing (68) or (69) above to be a singleton. This requirement leaves untouched the truth-conditions derived on the basis of (68) – but makes the truth-conditions associated with (69) above strictly stronger.

The PCDRT translation of the pronoun and the compositionally derived representation of sentence (1b) are provided in (70) and (71) below. For convenience, I provide the two complete representations of discourse (1) in (72) and (73) below.

70. she_{u_2} ⤳ $\lambda P_{\mathbf{et}}.$ [**unique**$\{u_2\}$]; $_{u_2}(P(u_2))$

71. [**unique**$\{u_2\}$, *very_pretty*$\{u_2\}$]

72. ($a^{\mathbf{wk:}\,u_2}$ *girl*>>*every*^{u_1} *convention*)

   $[u_2 \mid girl\{u_2\}]$; $_{u_2}(\mathbf{max}^{u_1}([convention\{u_1\}]))$;

   $[\,_{u_2}(court\_at\{Harvey, u_2, u_1\})$, **unique**$\{u_2\}$, *very_pretty*$\{u_2\}$]

73. (*every*^{u_1} *convention*>>$a^{\mathbf{wk:}\,u_2}$ *girl*)

   $\mathbf{max}^{u_1}([convention\{u_1\}])$;

   $[\,_{u_1}(u_2) \mid \,_{u_1}(girl\{u_2\}), \,_{u_1}(court\_at\{Harvey, u_2, u_1\})$, **unique**$\{u_2\}$, *very_pretty*$\{u_2\}$]

4.3. Quantifier Scope and Singular Anaphora, Intra-Sententially

In contrast, sentence (2b) contains the adverb of quantification *always*, which can take scope above or below the singular pronoun *she*; in the former case, the u_2-uniqueness requirement is weakened (and, basically, neutralized) by being relativized to u_1-conventions.

More precisely, I take the meaning of *always* to be universal quantification over an anaphorically retrieved restrictor, as shown in (74) below. Since *always* is basically interpreted as *every*, I provide a simplified translation that conflates the restrictor and nuclear scope dref's – much like the simplified translation for *every* in (65) above conflated them. The general format for the interpretation of quantifiers that anaphorically retrieve their restrictor set is provided in (75).

74. *always* $_{u_1}$ ⤳ $\lambda P_{\textbf{et}}.\ u_1(P(u_1))$

75. *det* $_u^{u' \sqsubseteq u}$ ⤳ $\lambda P_{\textbf{et}}.\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P(u'))); [\mathbf{DET}\{u, u'\}]$

The restrictor dref of *always* in (2b) is the nuclear scope dref of the quantifier *every* ^{u_1} *convention* in the preceding sentence (2a). To see that *always* is indeed anaphoric to the nuclear scope and not to the restrictor dref of *every*, we need to consider other determiners that do not effectively identity them, e.g. *most* in (76) below. In this case, it is intuitively clear that *always* quantifies over the conventions at which Harvey courts a girl (the nuclear scope dref) and not over all conventions (the restrictor dref).

76. **a.** Harvey courts a girl at most conventions.

    b. She always comes to the banquet with him.

The definite description *the banquet* in (2b) is intuitively a Russellian definite description (see (48) above), which contributes existence and a relativized (i.e. anaphoric) form of uniqueness: we are talking about a *unique* banquet *per convention*. The relevant meaning for the definite article is given in (77) below.

77. *the_sg* $_{u_1}^{u_3}$ ⤳ $\lambda P_{\textbf{et}}.\lambda P'_{\textbf{et}}.\ u_1(\mathbf{max}^{u_3}(_{u_3}(P(u_3))); [\mathbf{unique}\{u_3\}];\ _{u_3}(P'(u_3)))$

The relativized uniqueness is captured by the fact that the **unique**$\{u_3\}$ condition is within the scope of the $_{u_1}(\ldots)$ operator [22]. Thus, *the banquet* is in fact interpreted as a

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[22] Incidentally, note that the definite article *the_sg* $_{u_1}^{u_3}$ is anaphoric to the restrictor set u_1 of the *every*-quantification in the preceding sentence (2a) – unlike *always*, which is anaphoric to the nuclear scope set. To see this, we have to consider determiners like *most* that do not conflate their restrictor and nuclear scope

possessive definite description of the form *its u_1 u_3 banquet*, or, more explicitly, of the form *the u_1 u_3 banquet of it u_1*, where *it u_1* is anaphoric to u_1-conventions. The PCDRT translation of the definite description, obtained based on the translations in (77) and (78), is provided in (79) below.

78. *banquet of it u_1* ⤳ $\lambda v_\mathbf{e}.\,[banquet\{v\},\,of\{v,\,u_1\}]$

79. *the_sg u_1 u_3 banquet of it u_1* ⤳

$$\lambda P_\mathbf{et}.\;_{u_1}(\mathbf{max}^{u_3}\,([banquet\{u_3\},\,of\{u_3,\,u_1\}]);\,[\mathbf{unique}\{u_3\}];\;_{u_3}(P(u_3)))$$

However, to exhibit the interaction between the adverb *always u_1* and the pronoun *she u_2* in a simpler and more transparent way, I will assume that sentence (2b) contributes a dyadic relation of the form *come_with_Harvey_to_the_banquet_of* that relates girls and conventions. Just like *court_at*, this dyadic relation can be translated in two different ways, corresponding to the two possible relative scopes of *she u_2* and *always u_1* (that is, I employ the same scoping technique as the one used for sentence (1a/2a) in (62) and (63) above). The two different translations are provided in (80) and (81) below. The basic syntactic structure of sentence (2b) is provided in (82).

80. *come_to_banquet_of¹* ⤳ $\lambda Q_\mathbf{(et)t}.\lambda Q'_\mathbf{(et)t}.$

$$Q'(\lambda v'_\mathbf{e}.\,Q(\lambda v_\mathbf{e}.\,[come\_to\_banquet\_of\{v',\,v\}]))$$

81. *come_to_banquet_of²* ⤳ $\lambda Q_\mathbf{(et)t}.\lambda Q'_\mathbf{(et)t}.$

$$Q(\lambda v_\mathbf{e}.\,Q'(\lambda v'_\mathbf{e}.\,[come\_to\_banquet\_of\{v',\,v\}]))$$

82. *she [[always] come_to_banquet_of¹ᐟ²]*

The first lexical entry *come_to_banquet_of¹* gives the pronoun *she u_2* wide scope over the adverb *always u_1*, while the second lexical entry *come_to_banquet_of²* gives the pronoun narrow scope relative to the adverb. The corresponding, compositionally derived PCDRT representations are provided in (83) and (84) below.

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dref's. So, consider discourse (76) again: intuitively, there is a unique banquet at *every* convention, not only at the majority of conventions where Harvey courts a girl.

83. $(she_{u_2} >> always_{u_1})$

   $[\textbf{unique}\{u_2\},\ _{u_2}(come\_to\_banquet\_of\{u_2, u_1\})]$

84. $(always_{u_1} >> she_{u_2})$

   $[_{u_1}(\textbf{unique}\{u_2\}),\ _{u_1}(come\_to\_banquet\_of\{u_2, u_1\})]$

Thus, there are two possible representations for sentence (2a) – see (68) and (69) above – and two possible representations for sentence (2b) – given in (83) and (84) above. Hence, there are four possible representations for discourse (2) as a whole.

Out of the four possible combinations, three boil down to effectively requiring the indefinite $a^{\textbf{wk}:u_2}$ *girl* to take wide scope over the quantifier *every*^{u_1} *convention*. This can happen if: (**i**) we assign the representation in (68) to sentence (2a), in which case it does not matter which of the two representations in (83) and (84) we assign to sentence (2b), or (**ii**) we assign the representation in (83) to sentence (2b), which, as we have already shown for discourse (1) (see section **4.2** above), effectively identifies the two possible representations of sentence (2a).

We are left with the fourth combination (69) + (84), i.e. *every*^{u_1} *convention*$>>a^{\textbf{wk}:u_2}$ *girl* + *always*$_{u_1} >> she_{u_2}$, which is given in (85) below and which provides the desired "narrow-scope indefinite" reading that is available for discourse (2), but not for (1).

85. $\textbf{max}^{u_1}([convention\{u_1\}]);\ [_{u_1}(u_2)];\ [_{u_1}(girl\{u_2\}),\ _{u_1}(court\_at\{Harvey, u_2, u_1\})];$
   $[_{u_1}(\textbf{unique}\{u_2\}),\ _{u_1}(come\_to\_banquet\_of\{u_2, u_1\})]$

Intuitively, the PCDRT representation in (85) instructs us to modify the input info state $\{i_{\#}\}$ by introducing the set of all conventions relative to the dref u_1, followed by the introduction of a non-empty set of u_2-individuals relative to each u_1-convention. The remainder of the representation tests that, for each u_1-convention, the corresponding u_2-set is a singleton set consisting of a girl that is courted by Harvey at the u_1-convention currently under consideration and that comes with him at the banquet of said u_1-convention.

5. Summary

PCDRT enables us to formulate in classical type logic a compositional dynamic account of the intra- and cross-sentential interaction between generalized quantifiers, anaphora and number morphology exhibited by the quantificational subordination discourses in (1) and (2) above from Karttunen (1976).

The main proposal is that plural info states together with a suitable dynamic reformulation of independently motivated denotations for generalized determiners and number morphology in static Montague semantics enables us to account for quantificational subordination in terms of anaphora to quantifier domains and, consequently, for the contrast in interpretation between the discourses in (1) and (2) above.

The cross-sentential interaction between quantifier scope and anaphora, in particular the fact that a *singular* pronoun in the second sentence can disambiguate between the two readings of the first sentence, can be captured by plural information states because they enable us to store both quantifier domains (i.e. values) and quantificational dependencies (i.e. structure), pass them across sentential boundaries and further elaborate on them, e.g. by letting a pronoun constrain the cardinality of a previously introduced quantifier domain.

In the process, we were also able to show how the definite descriptions in sentences (2b) and (2c) can be analyzed and also how natural language quantifiers enter structured anaphoric connections as a matter of course, usually functioning simultaneously as both indefinites and pronouns.

6. Comparison with Alternative Approaches

6.1. Cross-Sentential Anaphora and Uniqueness

In this section (and the following one), I will briefly indicate some of the ways in which PCDRT relates to the previous literature on uniqueness effects associated with

singular anaphora (see Evans 1977, 1980, Parsons 1978, Cooper 1979, Heim 1982/1988, Kadmon 1987, 1990, Neale 1990 and Roberts 2003 among others).

As indicated in section **3.5** of the present chapter (see also section **3.4** of chapter **5**), the uniqueness enforced by the condition **unique**$\{u\}$ is weak in the sense that it is relativized to the current plural info state. However, we can require *strong* uniqueness, i.e. uniqueness relative to the entire model, by combining the **max**^{u} operator and the condition **unique**$\{u\}$ – as, for example, in the PCDRT translation for Russellian, non-anaphoric definite descriptions provided in (48) above.

The same **max**^{u} + **unique**$\{u\}$ strategy can be employed to capture the strong uniqueness intuitions associated with the "narrow-scope indefinite" reading of the quantificational subordination discourse in (2) above, i.e. the fact that discourse (2) as a whole implies that Harvey courts a *unique* girl *per convention*.

In more detail: we have assumed throughout this chapter (for simplicity) that the indefinite *a girl* in (2a) receives a weak reading – but, if we assume that the indefinite has a strong / maximal reading (see the translation in (38) above), we can capture the above mentioned uniqueness intuitions. The PCDRT representation of the "narrow-scope strong indefinite" reading is provided in (86) below, which differs from the representation in (85) above only with respect to the presence of the additional maximization operator **max**^{u_2} contributed by the strong indefinite.

86. **max**^{u_1} ([*convention*$\{u_1\}$]); $_{u_1}$ (**max**^{u_2} ([*girl*$\{u_2\}$, *court_at*$\{Harvey, u_2, u_1\}$]));

[$_{u_1}$ (**unique**$\{u_2\}$), $_{u_1}$ (*come_to_banquet_of*$\{u_2, u_1\}$)]

The strong uniqueness effect emerges as a consequence of the *combined* meanings assigned to the strong indefinite and the singular pronoun: the strong indefinite makes sure (by **max**^{u_2}) that, with respect to each u_1-convention, the dref u_2 stores all the girls courted by Harvey at that convention; the singular pronoun subsequently requires (by **unique**$\{u_2\}$) that the set of u_2-individuals stored relative to each u_1-individual is a singleton set. Together, the strong indefinite and the singular pronoun require that, at each u_1-convention, Harvey courts exactly one girl, which the dref u_2 stores.

Thus, PCDRT can capture the intuition that discourse (2) is interpreted as talking about conventions at which Harvey courts a unique girl (possibly different from convention to convention). Moreover, the fact that, in PCDRT, the uniqueness implications are a consequence of *combining* the meanings of the indefinite and of the singular anaphor captures the observation in Kadmon (1990): 279-280 that "[…] indefinite NP's don't always have unique referents. […] When anaphora is attempted, however, the uniqueness effect always shows up".

In a sense, this observation is literally captured in PCDRT: singular pronouns always contribute a **unique**$\{u\}$ condition. However, whether this condition yields strong uniqueness depends on the weak / strong reading of the antecedent indefinite. Against Kadmon, I take this variation to be a welcome prediction since it converges with the wavering uniqueness intuitions that native speakers have with respect to various cases of singular cross-sentential anaphora (I will return to this issue presently).

The very same ingredients employed in PCDRT to derive the (relativized) uniqueness effects in quantificational subordination also provide an account of the (absolute / non-relativized) uniqueness intuitions associated with the well-known example in (87) below.

87. There is a$^{\mathbf{str}:u}$ doctor in London and he$_u$ is Welsh.

   (Evans 1980: 222, (26)[23])

88. $\mathbf{max}^u([doctor\{u\}, in\_London\{u\}])$; $[\mathbf{unique}\{u\}, Welsh\{u\}]$

In contrast, the weak and strong readings for the indefinite article in example (89) below (from Heim 1982/1988: 28, (14a)) are truth-conditionally indistinguishable in PCDRT[24], i.e. there are no strong uniqueness implications – and correctly so. Thus, PCDRT can also account for the difference between the interpretations of (87) and (89).

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[23] Page references are to Evans (1985).

[24] The weak / strong contrast associated with an indefinite has truth-conditional effects only if there is anaphora to that indefinite.

89. There is a doctor**wk/str:***u* who is Welsh in London. [25]

Finally, given that indefinite articles are associated with both a weak and a strong meaning enables us to account for the observation in Heim (1982): 31 that singular cross-sentential anaphora is not necessarily associated with uniqueness implications, as shown by the narration-type example in (90) below (from Heim (1982): 31, (29)).

90. There was a**wk:***u* doctor in London. He*u* was Welsh…

Summarizing, the hypothesis that the indefinite article is ambiguous between a weak and a strong reading together with proposal that singular number morphology on pronouns contributes a **unique** condition enables PCDRT to capture the three-way contrast between (87), (89) and (90) above. In particular, the contrast between (87) and (90) is due to what reading is associated with the indefinite in each particular case. PCDRT does not have anything to say about this choice – and, I think, rightfully so: as much of the literature observes (Heim 1982/1988, Kadmon 1990, Roberts 2003 among others), the choice is sensitive to various factors that are pragmatic in nature and / or have related to the global structure of the discourse (e.g. that (90) is a narrative, while (87) is not).

Thus, unlike Heim (1982) and classical DRT / FCS / DPL in general, PCDRT can capture the uniqueness intuitions (sometimes) associated with cross-sentential singular anaphora – and the ingredients of the analysis, in particular the two meanings associated with the indefinite article, are independently motivated by mixed reading donkey sentences (see chapter **5** above).

Moreover, the overall account is compositional and the **unique**{*u*} condition contributed by singular number morphology on anaphors is a local constraint of the same kind as ordinary lexical relations, in contrast to the non-local and non-compositional[26]

[25] PCDRT also makes correct predictions with respect to the similar examples in (i) and (ii) below, due to Heim (1982): 28, (27) and (27a).

(i) A wine glass broke last night. It had been very expensive.

(ii) A wine glass which had been very expensive broke last night.

[26] At least, not compositional in any obvious way.

uniqueness condition proposed in Kadmon (1990) to account for such uniqueness effects[27].

Also, unlike Kadmon (1990) (see the contrast between the preliminary and the final version of the uniqueness condition in Kadmon 1990[28]), PCDRT captures the constrast between the *absolute* and *relativized* uniqueness effects instantiated by (87) (where the doctor is absolutely unique) and (2) above (where there is a unique girl per convention) without any additional stipulations.

In particular, relativized uniqueness is a consequence of the distributivity operators contributed by the quantifier taking scope over the singular pronoun – and these distributivity operators are independently motivated by the scopal interaction between multiple quantifiers and by the interaction between generalized quantification and donkey anaphora (see the discussion in section **3.3** above).

Finally, the fact that indefinite articles are analyzed in PCDRT as being associated with both a weak and a strong meaning (independently motivated by mixed reading donkey sentences) adds the needed flexibility to account for the observation that cross-sentential anaphora is not always associated with uniqueness implications, as shown by the contrast between (87) and (90) above.

6.2. Donkey Anaphora and Uniqueness

The uniqueness implications associated with intra-sentential singular donkey anaphora are, by and large, just as unstable as the ones associated with cross-sentential singular anaphora.

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[27] This is the preliminary (simpler) version of the uniqueness condition in Kadmon (1990): 284, (30): "A definite NP associated with a variable X in DRS K is used felicitously only if for every model M, for all embedding functions f, g verifying K relative to M, $f(X)=g(X)$".

[28] The preliminary version of the uniqueness condition is provided in fn. **27** above. The final version of the uniqueness condition is as follows: "Let α be a definite NP associated with a variable Y, let K_{loc} be the local DRS of α, and let K be the highest DRS s.t. K is accessible from K_{loc} and $Y \in U_K$. α is used felicitously only if for every model M, for all embedding functions f, g verifying K relative to M, if $\forall X \in B_K\, f(X)=g(X)$ then $f(Y)=g(Y)$" (Kadmon 1990: 293, (31)), where $B_K := \{X: \exists K'$ accessible from K s.t. $K' \neq K$ and $X \in U_{K'}\}$.

On the one hand, the examples in (91) and (92) below exhibit uniqueness effects –
more precisely: uniqueness effects relativized to each particular value of 'main'
generalized determiner of each sentence (i.e. *most*, *every* and *every* respectively).

91. Every man who has a^u son wills him$_u$ all his money.

   (Parsons 1978: 19, (4), attributed to B. Partee)

92. Every man who has a^u daughter thinks she$_u$ is the most beautiful girl in the world.

   (Cooper 1979: 81, (60))

On the other hand, the examples in (93), (94), (95) and (96) below do not seem to
exhibit uniqueness effects[29]. Note in particular that there are no uniqueness effects
associated even with the *weak* donkey anaphora *a$^{u'}$ credit card-it$_{u'}$* in (96) (for more
discussion of this observation, see chapter **5** above).

93. Every farmer who owns a^u donkey beats it$_u$.

94. Most people that owned a^u slave also owned his$_u$ offspring.

   (Heim 1990: 162, (49))

95. No parent with a^u son still in high school has ever lent him$_u$ the car on a
   weeknight.

   (Rooth 1987: 256, (48))

96. Every person who buys a^u TV and has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_u$.

In general, previous accounts of donkey anaphora are designed to account either for
the first set of examples, which exhibit uniqueness (e.g. Parsons 1978, Cooper 1979,
Kadmon 1990 among others), or for the second set of examples, which do not (e.g. Kamp
1981, Heim 1982/1988, 1990, Neale 1990, Kamp & Reyle 1993 among others). This is
not to say that these approaches cannot be amended to account for a broader range of data
– the point is only that the basic architecture of the theory is such that either uniqueness
or non-uniqueness follows from it.

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[29] Kadmon (1990): 307 takes examples like (93) and (94) above to exhibit uniqueness – see also example
(48) in Kadmon 1990: 307, repeated in (i) below. At the same time, Kadmon (1990): 308-309 mentions that
some informants disagree and "treat [(i)] as if it said 'at least one dog'; for them, [(i)] doesn't display a
uniqueness effect".

(i) Most women who own a dog talk to it.

In this section, I argue that the PCDRT combination of plural information states (plus maximization) on the one hand and the **unique** condition (plus distributivity) on the other hand makes for a flexible theory that can accommodate both kinds of donkey examples in a natural way. The main idea will be that all these resources enable us to 'partition' the restrictor of a generalized quantification in various ways and, depending on this 'partitioning', the morphologically singular anaphors in the nuclear scope of the generalized quantification contribute uniqueness or not.

The intuition that the uniqueness effects associated with donkey anaphora are dependent on how we 'think' about the restrictor of the generalized quantification is by no means new – it underlies the notion of *cases* in Lewis (1975), the use of *minimal situations* in Heim (1990) (among others) and the quantification over *instances* in Kadmon (1990) (see Kadmon 1990: 301). Thus, in this section, I argue that PCDRT enables us to formulate in a new and intuitive formalization of this familiar intuition.

Singular Donkey Anaphora Does Not Always Imply Uniqueness

The assumption that singular donkey anaphora can involve *non-singleton* sets has been repeatedly challenged because singular donkey anaphora seems to be intuitively associated with a kind of uniqueness implication[30]. Relative-clause donkeys in particular (like (1) and (2) above) are claimed to be associated with uniqueness presuppositions: some authors (e.g. Kanazawa 2001: 391, fn. 5) actually distinguish between relative-clause and conditional donkey sentences and claim that the former but not the latter contribute some form of uniqueness.

However, this is not the whole story. First, the uniqueness intuitions associated with relative-clause donkeys are much weaker (if at all present) when we consider examples with *multiple* donkey indefinites like (96) above, i.e., in a sense, relative-clause donkey sentences that are closer in form to conditional donkey sentences.

Second, even the proponents of uniqueness have to concede that donkey uniqueness is of a rather peculiar kind. One of the main debates revolves around the 'sage plant'

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[30] For recent discussion, see Kanazawa (2001) and Geurts (2002).

example in (97) below which, on the face of it, strongly argues against donkey uniqueness.

97. Everybody who bought a^u sage plant here bought eight others along with it$_u$.
   (Heim 1982/1988: 89, (12))

Kadmon (1990): 317 conjectures that the donkey anaphora in (97) still contributes a uniqueness presupposition, but the "speakers accept this example because it can't make any difference to truth conditions which sage plant the pronoun *it* stands for, out of all the sage plants that a buyer *x* bought (for each buyer *x*)".

But, as Heim (1990): 161 points out, Kadmon's 'supervaluation'[31] analysis makes incorrect predictions with respect to the example in (95) above from Rooth 1987: intuitively, sentence (95) is falsified by any parent who has a son in high school and who has lent him the car on a weeknight even if said parent has another son who never got the car – which is to say that it *does* make a difference in this case which son the pronoun *him$_u$* in (95) stands for[32].

This being said, example (91) above does seem to exhibit uniqueness implications – so, an empirically adequate account of donkey anaphora should be flexible enough to accommodate the wavering nature of the uniqueness intuitions associated with it.

Capturing the Wavering Nature of the Uniqueness Intuitions

As it now stands, the revised version of PCDRT introduced in this chapter predicts that donkey anaphora is associated with relativized uniqueness implications, i.e. it can account for the uniqueness intuitions associated with (91) above. As shown in (98) below, relativized uniqueness emerges as a consequence of the interaction between: (**i**) the distributivity operators contributed by selective generalized determiners, (**ii**) the maximization contributed by the strong reading of the indefinite and (**iii**) the **unique** condition contributed by the singular pronoun.

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[31] The connection with supervaluation treatments of vagueness is due to Mats Rooth – see Heim (1990): 160, fn. 11.

[32] For more discussion, see also Geurts (2002): 145 et seqq.

98. Every$^{u'}$ man who has a$^{\mathbf{str}:u}$ son wills him$_u$ all his money.

$\mathbf{max}^{u'}(_u([man\{u'\}]; \mathbf{max}^u([son\{u\}, have\{u', u\}])));$

$_u([\mathbf{unique}\{u\}, will\_all\_money\{u', u\}])$[33]

Parsons (1978) considers the uniqueness effects associated with the donkey sentence in (91) above and suggests two different ways to capture them. The above PCDRT analysis can be seen as an implementation of the first suggestion:

> "One might suggest that the feeling of inappropriateness [of sentence (91) when taken to be talking about men that have more than one son] comes explicitly from the use of the pronoun. How would that work? Well, one purported meaning of 'a' is 'one', in the sense of 'exactly one'. Usually this is thought to be a presupposition, implication, or implicature of the utterance rather than part of the content of what is said. But perhaps the use of a singular pronoun can make the import part of the official content.
> The suggestion then is that 'a' can mean either 'at least one' or 'exactly one'. Normally it means the former, but certain grammatical constructions force the latter reading. The former reading is the 'indefinite' one, and the latter is the 'definite' one."
> (Parsons 1978: 19)

Interestingly, Parson's second suggestion is the one that is taken up by D-/E-type approaches that take pronouns to be numberless Russellian definite descriptions (e.g. Neale 1990)[34].

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[33] An unfortunate consequence of the fact that the **unique**$\{u\}$ condition contributed by the pronoun is taken to be part of the assertion is that the PCDRT representation in (98) is true only if every man has exactly one son, while, intuitively, the quantification should be restricted to men that have only one son. That is, the intuitively correct representation for (91) is the one in (i) below, where the **unique**$\{u\}$ condition occurs in the restrictor. This representation can be obtained if we assume that the **unique**$\{u\}$ condition is presupposed and that presuppositions triggered in the nuclear scope of tripartite quantificational structures can be accommodated in the restrictor (both assumptions, i.e. that number morphology on pronouns is presuppositional and that nuclear scope presuppositions can be accommodated in the restrictor, are independently assumed and motivated in the literature – see for example Beaver & Zeevat 2006, Heim 2005 and references therein).

(i) $\mathbf{max}^{u'}(_u([man\{u'\}]; \mathbf{max}^u([son\{u\}, have\{u', u\}]); [\mathbf{unique}\{u\}])); _u(will\_all\_money\{u', u\}]).$

[34] "Sometimes 'the' doesn't mean 'exactly one', but rather 'at least one' or 'every'. It means 'at least one' in *everyone must pay the clerk five dollars* and it means 'every' in *you should always watch out for the other driver*. Or something like this. So perhaps the treatment of pronouns as paraphrases is correct, but we have to tailor the meaning of 'the' for the situation at hand. For example, in our sample sentence we need to read *the donkey he owns* as *every donkey he owns*. This response would involve specifying some method for determining which reading of *the* is appropriate in a given paraphrase; I haven't carried this out" (Parsons 1978: 20).

Thus, the version of PCDRT proposed in this chapter (chapter **6**) sides with the "uniqueness" approaches (e.g. Parsons 1978, Cooper 1979, Kadmon 1990 among others) – and therefore accounts for only one of the two sets of data. In contrast, the version of PCDRT proposed in the previous chapter (chapter **5**), which does not take singular pronouns to contribute a **unique** condition, sides with the "non-uniqueness" approaches (e.g. Kamp 1981, Heim 1982/1988, 1990, Neale 1990, Kamp & Reyle 1993 among others).

The trade-off is as follows. On the one hand, chapter **5** accounts for a variety of donkey sentences, i.e. cases of intra-sentential anaphora, including mixed reading examples like (96) above. On the other hand, chapter **6** accounts for a variety of uniqueness effects with cross-sentential and intra-sentential anaphora, i.e. forcing the "wide-scope indefinite" reading for discourse (1) above, deriving the relativized uniqueness effects for the "wide-scope indefinite" reading of discourse (2) and deriving the relativized uniqueness effects for the donkey sentence in (91) above.

I will now show that there is a straightforward way to recover the results of chapter **5** within the version of PCDRT introduced in the present chapter. The main observation is that **unique**$\{u\}$ conditions are vacuously satisfied under distributivity operators like **dist**$_u$, so, to cancel the uniqueness effects, we only need to assume that selective generalized determiners introduce such distributivity operators relative to their nuclear scope update.

The simplest such operator is the unselective distributivity operator defined in (99) below, which is used in the definition of generalized quantification in (103). Note that this definition of generalized quantification differs from the one introduced in (36) above (see section **3.3**) only with respect to the nuclear scope distributivity operator.

99. $\mathrm{dist}(D) := \lambda I_{st} J_{st}.\ \exists R_{s((st)t)}(I{=}\mathbf{Dom}(R) \wedge J{=}\cup\mathbf{Ran}(R)\ \wedge \forall {<}i_s, J_{st}{>}\in R(D\{i\}J),$

$\quad$ where D is of type $\mathbf{t} := (st)((st)t)$.

100. $\mathrm{dist}_u(D) := \lambda I_{st} J_{st}.\ uI{=}uJ \wedge \forall x_e\in uI(\mathrm{dist}(D)I_{u=x}J_{u=x})$

101. $_u\{D\} := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}{=}J_{u=\#} \wedge I_{u\neq\#}{\neq}\emptyset \wedge \mathrm{dist}_u(D)I_{u\neq\#}J_{u\neq\#}$

102. $_{\langle u\rangle}\{D\} := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}{=}J_{u=\#} \wedge (I_{u\neq\#}{=}\emptyset \to I{=}J) \wedge (I_{u\neq\#}{\neq}\emptyset \to \mathrm{dist}_u(D)I_{u\neq\#}J_{u\neq\#})$

103. $det^{u, u' \sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u({}_{\langle u \rangle}(P(u)));\ \mathbf{max}^{u' \sqsubseteq u}({}_{\langle u \rangle}\{P'(u')\});\ [\mathbf{DET}\{u, u'\}]$

The distributivity operator $\mathbb{dist}(D)$ is unselective because any input info state I is updated with the DRS D in a pointwise manner, i.e. we update each of the 'assignments' $i_s \in I$ with D.

This way of updating a set of 'assignments' is unselective in same sense as the generalized quantification over cases proposed in Lewis (1975) is unselective: the definition in (103) instructs us to take each 'assignment' delivered by the restrictor of the quantification separately and check that it satifies the nuclear scope of the quantification, where 'assignments' are also known as: "cases" in the terminology of Lewis (1975), "minimal situations" in the terminology of Heim (1990) and "instances" in the terminology of Kadmon (1990).

Note that the use of unselective distributivity in the definition of dynamic generalized quantification does not endanger our previous results: the definition in (103) does not have a proportion problem (because **DET** relates the relevant sets of inviduals) and can account for weak / strong ambiguities, including mixed reading donkey sentences. For example, sentence (96) is represented as shown in (104) below. The **unique**$\{u'\}$ and **unique**$\{u\}$ conditions contributed by the singular donkey pronouns $it_{u'}$ and it_u are vacuously satisfied because the unselective $\mathbb{dist}$ operator 'feeds' them only singleton informations states $\{i\}$[35].

104. Every$^{u''}$ person who buys a$^{\mathbf{str}:u}$ TV and has a$^{\mathbf{wk}:u'}$ c.card uses it$_{u'}$ to pay for it$_u$.

   $\mathbf{max}^{u''}({}_{u''}([pers\{u''\}];\ \mathbf{max}^u([\mathrm{TV}\{u\},\ buy\{u'',u\}]);\ [u' \mid c.card\{u'\},\ hv\{u'',u'\}]));$

   ${}_{u''}\{[\mathbf{unique}\{u'\},\ \mathbf{unique}\{u\},\ {}_u(use\_to\_pay\{u'',u',u\})]\}$

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[35] More precisely: the second conjunct of the definitions of **unique**$\{u'\}$ or **unique**$\{u\}$ is indeed vacuously satisfied with respect to any singleton info state $\{i\}$ whatsoever, but the first conjunct of their definitions fails for any $\{i\}$ such that $ui=\#$ or $u'i=\#$ (i.e. $\{i\}_{u' \neq \#} \neq \emptyset$ is false or $\{i\}_{u \neq \#} \neq \emptyset$ is false). Therefore, the nuclear scope update ${}_{u''}\{[\mathbf{unique}\{u'\}, \mathbf{unique}\{u\}, {}_u(use\_to\_pay\{u'',u',u\})]\}$ fails for any input info state I where there is at least one $i_s \in I_{u'' \neq \#}$ such that $ui=\#$ or $u'i=\#$. But this does not affect the truth-conditions derived by the representation in (104), which are the intuitively correct ones for mixed reading donkey sentence in (96). And, as far as the anaphoric potential goes (both with respect to value and with respect to structure), it seems to me that the above mentioned consequence of the ${}_{u''}\{\dots\}$ operator is in fact desirable.

Thus, our strategy is to neutralize the uniqueness effects associated with intra-sentential singular anaphora by introducing suitable distributivity operators that take the nuclear scope of the main generalized quantification as argument. This (as opposed to, for example, making the **unique** condition contributed by the singular pronoun optional) has the desirable consequence that we leave untouched the uniqueness effects associated with cross-sentential anaphora in general and with quantificational subordination in particular; that is, we preserve all the results previously obtained in this chapter (see sections **4** and **6.1** above).

Summarizing, the increased flexibility of the theoretical architecture of PCDRT (when compared to previous approaches) enables it to account for the unstable uniqueness intuitions associated with donkey anaphora. The account makes crucial use of plural info states and distributivity operators. More precisely, in any tripartite quantificational structure, we have a choice between selective and unselective nuclear scope distributivity. The decision to use one or the other depends on how we 'think about' the relation between the restrictor and the nuclear scope of the quantification on a particular occasion (which, in turn, is determined by the global discourse context, world knowledge etc., i.e. by various pragmatic factors):

- if we focus on the individuals contributed by the restrictor, we predicate the nuclear scope of each such individual separately, so we use a selective distributivity operator $\mathbf{dist}_u$ and we obtain uniqueness effects (relativized to u);
- if we focus on the (minimal) cases / situations contributed by the restrictor, we predicate the nuclear scope of each such case separately, so we use an unselective distributivity operator $\mathbb{dist}$ and we neutralize / cancel all uniqueness effects.

These two choices, i.e. $\mathbf{dist}_u$ and $\mathbb{dist}$, are the two extremes of a possibly much richer spectrum: if we use $\mathbf{dist}_u$, we are as coarse-grained as possible when we predicate the nuclear scope update; if we use $\mathbb{dist}$, we are as fine-grained as possible. In between these extremes, we can define a family of of *multiply selective* distributivity operators as

shown in (105) and (106) below (see also appendix **0** below). I leave their investigation for future research[36].

105. $_{u,u',...}(D) := \lambda I_{st}.\lambda J_{st}. \ (I_{u=\#}=J_{u=\#} \wedge I_{u'=\#}=J_{u'=\#} \wedge \ldots) \wedge \ I_{u\neq\#,u'\neq\#,...}\neq\emptyset \wedge$

$$\mathbf{dist}_{u,u',...}(D)I_{u\neq\#,u'\neq\#,...}J_{u\neq\#,u'\neq\#,...}$$

106. $_{\langle u,u',...\rangle}(D) := \lambda I_{st}.\lambda J_{st}. \ (I_{u=\#}=J_{u=\#} \wedge I_{u'=\#}=J_{u'=\#} \wedge \ldots) \wedge (I_{u\neq\#,u'\neq\#,...}=\emptyset \rightarrow I=J) \wedge$

$$(I_{u\neq\#,u'\neq\#,...}\neq\emptyset \rightarrow \mathbf{dist}_{u,u',...}(D)I_{u\neq\#,u'\neq\#,...}J_{u\neq\#,u'\neq\#,...})$$

6.3. Telescoping

The phenomenon of telescoping is exemplified by discourses (107) and (108) below, where a singular pronoun seems to be cross-sententially anaphoric to a quantifier. The term is due to Roberts (1987, 1989) and is meant to capture the fact that, in such discourses, "from a discussion of the general case, we zoom in to examine a particular instance" (Roberts 1987: 36).

107. **a.** Each^u candidate for the space mission meets all our requirements.

   b. He$_u$ has a PhD in Astrophysics and extensive prior flight experience.

   (Roberts 1987: 36, (38)[37])

108. **a.** Each^u degree candidate walked to the stage.

   b. He$_u$ took his$_u$ diploma from the Dean and returned to his$_u$ seat.

   (Roberts 1987: 36, (34), attributed to B. Partee)

The main observation about this phenomenon (which can be traced back to a pair of examples due to Fodor & Sag 1982 and to Evans 1980) is that "the possibility of anaphoric relations in such telescoping cases depends in part on the plausibility of some sort of narrative continuity between the utterances in the discourse" (Roberts 1987: 36). Thus, Evans (1980) observes that the discourse in (109) below is infelicitous. The examples in (110) and (111) from Poesio & Zucchi (1992) are similarly infelicitous.

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[36] The analysis of the interaction between donkey anaphora and quantificational adverbs (*always*, *usually* etc.) in conditionals might require such multiply selective distributivity operators.

[37] Page references to Roberts (1990).

109. #Every^u congressman came to the party and he$_u$ had a marvelous time.

(Evans 1980: 220, (21)[38])

110. #Every^u dog came in. It$_u$ lay down under the table.

(Poesio & Zucchi 1992: 347, (1))

111. #Each^u dog came in. It$_u$ lay down under the table.

(Poesio & Zucchi 1992: 360, (39c))

The challenge posed by telescoping is to account both for the felicity of (107) and (108) and for the infelicity of (109), (110) and (111), as Poesio & Zucchi (1992) and Roberts (1995, 1996) among others emphasize.

In this respect, DRT / FCS / DPL approaches (Kamp 1981, Heim 1982/1988, Kamp & Reyle 1993 among others) fail because they can account only for the infelicity of (109), (110) and (111), but not for the felicity of (107) and (108). This is a direct consequence of the fact that generalized quantifiers are externally static in this kind of systems (such systems also fail to account for the quantificational subordination discourse in (2) above). Dynamic Montague Grammar (DMG, see Groenendijk & Stokhof 1990) and systems based on it (e.g. Dekker 1993) define generalized quantifers as externally dynamic and, therefore, fail in the opposite way: they can account for the felicity of (107) and (108), but not for the infelicity of (109), (110) and (111). Moreover, DMG does not derive the correct truth-conditions for all telescoping and quantificational subordination discourses (see the discussion in Poesio & Zucchi (1992): 357-359).

The analyses of telescoping in Poesio & Zucchi (1992), Roberts (1995, 1996)[39] and Wang et al (2006) (among others – see the detailed discussion in Wang et al 2006) are more flexible and they can account for both kinds of examples. These accounts make crucial use of more general, pragmatic notions having to with world knowledge and global discourse structure: (**i**) accommodation (for Poesio & Zucchi 1992 and Roberts 1995, 1996) and (**ii**) rhetorical relations (for Wang et al 2006). These accounts differ with

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[38] Page references to Evans (1980).

[39] See also the modal subordination accounts in Geurts (1995/1999) and Frank (1996), which could be generalized to quantificational subordination following the same basic strategy as Poesio & Zucchi (1992) and Roberts (1995, 1996).

respect to their main strategy of analysis: Poesio & Zucchi (1992) and Roberts (1995, 1996) take the infelicitous examples as basic and then devise special mechanisms to account for the felicitous examples, which extract and pass on the relevant discourse information; Wang et al (2006) take the felicitous examples as basic, assume that the relevant discourse information is always available, but that it has to *accessed* in a particular way.

The PCDRT account of telescoping I will sketch below falls in the same category as the Wang et al (2006) account: plural information states ensure that the relevant information is always available, but the singular number morphology on the anaphoric pronoun constrains the way in which it can be accessed. At the same time, I will make limited use of accommodation – and, in this respect, the account is similar to Poesio & Zucchi (1992) and Roberts (1995, 1996).

The PCDRT account is a development of the suggestion made in Evans (1980): 220 with respect to the infelicity of (109). Evans conjectures that the infelicity is a consequence of a clash in *semantic* number between the antecedent and the anaphor (note that there is no clash in morphological number): on the one hand, the quantificational antecedent contributes a non-singleton condition on its restrictor set; on the other hand, the singular pronoun anaphoric to the restrictor set requires it to be a singleton.

I will formalize the non-singleton requirement contributed by selective generalized determiners by means of the **non-unique** condition defined in (112) below[40].

112. **non-unique**$\{u\} := \lambda I_{st}.\ I_{u\neq\#}\neq\emptyset \wedge \exists i_s, i'_s \in I_{u\neq\#}(ui\neq ui')$

In addition, I will make use of two ingredients independently motivated by the uniqueness effects associated with donkey anaphora (see the previous section), namely: (**i**) the **unique**$\{u\}$ condition contributed by the singular number morphology on a

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[40] Green (1989) and Chierchia (1995) (among others) argue that this non-singleton condition has presuppositional status. In contrast, Neale (1990) suggests that it is in fact an implicature. I find the arguments in Green (1989) more persuasive, but I leave a more careful investigation of this issue for future research. For simplicity, I will take the **non-unique**$\{u\}$ condition contributed by generalized determiners to be part of the assertion.

pronoun and **(ii)** the fact that a **dist**$_u$ or 𝖽𝗂𝗌𝗍 operator that takes scope over such a condition ensures that it is vacuously satisfied.

More concretely, I will assume that the global accommodation of a distributivity operator $_u(\ldots)$ is licensed in the case of the felicitous example analyzed in (113) below, but it is not licensed in the case of the infelicitous example analyzed in (114).

113. Each^u candidate meets all our requirements. $_u($ He$_u$ has a PhD in Astrophysics).
     max^u([*candidate*{u}]); $_u$([*meet_requirements*{u}]); [**non-unique**{u}];
     $_u$([**unique**{u}, *have_PhD*{u}])

114. Each^u dog came in. #It$_u$ lay down under the table.
     max^u([*dog*{u}]); $_u$([*come_in*{u}]); [**non-unique**{u}];
     [**unique**{u}, *lay_under_table*{u}]

Of course, nothing in the above analysis specifies when we can and when we cannot accommodate such a distributivity operator. I will return to this issue below. For now, note only that accommodating such an operator should not come for free because we introduce a new meaning component in the discourse representation that is not associated with any morpho-syntactic realization.

The account of the felicitous example in (113) above captures in a direct way the 'telescoping' intuitions associated with it, i.e. the fact that, as Roberts (1987) puts it, the second sentence "zooms in" from a discussion of the general case to a particular instance: the distributivity operator partitions a particular domain of quantification and each cell of the partition is associated with a particular individual; after the domain is partitioned in this way, we update each cell in the partition separately, i.e. "instance by instance".

Note also that the PCDRT account correctly predicts that telescoping cases with plural pronouns are felicitous (or at least better than their singular counterparts), as shown by (115) and (116) below. The reason is that plural pronouns like *they*$_u$ do not contribute a **unique**{u} condition, hence there is no need to accommodate a distributivity operator $_u(\ldots)$ to neutralize / cancel the effects of such a condition.

115. **a.** Every^u dog came in. **b.** (?)They$_u$ lay down under the table.

116. **a.** Every^u congressman came to the party. **b.** (?)They^u had a marvelous time.

(Evans 1980: 220, (22))

Similarly, PCDRT can account for the plural anaphora example in (117), which combines quantificational subordination and telescoping – and, also, for the variation on this example in (118). Note in particular that PCDRT can capture the relativized uniqueness effects in (118), i.e. the fact that, intuitively, every man loves exactly one woman; this is due to the fact that the **unique**$\{u'\}$ condition contributed by the singular pronoun *her$_{u'}$* is within the scope of the distributivity operator $_u(\dots)$ contributed by the pronoun *they$_u$*.

117. **a.** Every^u man loves a$^{u'}$ woman. **b.** They$_u$ bring them$_{u'}$ flowers to prove this.

(van den Berg 1996a: 168, (16))

118. **a.** Every^u man loves a$^{u'}$ woman. **b.** They$_u$ bring her$_{u'}$ flowers.

(Wang et al 2006: 7, (20))

Moreover, PCDRT can capture the relativized uniqueness associated with the cross-sentential anaphora *a$^{u'}$ spare pawn-it$_{u'}$* in example (119) below from Sells (1985) (see also Kadmon 1990 for discussion). We only need to assume that a distributivity operator $_u(\dots)$ with scope over the second sentence in (119) is accommodated. At the same time, PCDRT correctly predicts that the restrictive relative clause example in (120) (also from Sells 1985) does not have relativized uniqueness implications associated with it.

119. **a.** Every^u chess set comes with a$^{\mathbf{str}:u'}$ spare pawn.

b. $_u$(It$_{u'}$ is taped to the top of the box).

120. Every^u chess set comes with a$^{\mathbf{wk/str}:u'}$ spare pawn that is taped to the top of the box.

Using the same ingredients, PCDRT can also account for the contrast in acceptability between (121) and (122) below, from Roberts (1996) (examples (1) and (1') on p. 216) – we only need to assume that the accommodation of a distributivity operator $_u(\dots)$ is possible in the case of (122) but not in the case of (121).

121. **a.** Every^u frog that saw an$^{u'}$ insect ate it$_{u'}$. #It$_{u'}$ was a fly.

122. **a.** Every$^{u'}$ frog that saw an$^{u'}$ insect ate it$_{u'}$. **b.** $_u$(It$_{u'}$ disappeared forever).

The infelicity of (121) is derived as follows: given that it is not possible for multiple frogs to eat the same insect (this is world knowledge), after we process the update contributed by the first sentence in (121), there should be at least as many eaten insects as there are frogs. But the **unique**$\{u'\}$ condition contributed by the pronoun *it*$_{u'}$ in the second sentence of (121), which is not in the scope of any distributivity operator, requires that there is only one such eaten insect (which, by the way, was a fly). Since there are at most as many insects as there are frogs, this means that the set of frogs is (at most) a singleton, which contradicts the **non-unique**$\{u\}$ condition contributed by the determiner *every*^{u}.

Finally, the same ingredients also enable us to account for the examples in (123) and (124) below from Wang et al (2006) (examples (2) on p. 1 and (19) on p. 7 respectively) – and for the relativized uniqueness effects associated with (123).

123. **a.** Every^u hunter that saw a$^{u'}$ deer shot it$_{u'}$. **b.** $_u$(It$_{u'}$ died immediately.)

124. **a.** Every^u hunter that saw a$^{u'}$ deer shot it$_{u'}$. **b.** They$_{u'}$ died immediately.

The problem left unaddressed by the account sketched above is how to decide when we can and when we cannot accommodate such distributivity operators. – and which distributivity operator it is, i.e. which quantificational domain we "zoom in". PCDRT, which is a semantic framework, does not (have to) say any thing about this – but I want to suggest that it offers the two things that we can expect from a semantic theory, namely: (**i**) it provides a precisely circumscribed way in which a more general pragmatic theory can interface with the semantic theory and (**ii**) when the pragmatic 'parameters' / factors are specified, it delivers the intuitively correct truth-conditions.

The previous literature uncovered two important factors that determine whether a distributivity operator can be accommodated or not in PCDRT: (**i**) the rhetorical structure of the discourse – see Wang et al (2006) and (**ii**) general world knowledge – see the notion of script in Poesio & Zucchi (1992). As I have suggested, PCDRT needs to be supplemented with the same kind of pragmatic theory that these alternative approaches assume; there are, however, certain differences between PCDRT and these alternative approaches.

Compared to the accommodation theories proposed in Poesio & Zucchi (1992) and Roberts (1995, 1996)[41], which involve accommodation of discourse referents, conditions, DRS's etc. (triggered by the presuppositional nature of quantifier domain restriction in the case of Roberts 1995, 1996), the PCDRT accommodation procedure is much simpler and involves a clearly circumscribed alteration of the discourse representation, namely: the global accommodation of a distributivity operator with the purpose of satisfying the **unique**$\{u\}$ presupposition[42] contributed by singular number morphology on pronouns. Therefore, I expect that the over-generation problem faced by PCDRT is milder than the one faced by these theories.

Compared to Wang et al (2006) and van den Berg (1996a) (and also Poesio & Zucchi 1992 and Roberts 1987, 1989, 1995, 1996), PCDRT has the advantage that, given its underlying type logic, a Montagovian compositional interpretation procedure can be easily specified, as the present chapter and the previous one have shown.

Moreover, PCDRT simplifies the system in van den Berg (1996a) both with respect to the underlying logic (which is not partial anymore) and with respect to various definitions (e.g. the definitions of the maximization and distributivity operatos) and translations (e.g. the translation of indefinite articles and pronouns).

Finally, unlike the account proposed in Wang et al (2006) (see p. 17 et seqq), the PCDRT account of telescoping is more modular, in the sense that its semantic interpretation procedure (i.e. type-driven translation) is separated from the more global pragmatics of discourse (which involves world knowledge, rhetorical relations etc.). The separation of the semantic and pragmatic interpretive components in PCDRT enables us to simplify multiple aspects of the semantic theory: its underlying logic, the notion of

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[41] Roberts (1995, 1996) build on the more explicit account in Roberts (1987, 1989), which involves accommodation of DRS's.

[42] The condition **unique**$\{u\}$ contributed by number morphology on pronouns is clearly presuppositional – I treat it as an assertion throughout the present dissertation only for simplicity.

See Heim (2005) and reference therein for more discussion of the presuppositional contributions of pronominal morphology. See Beaver & Zeevat (2006) for a recent discussion of accommodation. See Kramer (1998) for a systematic investigation of presupposition in a framework based on CDRT (Muskens 1996), hence closely related to PCDRT.

info state that we use, the operators that we need to access the information stored in these states and the translations given for various lexical items.

I will conclude with the observation that the brief comparison with the previous literature in the last three sections can only be preliminary – and for at least three reasons:

- uniqueness implications are taken to have presuppositional status in much of the previous literature (and for good reason), while I have assumed (for simplicity) that the **unique**$\{u\}$ condition is part of the assertion; thus, a more thorough comparison will be possible only when PCDRT is extended with a theory of presupposition (see Krahmer 1998 for an extensive investigation of presupposition within a framework that also builds on the CDRT of Muskens 1996);

- the import of various design choices specific to different theoretical architectures can be properly evaluated only in the context of a precise investigation of the factors that affect uniqueness in particular instances of singular intra- and cross-sentential anaphora – and such an investigation is beyond the scope of the present dissertation (but see Roberts 2003 and Wang et al 2006 for two recent discussions);

- the uniqueness implications associated with singular cross-sentential anaphora are closely related to the maximality implications associated with plural cross-sentential anaphora – and a proper comparison needs to take into account how any given theory fares with respect to both of them; the present investigation, however, focuses on *morphologically singular* anaphora and on the arguments it provides for a notion of *plural* information state[43].

Given the primarily foundational purpose of the present investigation, such issues can be addressed only partially – but I hope to have at least shown that PCDRT provides a promising framework within which it is possible to formulate simpler and, in certain respects, better analyses of quantificational subordination, donkey anaphora and telescoping and the uniqueness effects associated with them.

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[43] For more discussion of the distinction between plural information states and morphologically plural anaphora, see chapter **8** below and Brasoveanu 2006c.

Appendix

A1. Extended PCDRT: The New Definitions and Translations

125. **Structured Inclusion, Maximization and Distributivity Operators.**

    a. $u' \Subset u := \lambda I_{st}.\ \forall i_s \in I(u'i{=}ui \lor u'i{=}\#)$

    b. $u' \sqsubseteq u := \lambda I_{st}.\ (u' \Subset u)I \land \forall i_s \in I(ui \in u'I_{u' \neq \#} \to ui{=}u'i)$

    c. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \land \forall K_{st}(([u]; D)IK \to uK_{u \neq \#} \subseteq uJ_{u \neq \#})$

    d. $\mathbf{max}^{u' \sqsubseteq u}(D) := \mathbf{max}^{u'}([u' \sqsubseteq u]; D)$

    e. $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ \forall x_e(I_{u=x} \neq \emptyset \leftrightarrow J_{u=x} \neq \emptyset) \land \forall x_e(I_{u=x} \neq \emptyset \to DI_{u=x}J_{u=x})$,

        i.e. $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ uI{=}uJ \land \forall x_e \in uI(DI_{u=x}J_{u=x})$,

        where $I_{u=x} := \{i_s \in I: ui{=}x\}$

    f. $_u(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}{=}J_{u=\#} \land I_{u \neq \#} \neq \emptyset \land \mathbf{dist}_u(D)I_{u \neq \#}J_{u \neq \#}$

    g. $_{\langle u \rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}{=}J_{u=\#} \land (I_{u \neq \#}{=}\emptyset \to I{=}J) \land (I_{u \neq \#} \neq \emptyset \to \mathbf{dist}_u(D)I_{u \neq \#}J_{u \neq \#})$

    h. $_u(C) := \lambda I_{st}.\ I_{u \neq \#} \neq \emptyset \land \forall x \in uI_{u \neq \#}(CI_{u=x})$,   where C is a condition (of type $(st)t$)

    i. $_u(u_1, \ldots, u_n) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#}{=}J_{u=\#} \land I_{u \neq \#}[u_1, \ldots, u_n]J_{u \neq \#}$,

        where $u \notin \{u_1, ..., u_n\}$ and $[u_1, ..., u_n] := [u_1]; ...; [u_n]$

126. **Distributivity-based Equivalences.**

    a. $_u([C_1, \ldots, C_m]) = [_u(C_1), \ldots, _u(C_m)]$

    b. $_u([u_1, ..., u_n \mid C_1, \ldots, C_m]) = [_u(u_1, ..., u_n) \mid _u(C_1), \ldots, _u(C_m)])$

127. **Atomic Conditions.**

    a. $R\{u_1, ..., u_n\} := \lambda I_{st}.\ I_{u_1 \neq \#, ..., u_n \neq \#} \neq \emptyset \land$

                $\forall i_s \in I_{u_1 \neq \#, ..., u_n \neq \#}(R(u_1 i, \ldots, u_n i))$,

$$\text{where } I_{u_1 \neq \#,...,u_n \neq \#} := \{i_s \in I: u_1 i \neq \# \wedge \ldots \wedge u_n i \neq \#\}$$

b. DET$\{u, u'\} := \lambda I_{st}.$ **DET**$(uI_{u\neq\#}, u'I_{u'\neq\#}),$ where **DET** is a static determiner.

c. unique$\{u\} := \lambda I_{st}. I_{u\neq\#} \neq \emptyset \wedge \forall i_s, i'_s \in I_{u\neq\#}(ui = ui')$

128. **Translations.**

a. $det^{u,u' \sqsubseteq u} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ **max**$^{u}(_{\langle u \rangle}(P(u)));$ **max**$^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')));$ [**DET**$\{u, u'\}$]

b. $det_u{}^{u' \sqsubseteq u} \rightsquigarrow \lambda P_{et}.$ **max**$^{u' \sqsubseteq u}(_{\langle u' \rangle}(P(u')));$ [**DET**$\{u, u'\}$]

c. $a^{\mathsf{wk}:u} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ $[u];$ $_u(P(u));$ $_u(P'(u))$

d. $a^{\mathsf{str}:u} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ **max**$^{u}(_u(P(u));$ $_u(P'(u)))$

e. $he_u \rightsquigarrow \lambda P_{et}.$ [**unique**$\{u\}$]; $_u(P(u))$

f. $they_u \rightsquigarrow \lambda P_{et}.$ $_u(P(u))$

g. $the\_sg_u \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ [**unique**$\{u\}$]; $_u(P(u));$ $_u(P'(u))$

h. $the\_pl_u \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ $_u(P(u));$ $_u(P'(u))$

i. $the\_sg^u \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ **max**$^{u}(_u(P(u)));$ [**unique**$\{u\}$]; $_u(P'(u))$

j. $the\_pl^u \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ **max**$^{u}(_u(P(u)));$ $_u(P'(u))$

k. $the\_sg_u{}^{u'} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ $_u($**max**$^{u'}(_{u'}(P(u')));$ [**unique**$\{u'\}$]; $_{u'}(P'(u')))$

l. $Harvey^u \rightsquigarrow \lambda P_{et}.$ $[u \mid u \in Harvey];$ $_u(P(u)),$ where $Harvey := \lambda i_s.$ $harvey_e$

m. $every^u \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.$ **max**^{u} $(_u(P(u)));$ $_u(P'(u))$

n. $always_u \rightsquigarrow \lambda P_{et}.$ $_u(P(u))$

A2. Generalized Selective Distributivity

First, we need to generalize our abbreviation for partition cells induced by dref's over plural information states, as shown in (129) below.

129. $I_{u=x} := \{i_s \in I_{st}: ui=x\}$ and $I_{p=w} := \{i_s \in I_{st}: pi=w\}$.

In general:

$$I_{\alpha_1=f_1,...,\alpha_n=f_n} := \{i_s \in I_{st}: \alpha_1 i=f_1 \wedge ... \wedge \alpha_n i=f_n\},$$

where the types of the terms $\alpha_1,...,\alpha_n$ are in **DRefTyp** and

for each m s.t. $1 \leq m \leq n$, if the type of α_m is $(s\tau)$, then f_m is of type $\tau \in$ **STyp**.

Second, we generalize DRS-level distributivity to multiple dref's, as shown in (130) below.

130. **DRS-level selective distributivity (i.e. distributivity over type $t := (st)((st)t)$).**

$$\mathbf{dist}_u(D) := \lambda I_{st} J_{st}. \ uI=uJ \wedge \forall x_e \in uI(DI_{u=x}J_{u=x}),$$

where u is of type $\mathbf{e} := se$ and D is of type $\mathbf{t} := (st)((st)t)$.

In general:

$$\mathbf{dist}_{\alpha_1,...,\alpha_n}(D) := \lambda I_{st} J_{st}. \ (\alpha_1 I=\alpha_1 J \wedge ... \wedge \alpha_n I=\alpha_n J) \wedge$$

$$\forall f_1 \in \alpha_1 I ... \forall f_n \in \alpha_n I(I_{\alpha_1=f_1,...,\alpha_n=f_n} \neq \emptyset \leftrightarrow J_{\alpha_1=f_1,...,\alpha_n=f_n} \neq \emptyset) \wedge$$

$$\forall f_1 \in \alpha_1 I ... \forall f_n \in \alpha_n I(I_{\alpha_1=f_1,...,\alpha_n=f_n} \neq \emptyset \rightarrow DI_{\alpha_1=f_1,...,\alpha_n=f_n} J_{\alpha_1=f_1,...,\alpha_n=f_n}),$$

where the types of the terms $\alpha_1, ..., \alpha_n$ are in **DRefTyp**

and for each m s.t. $1 \leq m \leq n$, if the type of α_m is $(s\tau)$, then f_m is of type $\tau \in$ **STyp**

and D is of type $\mathbf{t} := (st)((st)t)$.

The general version of DRS-level selective distributivity is more complicated because we work simultaneously with n partitions induced by the drefs $\alpha_1, \ldots, \alpha_n$ on both the input state I and the output state J. The intersection of two partitions is another partition, but we are not guaranteed than the intersection of any two cells in the two partitions is non-empty – hence the antecedent of the conditional in the third conjunct of the generalized definition in (130), i.e. $I_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset$.

Moreover, we want to ensure that there is a bijection between the intersection of the n partitions over the input state I and the intersection of the n partitions over the output state J, hence the first two conjuncts in the generalized definition in (130): the first one ensures that the values of the n drefs that we distribute over are the same; the second conjunct ensures that there is a bijection between the non-empty, n-distributive cells in the input state partition and the non-empty, n-distributive cells in the output state partition.

Note that the first two conjuncts in the generalized definition in (130) could be replaced with the biconditional $\forall f_1 \ldots \forall f_n (I_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset \leftrightarrow J_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset)$, which would make clear the parallel between the general case $\mathbf{dist}_{\alpha_1,\ldots,\alpha_n}(D)$ and the special case $\mathbf{dist}_u(D)$ – since the first conjunct of the special case definition in (130) can be replaced with $\forall x_e (I_{u=x} \neq \emptyset \leftrightarrow J_{u=x} \neq \emptyset)$[44]. We can now easily see that the identity in (131) below holds.

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[44] Thus, the two most compact (and completely parallel) definitions are:

(i) $\mathbf{dist}_u(D) := \lambda I_{st} J_{st}. \ \forall x_e (I_{u=x} \neq \emptyset \leftrightarrow J_{u=x} \neq \emptyset) \land \forall x_e (I_{u=x} \neq \emptyset \rightarrow DI_{u=x} J_{u=x})$

(ii) $\mathbf{dist}_{\alpha_1,\ldots,\alpha_n}(D) := \lambda I_{st} J_{st}. \ \forall f_1 \ldots \forall f_n (I_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset \leftrightarrow J_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset) \land$

$\qquad\qquad\qquad \forall f_1 \ldots \forall f_n (I_{\alpha_1=f_1,\ldots,\alpha_n=f_n} \neq \emptyset \rightarrow DI_{\alpha_1=f_1,\ldots,\alpha_n=f_n} J_{\alpha_1=f_1,\ldots,\alpha_n=f_n})$

131. $\mathbf{dist}_\alpha(\mathbf{dist}_{\alpha'}(D)) = \mathbf{dist}_{\alpha,\alpha'}(D)^{45}$

(in more detail: $\mathbf{dist}_\alpha(\mathbf{dist}_{\alpha'}(D)) = \mathbf{dist}_{\alpha,\alpha'}(D) = \mathbf{dist}_{\alpha',\alpha}(D) = \mathbf{dist}_{\alpha'}(\mathbf{dist}_\alpha(D)))$

Finally, we define *generalized* selective distributivity, i.e. distributivity generalized to arbitrary distributable types as shown in (132) below. The distributable types are the same as the dynamically conjoinable types **DCTyp** (see definition (62) in section **4** of chapter **5**).

132. **Generalized Selective Distributivity.**

For any term β of type τ, for any $\tau \in$ **DCTyp**:

$$\delta_{:\{\alpha_1,...,\alpha_n\}}\beta := \mathbf{dist}_{\alpha_1,...,\alpha_n}(\beta)$$

if $\tau = \mathbf{t}$ and the types of the terms $\alpha_1,...,\alpha_n$ are in **DRefTyp**.

$$\delta_{:\{\alpha_1,...,\alpha_n\}}\beta := \lambda v_{n+1}. \; \delta_{:\{\alpha_1,...,\alpha_n,v_{n+1}\}}\beta(v_{n+1})$$

if $\tau = (\sigma\rho)$, v_{n+1} is of type σ and $\sigma \in$ **DRefTyp**.

Abbreviation. $\delta_{\emptyset}\beta := \delta\beta$

To understand the intuition behind the above definition of generalized distributivity, we need to begin with the end, i.e. with the abbreviation. Let us assume that our term β is a dynamic property $P_{\mathbf{et}}$, i.e. an object that can be an argument for an extensional generalized determiner. We want to distribute over this property P, i.e. we want to define a *distributed* property δP of type **et** based on property P.

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[45] Proof. I use the definitions of $\mathbf{dist}_\alpha(D)$ and $\mathbf{dist}_{\alpha,\alpha'}(D)$ in the immediately preceding footnote.

$\mathbf{dist}_\alpha(\mathbf{dist}_{\alpha'}(D))IJ = \forall f(I_{\alpha=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f}{\neq}\emptyset) \wedge \forall f(I_{\alpha=f}{\neq}\emptyset \rightarrow \mathbf{dist}_{\alpha'}(D)I_{\alpha=f}J_{\alpha=f}) =$

(since $\mathbf{dist}_{\alpha'}(D)I_{\alpha=f}J_{\alpha=f} = \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f,\alpha'=f}{\neq}\emptyset) \wedge \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \rightarrow DI_{\alpha=f,\alpha'=f}J_{\alpha=f,\alpha'=f}))$

$\forall f(I_{\alpha=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f}{\neq}\emptyset) \wedge \forall f(I_{\alpha=f}{\neq}\emptyset \rightarrow \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f,\alpha'=f}{\neq}\emptyset) \wedge \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \rightarrow DI_{\alpha=f,\alpha'=f}J_{\alpha=f,\alpha'=f})) =$

$\forall f \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f,\alpha'=f}{\neq}\emptyset) \wedge \forall f(I_{\alpha=f}{\neq}\emptyset \rightarrow \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \rightarrow DI_{\alpha=f,\alpha'=f}J_{\alpha=f,\alpha'=f}) =$

$\forall f \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \leftrightarrow J_{\alpha=f,\alpha'=f}{\neq}\emptyset) \wedge \forall f \forall f'(I_{\alpha=f,\alpha'=f}{\neq}\emptyset \rightarrow DI_{\alpha=f,\alpha'=f}J_{\alpha=f,\alpha'=f}) = \mathbf{dist}_{\alpha,\alpha'}(D)IJ.$ $\square$

By the second clause of definition (132), we have that:

133. $\delta P = \delta_{\varnothing} P = \lambda v_{\mathbf{e}}.\ \delta_{\{v\}} P(v),\quad$ where $P(v)$ is a DRS.

Since $P(v)$ is a DRS, i.e. of type $\mathbf{t}$, we can apply the first clause of definition (132). Therefore:

134. $\delta P = \delta_{\varnothing} P = \lambda v_{\mathbf{e}}.\ \delta_{\{v\}} P(v) = \lambda v_{\mathbf{e}}.\ \mathbf{dist}_v(P(v))$

Thus, the distributed property δP is obtained by distributing over the DRS $P(v)$ with respect to the dref variable v. For example, if we distribute over the extensional properties denoted by *man* and *leave*, we obtain the distributed properties in (135) below.

135. $\delta man = \delta(\lambda v_{\mathbf{e}}.\ [man_{et}\{v\}]) = \lambda v_{\mathbf{e}}.\ \mathbf{dist}_v([man_{et}\{v\}])$

$\delta leave = \delta(\lambda v_{\mathbf{e}}.\ [leave_{et}\{v\}]) = \lambda v_{\mathbf{e}}.\ \mathbf{dist}_v([leave_{et}\{v\}])$

A3. DRS-Level Selective Distributivity: Formal Properties

This appendix investigates the basic formal properties of DRS-level selective distributivity. Crucially, I will assume throughout this chapter the simpler PCDRT system introduced in chapter **5** that does not countenance the dummy individual #. The simpler PCDRT system assigns semantic values to atomic conditions, DRS's etc. that are formally much better behaved than the ones assigned by the PCDRT system of chapter **6** which has to introduce the dummy individual # in order to define structured inclusion.

Let us first define what it means for a DRS D to be closed under arbitrary unions.

136. The union $\cup\mathbb{D}$ of a set $\mathbb{D}$ of pairs of info states $<I, J>$ is defined as the pair of info states $<\cup\mathbf{Dom}(\mathbb{D}), \cup\mathbf{Ran}(\mathbb{D})>$.

137. A DRS D (of type $\mathbf{t} := (st)((st)t)$) is *closed under arbitrary unions* iff, given a set $\mathbb{D}$ of info state pairs s.t. $\mathbb{D}{\subseteq}D$, we have that $D(\cup\mathbf{Dom}(\mathbb{D}))(\cup\mathbf{Ran}(\mathbb{D}))$, i.e. $\cup\mathbb{D}\in D$.

The following kinds of DRS's are closed under arbitrary unions – again, if we assume their simpler definitions according to the PCDRT system of chapter **5** that does not countenance the dummy individual:

138. **a.** Tests that contain only conditions denoting c-ideals (e.g. atomic conditions, dynamic negations of DRS's whose domains are c-ideals etc.) are closed under arbitrary unions since c-ideals are closed under arbitrary unions.

 b. A DRS D of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, is closed under arbitrary unions[46].

 c. A DRS $\mathbf{max}^u(D)$, where D is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ and the conditions $C_1, \ldots, C_m$ are c-ideals, is closed under arbitrary unions[47].

---

[46] Proof. Recall that the denotation of a DRS D of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, can be defined as shown in (ii) below based on the relation in (i).

(i) $\mathbb{R}^D := \{<i_s, j_s>: i[u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)\}$;

(ii) $D = \{<I_{st}, J_{st}>: \exists \mathbb{R}_{s(st)} \neq \emptyset (I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^D)\}$.

Now take an arbitrary set $\mathbb{D}$ of info state pairs s.t. $\mathbb{D} \subseteq D$. For any pair of info states $<I, J> \in \mathbb{D}$, there is some $\mathbb{R} \subseteq \mathbb{R}^D$ s.t. $I = \mathbf{Dom}(\mathbb{R})$ and $J = \mathbf{Ran}(\mathbb{R})$. If we take the union of all such relations $\mathbb{R}$, we will obtain a relation $\mathbb{R}^*$ s.t. $\mathbb{R}^* \subseteq \mathbb{R}^D$ and s.t. $\cup \mathbf{Dom}(\mathbb{D}) = \mathbf{Dom}(\mathbb{R}^*)$ and $\cup \mathbf{Ran}(\mathbb{D}) = \mathbf{Ran}(\mathbb{R}^*)$. Hence, we have that $D(\cup \mathbf{Dom}(\mathbb{D}))(\cup \mathbf{Ran}(\mathbb{D}))$. □

[47] Proof. Consider a DRS of the form $\mathbf{max}^u(D)$, where D is of the form in the immediately preceding proof. Then the DRS $D' = ([u]; D) = [u, u_1, \ldots, u_n \mid C_1, \ldots, C_m]$ is of the same form and has a similar kind of denotation in terms of the relation $\mathbb{R}^{D'}$ defined in (i) below.

(i) $\mathbb{R}^{D'} := \{<i_s, j_s>: i[u, u_1, \ldots, u_n]j \wedge j \in (\cup C_1) \cap \ldots \cap (\cup C_m)\}$

Note that, in this case, the following identities hold: $\mathbf{Dom}(\mathbf{max}^u(D)) = \mathbf{Dom}([u]; D) = \mathbf{Dom}(D')$ – because, for any info state $I \in \mathbf{Dom}([u]; D)$, there is a maximal state J in the set of output states $([u]; D)I$: this maximal state is the image of I under the relation $\mathbb{R}^{D'}$; since J is the supremum info state, it follows that uJ is also the supremum set of individuals.

Now take an arbitrary set $\mathbb{D}$ of info state pairs s.t. $\mathbb{D} \subseteq \mathbf{max}^u(D)$. We show that $\mathbf{max}^u(D)(\cup \mathbf{Dom}(\mathbb{D}))(\cup \mathbf{Ran}(\mathbb{D}))$, i.e.: (i) $D'(\cup \mathbf{Dom}(\mathbb{D}))(\cup \mathbf{Ran}(\mathbb{D}))$ and (ii) $\forall K(D'(\cup \mathbf{Dom}(\mathbb{D}))K \rightarrow uK \subseteq u(\cup \mathbf{Ran}(\mathbb{D})))$.

We know that $\mathbf{max}^u(D) \subseteq D'$, therefore $\mathbb{D} \subseteq D'$ and (i) follows because D' is closed under arbitrary unions (by the previous proof).

Now suppose (ii) does not hold, i.e. there is a K s.t. $D'(\cup \mathbf{Dom}(\mathbb{D}))K$ and s.t. $uK \not\subseteq u(\cup \mathbf{Ran}(\mathbb{D}))$. Based on the observation above, the set of output states corresponding to $\cup \mathbf{Dom}(\mathbb{D})$, i.e. the set $D'(\cup \mathbf{Dom}(\mathbb{D}))$, has a supremum info state, i.e. the image of $\cup \mathbf{Dom}(\mathbb{D})$ under the relation $\mathbb{R}^{D'}$. Let's abbreviate it as $\mathbb{J}^*$. Now, since $\mathbb{J}^*$ is the supremum info state, the set $u\mathbb{J}^*$ is also the supremum set of individuals, so $uK \subseteq u\mathbb{J}^*$ and, therefore, $u\mathbb{J}^* \not\subseteq u(\cup \mathbf{Ran}(\mathbb{D}))$.

I will now show that $u\mathbb{J}^* = u(\cup \mathbf{Ran}(\mathbb{D}))$, which yields a contradiction. Consider an arbitrary pair of info states $<I, J> \in \mathbb{D}$; given that $\mathbb{D} \subseteq \mathbf{max}^u(D)$, we have that $\mathbf{max}^u(D)IJ$, i.e. that $\forall K(D'IK \rightarrow uK \subseteq uJ)$. In

d. A DRS D; D' is closed under arbitrary unions if D and D' are closed under arbitrary unions, i.e. dynamic conjunction preserves closure under arbitrary unions[48].

e. A DRS $\textbf{dist}_\alpha(D)$ is closed under arbitrary unions for any dref α if D is closed under arbitrary unions[49].

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particular, we have that $uJ = u\mathbb{J}^I$, where $\mathbb{J}^I$ is the image of I under the relation $\mathbb{R}^{D'}$, i.e. the supremum output state in the set of output states $D'I$. The union of all such supremum output states $\mathbb{J}^I$ corresponding to some input state $I \in \textbf{Dom}(\mathbb{D})$ is precisely $\mathbb{J}^*$, i.e. $\mathbb{J}^* = \cup_{I \in \textbf{Dom}(\mathbb{D})}\mathbb{J}^I$ and, therefore, $u\mathbb{J}^*$ is the union of all the sets $u\mathbb{J}^I$. Thus, we have that $u\mathbb{J}^* = \cup_{I \in \textbf{Dom}(\mathbb{D})}u\mathbb{J}^I = \cup_{J \in \textbf{Ran}(\mathbb{D})}uJ = u(\cup_{J \in \textbf{Ran}(\mathbb{D})}J) = u(\cup\textbf{Ran}(\mathbb{D}))$. Contradiction. □

[48] Proof. Take an arbitrary set $\mathbb{D}$ of info state pairs s.t. $\mathbb{D} \subseteq (D; D')$. This means that for any $<I, J> \in \mathbb{D}$, there is an H s.t. DIH and $D'HJ$. For every pair $<I, J> \in \mathbb{D}$, choose two other pairs IH and HJ s.t. DIH and $D'HJ$. Abbreviate the union of all the IH pairs $\mathbb{D}'$ and the union of all the HJ pairs $\mathbb{D}''$. We have that $\textbf{Dom}(\mathbb{D}) = \textbf{Dom}(\mathbb{D}')$, $\textbf{Ran}(\mathbb{D}) = \textbf{Ran}(\mathbb{D}'')$ and $\textbf{Ran}(\mathbb{D}') = \textbf{Dom}(\mathbb{D}'')$.

Since $\mathbb{D}' \subseteq D$ and $\mathbb{D}'' \subseteq D'$ and D and D' are closed under arbitrary unions, we have that $D(\cup\textbf{Dom}(\mathbb{D}'))(\cup\textbf{Ran}(\mathbb{D}'))$ and $D'(\cup\textbf{Dom}(\mathbb{D}''))(\cup\textbf{Ran}(\mathbb{D}''))$. Given that $\textbf{Ran}(\mathbb{D}') = \textbf{Dom}(\mathbb{D}'')$, we have that $(D; D')(\cup\textbf{Dom}(\mathbb{D}'))(\cup\textbf{Ran}(\mathbb{D}''))$, i.e. $(D; D')(\cup\textbf{Dom}(\mathbb{D}))(\cup\textbf{Ran}(\mathbb{D}))$, i.e. $D; D'$ is closed under arbitrary unions. □
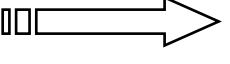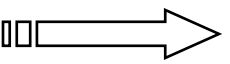
[49] Proof. First note that, in general, $\textbf{dist}_\alpha(D)$ is not closed under arbitrary unions; selective distributivity is based on unions, but *not* on arbitrary unions of info states. Assume, for example, that we have two pairs $<I, J> \in D$ and $<I', J'> \in D$ s.t. $\alpha I = \alpha J$, $\alpha I' = \alpha J'$, $|\alpha I| = |\alpha I'| = 1$ and, in addition, $\alpha I = \alpha I'$. Both pairs will be in $\textbf{dist}_\alpha(D)$, but the union of these two pairs, i.e. $<I \cup I', J \cup J'>$ is not necessarily in the distributed DRS $\textbf{dist}_\alpha(D)$ – not unless it is in the DRS D itself. This is where the assumption that D is closed under arbitrary unions becomes useful: it entails that $<I \cup I', J \cup J'> \in D$ and, since $\alpha(I \cup I') = \alpha(J \cup J') = \alpha I$ (because we know that $\alpha I = \alpha J = \alpha I' = \alpha J'$) and $|\alpha(I \cup I')| = |\alpha(J \cup J')| = 1$, we have that $<I \cup I', J \cup J'> \in \textbf{dist}_\alpha(D)$. The proof generalizes this observation to arbitrary sets of pairs (i.e. there is no more insight to be gained from it). I provide it here for completeness.

Suppose we have a set $\mathbb{D}$ of info states pairs s.t. $\mathbb{D} \subseteq \textbf{dist}_\alpha(D)$. We have to show that $<\cup\textbf{Dom}(\mathbb{D}), \cup\textbf{Ran}(\mathbb{D})> \in \textbf{dist}_\alpha(D)$. By the definition of $\textbf{dist}_\alpha(D)$, any pair $<I, J> \in \mathbb{D}$ has one of the following two forms: (i) $<I, J> \in D$, $\alpha I = \alpha J$ and $|\alpha I| = 1$; (ii) arbitrary unions of sets of pairs of the kind specified in (i), under the condition that, for any two such pairs $<I, J>$ and $<I', J'>$, $\alpha I \neq \alpha I'$. Therefore, $<\cup\textbf{Dom}(\mathbb{D}), \cup\textbf{Ran}(\mathbb{D})>$ is the result of taking the union of some arbitrary set of pairs of info states of the kind specified in (i), i.e. pairs of the form $<I, J>$ s.t. $<I, J> \in D$, $\alpha I = \alpha J$ and $|\alpha I| = 1$.

We will partition this sets of pairs into equivalence classes as follows: the equivalence class of a given pair $<I, J>$ is the set $\mathbb{D}^{<I,J>} = \{<I', J'> \in D: \alpha I' = \alpha J' \wedge |\alpha I'| = 1 \wedge \alpha I = \alpha I'\}$. For each such equivalence class of pairs $\mathbb{D}^{<I,J>}$, we take its union $\cup\mathbb{D}^{<I,J>}$, where the union is defined as in (136) above, i.e. as the pair of info states $\cup\mathbb{D}^{<I,J>} = <\cup\textbf{Dom}(\mathbb{D}^{<I,J>}), \cup\textbf{Ran}(\mathbb{D}^{<I,J>})>$ This pair is in the denotation of the DRS D, i.e. $<\cup\textbf{Dom}(\mathbb{D}^{<I,J>}), \cup\textbf{Ran}(\mathbb{D}^{<I,J>})> \in D$, because $\mathbb{D}^{<I,J>} \subseteq D$ and, by assumption, D is closed under arbitrary unions. Moreover, each pair $<\cup\textbf{Dom}(\mathbb{D}^{<I,J>}), \cup\textbf{Ran}(\mathbb{D}^{<I,J>})>$ satisfies the conditions $\alpha(\cup\textbf{Dom}(\mathbb{D}^{<I,J>}) = \alpha(\cup\textbf{Ran}(\mathbb{D}^{<I,J>}))$ (because $\alpha(\cup\textbf{Dom}(\mathbb{D}^{<I,J>})) = \alpha I = \alpha J = \alpha(\cup\textbf{Ran}(\mathbb{D}^{<I,J>}))$) and $|\alpha(\cup\textbf{Dom}(\mathbb{D}^{<I,J>}))| = 1$ (because $|\alpha I| = 1$). Therefore, for each pair $<I, J>$, we have that $<\cup\textbf{Dom}(\mathbb{D}^{<I,J>}), \cup\textbf{Ran}(\mathbb{D}^{<I,J>})> \in \textbf{dist}_\alpha(D)$. Moreover, for any two distinct equivalence classes of pairs $\mathbb{D}^{<I,J>}$ and $\mathbb{D}^{<K,L>}$, their unions $<\cup\textbf{Dom}(\mathbb{D}^{<I,J>}), \cup\textbf{Ran}(\mathbb{D}^{<I,J>})>$ and $<\cup\textbf{Dom}(\mathbb{D}^{<K,L>}), \cup\textbf{Ran}(\mathbb{D}^{<K,L>})>$ satisfy the additional condition $\alpha(\cup\textbf{Dom}(\mathbb{D}^{<I,J>})) \neq \alpha(\cup\textbf{Dom}(\mathbb{D}^{<K,L>}))$. Therefore, the union of such pairs (i.e. of all pairs resulting from unions of equivalence classes) is also in

For example, the DRS $\mathbf{max}^{u'}([happy\_for\{u', u\}])$ is closed under arbitrary unions in the following sense. Suppose that this DRS contains the pairs of info states $<I_1, J_1>$ and $<I_2, J_2>$ in (139) below. The two pairs of info states record the following: given an input state I_1 such that uI_1 is John, the set of individuals that are happy for him are Jessica, Mary and Sue; similarly, given an input state I_2 such that uI_2 is Bill, the set of individuals that are happy for him are Jane and Jessica. Then, the DRS $\mathbf{max}^{u'}([happy\_for\{u', u\}])$ also contains the pair of info states $<I_1\cup I_2, J_1\cup J_2>$, since, given the set of individuals $u(I_1\cup I_2)$, i.e. John and Bill, the set of individuals that are happy for at least one of the two is Jane, Jessica, Mary and Sue.

139. $\mathbf{max}^{u'}([happy\_for\{u', u\}])$ is closed under arbitrary unions.

$\mathbf{max}^{u'}([h.f\{u', u\}])I_1J_1$	Output state J_1	...	u	u'	...
	j_1	...	john	jess	...
	j_2	...	john	mary	...
	j_3	...	john	sue	...

Input state I_1	...	u	...
i_1		john	

$\mathbf{max}^{u'}([h.f\{u', u\}])I_2J_2$	Output state J_2	...	u	u'	...
	j_4	...	bill	jane	...
	j_5	...	bill	jess	...

Input state I_2	...	u	...
i_2	...	bill	...

We can now state the following observation.

140. **Selective distributivity and closure under arbitrary unions.**

If a DRS D is closed under arbitrary unions, then $\mathbf{dist}_\alpha(D)\subseteq D$, for any term α whose type is in **DRefTyp**[50].

$\mathbf{dist}_\alpha(D)$. But this big union is precisely $<\cup\mathbf{Dom}(\mathbb{D}), \cup\mathbf{Ran}(\mathbb{D})>$, i.e. $<\cup\mathbf{Dom}(\mathbb{D}), \cup\mathbf{Ran}(\mathbb{D})>\in \mathbf{dist}_\alpha(D)$ and we have that $\mathbf{dist}_\alpha(D)$ is closed under arbitrary unions. □

[50] Proof. It follows directly from the observation about $\mathbf{dist}_\alpha(D)$ in (i) below and the assumption that D is closed under arbitrary unions. □

(i) The denotation of a DRS $\mathbf{dist}_\alpha(D)$ contains all and only:

   (a) those pairs $<I, J>\in D$ such that $\alpha I=\alpha J$ and $|\alpha I|=1$;

More generally (since $\mathbf{dist}_\alpha(D)$ is closed under arbitrary unions for any dref α if D is closed under arbitrary unions – see (138e) above):

If a DRS D is closed under arbitrary unions, then $\mathbf{dist}_{\alpha_1,...,\alpha_n}(D) \subseteq D$, for any terms $\alpha_1, ..., \alpha_n$ whose types are in **DRefTyp**.

The inclusion $\mathbf{dist}_\alpha(D) \subseteq D$ can be strengthened to equality, i.e. we can also show that $D \subseteq \mathbf{dist}_\alpha(D)$, if we require closure under subsets over and above closure under unions.

141. A DRS D (of type $\mathbf{t} := (st)((st)t)$) is *closed under subsets* iff, for any pair of info states $<I, J> \in D$, there is a set $\mathbb{D} \subseteq D$ of info state pairs such that:

(**i**) all the pairs in $\mathbb{D}$ are of the form $<\{i_s\}, \{j_s\}>$, i.e. they contain only singleton info states;

(**ii**) $\cup\mathbb{D} = <I, J>$, where $\cup\mathbb{D} = <\cup\mathbf{Dom}(\mathbb{D}), \cup\mathbf{Ran}(\mathbb{D})>$ (see (136) above), i.e. $I = \cup\mathbf{Dom}(\mathbb{D})$ and $J = \cup\mathbf{Ran}(\mathbb{D})$;

(**iii**) for any set of info state pairs $\mathbb{D}' \subseteq \mathbb{D}$, we have that $D(\cup\mathbf{Dom}(\mathbb{D}'))(\cup\mathbf{Ran}(\mathbb{D}'))$, i.e. $\cup\mathbb{D}' \in D$ (note that this condition follows automatically if D is also closed under unions).

142. The following kinds of DRS's are closed under subsets (if we assume their denotations according to the PCDRT system of chapter **5**):

a. Tests that contain only conditions denoting c-ideals (e.g. atomic conditions, dynamic negations of DRS's whose domains are c-ideals etc.) are closed under subsets since c-ideals are closed under subsets.

b. A DRS D of the form $[u_1, ..., u_n \mid C_1, ..., C_m]$, where the conditions $C_1, ..., C_m$ are c-ideals, is closed under subsets[51].

(b) arbitrary unions of sets of pairs of the kind specified in (a) above, under the condition that, for any two such pairs $<I, J>$ and $<I', J'>$, $\alpha I \neq \alpha I'$ – hence, given that $\alpha I = \alpha J$, $\alpha I' = \alpha J'$ and $|\alpha I| = |\alpha I'| = 1$, we have that I and I' are disjoint and J and J' are disjoint.

[51] Proof: Recall that the denotation of a DRS D of the form $[u_1, ..., u_n \mid C_1, ..., C_m]$, where the conditions $C_1, ..., C_m$ are c-ideals, can be defined as shown in (ii) below based on the relation in (i).

(i) $\mathbb{R}^D := \{<i_s, j_s>: i[u_1, ..., u_n]j \wedge j \in (\cup C_1) \cap ... \cap (\cup C_m)\}$;

We can now state the very useful observation in (143) below, which shows that PCDRT with selective distributivity properly extends the PCDRT system of chapter **5** without selective distributivity only when maximization operators or generalized determiners are involved.

143. **Selective distributivity and closure under arbitrary unions and subsets.**

If a DRS D is closed under arbitrary unions and subsets, then $\textbf{dist}_\alpha(D)=D$, for any term α whose type is in **DRefTyp** and any DRS D s.t. $\forall<I, J>\in D(\alpha I=\alpha J)$[52].

More generally (this follows directly from the special case $\textbf{dist}_\alpha(D)=D$ and from the fact that $\textbf{dist}_{\alpha,\alpha'}(D) = \textbf{dist}_\alpha(\textbf{dist}_{\alpha'}(D))$ – see (131) above):

If a DRS D is closed under arbitrary unions and under subsets, then

$\textbf{dist}_{\alpha_1,...,\alpha_n}(D)=D$, for any terms α_1, …, α_n whose types are in **DRefTyp** and any

DRS D s.t. $\forall<I, J>\in D\ (\alpha_1 I=\alpha_1 J \wedge \dots \wedge \alpha_n I=\alpha_n J)$.

(ii) $D = \{<I_{st}, J_{st}>: \exists \mathbb{R}_{s(st)}\neq\emptyset(I=\textbf{Dom}(\mathbb{R}) \wedge J=\textbf{Ran}(\mathbb{R}) \wedge \mathbb{R}\subseteq\mathbb{R}^D)\}$.

Now take an arbitrary pair of info states $<I, J>\in D$; by (ii), there is some $\mathbb{R}\subseteq\mathbb{R}^D$ s.t. $I=\textbf{Dom}(\mathbb{R})$ and $J=\textbf{Ran}(\mathbb{R})$. Take the set $\mathbb{D}$ of info states pairs to be as follows $\mathbb{D} := \{<\{i_s\}, \{j_s\}>: <i_s, j_s>\in\mathbb{R}\}$. For every pair of info states $<\{i\}, \{j\}>\in\mathbb{D}$, there is the singleton relation $\{<i, j>\}\subseteq\mathbb{R}\subseteq\mathbb{R}^D$ s.t. $\{i\}=\textbf{Dom}(\{<i, j>\})$ and $\{j\}=\textbf{Ran}(\{<i, j>\})$, therefore $\mathbb{D}\subseteq D$. Moreover, $\cup\mathbb{D} = <\cup\textbf{Dom}(\mathbb{D}), \cup\textbf{Ran}(\mathbb{D})> = <\textbf{Dom}(\mathbb{R}), \textbf{Ran}(\mathbb{R})> = <I, J>$. The last condition, namely that for any set of info state pairs $\mathbb{D}'\subseteq\mathbb{D}$, it is the case that $D(\cup\textbf{Dom}(\mathbb{D}'))(\cup\textbf{Ran}(\mathbb{D}'))$, i.e. that $\cup\mathbb{D}'\in D$, follows directly from (ii), the fact that, for any $\mathbb{R}'\subseteq\mathbb{R}$, $<\textbf{Dom}(\mathbb{R}'), \textbf{Ran}(\mathbb{R}')>\in D$. $\square$

[52] Proof. Since D is closed under arbitrary unions, we have that $\textbf{dist}_\alpha(D)\subseteq D$ by observation (140). We just have to prove that $D\subseteq \textbf{dist}_\alpha(D)$.

Take an arbitrary pair $<I, J>\in D$. Since D is closed under subsets, we know that there is a $\mathbb{D}\subseteq D$ s.t. $\cup\mathbb{D} =<\cup\textbf{Dom}(\mathbb{D}), \cup\textbf{Ran}(\mathbb{D})> = <I, J>$ and $\mathbb{D}$ contains only info state pairs of the form $<\{i_s\}, \{j_s\}>$. Take a pair $<\{i\}, \{j\}>\in\mathbb{D}\subseteq D$. We know that $\alpha\{i\}=\alpha\{j\}$ because, by assumption, $\forall<I, J>\in D(\alpha I=\alpha J)$. Moreover, $|\alpha\{i\}|=|\{\alpha i\}|=1$. Therefore, any pair $<\{i\}, \{j\}>\in\mathbb{D}$ is s.t. $<\{i\}, \{j\}>\in \textbf{dist}_\alpha(D)$.

We now apply the same technique as the one we used in the proof of (138e). We partition the set $\mathbb{D}$ of pairs into equivalence classes; the equivalence class of a pair $<\{i\}, \{j\}>\in\mathbb{D}$ is $\mathbb{D}^{<\{i\},\{j\}>} := \{<\{i'\}, \{j'\}>\in\mathbb{D}: \alpha i'=\alpha i\}$; thus, $\cup\mathbb{D}^{<\{i\},\{j\}>}=<\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>}), \cup\textbf{Ran}(\mathbb{D}^{<\{i\},\{j\}>})>$ and, since $\mathbb{D} = \cup_{<\{i\},\{j\}>\in\mathbb{D}}\mathbb{D}^{<\{i\},\{j\}>}$, we have that $\cup\mathbb{D}$, i.e. $<I, J>$, is the union of the set of pairs formed $\cup\mathbb{D}^{<\{i\},\{j\}>}$.

Since D is closed under arbitrary unions and $\mathbb{D}^{<\{i\},\{j\}>}\subseteq\mathbb{D}\subseteq D$, we have that $\cup\mathbb{D}^{<\{i\},\{j\}>}=<\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>}), \cup\textbf{Ran}(\mathbb{D}^{<\{i\},\{j\}>})>\in D$. Moreover, $\alpha(\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>})) = \alpha(\{i\}) = \alpha\{j\} = \alpha(\cup\textbf{Ran}(\mathbb{D}^{<\{i\},\{j\}>}))$ and, therefore, we also have that $|\alpha(\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>}))|=1$. Thus, $\cup\mathbb{D}^{<\{i\},\{j\}>}=<\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>}), \cup\textbf{Ran}(\mathbb{D}^{<\{i\},\{j\}>})>\in \textbf{dist}_\alpha(D)$ for any pair $<\{i\}, \{j\}>\in\mathbb{D}$. Moreover, since for any two distinct equivalence classes $\mathbb{D}^{<\{i\},\{j\}>}$ and $\mathbb{D}^{<\{i'\},\{j'\}>}$, we have that $\alpha(\cup\textbf{Dom}(\mathbb{D}^{<\{i\},\{j\}>}))\neq\alpha(\cup\textbf{Dom}(\mathbb{D}^{<\{i'\},\{j'\}>}))$, the union of all $\cup\mathbb{D}^{<\{i\},\{j\}>}$ is also in $\textbf{dist}_\alpha(D)$. But this big union is precisely $\cup\mathbb{D} = <\cup\textbf{Dom}(\mathbb{D}), \cup\textbf{Ran}(\mathbb{D})> = <I, J>$. Thus, $<I, J>\in \textbf{dist}_\alpha(D)$ and therefore $D\subseteq\textbf{dist}_\alpha(D)$. $\square$

It follows from the observation in (143) above that selective distributive operators are vacuous when applied to tests containing only conditions denoting c-ideals or to DRS's of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals – if, in the latter case, we distribute over a dref α different from $u_1, \ldots, u_n$.

The equivalence in (144) below shows that, in PCDRT / IP-CDRT, selective distributivity operators distribute over dynamic conjunction.

144. $\mathbf{dist}_\alpha(D; D') = \mathbf{dist}_\alpha(D); \mathbf{dist}_\alpha(D')$,

for any term α whose type is in **DRefTyp** and any DRS's D and D' s.t.

$\forall <I, J> \in D(\alpha I = \alpha J)$ and $\forall <I, J> \in D'(\alpha I = \alpha J)$[53]

Finally, we show that we can extend our previous results about the reduction of multiply embedded $\mathbf{max}^u$ operators to more complex representations involving selective distributivity in addition to embedded $\mathbf{max}^u$ operators. In particular, the statement in (145) below is a theorem of PCDRT (or IP-CDRT) with selective distributivity. The conditions are identical to the ones needed to reduce structures with multiply embedded $\mathbf{max}^u$ operators that do not contain selectively distributive operators (see the Appendix to the previous chapter).

145. **Simplifying 'Max-under-Max' Representations with selective distributivity:**

$\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D'))) = \mathbf{max}^u(D; \mathbf{dist}_u([u']; D')); \mathbf{dist}_u(\mathbf{max}^{u'}(D'))$,

if the following three conditions obtain:

a. u is not reintroduced in D';

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[53] Proof:

$(\mathbf{dist}_\alpha(D); \mathbf{dist}_\alpha(D'))IJ = \exists H(\mathbf{dist}_\alpha(D)IH \wedge \mathbf{dist}_\alpha(D')HJ)$

$= \exists H(\alpha I = \alpha H \wedge \forall f \in \alpha I(DI_{\alpha=f}H_{\alpha=f}) \wedge \alpha H = \alpha J \wedge \forall f \in \alpha H(D'H_{\alpha=f}J_{\alpha=f}))$

$= \exists H(\alpha I = \alpha H = \alpha J \wedge \forall f \in \alpha I(DI_{\alpha=f}H_{\alpha=f} \wedge D'H_{\alpha=f}J_{\alpha=f}))$

$= $ (given that $\forall <I, J> \in D(\alpha I = \alpha J)$ and $\forall <I, J> \in D'(\alpha I = \alpha J)$) $\alpha I = \alpha J \wedge \forall f \in \alpha I(\exists H(DI_{\alpha=f}H \wedge D'HJ_{\alpha=f}))$

$= \alpha I = \alpha J \wedge \forall f \in \alpha I((D; D')I_{\alpha=f}J_{\alpha=f}) = \mathbf{dist}_\alpha(D; D')IJ.$ □

b. $\forall I_{st}\forall X_{et}\,(\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ));$

c. $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$[54].

---

[54] Proof.

Claim1. If $\forall I_{st}\forall X_{et}(\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ))$, then $\forall I_{st}\forall X_{et}(\exists J_{st}(\mathbf{dist}_u([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{dist}_u(\mathbf{max}^{u'}(D'))IJ \wedge X=uJ))$.

Proof of Claim1. $\forall I_{st}\forall X_{et}(\exists J_{st}(\mathbf{dist}_u([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{dist}_u(\mathbf{max}^{u'}(D'))IJ \wedge X=uJ)) =$

$\forall I_{st}\forall X_{et}(\exists J_{st}(uI=uJ \wedge \forall x_e\in uI(([u']; D')I_{u=x}J_{u=x}) \wedge X=uJ) \leftrightarrow \exists J_{st}(uI=uJ \wedge \forall x_e\in uI(\mathbf{max}^{u'}(D')I_{u=x}J_{u=x}) \wedge X=uJ)) =$

$\forall I_{st}(\exists J_{st}(uI=uJ \wedge \forall x_e\in uI(([u']; D')I_{u=x}J_{u=x})) \leftrightarrow \exists J_{st}(uI=uJ \wedge \forall x_e\in uI(\mathbf{max}^{u'}(D')I_{u=x}J_{u=x})))$

LR→: Assume that, for an arbitrary I, we find some J s.t. $uI=uJ \wedge \forall x_e\in uI(([u']; D')I_{u=x}J_{u=x})$. Pick an arbitrary x; we have that $([u']; D')I_{u=x}J_{u=x}$. By hypothesis, we have that $\forall I_{st}\forall X_{et}(\exists J_{st}(([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')IJ \wedge X=uJ))$. Instantiate I with $I_{u=x}$ and X with $\{x\}$. We therefore have that:

$\exists J_{st}(([u']; D')I_{u=x}J \wedge \{x\}=uJ) \leftrightarrow \exists J_{st}(\mathbf{max}^{u'}(D')I_{u=x}J \wedge \{x\}=uJ)$

The left hand-side is true because $([u']; D')I_{u=x}J_{u=x}$ is true and, obviously, $uJ_{u=x}=\{x\}$. We can therefore find a state J^x s.t. $\mathbf{max}^{u'}(D')I_{u=x}J^x \wedge \{x\}=uJ^x$. Thus, for all $x_e\in uI$, there is some J^x s.t. $\mathbf{max}^{u'}(D')I_{u=x}J^x \wedge \{x\}=uJ^x$. Take the union of all these states, i.e. $\cup_{x\in uI}J^x$. Clearly, $uI=u(\cup_{x\in uI}J^x)$ and $\forall x_e\in uI(\mathbf{max}^{u'}(D')I_{u=x}J^x)$. Therefore $\exists J_{st}(uI=uJ \wedge \forall x_e\in uI(\mathbf{max}^{u'}(D')I_{u=x}J_{u=x}))$.

RL→: the reasoning is parallel. **End of proof of Claim1.**

Thus, $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ =$

$\exists H(([u]; D)IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ \wedge \forall K(\exists H(([u]; D)IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HK) \rightarrow uK\subseteq uJ)$

By condition (145b) and **Claim1**, we have that: $\forall I_{st}\forall X_{et}(\exists J_{st}(_u([u']; D')IJ \wedge X=uJ) \leftrightarrow \exists J_{st}(\mathbf{dist}_u(\mathbf{max}^{u'}(D'))IJ \wedge X=uJ))$.

Therefore, $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ =$

$\exists H(([u]; D)IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ \wedge \forall K(\exists H(([u]; D)IH \wedge \mathbf{dist}_u([u']; D')HK) \rightarrow uK\subseteq uJ) =$

$\exists H(([u]; D)IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ \wedge \forall K(([u]; D; \mathbf{dist}_u([u']; D'))IK \rightarrow uK\subseteq uJ)$

We have that $\mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D')$ (condition (145c)). Hence: $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ =$

$\exists H(([u]; D)IH \wedge \mathbf{dist}_u([u']; D'; \mathbf{max}^{u'}(D'))HJ \wedge \forall K(([u]; D; \mathbf{dist}_u([u']; D'))IK \rightarrow uK\subseteq uJ)$

Since u is not reintroduced in D' (condition (145a)), we have by fact (144) that $\mathbf{dist}_u([u']; D'; \mathbf{max}^{u'}(D')) = \mathbf{dist}_u([u']; D'); \mathbf{dist}_u(\mathbf{max}^{u'}(D'))$. Therefore: $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ =$

$\exists H(([u]; D; \mathbf{dist}_u([u']; D'))IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ \wedge \forall K(([u]; D; \mathbf{dist}_u([u']; D'))IK) \rightarrow uK\subseteq uJ) =$

$\exists H(([u]; D; \mathbf{dist}_u([u']; D'))IH \wedge \forall K(([u]; D; \mathbf{dist}_u([u']; D'))IK) \rightarrow uK\subseteq uJ) \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ$

Since u is not reintroduced in D' (condition (145a)), we have that $uJ=uH$. Hence: $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ =$

$\exists H(([u]; D; \mathbf{dist}_u([u']; D'))IH \wedge \forall K(([u]; D; \mathbf{dist}_u([u']; D'))IK) \rightarrow uK\subseteq uH) \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ =$

$\exists H(\mathbf{max}^u(D; \mathbf{dist}_u([u']; D'))IH \wedge \mathbf{dist}_u(\mathbf{max}^{u'}(D'))HJ = (\mathbf{max}^u(D; \mathbf{dist}_u([u']; D')); \mathbf{dist}_u(\mathbf{max}^{u'}(D')))IJ.$ □

Just as before, we can further simplify the three conditions in (145). First, given the first condition, i.e. (145a), the second condition is equivalent to $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$. Moreover, based on the two facts in (146) (see the appendix of chapter **5** for their proofs), we can further simplify condition (145c).

146. **a.** If D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$,

$$\text{then } \forall I_{st} J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J).$$

    b. If $\forall I_{st} J_{st}(([u']; D')IJ \rightarrow ([u']; D')I=([u']; D')J)$,

$$\text{then } \mathbf{max}^{u'}(D') = [u']; D'; \mathbf{max}^{u'}(D').$$

Thus, we have the corollary in (147) below.

147. **Simplifying 'Max-under-Max' Representations with selective distributivity (corollary):**

    $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D'))) = \mathbf{max}^u(D; \mathbf{dist}_u([u']; D')); \mathbf{dist}_u(\mathbf{max}^{u'}(D'))$,

    if the following three conditions obtain:

      a. u is not reintroduced in D';

      b. $\mathbf{Dom}([u']; D') = \mathbf{Dom}(\mathbf{max}^{u'}(D'))$;

      c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$.

The right handside of the identity can be further simplified if the DRS $[u']; D'$ is closed under unions and subsets, in which case we can omit the distributive operator embedded under $\mathbf{max}^u$ since $\mathbf{dist}_u([u']; D') = [u']; D'$ – this holds because, by (147a), u is not reintroduced in D' and, therefore, $\forall \langle I, J \rangle \in ([u']; D')(uI=uJ)$. Consequently, we have the additional corollary in (148) below, which is useful for the simplification of derivations.

148. **Simplifying 'Max-under-Max' Representations with selective distributivity (corollary2):**

    $\mathbf{max}^u(D; \mathbf{dist}_u(\mathbf{max}^{u'}(D'))) = \mathbf{max}^u(D; [u']; D'); \mathbf{dist}_u(\mathbf{max}^{u'}(D'))$,

    if the following three conditions obtain:

      a. u is not reintroduced in D';

b. Dom$([u']; D') =$ **Dom**$(\mathbf{max}^{u'}(D'))$;

c. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$

and $C_1, \ldots, C_m$ are c-ideals.

Moreover, (148b) actually follows from (148c) because $C_1, \ldots, C_m$ are c-ideals.

Chapter 7. Structured Modal Reference: Modal Anaphora and Subordination

1. Introduction

This chapter shows that PCDRT can be extended to analyze structured discourse reference in the modal domain. In particular, adding a new type **w** for possible worlds is the only extension to our underlying logic Dynamic Ty2 that is needed to account for the discourse in (1) below, i.e. to derive its intuitively correct truth-conditions and explicitly capture the individual-level and modal anaphoric connections established in it.

1. **a.** [A] man cannot live without joy.

   b. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures

   (Thomas Aquinas[1]).

We will focus on only one of the meaning dimensions of this discourse, namely the *entailment* relation established by *therefore* between the modal premise (1a) and the modal conclusion in (1b)[2]. We are interested in the following features of this discourse. First, we want to capture the meaning of the entailment particle *therefore*, which relates the *content* of the premise (1a) and the *content* of the conclusion (1b) and requires the latter to be entailed by the former. I take the content of a sentence to be *truth-conditional* in nature, i.e. to be the set of possible worlds in which the sentence is true, and entailment to be *content inclusion*, i.e. (1a) entails (1b) iff for any world *w*, if (1a) is true in *w*, so is (1b)[3].

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[1] Attributed to Thomas Aquinas, http://en.wikiquote.org/wiki/Thomas_Aquinas#Attributed.

[2] For the multi-dimensionality of the meaning of *therefore*-discourses, see for example Grice (1975) and Potts (2003).

[3] I am grateful to a *Logic & Language 9* reviewer for pointing out that modeling the entailment relation expressed by *therefore* as a truth-conditional relation, i.e. as requiring inclusion between two sets of possible worlds, cannot account for the fact that the discourse *Pi is an irrational number, therefore Fermat's last theorem is true* is not intuitively acceptable as a valid entailment and it cannot be accepted as a mathematical proof despite the fact that both sentences are necessary truths (i.e. they are true in every possible world). I think that at least some of the available accounts of hyper-intensional phenomena are compatible with my proposal, so I do not see this as an insurmountable problem.

Second, we are interested in the *meanings* of (1a) and (1b). I take meaning to be *context-change potential*, i.e. to encode both content (truth-conditions) and anaphoric potential. Thus, on the one hand, we are interested in the contents of (1a) and (1b). They are both modal quantifications: (1a) involves a circumstantial modal base (to use the terminology introduced in Kratzer 1981) and asserts that, in view of the circumstances, i.e. given that God created man in a particular way, as long as a man is alive, he must find some thing or other pleasurable; (1b) involves the same modal base and elaborates on the preceding modal quantification: in view of the circumstances, if a man is alive and has no spiritual pleasure, he must have a carnal pleasure. Note that we need to make the contents of (1a) and (1b) accessible in discourse so that the entailment particle *therefore* can relate them.

On the other hand, we are interested in the anaphoric potential of (1a) and (1b), i.e. in the anaphoric connections between them. These connections are explicitly represented in discourse (2) below, which is intuitively equivalent to (1) albeit more awkwardly phrased.

2. **a.** If a^{u_1} man is alive, he$_{u_1}$ must find something^{u_2} pleasurable / he$_{u_1}$ must have a^{u_2} pleasure.

   b. Therefore, if he$_{u_1}$ doesn't have any^{u_3} spiritual pleasure, he$_{u_1}$ must have a^{u_4} carnal pleasure.

Note in particular that the indefinite *a^{u_1} man* in the antecedent of the conditional in (2a) introduces the dref u_1, which is anaphorically retrieved by the pronoun *he$_{u_1}$* in the antecedent of the conditional in (2b). This is an instance of *modal subordination* (Roberts 1989), i.e. an instance of simultaneous modal and invididual-level anaphora (see Geurts 1995/1999, Frank 1996 and Stone 1999): the interpretation of the conditional in (2b) is such that it seems to covertly duplicate the antecedent of the conditional in (2a), i.e. the conditional in (2b) asserts that, if *a man is alive* and doesn't have any spiritual pleasure, he must have a carnal one.

I will henceforth analyze the simpler and more transparent discourse in (2) instead of the naturally occurring discourse in (1). The challenge posed by (2) is that, when we *compositionally* assign meanings to (**i**) the modalized conditional in (2a), i.e. the premise, (**ii**) the modalized conditional in (2b), i.e. the conclusion, and (**iii**) the entailment particle *therefore*, which relates the premise and the conclusion, we have to capture both the intuitively correct *truth-conditions* of the whole discourse and the modal and individual-level *anaphoric connections* between the two sentences of the discourse and within each one of them.

The structure of the chapter is the following. Section **2** outlines the proposed account of the Aquinas discourse in (1/2) above. The discourse is basically analyzed as a network of structured anaphoric connections and the meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora.

Section **3** defines the formal system, dubbed Intensional PCDRT (IP-CDRT), i.e. the extension of PCDRT with (dref's for) possible worlds. Section **4** shows how modalized conditionals and the entailment particle *therefore* are analyzed in IP-CDRT, while section **5** introduces the IP-CDRT analysis of modal subordination: modal subordination is basically analyzed as an instance of restricting the domain of modal quantifiers via structured modal anaphora; that is, the antecedent of (2b) is simultaneously anaphoric to the set of worlds and the set of individuals introduced by the the antecedent of (2a) and, also, to the quantificational dependency established between these two sets.

In order to make the presentation simpler and, hopefully, clearer, the development of Intensional PCDRT in sections **3**, **4** and **5** builds on the simpler PCDRT system introduced in chapter **5**, which does not contain all the extensions introduced in chapter **6** for the PCDRT analysis of quantificational subordination (e.g. the dummy individual, distributivity operators over individual dref's etc.).

It is only in section **6** that I revise the analysis of modal quantification, modal anaphora and modal subordination within an intensional system that incorporates and

extends the PCDRT system of chapter **6**. The revised analysis introduced in section **6** will explicitly and systematically capture the intuitive parallel between quantificational subordination and modal subordination – in particular, the intuitive parallel between the quantificational subordination discouse *Harvey courts a^u girl at every convention. She$_u$ always comes to the banquet with him* (Karttunen 1976) and the modal subordination discourse *A^u wolf might come in. It$_u$ would attack Harvey first* (based on Roberts 1989).

The final section (section **7**) compares IP-CDRT with alternative analyses of modalized conditionals and modal subordination.

2. Structured Reference across Domains

This section outlines the account of the Aquinas discourse in (1/2) above. I first show how to extend Plural Compositional DRT (PCDRT) with (dref's for) possible worlds (**2.1**). The extension enables us to analyze the discourse in (1/2) as a network of structured anaphoric connections. The meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora (**2.2**).

2.1. Extending PCDRT with Possible Worlds

To analyze discourse (1/2), I will extend Dynamic Ty2 (and PCDRT) with a new basic type **w** for possible worlds. Thus, we will work with a Dynamic Ty3 logic with four basic types: *t* (truth-values), *e* (individuals; variables: *x*, *x'* etc.) and **w** (possible worlds; variables: *w*, *w'* etc.) and *s* ('variable assignments'; variables: *i*, *j*, *i'*, *j'* etc.). The only modifications we have to make to the Dynamic Ty2 logic introduced in chapter **3** are: (**i**) resetting the set of basic static types **BasSTyp** to {*t*, *e*, **w**} and (**ii**) redefining the notion of standard frame for Dynamic Ty3 so that D_t, D_e, D_w and D_s are non-empty and pairwise disjoint sets. In particular, the set of four axioms that ensures that the objects in the domain D_s actually behave like variable assignments in the relevant respects remain the same.

In the spirit of Stone (1999), I will analyze modal anaphora by means of dref's for *static* modal objects; in this way, we will explicitly capture the intuitive parallel between anaphora and quantification in the individual and modal domains argued for in Geurts

(1995/1999), Frank (1996), Stone (1999), Bittner (2001) and Schlenker (2005) among others. I will call the resulting system Intensional Plural CDRT (IP-CDRT). IP-CDRT takes the research program in Muskens (1996), i.e. the unification of Montague semantics and DRT, one step further: IP-CDRT unifies – in classical type logic – the static Lewis (1973) / Kratzer (1981) analysis of modal quantification and van den Berg's Dynamic Plural Logic.

Throughout this chapter, I will continue to subscript terms with their types, e.g. x_e, $w_\mathbf{w}$, i_s. I will also subscript lexical relations with their world variable, e.g. $see_w(x, y)$ is meant to be interpreted as x sees y in world w.

Just as in CDRT+GQ and PCDRT, a dref for individuals u will be a function of type se from 'assignments' i_s to individuals x_e; intuitively, the individual $u_{se}i_s$ is the individual that i assigns to the dref u. In addition, IP-CDRT has dref's for possible worlds p, p', …, p_1, p_2, which are functions of type $s\mathbf{w}$ from 'assignments' i_s to possible worlds $w_\mathbf{w}$; intuitively, the world $p_{s\mathbf{w}}i_s$ is the world that i assigns to the dref p.

As in PCDRT, dynamic info states are sets of 'variable assignments', i.e. terms I, J etc. of type st. A sentence is still interpreted as a DRS, i.e. a relation of type $(st)((st)t)$ between an input and an output info state. An individual dref u stores a set of individuals with respect to an info state I, abbreviated $uI := \{u_{se}i_s: i_s \in I_{st}\}$ (that is, uI is the image of the set of 'assignments' I under the function u). A dref p stores a set of worlds, i.e. a *proposition*, with respect to an info state I, abbreviated $pI := \{p_{s\mathbf{w}}i_s: i_s \in I_{st}\}$ (that is, pI is the image of the set of 'assignments' I under the function p).

Propositional dref's have two uses: (**i**) they store contents, e.g. the content of the entire conditional in (2a) (i.e. the content of the premise of the Aquinas argument); (**ii**) they store possible scenarios (in the sense of Stone 1999), e.g. the set of worlds introduced by the conditional antecedent in (2a), i.e. a possible scenario containing a man that is alive and on which the consequent of the conditional in (2a) further elaborates

As before, we use plural info states to store sets of individuals and propositions instead of simply using dref's for sets of individuals or possible worlds (their types would be $s(et)$ and $s(\mathbf{w}t)$) because we need to store in our discourse context (i.e. in our

information states) both the *values* assigned to various dref's and the *structure* associated with those values, as shown in (3) below.

3.

Info State I	...	u	u'	p	p'	...
i_1	...	x_1 (i.e. ui_1)	y_1 (i.e. $u'i_1$)	w_1 (i.e. pi_1)	v_1 (i.e. $p'i_1$)	...
i_2	...	x_2 (i.e. ui_2)	y_2 (i.e. $u'i_2$)	w_2 (i.e. pi_2)	v_2 (i.e. $p'i_2$)	...
i_3	...	x_3 (i.e. ui_3)	y_3 (i.e. $u'i_3$)	w_3 (i.e. pi_3)	v_3 (i.e. $p'i_3$)	...
...	...	...	...	...	...	...

Values (sets of individuals or worlds): $\{x_1, x_2, x_3, \ldots\}$, $\{w_1, w_2, w_3, \ldots\}$ etc. **Structure (relations between individuals and / or worlds):** $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \ldots\}$, $\{<x_1, y_1, w_1>, <x_2, y_2, w_2>, <x_3, y_3, w_3>, \ldots\}$, $\{<w_1, v_1>, <w_2, v_2>, <w_3, v_3>, \ldots\}$ etc.

Mixed reading donkey sentences, donkey anaphora to structure (both analyzed in chapter **5**) and quantificational subordination (analyzed in chapter **6**) provide empirical motivation for plural info states. The example of modal subordination in (5) below, which is intuitively parallel to the example of quantificational subordination in (4), provides independent empirical support.

4. **a.** Every ^{u_1} man saw a ^{u_2} woman. **b.** They $_{u_1}$ greeted them $_{u_2}$.

5. **a.** A ^{u_1} wolf might ^{p_1} enter the cabin. **b.** It $_{u_1}$ would $_{p_1}$ attack John.

In both discourses, we do not simply have anaphora to sets of values (individuals and / or possible worlds), but anaphora to *structured* sets.

In particular, if man m_1 saw woman n_1 and m_2 saw n_2, (4b) is interpreted as asserting that m_1 greeted n_1, not n_2, and that m_2 greeted n_2, not n_1; the structure of the greeting is the same as the structure of the seeing[4]. Similarly, (5b) is interpreted as asserting that, if a wolf entered the cabin, it would attack John, i.e. if a black wolf x_1 enters the cabin in world w_1 and a white wolf x_2 enters the cabin in world w_2, then x_1 attacks John in w_1, not in w_2, and x_2 attacks John in w_2, not in w_1.

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[4] The fact that *correspondence* interpretation of discourse (4) – in which the structure of the greeting is the same as the structure of the seeing – is a distinct reading for this discourse and not simply a particular understanding of a vague / underspecified *cumulative*-like reading is argued for in Krifka (1996b) and Nouwen (2003).

A plural info state I stores the *quantificational structure* associated with sets of individuals and possible worlds: (4a) requires each variable assignment $i \in I$ to be such that the man u_1i saw the woman u_2i; (4b) elaborates on this structured dependency by requiring that, for each $i \in I$, the man u_1i greeted the woman u_2i. The structured dependency can be represented in the (by now) familiar way, i.e. by means of a matrix like the one in (6) below.

6. Info state I	...	u_1 (**men**)	u_2 (**women**)	...
i_1	...	$m_1 (=u_1i_1)$	$n_1 (=u_2i_1)$	...

<center>*man m_1 saw woman n_1*</center>

i_2	...	$m_2 (=u_1i_2)$	$n_2 (=u_2i_2)$	...
i_3	...	$m_3 (=u_1i_3)$	$n_3 (=u_2i_3)$	...
...	...	...	...	...

Similarly, (5a) outputs an info state I such that, for each $i \in I$, the wolf u_1i enters the cabin in the world p_1i; (5b) elaborates on this structured dependency: for each assignment $i \in I$, it requires the wolf u_1i to attack John in world p_1i.

7. Info state I	...	u_1 (**wolves**)	p_1 (**worlds**)	...
i_1	...	$x_1 (=u_1i_1)$	$w_1 (=p_1i_1)$	...

<center>*wolf x_1 enters the cabin in world w_1*</center>

i_2	...	$x_2 (=u_1i_2)$	$w_2 (=p_1i_2)$	...
i_3	...	$x_3 (=u_1i_3)$	$w_3 (=p_1i_3)$	...
...	...	...	...	...

Moreover, we need plural info states to capture structured anaphora between the premise(s) and the conclusion of entailment discourses like (1/2) above or (8) and (9) below.

8. **a.** Every^{u_1} man saw a^{u_2} woman. **b.** Therefore, they$_{u_1}$ noticed them$_{u_2}$.

9. **a.** A^{u_1} wolf might^{p_1} enter the cabin. **b.** It$_{u_1}$ would$_{p_1}$ see John^{u_2}.

   c. Therefore, it$_{u_1}$ would$_{p_1}$ notice him$_{u_2}$.

2.2. Structured Reference in Modal Discourse

Let us return now to discourse (2), which is analyzed as shown in (10) below.

10. **CONTENT**^{p_1} **:** **if**^{p_2} (a^{u_1} man$_{p_2}$ is alive$_{p_2}$);

   must^{p_3} $_{p_1, \mu, \omega}$ (p_2, p_3); he$_{u_1}$ has$_{p_3}$ a^{u_2} pleasure$_{p_3}$.

   THEREFORE^{p_4} $_{p*, \mu*, \omega*}$ (p_1, p_4)**:**

   if^{p_5} ($p_5 \subseteq p_2$; **not**(he$_{u_1}$ has$_{p_5}$ a^{u_3} spiritual pleasure$_{p_5}$));

   must^{p_6} $_{p_4, \mu, \omega}$ (p_5, p_6); he$_{u_1}$ has$_{p_6}$ a^{u_4} carnal pleasure$_{p_6}$.

The representation in (10) is basically a network of structured anaphoric connections. Consider the conditional in (2a) first. The morpheme *if* introduces a dref p_2 that stores the content of the antecedent – we need this distinct dref because the antecedent in (2b) is anaphoric to it (due to modal subordination). The indefinite *a man* introduces an individual dref u_1, which is later retrieved: (**i**) by the pronoun *he* in the consequent of (2a), i.e. by donkey anaphora, and (**ii**) by the pronoun *he* in the antecedent of (2b), i.e. by modal subordination.

The modal verb *must* in the consequent of (2a) contributes a tripartite quantificational structure and it relates three propositional dref's. The dref p_1 stores the content of the whole modalized conditional. The dref p_2, which was introduced by the antecedent and which is anaphorically retrieved by *must*, provides the restrictor of the modal quantification. Finally, p_3 is the nuclear scope of the modal quantification; it is introduced by the modal *must*, which constrains it to contain the set of *ideal* worlds among the p_2-worlds – ideal relative to the p_1-worlds, a *circumstantial* modal base μ and an *empty* ordering source ω. Finally, we test that the set of ideal worlds stored in p_3 satisfies the remainder of the consequent.

Consider now the entailment particle *therefore*. I take it to relate *contents* and not meanings. We can see this by examining the discourses in (8) and (9) above: in both cases, the contents (i.e. truth-conditions) of the premise(s) and the conclusion stand in an inclusion relation, but not their meanings (i.e. context change potentials). Further support is provided by the fact that the felicity of *therefore*-discourses is *context-dependent* – which is expected if *therefore* relates contents because contents are determined in a context-sensitive way. Consider, for example, the discourse in (11) below: entailment obtains if (11) is uttered on a Thursday in a discussion about John, but not otherwise.

11. **a.** He*John* came back three days ago*Thursday*.

 b. Therefore, John came back on a Monday.

Moreover, I propose that *therefore* in (2b) should be analyzed as a modal relation, in particular, as expressing *logical consequence*; thus, I analyze discourse (1/2) as a modal quantification that relates two embedded modal quantifications, the second of which is modally subordinated to the first. Just as the modal *must*, *therefore* contributes a necessity modal relation and introduces a tripartite quantificational structure: the restrictor is p_1 (the content of the premise) and the nuclear scope is the newly introduced dref p_4, which stores the set of ideal p_1-worlds – ideal relative to the dref p^* (the designated dref for the actual world w^*), an *empty* modal base μ^* and an *empty* ordering source ω^* (the modal base μ^* and the ordering source ω^* are empty because *therefore* is interpreted as logical consequence). Since μ^* and ω^* are empty, the dref p_4 is identical to p_1.

Analyzing *therefore* as an instance of modal quantification makes at least two welcome predictions. First, it predicts that we can interpret it relative to different modal bases and ordering sources – and this prediction is borne out. *Therefore* expresses *causal* consequence in (12) below and it seems to express a form of *practical inference* in (13).

12. Reviewers are usually people who would have been poets, historians, biographers, etc., if they could; they have tried their talents at one or the other, and have failed; therefore they turn critics.

 (Samuel Taylor Coleridge, *Lectures on Shakespeare and Milton*)

13. We cannot put the face of a person on a stamp unless said person is deceased. My suggestion, therefore, is that you drop dead.

(attributed to J. Edward Day; letter, never mailed, to a petitioner who wanted himself portrayed on a postage stamp)

Second, it captures the intuitive equivalence between the *therefore* discourse *A man saw a woman, therefore he noticed her* and the modalized conditional *If a man saw a woman, he (obviously / necessarily) noticed her* (they are equivalent provided we add the premise *A man saw a woman* to the conditional).

The conditional in (2b) is interpreted like the conditional in (2a), with the additional complication that its antecedent is anaphoric to the antecedent of the conditional in (2a), i.e. to the dref p_2. The dref p_5 is a *structured* subset of p_2, symbolized as $p_5 \Subset p_2$. We need *structured inclusion* because we want p_5 to preserve the structure associated with the p_2-worlds, i.e. to preserve the quantificational correspondence between the p_2-worlds and the u_1-men that are alive in them. The modal verb *must* in (2b) is anaphoric to p_5, it introduces the set of worlds p_6 containing all the p_5-worlds that are ideal relative to the p_4-worlds, μ and ω (the same as the modal base and ordering source in the premise (2a)) and it checks that, in each ideal p_6-world, all its associated u_1-men have a carnal pleasure.

3. Intensional Plural CDRT (IP-CDRT)

In an intensional Fregean / Montagovian framework, the compositional aspect of interpretation is largely determined by the types for the extensions of the 'saturated' expressions, i.e. names and sentences, plus the type that enables us to build intensions out of these extensions. Let us abbreviate them as **e**, **t** and **s**, respectively. In IP-CDRT, we assign the following dynamic types to the 'meta-types' **e**, **t** and **s**: a sentence is interpreted as a DRS, i.e. as a relation between info states, hence $\mathbf{t} := (st)((st)t)$ (the same as in PCDRT); a name is interpreted as an individual dref, hence $\mathbf{e} := se$ (again, the same as in PCDRT). Finally, $\mathbf{s} := s\mathbf{w}$, i.e. we use the type of propositional dref's to build intensions.

To interpret a noun like 'man', we define an atomic $man_p\{u\}$ based on the static one $man_w(x)$, as shown in (14) below. The IP-CDRT atomic conditions are the obvious intensionalized versions of the corresponding PCDRT conditions.

14. **Atomic conditions – first attempt.**

$$man_p\{u\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I(man_{pi}(ui)).$$

In general, the IP-CDRT basic meanings for lexical items are the usual intensionalized versions of the corresponding extensional PCDRT meanings, as shown in table (45) below. I use the following notational conventions:

- u, u' etc. for dref's of type $\mathbf{e}{:=}se$ (recall that they are constants in our Dynamic Ty3 logic) and v, v' etc. for variables of type $\mathbf{e}{:=}se$;

- p, p' etc. for dref's of type $\mathbf{s}{:=}s\mathbf{w}$ (which are also constants in our Dynamic Ty3 logic) and q, q' etc. for variables of type $\mathbf{s}{:=}s\mathbf{w}$;

- $\mathbb{P}$, $\mathbb{P}'$ etc. for variables over dynamic propositions of type $\mathbf{st}$, where $\mathbf{s}{:=}s\mathbf{w}$ and $\mathbf{t}{:=}(st)((st)t)$;

- P, P' etc. for variables over dynamic intensional properties of type $\mathbf{e(st)}$, where $\mathbf{e}{:=}se$;

- Q, Q' etc. for variables over dynamic intensional quantifiers of type $\mathbf{(e(st))(st)}$.

15. **TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
$[sleep]_{V_{in}}$	$\leadsto \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [sleep_q\{v\}]$, where $sleep$ is of type $e(\mathbf{w}t)$	$\mathbf{e(st)}$
$[own]_{V_{tr}}$	$\leadsto \lambda Q_{\mathbf{(e(st))(st)}}.\ \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ Q(\lambda v'_{\mathbf{e}}.\ \lambda q'_{\mathbf{s}}.\ [own_{q'}\{v, v'\}])(q)$, where own is of type $e(e(\mathbf{w}t))$	$\mathbf{((e(st))(st))(e(st))}$
	equivalently: $\leadsto$ $\lambda Q_{\mathbf{(e(st))(st)}}.\ \lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [own_q\{v, v'\}])$	
$[buy]_{V_{di}}$	$\leadsto \lambda Q'_{\mathbf{(e(st))(st)}}.\ \lambda Q_{\mathbf{(e(st))(st)}}.\ \lambda v_{\mathbf{e}}.$ $\quad Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v''_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [buy_q\{v, v', v''\}]))$, where buy is of type $e(e(e(\mathbf{w}t)))$	$\mathbf{(e(st)(st))((e(st)(st))}$ $\mathbf{(e(st)))}$
$[house\text{-}elf]_{N}$	$\leadsto \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [house\_elf_q\{v\}]$, where $house\_elf$ is of type $e(\mathbf{w}t)$	$\mathbf{e(st)}$

15. **TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
$[he_u]_{\text{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ P(u_{\mathbf{e}})$	$(\mathbf{e(st)})(\mathbf{st})$
$[the_u]_{\text{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [\mathbf{unique}_q\{u\}];\ P'(u)(q);\ P(u)(q),$ where $\mathbf{unique}_q\{u\} :=$ $\quad \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(qi{=}qi' \to ui{=}ui'),$ i.e. anaphoric and 'weakly' unique. $\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ P'(u)(q);\ P(u)(q),$ i.e. anaphoric.	$(\mathbf{e(st)})((\mathbf{e(st)})(\mathbf{st}))$
$[t_v]_{\text{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ P(v_{\mathbf{e}})$	$(\mathbf{e(st)})(\mathbf{st})$
$[he_{Dobby}]_{\text{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ P(Dobby_{\mathbf{e}})$	$(\mathbf{e(st)})(\mathbf{st})$
$[Dobby^u]_{\text{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u \mid u{=}Dobby];\ P(u)(q)$	$(\mathbf{e(st)})(\mathbf{st})$
$[who]_{\text{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ P$	$(\mathbf{e(st)})(\mathbf{e(st)})$
$[\emptyset]_{\text{I}}/[\text{-}ed]_{\text{I}}/[\text{-}s]_{\text{I}}$	$\rightsquigarrow \lambda \mathbb{P}_{\mathbf{(st)}}.\ \mathbb{P}$	$(\mathbf{st})(\mathbf{st})$
$[doesn't]_{\text{I}}/[didn't]_{\text{I}}$	$\rightsquigarrow \lambda \mathbb{P}_{\mathbf{(st)}}.\ \lambda q_{\mathbf{s}}.\ [{\sim}\mathbb{P}(q)],\quad$ where: ${\sim}D := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall H_{st}(H{\neq}\emptyset \wedge H{\subseteq}I \to \neg\exists K_{st}(DHK)),$ where D is a DRS (type $\mathbf{t}$)	$(\mathbf{st})(\mathbf{st})$
$[a^{\mathbf{wk}:u}]_{\text{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u];\ P'(u)(q);\ P(u)(q),$ i.e. $\lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \exists u(P'(u)(q);\ P(u)(q)),$ where $\exists u(D) := [u];\ D$	$(\mathbf{e(st)})((\mathbf{e(st)})(\mathbf{st}))$
$[a^{\mathbf{str}:u}]_{\text{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q);\ P(u)(q)),\quad$ where: $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \to uK{\subseteq}uJ),$ i.e. $\lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \exists^{\mathbf{m}}u(P'(u)(q);\ P(u)(q)),$ where $\exists^{\mathbf{m}}u(D) := \mathbf{max}^u(D)$	$(\mathbf{e(st)})((\mathbf{e(st)})(\mathbf{st}))$
$[the^u]_{\text{D}}$	$\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q));\ [\mathbf{unique}_q\{u\}];\ P(u)(q),$ where $\mathbf{unique}_q\{u\} :=$ $\quad \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I\forall i'_s{\in}I(qi{=}qi' \to ui{=}ui')$ and $\mathbf{max}^u(D) := \lambda I_{st}J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \to uK{\subseteq}uJ),$ i.e. $\lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \exists^{\mathbf{m}}u(P'(u)(q));\ [\mathbf{unique}_q\{u\}];\ P(u)(q),$ i.e. existence and uniqueness – the Russellian analysis $\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ \mathbf{max}^u(P'(u)(q));\ P(u)(q),\quad$ where:	$(\mathbf{e(st)})((\mathbf{e(st)})(\mathbf{st}))$

15. **TR 0: IP-CDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
	$\mathbf{max}^u(D) := \lambda I_{st} J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \to uK \subseteq uJ),$ i.e. $\lambda P'_{e(st)}.\ \lambda P_{e(st)}.\ \lambda q_s.\ \exists^{\mathbf{m}} u(P'(u)(q));\ P(u)(q),$ i.e. existence and maximality	
$[det^u]_D$	$\leadsto \lambda P'_{e(st)}.\ \lambda P_{e(st)}.\ \lambda q_s.\ [_q\mathbf{det}_u(P'(u)(q), P(u)(q))],$ where:	$(\mathbf{e(st)})((\mathbf{e(st)})(\mathbf{st}))$
e.g. *every^u*, *no^u*, *most^u*… (but not *a$^{\mathbf{wk}:u}$*, *a$^{\mathbf{str}:u}$*, *the$_u$* or *the^u*)	$_p\mathbf{det}_u(D_1, D_2) := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(u_p[D_1 I],\ u_p[(D_1; D_2)I]),$ where $u_p[DI] := \cup \{uJ: ([u \mid \mathbf{unique}_p\{u\}]; D)IJ\}$ and $\mathbf{unique}_p\{u\} :=$ $\qquad \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(pi=pi' \to ui=ui')$ and $\mathbf{DET}$ is the corresponding static determiner	
$[and]_{Conj}$	$\leadsto \lambda v_1.\ \dots\ \lambda v_n.\ v_1 \sqcap \dots \sqcap v_n$	$\tau(\dots(\tau\tau)\dots)$
$[or]_{Conj}$	$\leadsto \lambda v_1.\ \dots\ \lambda v_n.\ v_1 \sqcup \dots \sqcup v_n$	$\tau(\dots(\tau\tau)\dots)$

The IP-CDRT definitions of generalized conjunction $\sqcap$ and generalized disjunction $\sqcup$ are the same as the PCDRT ones.

3.1. An Example: Indicative Sentences in IP-CDRT

Let us now look at the IP-CDRT analysis of a simple indicative sentence like the one in (16) below. I will assume that the LF of such a sentence contains an indicative mood morpheme in the complementizer head *C*, whose meaning is provided in (1) below: the indicative mood stakes the dynamic proposition $\mathbb{P}_{\mathbf{st}}$ denoted by the remainder of the sentence and applies it to the designated dref for the actual world *p**. We capture the fact that the dref *p** refers to the actual world *w** by requiring that *p*I*={*w**}, where *I* is the input information state relative to which the sentence is interpreted.

Furthermore, I assume that *alive* functions as an intransitive verb and that *is* functions as a semantically vacuous inflectional head *I*, much like $[\emptyset]_I$/$[\text{-}ed]_I$/$[\text{-}s]_I$ – and it is assigned the same kind of meaning, i.e. an identity function over dynamic propositions: $\lambda \mathbb{P}_{(\mathbf{st})}.\ \mathbb{P}$.

16. A $^{\mathbf{wk:}\, u_1}$ man is alive.

17. $[\mathbf{ind}_{p^*}]_C \;\rightsquigarrow\; \lambda\mathbb{P}_{\mathbf{st}}.\ \mathbb{P}(p^*)$

18. $a^{\mathbf{wk:}\, u_1} \rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\ \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u_1];\ P'(u_1)(q);\ P(u_1)(q)$

   $man \rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [man_q\{v\}]$

   $a^{\mathbf{wk:}\, u_1}\ man \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\ \lambda q_{\mathbf{s}}.\ [u_1\mid man_q\{u_1\}];\ P(u_1)(q)$

   $alive \rightsquigarrow \lambda v_{\mathbf{e}}.\ \lambda q_{\mathbf{s}}.\ [alive_q\{v\}]$

   $a^{\mathbf{wk:}\, u_1}\ man\ is\ alive \rightsquigarrow \lambda q_{\mathbf{s}}.\ [u_1\mid man_q\{u_1\},\ alive_q\{u_1\}]$

   $\mathbf{ind}_{p^*}\ a^{\mathbf{wk:}\, u_1}\ man\ is\ alive \rightsquigarrow [u_1\mid man_{p^*}\{u_1\},\ alive_{p^*}\{u_1\}]$

Note that, before introducing the meaning of the indicative mood morpheme $\mathbf{ind}_{p^*}$, the composition makes available the dynamic proposition (of type $\mathbf{st}$) $\lambda q_{\mathbf{s}}.\ [u_1\mid man_q\{u_1\},$ $alive_q\{u_1\}]$ and it is based on this proposition that the meaning of the conditional antecedent in (2a) is obtained – as the following section endeavors to show.

4. *Conditionals, Modals and* Therefore *in IP-CDRT*

In this section, I show how to compositionally analyze in Intensional Plural CDRT (IP-CDRT):

- modalized conditionals, i.e. the meaning of the particle *if* (**4.1**) and the meaning of modals (**4.2**);
- the entailment particle *therefore* (**4.3**).

4.1. *If*

To interpret the conditional in (2a) above, we need to: (**i**) extract the content of the antecedent of the conditional and store it in a propositional dref p_2 and (**ii**) define a dynamic notion of *structured* subset of a set of worlds.

We will first see how to extract the content of the antecedent of the conditional. For this purpose, I define two operators over a propositional dref p and a DRS D: a

maximization operator $\mathbf{max}^p(D)$ and a distributivity operator $\mathbf{dist}_p(D)$. The maximization operator over propositional dref's, defined in (19) below, is identical to the maximization operator over individual dref's in PCDRT.

19. $\mathbf{max}^p(D) := \lambda I_{st}\lambda J_{st}. ([p]; D)IJ \wedge \forall K_{st}(([p]; D)IK \rightarrow pK \subseteq pJ)$

The definition of the distributivity operator in (20) below follows the basic format (but not the exact implementation) of the corresponding operator over individual dref's in van den Berg (1994, 1996a) and incorporates an amendment of van den Berg's definition proposed in Nouwen (2003)[5]. Just like $\mathbf{max}^p$, the $\mathbf{dist}_p$ operator is an operator over DRS's: its argument is a DRS D, i.e. a term of type $\mathbf{t} := (st)((st)t)$ and its value is another DRS (of type $\mathbf{t}$), i.e. $\mathbf{dist}_p(D)$.

20. **Selective distributivity over modal dref's in IP-CDRT.**

$\mathbf{dist}_p(D) := \lambda I_{st}\lambda J_{st}. pI = pJ \wedge \forall w \in pI(DI_{p=w}J_{p=w}),$

where $I_{p=w} := \{i_s \in I: pi = w\}$

and p is of type $\mathbf{s} := s\mathbf{w}$ and D is of type $\mathbf{t} := (st)((st)t)$.

The basic idea behind *distributively* updating an input info state I with a DRS D is that we first partition the info state I and then *separately* update each partition cell (i.e. subset of I) with D. Moreover, the partition of the info state I is induced by a dref p as follows: consider the set of worlds $pI := \{pi: i \in I\}$; each world w in the set pI generates one cell in the partition of I, namely the subset $\{i \in I: pi = w\}$. Clearly, the family of sets $\{\{i \in I: pi = w\}: w \in pI\}$ is a partition of the info state I: the union of the family of sets is the info state I and, for any two distinct worlds w and w' in pI, the sets $\{i \in I: pi = w\}$ and $\{i \in I: pi = w'\}$ are disjoint.

Thus, updating an info state I with a DRS D *distributively* over a dref p means updating each cell in the p-partition of I with the DRS D and then taking the union of the resulting output info states. The first conjunct in definition (20) above, i.e. $pI = pJ$, is required to ensure that there is a bijection between the partition cells induced by the dref

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[5] See van den Berg (1994): 14, (43), van den Berg (1996a): 145, (18) and Nouwen (2003): 87, (4.17).

p over the input state I and the partition cells induced by p over the output state J; without this requirement, we could introduce arbitrary new values for p in the output state J, i.e. arbitrary new partition cells[6,7].

The second conjunct, i.e. $\forall w \in pI(DI_{p=w}J_{p=w})$, is the one that actually defines the distributive update: every partition cell in the input info state I is related by the DRS D to the corresponding partition cell in the output state J. The figure in (21) below schematically represents how the input state I is p-distributively updated with the DRS D.

21. **Updating the info state I with the DRS D distributively over the dref p.**

Input state I   – *update with D distributively over p* $\rightarrow$ *Output state J*
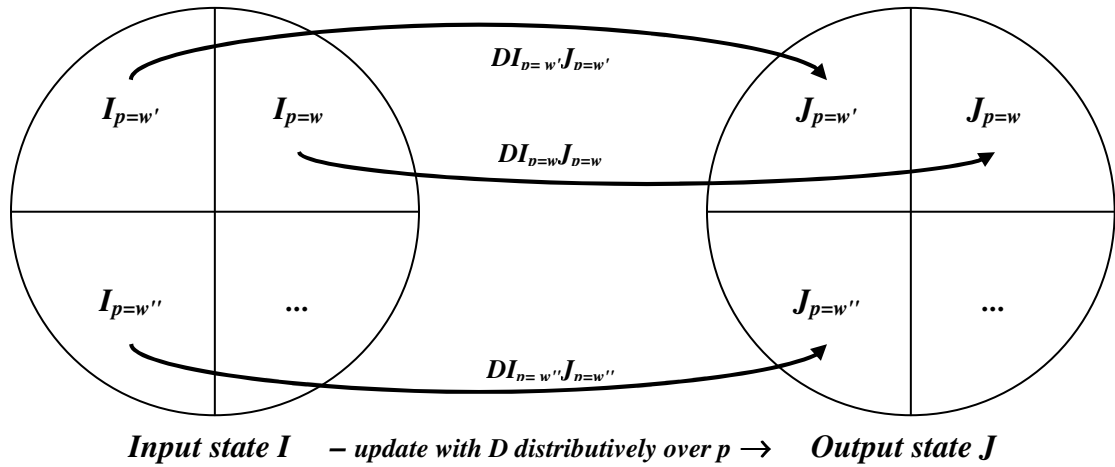
The Appendix to the chapter studies in more detail the formal properties of selective distributuity, generalizes it to distributivity over multiple dref's and defines distributivity operators over arbitrary distributable types over and above the basic distributable type **t** $:= (st)((st)t)$.

The operators $\mathbf{max}^p(D)$ and $\mathbf{dist}_p(D)$ enable us to 'dynamize' λ-abstraction over possible worlds, i.e. to extract and store contents: the $\mathbf{dist}_p(D)$ update checks one world at a time that the set of worlds stored in p satisfies the DRS D and the $\mathbf{max}^p(D)$ update collects in p all the worlds that satisfy D. I will analyze *if* as a dynamic λ-abstractor over

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[6] See Nouwen (2003): 87.

[7] Note that the first conjunct could be replaced with the biconditional $\forall w(I_{p=w} \neq \emptyset \leftrightarrow J_{p=w} \neq \emptyset)$.

possible worlds, i.e. as a morpheme that extracts the *content* of a dynamic proposition $\mathbb{P}_{\mathbf{st}}$ and stores it in a newly introduced propositional dref p, as shown in (22) below. The representation in (23) shows how the meaning of *if* combines with the dynamic proposition contributed by $A^{\mathbf{wk:}\, u_I}$ *man is alive* and stores its content in the dref p_2.

22. *if* $^p \leadsto \lambda \mathbb{P}_{\mathbf{st}}.\ \mathbf{max}^p(\mathbf{dist}_p(\mathbb{P}(p)))$

23. $a^{\mathbf{wk:}\, u_I}$ *man is alive* $\leadsto \lambda q_{\mathbf{s}}.\ [u_I \mid man_q\{u_I\},\ alive_q\{u_I\}]$

   if $^{p_2}\ a^{\mathbf{wk:}\, u_I}$ *man is alive* $\leadsto \mathbf{max}^{p_2}(\mathbf{dist}_{p_2}([u_I \mid man_{p_2}\{u_I\},\ alive_{p_2}\{u_I\}]))$

We need one last thing to translate the antecedent in (2a). The donkey indefinite *a man* receives a *strong* reading, i.e. the conditional in (2a) is interpreted as asserting that *every* (and not only *some*) man that is alive must have a pleasure. Thus, the antecedent of (2a) is translated in IP-CDRT as shown in (24) below[8].

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[8] Thus, I assume that the strong reading associated with the indefinite *a man* is contributed by the indefinite article itself and not by the modal verb *must* (and / or the morpheme *if*). I have chosen this analysis because it is parallel to the analysis of weak / strong readings of relative-clause donkey sentences in chapter **5** above. However, it might very well be that modal verbs in modalized conditionals might bind certain indefinites in the antecedent of the conditional, i.e. they might be instances of *multiply selective quantification*. See, for example, Chierchia (1995) for the use of the notion of dynamic multiply selective quantification in the analysis of extensional conditionals with adverbs of quantification like *always*, *usually* etc.

It seems clear to me that the analysis of conditional donkey sentences like *If a man buys a book on amazon.com and has a credit card, he always / usually uses it to pay for it* should allow for more readings than the corresponding relative-clause donkey sentences, i.e. *Every man who buys a book on amazon.com and has a credit card uses it to pay for it / Most men who buy a book on amazon.com and have a credit card use it to pay for it*. The *usually* donkey sentence has a reading in which we consider most cases in which a man buys a book, while the *most* donkey sentence seems to lack this reading – or, in any case, it is a lot less clear that the *most* donkey sentence has such a reading (see also the contrast between *If a farmer owns a donkey, he usually beats it* and *Most farmers who own a donkey beat it*).

Since conditional donkey sentences allow for more readings than the corresponding relative-clause donkey examples, it seems clear that this is due to the conditional structure itself, i.e. to the adverb of quantification together with the morpheme *if* – and I am inclined at this point to allow for a multiply selective analysis of such donkey conditionals in which the adverb binds indefinites in the antecedent (the analysis of *always* proposed in chapter **6** can be fairly easily extended to accomplish this).

It is not as clear to me that modal verbs in modalized conditionals should receive a similar, multiply selective interpretation, i.e. it is not at all clear to me that modal verbs and adverbs of quantification should be analyzed in parallel (I am indebted to Maribel Romero, p.c., for emphasizing the importance of this issue). Heim (1982), for example, proposes such a parallel analysis; note also that such a parallel analysis is an almost immediate consequence of a situation-based D-/E-type approach to donkey anaphora, since the

24. *if*^{p_2} *a*$^{\mathbf{str:}\,u_1}$ *man is alive* $\rightsquigarrow$ $\mathbf{max}^{p_2}(\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}([man_{p_2}\{u_1\},\,alive_{p_2}\{u_1\}])))$

The IP-CDRT representation in (24) provides the empirical motivation for the introduction of *selective* distributivity: we need the $\mathbf{dist}_{p_2}$ operator over and above the *unselective* distributivity built into the atomic conditions because, in the standard Kripke-style modal system that I assume, the same individual may exist in multiple worlds (though not necessarily in all of them). Therefore, it can be possible for a man to be alive in two distinct possible worlds – in which case, we want to introduce this man with respect to *each* of the possible worlds in which he is alive – and this is what the selective distributivity operator over the modal dref p_2 achieves: $\mathbf{dist}_{p_2}$ ensures that we separately consider every possible world stored in p_2 and relate it to all the men that are alive in it. Should we omit the selective distributive operator, we could introduce *all* the men that are alive in some world or other, but we might fail to introduce *each* man with respect to *each* possible world in which he is alive.

Thus, at least for the particular example we are considering, the need for selective distributivity is partly due to the assumed underlying ontology[9]. However, the introduction of selective distributivity has a more general motivation, namely the parallel treatment of the dynamics of values and structure in PCDRT and IP-CDRT. More precisely, maximization together with *selective* distributivity enables us to 'dynamize' λ-abstraction over *structure* as well as over values: one the one hand, selective distributive operators, e.g. $\mathbf{dist}_{p_2}$ in (24) above, enable us to λ-abstract one value at a time; on the other hand, selective maximization operators make it possible to extract the desired set and, when we maximize *under* the scope of a selective distributive operator, e.g.

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same domain of situations is used for modal quantification (see Kratzer 1989) and anaphora (see Heim 1990 among others).

However, conditionals with adverbs of quantification are intuitively extensional, while conditionals with modal verbs are intensional, so it is not obvious that we should have a parallel analysis of the two. Unfortunately, I have to leave the investigation of these crucial issues for future research.

[9] Had we used a counterpart-based system of the kind proposed by Lewis (see Lewis 1968 among others), we wouldn't have needed selective distributivity over modal dref's because, in such a system, an individual exists in exactly one possible world.

$\mathbf{dist}_{p_2}(\mathbf{max}^{u_1}(...))$ in (24) above, we are able to extract the full u_1-structure associated with each single p_2-value.

4.2. Modals

We have seen in the previous section how to extract the content of the antecedent of the conditional and store it in a propositional dref p_2. We turn now to the second notion needed for the interpretation of the conditional in (2a), namely the definition of a dynamic notion of *structured* subset of a set of worlds. We need a notion of structured inclusion because:

- the modal *must* and the donkey pronoun *he* in the consequent of (2a) are simultaneously anaphoric to the p_2-worlds and the u_1-men and we need to preserve the structured dependencies between them;
- the modally subordinated antecedent of the conditional in (2b) is also anaphoric to p_2 and u_1 in a structured way.

In the spirit of van den Berg's (extensional) Dynamic Plural Logic, who makes use of a dummy / 'undefined' individual #, I will assume that there is a dummy world # (of type **w**) relative to which all lexical relations are false (the dummy world # can be thought of as the world in which no individual whatsoever exists) and I will use this world to define the *structured inclusion* condition in (25) below [10].

25. $p \Subset p' := \lambda I_{st}.\ I \neq \emptyset \land \forall i_s \in I(pi = p'i \lor pi = \#).$

However, unlike van den Berg, who makes use of the dummy individual # within a partial logic (the dummy individual yields undefinedness), we will continue to work with a classical (bivalent, total) type logic and assume that the dummy world # yields falsity (i.e. any lexical relation of the form $R_w(x_1, ..., x_2)$ is false if w is #). We can think of the dummy world $\#_\mathbf{w}$ as the world where no individual whatsoever exists, hence all the

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[10] The corresponding notion of structured inclusion in the individual domain is defined and justified in section **3.2** of chapter **6** above.

lexical relations are false because a relation between individuals obtains at a particular world only if the individuals exist in that possible world.

The dummy world # is used to signal that an 'assignment' i such that pi=# is irrelevant for the evaluation of conditions, so we need to slightly modify the definition of atomic conditions as shown in (26) below. The matrix in (27) represents an info state I that satisfies the structured inclusion requirement $p \in p'$ and the atomic condition $man_p\{u\}$. The shaded rows i_2 and i_4 represent the 'assignments' that are discarded in the evaluation of the atomic condition $man_p\{u\}$ – and they are discarded because they both assign the dummy world # to the propositional dref p, i.e. $pi_2=pi_4$=#.

26. **Atomic conditions.**

$$man_p\{u\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s \in I_{p\neq\#}(man_{pi}(ui)),$$

$$\text{where } I_{p\neq\#} := \{i_s \in I:\ pi\neq\#\}.$$

27. Info state I: $p \in p'$ and $man_p\{u\}$	p' (**superset worlds**)	p (**subset worlds**)	u (**men**)
i_1	$w_1\ (=p'i_1)$	$w_1\ (=pi_1)$	$x_1\ (=ui_1)$
i_2	$w_1\ (=p'i_2)$	$\#\ (=pi_2)$	$x_2\ (=ui_2)$
i_3	$w_1\ (=p'i_3)$	$w_1\ (=pi_3)$	$x_3\ (=ui_3)$
i_4	$w_2\ (=p'i_4)$	$\#\ (=pi_4)$	$x_1\ (=ui_4)$
i_5	$w_2\ (=p'i_5)$	$w_2\ (=pi_5)$	$x_4\ (=ui_5)$

In a similar vein, we need to slightly modify the way we make use of selective distributivity: we will discard the 'dummy' partition cell $I_{p=\#}$ when we distributively update with the DRS D, which is formally captured by the first conjunct in definition (28) below, which requires the equality of the input and output 'dummy' partition cells. The second conjunct $I_{p\neq\#}\neq\emptyset$ is needed to rule out the degenerate case in which the distributive update $\mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$ is vacuously satisfied.

28. **Selective distributivity modulo the dummy world #$_w$.**

$$_p(D) := \lambda I_{st}\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$$

where $I_{p=\#} := \{i_s \in I: pi=\#\}$, $I_{p\neq\#} := \{i_s \in I: pi\neq\#\}$,

p is of type $\mathbf{s} := s\mathbf{w}$ and D is of type $\mathbf{t} := (st)((st)t)$ [11].

Finally, we also need to slightly modify the definition of the maximization operator, as shown in (29) below.

29. **Selective maximization modulo the dummy world #$_w$.**

$$\mathbf{max}^p(D) := \lambda I_{st}.\lambda J_{st}. ([p]; D)IJ \wedge \forall K_{st}(([p]; D)IK \rightarrow pK_{p\neq\#} \subseteq pJ_{p\neq\#})$$

We are now ready to give the lexical entries for modal verbs. The modal verb *must* is interpreted in terms of a modal condition $\mathbf{nec}_{p,\mu,\omega}(p', p'')$, defined in (30) below. The condition is relativized to: (**i**) a propositional dref p storing the content of the entire modal quantification, (**ii**) an modal base dref μ and (**iii**) an ordering source dref ω.

30. $\mathbf{nec}_{p,\mu,\omega}(p', p'') := \lambda I_{st}. I_{p\neq\#} \neq \emptyset \wedge$

$$\forall w \in pI_{p\neq\#}(\mathbf{NEC}_{\mu I_{p=w,\mu\neq\{\#\}}, \omega I_{p=w,\omega\neq\{\#\}}}(p'I_{p=w,p'\neq\#}, p''I_{p=w,p''\neq\#}) \wedge$$

$$(p'' \sqsubseteq p')I_{p=w})$$

The definition crucially relies on the notion of structured inclusion defined in (25) above. However, we need to strengthen this notion of structured inclusion as shown in (31) below. The reason is that the notion of structured inclusion in (25) merely requires the subset dref to store *only* the superset structure, but modal quantifications in general additionally require the subset dref to store *all* the superset structure – which is what the second conjunct in (31) ensures. To see that we need to store all the superset structured, consider example (32) below, which is interpreted as asserting that, in every deontically

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[11] Strictly speaking, we should also modify the translation of the indicative morpheme from the one in (17) above to the one in (i) below, which makes use of the $_{p*}(\ldots)$ operator. However, I will ignore this complication throughout most of the chapter (more precisely, until section **6**, where the parallel between singular pronouns and the indicative morpheme is explicitly captured). This simplification will not affect any of the analyses in this chapter. Indeed, the translation in (17) and the one in (i) below are equivalent with respect to any input info state I such that $p*I$ is a singleton set, namely the singleton set containing only the actual world, i.e. $\{w*\}$. We can in fact achieve this (and therefore preserve the simpler translation of the indicative morpheme in (17)) by assuming that any discourse starts with a default update of the form $[p* | p*=w*]$, where $p*=w* := \lambda I_{st}. p*I_{p*\neq\#}=\{w*\}$.

(i) $[\mathbf{ind}_{p^i}]_C \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}. {}_{p*}(\mathbb{P}(p*))$.

ideal world among the worlds in which there is a murder, for *each and every* murder (and not merely some of the murders) in said ideal world, the murder is investigated in that world[12].

31. **Structured inclusion for dynamic modal quantification.**

$$p''\sqsubseteq p' := \lambda I_{st}. \ (p''\subseteq p')I \wedge \forall i\in I(p'i\in p''I_{p''\neq\#} \to p'i=p''i)$$

32. If$^{p'}$ there is a^u murder, it$_u$ must$_p^{p''\sqsubseteq p'}$ be investigated.

Both μ and ω are dref's for sets of worlds, i.e. they are of type $s(\mathbf{w}t)$[13], a significant simplification compared to the type of static modal bases and ordering sources in Kratzer (1981), i.e. $\mathbf{w}((\mathbf{w}t)t)$. We can simplify these types in IP-CDRT because we have plural info states: every world $w\in pI$ is associated with a sub-state $I_{p=w}$ and we can use this sub-state to associate a set of propositions with the world w, e.g. the set of propositions $\{\mu i: i_s\in I_{p=w}\}$, where each μi is of type $\mathbf{w}t$. A similar procedure enables us to associate an ordering source ω with each p-world.

NEC is the static modal relation, basically defined as in Lewis (1973) and Kratzer (1981). In particular, the dref's μ and ω in (30) above associate with each p-world two sets of propositions M and O of type $(\mathbf{w}t)t$: for each world $w\in pI_{p\neq\#}$, the set of propositions M is the modal base $\{\mu i: i\in I_{p=w}\}$ and the set of propositions O is the ordering source $\{\omega i: i\in I_{p=w}\}$. The set of propositions O induces a strict partial order $<_O$ on the set of all possible worlds as shown in (33) below.

33. $w<_O w'$ iff $\forall W_{\mathbf{w}t}\in O(w'\in W \to w\in W) \wedge \exists W_{\mathbf{w}t}\in O(w\in W \wedge w'\notin W)$

I assume that all the strict partial orders of the form $<_O$ satisfy the Generalized Limit Assumption in (34) – therefore, the **Ideal** function in (35) is well-defined. This function extracts the subset of O-ideal worlds from the set of worlds W.

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[12] See the corresponding strengthened notion of structured inclusion in the individual domain defined in section **3.2** of chapter **6** above and its parallel justification.

[13] I take the dummy value for modal base and OS dref's to be the singleton set whose member is the dummy world, i.e. {#}.

34. **Generalized Limit Assumption.**

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$,

$$\forall w \in W \exists w' \in W((w' <_O w \vee w' = w) \wedge \neg \exists w'' \in W(w'' <_O w'))$$

35. **The Ideal function.**

For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\mathbf{Ideal}_O(W) := \{w \in W: \neg \exists w' \in W(w' <_O w)\}$$

Possibility modals are interpreted in the same way, we only need to replace the static modal relation **NEC** with **POS**; both static modal relations are defined in (36) below. The definition of the dynamic modal relation **pos**, parallel to the definition of the dynamic relation **nec** in (30) above, is given in (37).

36. $\mathbf{NEC}_{M,O}(W_1, W_2) := W_2 = \mathbf{Ideal}_O((\cap M) \cap W_1)$

   $\mathbf{POS}_{M,O}(W_1, W_2) := W_2 \neq \emptyset \wedge W_2 \subseteq \mathbf{Ideal}_O((\cap M) \cap W_1)$

37. $\mathbf{pos}_{p,\mu,\omega}(p', p'') := \lambda I_{st}. I_{p \neq \#} \neq \emptyset \wedge$

$$\forall w \in p I_{p \neq \#}(\mathbf{POS}_{\mu I_{p=w, \mu \neq \{\#\}}, \omega I_{p=w, \omega \neq \{\#\}}}(p' I_{p=w, p' \neq \#}, p'' I_{p=w, p'' \neq \#}) \wedge$$

$$(p'' \sqsubseteq p') I_{p=w})$$

The dref p' is the restrictor of the dynamic modal quantification and the dref p'' is the nuclear scope, containing the ideal worlds among the p'-worlds – this is ensured by the second conjunct in (30) and (37) above, which takes care of the *values* (i.e. sets of worlds) associated with the dref's p' and p''. The third and fourth conjuncts make sure that we associate the correct *structure* with these dref's: the third conjunct (i.e. structured inclusion) requires that p'' (the set of ideal worlds) stores *only* the p'-structure, while the fourth conjunct ensures that p'' stores *all* the p'-structure associated with the ideal worlds, i.e. for any assignment i such that $p'i$ is an ideal world, we require p'' to store the same ideal world, thereby ruling out the possibility that p'' stores the dummy world #.

The structural requirements are necessary if we want to capture donkey anaphora between the nuclear scope, i.e. the consequent, and the restrictor, i.e. the antecedent of the modalized conditional in (2a): storing in p'' all and only the structure in p' boils down

in this case to the requirement that each ideal world should be associated with all the men that are alive in it.

The matrix in (38) below shows an info state I satisfying the modal relation **nec** $_{p_1,\mu,\omega}$ (p_2, p_3): w_1 is an ideal world among the p_2-worlds, so p_3 inherits all the p_2-rows (i.e. 'assignments') that store w_1, i.e. p_3 inherits all the structure associated with w_1 by the dref p_2. In contrast, w_2 is not an ideal world among the p_2-worlds, so p_3 stores the 'dummy' world in all the p_2-rows that store w_2; all these rows are shaded because we discard all of them when we compute atomic conditions that contain the dref p_3.

38. Info state I: **nec** $_{p_1,\mu,\omega}$ (p_2, p_3)	p_2 (**antecedent worlds**)	u_1 (**men**)	p_3 (**consequent worlds, i.e. ideal worlds**)
i_1	w_1 $(=p_2 i_1)$	x_1 $(=u_1 i_1)$	w_1 $(=p_3 i_1)$

$$\overbrace{\qquad\qquad\qquad}$$
man x_1 is alive in world w_1

i_2	w_1 $(=p_2 i_2)$	x_2 $(=u_1 i_2)$	w_1 $(=p_3 i_2)$
i_3	w_1 $(=p_2 i_3)$	x_3 $(=u_1 i_3)$	w_1 $(=p_3 i_3)$
i_4	w_2 $(=p_2 i_4)$	x_2 $(=u_1 i_4)$	# $(=p_3 i_4)$
i_5	w_2 $(=p_2 i_5)$	x_4 $(=u_1 i_5)$	# $(=p_3 i_5)$
…	…	…	…

Thus, the modal verb *must* in (2a) above is translated as shown in (39) below. Note that the type of its denotation is (**st**)**t**, which is parallel to the type of modal quantifiers in static Montague semantics.

39. $must^{p_3 \hat{} p_2}$ $_{p_1,\mu,\omega}$ $\rightsquigarrow$ $\lambda \mathbb{P}_{st}.$ $[\mu, \omega \mid$ **circumstantial**$_{p*}\{p_1, \mu\}$, **empty**$\{p_1, \omega\}]$;

$[p_3 \mid$ **nec** $_{p_1,\mu,\omega}(p_2, p_3)]$; $_{p_3}(\mathbb{P}(p_3))$ [14]

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[14] I assume, for simplicity, that the modal base and ordering source dref's μ and ω are introduced by the modal verb *must*. As Kratzer (1981) argues, they are in fact contextually supplied, i.e. the modal *must* is, in this respect, very much like the deictic pronouns discussed in section **3.7** of chapter **6** above. The update $[\mu, \omega \mid$ **circumstantial**$_{p*}\{p_1, \mu\}$, **empty**$\{p_1, \omega\}]$ is, therefore, either contributed by the 'deixis' associated with

Let us examine the translation in (39) in more detail. First, we introduce the modal base μ and the ordering source ω and relate them to the dref p_1 (which stores the content of the entire modalized conditional) by the **circumstantial** and **empty** conditions defined in (40) below. The **circumstantial**$_{p*}\{p_1, \mu\}$ condition is *context-dependent*, i.e. it is relativized to the dref for the actual world $p*$; we need this because the argument put forth by Aquinas in discourse (1/2) goes through only if we add another premise to the one explicitly stated, namely that pleasures are either spiritual or carnal.

Thus, the condition **circumstantial**$_{p*}\{p_1, \mu\}$ is meant to contrain the modal quantification expressed by the modalized conditional in (2a) so that it is evaluated only with respect to worlds whose circumstances are identical to the actual world $w*$ in the relevant respects – in particular, the proposition in (41) below has to be true in all the p_1-worlds just as it is in $w*$.

40. **circumstantial**$_p\{p', \mu\} := \lambda I_{st}.\ I_{p \neq \#, p' \neq \#} \neq \varnothing \ \wedge$

$$\forall w \in pI_{p \neq \#}(\forall w' \in p'I_{p=w, p' \neq \#}(\textbf{circumstantial}_w(w', \mu I_{p=w, p'=w'})))$$

  empty$\{p, \omega\} := \lambda I_{st}.\ I_{p \neq \#} \neq \varnothing \ \wedge\ \forall i_s \in I(\omega i = \{\#\})$

  empty$\{p, \mu\} := \lambda I_{st}.\ I_{p \neq \#} \neq \varnothing \ \wedge\ \forall i_s \in I(\mu i = \{\#\})$

41. $\{w_\mathbf{w}: \forall x_e(pleasure_w(x) \rightarrow spiritual_w(x) \vee carnal_w(x))\}$

The remainder of the lexical entry in (39) ensures that the propositional dref p_3 stores all and only the ideal p_2-worlds and then checks that the dynamic proposition $\mathbb{P}$ contributed by the consequent of the conditional in (2a) is satisfied in each such ideal world.

In sum, the modalized conditional in (2a) above is translated in IP-CDRT as shown in (42) below. Since the contrast between the weak and the strong reading of the

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the use of the modal verb *must* or, alternatively, it is accommodated to satisfy the requirements that this 'deixis' places on its (local) discourse context.

indefinite *a pleasure* is irrelevant for the discourse as a whole[15], I will take the indefinite to have the formally simpler weak reading.

42. *if* ^{p_2} *a* $^{\mathbf{str:}\, u_1}$ *man is alive, he* $_{u_1}$ *must* $^{p_3 \hat{}\, p_2}$ $_{p_1, \mu, \omega}$ *have a* $^{\mathbf{wk:}\, u_2}$ *pleasure*

$$\rightsquigarrow \mathbf{max}^{p_2}(_{p_2}(\mathbf{max}^{u_1}([man_{p_2}\{u_1\}, alive_{p_2}\{u_1\}])));$$

$$[\mu, \omega \,|\, \mathbf{circumstantial}_{p*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}];$$

$$[p_3 \,|\, \mathbf{nec}\,_{p_1, \mu, \omega}(p_2, p_3)];\; _{p_3}([u_2 \,|\, pleasure_{p_3}\{u_2\}, have_{p_3}\{u_1, u_2\}])\;^{16}$$

The IP-CDRT representation in (42) encodes the following sequence of updates: consider all the worlds in which at least one man is alive and consider all the men that are alive in these worlds; store them in p_2 and u_1 respectively. Now consider a circumstantial modal base μ and an empty ordering source ω. Then, every p_2-world that is ideal relative to μ and ω (these ideal worlds are stored in p_3) is such that each of its corresponding u_1-men have some pleasure or other.

4.3. Therefore

Like *must*, the particle *therefore* introduces a necessity quantificational structure, as shown in (43) below. Since *therefore* expresses logical consequence, both its modal base μ^* and its ordering source ω^* are empty.

43. *therefore* $^{p_4 \hat{}\, p_1}$ $_{p*, \mu*, \omega*} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.\, [\mu^*, \omega^* \,|\, \mathbf{empty}\{p^*, \mu^*\}, \mathbf{empty}\{p^*, \omega^*\}];$

$$[p_4 \,|\, \mathbf{nec}\,_{p*, \mu*, \omega*}(p_1, p_4)];\; _{p_4}(\mathbb{P}(p_4))$$

The effect of the update is that the dref p_4 is identical to p_1 both in its value and in its structure, i.e., if J is the output state after processing the **nec** condition in (43) above,

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[15] The weak vs. strong constrast is irrelevant in this case because there is no subsequent anaphora to the indefinite *a pleasure* and both readings yield the same truth-conditions for the discourse as a whole.

[16] The use of the operator $_p(\ldots)$ in the definition of modal quantification builds an existential commitment into its meaning – see the corresponding discussion for individual-level quantification in section **3.4** of chapter **6**. The revised definition of modal quantification in section **6** below will employ the operator $_{\langle p \rangle}(\ldots)$ and solve this problem.

we have that $p_1j=p_4j$ for any 'assignment' $j \in J$. Consequently, p_1 can be freely substituted for p_4 and we can simplify the translation of *therefore* as shown in (44) below [17].

44. *therefore* $^{p_4 \hat{\delta} \, p_1}$ $_{p^*, \mu^*, \omega^*}$ $\rightsquigarrow$ $\lambda \mathbb{P}_{\mathbf{st}}.$ $_{p_1}(\mathbb{P}(p_1))$

I assume that the anaphoric nature of the entailment particle *therefore*, which requires a propositional dref p_1 as the restrictor of its quantification, triggers the accommodation of a *covert* 'content-formation' morpheme *if* ^{p_1} that takes scope over the entire modalized conditional in (2a), i.e. the premise of the Aquinas argument, and stores its content in p_1.

5. Modal Subordination in IP-CDRT

The conditional in (2b) is different from the one in (2a) in three important respects. First, given that (2b) elaborates on the modal quantification in (2a), the modal verb *must* in (2b) is anaphoric to the previously introduced modal base μ (circumstantial) and ordering source ω (empty), so it is translated as shown in (45) below.

45. *must* $^{p_6 \hat{\delta} \, p_5}$ $_{p_1, \mu, \omega}$ $\rightsquigarrow$ $\lambda \mathbb{P}_{\mathbf{st}}.$ $[p_6 \mid \mathbf{nec} \, _{p_1, \mu, \omega} (p_5, p_6)];$ $_{p_6}(\mathbb{P}(p_6))$

Second, the negation in the antecedent of (2b) is translated as in table (45) above, i.e. **not** $\rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}. \, \lambda q_{\mathbf{s}}. \, [\sim \mathbb{P}(q)].$

Finally and most importantly, the modally subordinated antecedent in (2b) is translated in terms of an update requiring the newly introduced dref p_5 to be a *maximal structured* subset of p_2, as shown in (46) below. Thus, modal subordination is capture by establishing a *modal* anaphoric connection that is parallel to the individual-level anaphora between the pronoun *he* in the antecedent of (2b) and the strong donkey indefinite *a man* in the antecedent of (2a).

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[17] See the parallel simplification of the meaning of the generalized quantifier *every* in chapter **6**, section **4.1**, definition (65).

46. $if^{p_5 Ð p_2} \rightsquigarrow \lambda \mathbb{P}_{st}.\ \mathbf{max}^{p_5 Ð p_2}\ (_{p_5}(\mathbb{P}(p_5)))$

The crucial component of the modally subordinated, i.e. modally anaphoric, $if^{p_5 Ð p_2}$ is the maximization operator $\mathbf{max}^{p_5 Ð p_2}$, which is defined in (47) below and which maximizes *both value and structure*. This makes the $\mathbf{max}^{p \in p'}$ operator crucially different from the simpler $\mathbf{max}^p$ operator defined in (19) above and which maximizes only values.

47. $\mathbf{max}^{p \in p'}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H([p \mid p \in p']IH \wedge DHJ\ \wedge$

$$\forall K_{st}([p \mid p \in p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p \neq \#} \subseteq H_{p \neq \#}))$$

We need *structure* maximization over and above value maximization in the analysis of modal subordination because the antecedent of the modalized conditional in (2b) is interpreted as quantifying over *all* the p_2-worlds in which there is at least one u_1-man without spiritual joys and over *all* such u_1-men, i.e. over the maximal structure associated with these p_2-worlds that satisfies the antecedent of (2b).

The effect of the $\mathbf{max}^{p_5 Ð p_2}$ operator is represented by the matrix in (48) below. Note that we can keep in p_5 some of the rows (i.e. 'assignments') associated with a particular possible world and shade (i.e. discard) other rows associated with the same world. This contrasts with the structured inclusion required by dynamic modal relations (see in particular the matrix in (38) above) where, if a row with a given possible world is shaded / discarded, then all the other rows in the matrix with that possible world also have to be shaded / discarded[18].

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[18] Why do we need to use $\mathbf{max}^{p \in p'}(D)$ instead of the simpler $\mathbf{max}^p([p \in p']; D)$? The reason is that the latter has value maximization (due to $\mathbf{max}^p$) and structured inclusion (due to $p \in p'$), but it does not also have *structure maximization*, which we get in (47) by the *info state* inclusion requirement $K_{p \neq \#} \subseteq H_{p \neq \#}$. And, to derive the correct truth-conditions for (15b), we need *structure* (and not only value) maximization: if a man is alive and he doesn't have any spiritual pleasure, he must have a carnal pleasure, i.e. we look at *every p_2-world* and at *every u_1-man* in it that is deprived of spiritual joys, then we select the ideal subset among these worlds and check that *every u_1-man* in each ideal world has a carnal pleasure. Thus, the antecedent of the conditional in (15b) has to introduce all the p_2-worlds where some u_1-man is alive and without spiritual joy and *all the structure* associated with these worlds, i.e., *all the u_1-men* in question, so that we can check in

48. Info state I	p_2 (**premise worlds**)	u_1 (**men**)	p_5 (**conclusion worlds, i.e. modally subordinated worlds**)
i_1	$w_1 \, (=p_2i_1)$	$x_1 \, (=u_1i_1)$	$w_1 \, (=p_3i_1)$

$$\underbrace{\qquad\qquad\qquad\qquad}$$

man x_1 is alive in world w_1

i_2	$w_1 \, (=p_2i_2)$	$x_2 \, (=u_1i_2)$	$w_1 \, (=p_3i_2)$
i_3	$w_1 \, (=p_2i_3)$	$x_3 \, (=u_1i_3)$	$\# \, (=p_3i_3)$
i_4	$w_2 \, (=p_2i_4)$	$x_2 \, (=u_1i_4)$	$w_2 \, (=p_3i_4)$
i_5	$w_2 \, (=p_2i_5)$	$x_4 \, (=u_1i_5)$	$\# \, (=p_3i_5)$
…	…	…	…

In sum, the antecedent of the modalized conditional in (2b) is translated as shown in (49) below. Just as before, the weak vs. strong contrast is otiose with respect to the indefinite *any spiritual pleasure*, so I interpret it as weak.

49. *if* $^{p_5 Ð p_2}$ *he* $_{u_1}$ *doesn't have any* $^{\mathbf{wk:} \, u_3}$ *spiritual pleasure*

$$\rightsquigarrow \mathbf{max}^{\, p_5 Ð p_2} (\, _{p_5}([\sim[u_3 \mid spiritual_{\, p_5}\{u_3\}, pleasure_{\, p_5}\{u_3\}, have_{\, p_5}\{u_1, u_3\}]]))$$

The translation of the consequent of (2b) is parallel to the translation of the consequent of (2a) – hence, the entire modalized conditional in (2b) is translated in IP-CDRT as shown in (50) below. The representation in (50) shows that modal subordination is basically analyzed in IP-CDRT as *quantifier domain restriction via structured modal anaphora*; that is, the antecedent of (2b) is simultaneously anaphoric to the set of worlds and the set of individuals introduced by the the antecedent of (2a) and, also, to the quantificational dependency established between these two sets.

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the consequent that each and every such man has a carnal pleasure. The update $\mathbf{max}^p([p \in p']; D)$ would introduce *all* the relevant p_2-worlds, but only *some* of the relevant u_1-men.

Moreover, the update $\mathbf{max}^p([p \sqsubseteq p']; D)$ would also be inadequate because it would store in p only the worlds in which each and every u_1-man that is alive has no spiritual pleasure, while incorrectly discarding all the worlds in which only some of the u_1-men that are alive have no spiritual pleasure, but not all of them.

50. *if* ^{p_5Ðp_2} *he*$_{u_1}$ *doesn't have any*$^{\mathbf{wk:}\,u_3}$ *spiritual pleasure, he*$_{u_1}$ *must*$^{p_6\hat{o}\,p_5}$ $_{p_1,\mu,\omega}$ *have*

a $^{\mathbf{wk:}\,u_4}$ *carnal pleasure*

⤳ $\mathbf{max}^{\,p_5Ðp_2}(\,_{p_5}([\sim[u_3\mid spiritual\,_{p_5}\{u_3\}, pleasure\,_{p_5}\{u_3\}, have\,_{p_5}\{u_1,u_3\}]]));$

$[p_6\mid\mathbf{nec}\,_{p_1,\mu,\omega}(p_5,p_6)];\;_{p_6}([u_4\mid carnal\,_{p_6}\{u_4\}, pleasure\,_{p_6}\{u_4\}, have\,_{p_6}\{u_1,u_4\}])$

One final observation before providing the IP-CDRT translation of the entire Aquinas discourse: the IP-CDRT analysis of modal subordination requires us to assign two different translations to the antecedent of the conditional in (2a) and the modally subordinated antecedent in (2b). Note, however, that the discourse-initial antecedent in (2a) can also be assigned a translation of the form $\mathbf{max}^{p\in p'}(D)$; since the conditional is discourse initial, the superset dref p' will have to be accommodated and it will be completely unrestricted, i.e. it will store the set of all possible worlds $\boldsymbol{D_w}^M$ [19]. Hence, this more complex translation will ultimately be equivalent to the simpler one in (23) above.

The entire translation of the Aquinas discourse is provided in (51) below. The reader can check that, given the PCDRT definition of truth, which is repeated in (52), we assign the intuitively correct truth-conditions to this discourse. And, according to the translation in (51), the argument does indeed go through: the premise (2a) establishes that the set of ideal worlds among the p_2-worlds is such that any man x has at least one pleasure y. The conclusion follows because in all the ideal p_2-worlds pleasures are spiritual or carnal (just as in the actual world w^*) and any man has at least one pleasure: hence, if a man x has no spiritual pleasure, he must have at least one carnal pleasure y.

51. *If*^{p_2} *a*$^{\mathbf{str:}\,u_1}$ *man is alive, he*$_{u_1}$ *must*$^{p_3\hat{o}\,p_2}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk:}\,u_2}$ *pleasure.*

Therefore$^{p_4\hat{o}\,p_1}$ $_{p^*,\mu^*,\omega^*}$, *if*^{p_5Ðp_2} *he*$_{u_1}$ *doesn't have any*$^{\mathbf{wk:}\,u_3}$ *spiritual pleasure, he*$_{u_1}$

must$^{p_6\hat{o}\,p_5}$ $_{p_1,\mu,\omega}$ *have a*$^{\mathbf{wk:}\,u_4}$ *carnal pleasure*

⤳ $\mathbf{max}^{\,p_1}(\,_{p_1}(\mathbf{max}^{\,p_2}(\,_{p_2}\mathbf{max}^{\,u_1}([man\,_{p_2}\{u_1\}, alive\,_{p_2}\{u_1\}]))));$

$$[\mu, \omega \mid \mathbf{circumstantial}_{p*}\{p_1, \mu\}, \mathbf{empty}\{p_1, \omega\}];$$

$$[p_3 \mid \mathbf{nec}_{p_1}, \mu, \omega(p_2, p_3)];\; _{p_3}([u_2 \mid pleasure_{p_3}\{u_2\}, have_{p_3}\{u_1, u_2\}]);$$

$$_{p_1}(\mathbf{max}^{p_5 \ni p_2}(_{p_5}([\sim[u_3 \mid spiritual_{p_5}\{u_3\}, pleasure_{p_5}\{u_3\}, have_{p_5}\{u_1, u_3\}]])));$$

$$[p_6 \mid \mathbf{nec}_{p_1}, \mu, \omega(p_5, p_6)];\; _{p_6}([u_4 \mid carnal_{p_6}\{u_4\}, pleasure_{p_6}\{u_4\}, have_{p_6}\{u_1, u_4\}])$$

52. **Truth:** A DRS D (type **t**) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

6. *A Parallel Account of Modal and Quantificational Subordination*

In this section, I will slightly revise the analysis of modal quantification proposed in section **4** above and make it parallel to the analysis of individual-level quantification proposed in chapter **6**. The benefit of the revised analysis is that we can give a compositional account of modal subordination examples like the one in (53) below (based on an example in Roberts 1989) that is completely parallel to the analysis proposed in chapter **6** of the quantificational subordination example in (54) below (from Karttunen 1976).

53. **a.** A^u wolf might come in. **b.** It$_u$ would attack Harvey first.

54. **a.** Harvey courts a^u girl at every convention.

   b. She$_u$ always comes to the banquet with him.

   [**c.** The$_u$ girl is usually also very pretty.]

Under its most salient interpretation, discourse (53) asserts that, for all the speaker knows, it is a possible that a wolf comes in. Moreover, for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first.

The modal subordination discourse in (53) is parallel to the quantificational subordination discourse in (54) because the interaction between the indefinite *a^u wolf* and the modal *might* on the one hand and the singular pronoun *it$_u$* and the modal *would* on the other hand is parallel to the interaction between *a^u girl-every convention* and *she$_u$-always*.

6.1. Redefining Modal Quantification

This section introduces the main definitions and abbreviations needed for the revised definition of dynamic modal quantification. They are point-for-point parallel to the definitions given in chapter **6** for individual-level quantification (see appendix **0** of chapter **6**).

As already indicated in section **4.2** above, we need a notion of structured inclusion to define dynamic modal quantification and we need to introduce a dummy / exception world $\#_{\mathbf{w}}$ to be able to define structured inclusion. The dummy world $\#_{\mathbf{w}}$ makes every lexical relation false, much like the dummy / exception individual $\#_e$ introduced in chapter **6** yields falsity.

Just as before, the new definition of intensional atomic conditions – provided in (55) below – relies on static lexical relations $R_w(x_1, \ldots, x_n)$ of the expected intensional type $e^n(\mathbf{w}t)^{20}$. The definition in (55), however, is different from the corresponding definition of lexical relations in section **3** because now we also have to take into account the dummy individual $\#_e$ over and above the dummy world $\#_{\mathbf{w}}$ (since the intensional system introduced in this section builds on the extended PCDRT system in chapter 6, which makes use of the dummy / exception individual $\#_e$).

The definitions in (56) through (61) are identical to the corresponding definitions introduced in section **4** above and they are repeated here only to make the comparison with the individual-level definitions in chapter **6** easier.

55. $R_p\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{\,p \,\neq\, \#,\, u_1 \,\neq\, \#,\, \ldots,\, u_n \,\neq\, \#} \neq \emptyset \ \wedge$

$$\forall i_s \in I_{\,p \,\neq\, \#,\, u_1 \,\neq\, \#,\, \ldots,\, u_n \,\neq\, \#}(R_{pi}(u_1 i, \ldots, u_n i))$$

56. $[p] := \lambda I_{st} J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[p]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[p]j)))$

57. $p' \Subset p := \lambda I_{st}.\ \forall i_s \in I(p'i = pi \vee p'i = \#)$

58. $p' \sqsubseteq p := \lambda I_{st}.\ (p' \Subset p)I \wedge \forall i_s \in I(pi \in p'I_{p' \neq \#} \to pi = p'i)$

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[20] Where $e^n \tau$ (for any type τ) is defined as in Muskens (1996): 157-158, i.e. $e^0 \tau := \tau$ and $e^{m+1} \tau := e(e^m \tau)$.

59. $\mathbf{max}^p(D) := \lambda I_{st}.\lambda J_{st}.\ ([p]; D)IJ \wedge \forall K_{st}(([p]; D)IK \to pK_{p\neq\#}\subseteq pJ_{p\neq\#})$

60. $\mathbf{max}^{p'\sqsubseteq p}(D) := \mathbf{max}^{p'}([p'\sqsubseteq p]; D)$

61. $\mathbf{dist}_p(D) := \lambda I_{st}.\lambda J_{st}.\ \forall w_\mathbf{w}(I_{p=w}\neq\emptyset \leftrightarrow J_{p=w}\neq\emptyset) \wedge \forall w_\mathbf{w}(I_{p=w}\neq\emptyset \to DI_{p=w}J_{p=w})$,

$\quad\quad$ i.e. $\mathbf{dist}_p(D) := \lambda I_{st}J_{st}.\ pI=pJ \wedge \forall w_\mathbf{w}\in pI(DI_{p=w}J_{p=w})$.

The most important novelties introduced in this section are the definition of modal quantification and the definition of the indicative mood in (68) and (69) below.

Just as the generalized determiners in chapter **6** above relate dynamic properties P, P' etc. of type **et**, modal verbs relate dynamic propositions $\mathbb{P}$, $\mathbb{P}'$ etc. of type **st**, as shown in (68).

Moreover, just as a singular pronoun anaphorically retrieves an individual dref, requires it to be unique and makes sure that a dynamic property holds of that dref (see the translation of *he_u* in chapter **6**), the indicative mood anaphorically retrieves $p*$, which is the designated dref for the actual world, requires it to be unique (since there is a unique actual world) and makes sure that a dynamic proposition holds of $p*$, as shown in (69).

62. $_p(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$

63. $_{\langle p\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge (I_{p\neq\#}=\emptyset \to I=J) \wedge (I_{p\neq\#}\neq\emptyset \to \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#})$

64. $\mathbf{unique}\{p\} := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall i_s,i'_s\in I_{p\neq\#}(pi=pi')$

65. $\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I \wedge$

$\quad\quad\quad\quad \mathbf{MODAL}_{\mu I\neq\{\#\}, \omega I\neq\{\#\}}\{pI_{p\neq\#}, p'I_{p'\neq\#}\}$,

$\quad\quad$ where μ and ω (dref's for a modal base and an ordering source

$\quad\quad$ respectively) are of type $s(\mathbf{w}t)^{21}$.

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[21] Note that the first two conjuncts in (65), i.e. $I_{q=\#}=\emptyset \wedge \mathbf{unique}\{q\}I$, entail that qI is a singleton set $\{w\}$, where w cannot be the dummy world $\#_\mathbf{w}$.

The third conjunct in (65) is of the form $\mathbf{MODAL}_{M,O}\{W, W'\}$, where w is a possible world (of type $\mathbf{w}$), M and O are sets of sets of worlds (of type $(\mathbf{w}t)t$), i.e. a modal base and ordering source respectively, and W and W' are sets of possible worlds (of type $\mathbf{w}t$), i.e. the restrictor and the nuclear scope of the modal quantification. The formula $\mathbf{MODAL}_{M,O}\{W, W'\}$ is defined following the Lewis (1973) / Kratzer (1981) semantics for modal quantification (see section section **4.2** above for more details).

66. **Example – the necessity condition (type** $(st)t$**):**

$\textbf{NEC}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \textbf{unique}\{q\}I \wedge \textbf{NEC}_{\mu I\neq\{\#\},\omega I\neq\{\#\}}\{pI_{p\neq\#},\ p'I_{p'\neq\#}\},$

$\qquad$ where $\textbf{NEC}_{M,O}(W_1, W_2) := \textbf{Ideal}_O((\cap M)\cap W_1)\subseteq W_2$ [22]

67. **Example – the possibility condition (type** $(st)t$**):**

$\textbf{POS}_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#}=\emptyset \wedge \textbf{unique}\{q\}I \wedge \textbf{POS}_{\mu I\neq\{\#\},\omega I\neq\{\#\}}\{pI_{p\neq\#},\ p'I_{p'\neq\#}\},$

$\qquad$ where $\textbf{POS}_{M,O}(W_1, W_2) := \textbf{Ideal}_O((\cap M)\cap W_1)\cap W_2\neq\emptyset$

68. $if^p + modal_{\mu,\omega}{}^{p'\sqsubseteq p}\ \rightsquigarrow$

$\qquad \lambda\mathbb{P}_{\textbf{st}}.\lambda\mathbb{P}'_{\textbf{st}}.\lambda q_{\textbf{s}}.\ \textbf{max}^p(_{\langle p\rangle}(\mathbb{P}(p)));\ \textbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\textbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

69. $\textbf{indicative}_{p^*}\ \rightsquigarrow \lambda\mathbb{P}_{\textbf{st}}.\ [\textbf{unique}\{p^*\}];\ _{p^*}(\mathbb{P}(p^*)),$

$\qquad$ where p^* is the dref for the actual world.

Note that the definition in (68) can be easily modified to allow for the kind of modal quantification instantiated by the second conditional in our Aquinas discourse (i.e. by the conditional in (2b) above). As shown in (70) below, we only need to make use of the maximization operator $\textbf{max}^{p\in p'}(D)$ introduced in section **5** above, whose definition is repeated in (71) for convenience.

70. $if^{p\in p''} + modal_{\mu,\omega}{}^{p'\sqsubseteq p}\ \rightsquigarrow$

$\qquad \lambda\mathbb{P}_{\textbf{st}}.\lambda\mathbb{P}'_{\textbf{st}}.\lambda q_{\textbf{s}}.\ \textbf{max}^{p\in p''}(_{\langle p\rangle}(\mathbb{P}(p)));\ \textbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\textbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

71. $\textbf{max}^{p\in p'}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H([p \mid p\in p']IH \wedge DHJ \wedge$

$\qquad\qquad \forall K_{st}([p \mid p\in p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p\neq\#}\subseteq H_{p\neq\#}))$

The most important difference between the definition of modal quantification in (68) and the definition in section **4** above is that we now introduce the *maximal* nuclear scope set of worlds *unrestricted / not parametrized by a modal base or an ordering source*. The modal parametrization comes in only later on, in the modal condition relating

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[22] The definitions of $\textbf{NEC}_{M,O}(W_1,\ W_2)$ and $\textbf{POS}_{M,O}(W_1,\ W_2)$ differ slightly from the corresponding definitions in section **4.2** above, but they still rely on the **Ideal** function defined in that section.

the unparametrized maximal restrictor set and the unparametrized maximal nuclear scope set.

In contrast, the definition in section **4** introduces only the maximal restrictor set of worlds and, if the modal relation is necessity (**NEC**), it also introduces the maximal set of ideal worlds among the restrictor worlds. That is, the old definition introduces a maximal nuclear scope set only in some cases and, even then, it is a *parametrized* nuclear scope set (parametrized by a modal base and an ordering source).

Thus, what distinguishes the definition of dynamic modal quantification in (68) from the previous one – and, to my knowledge, from any other analysis of modal quantification in the previous dynamic literature[23] – is that: (**i**) it introduces maximal restrictor and nuclear scope sets and (**ii**) these maximal sets are unparametrized by modal bases or ordering sources. As we will see in the next section, the new definition has several theoretical and empirical advantages over the definition in section **4** above and the definitions in the previous dynamic semantics literature.

6.2. Advantages of the Novel Definition

The novel definition of modal quantification, which introduces the maximal unparametrized (i.e. not restricted by any modal base or ordering source) nuclear scope set of worlds over and above the maximal restrictor set of worlds, has several advantages over the definition in section **4** above (which introduces only the maximal restrictor set) – and over various other definitions proposed in the previous dynamic semantics literature.

For ease of comparison, I will restate the old definition of dynamic modal quantification using the new format (i.e. the format of the definition in (68) above), as shown in (72) below.

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[23] Most previous analyses of modal quantification differ from the new IP-CDRT analysis because either they did not have any modal dref's at all (Roberts 1987, 1989) or, if they had, the dref's had dynamic objects as values, e.g. <world, variable assignment> pairs (see, for example, Geurts 1995/1999 and Frank 1996 among others). Stone (1999) does propose an analysis of modal quantification that relates dref's for static objects (in particular, dref's for accessibility relations of type $s(\mathbf{w}(\mathbf{w}t))$, but his restrictor and nuclear scope sets, which are introduced by means of an *if*-update (see Stone 1999: 17, (34)), are parametrized – their maximality is relativized to a Lewis-style similarity ordering source built into the *if*-update.

72. **The previous definition of modal quantification** (see section **4** above)**.**

$if^p + modal_{\mu,\omega}{}^{p' \sqsubseteq p} \rightsquigarrow$

$$\lambda \mathbb{P}_{st}.\lambda \mathbb{P}'_{st}.\lambda q_s.\ \mathbf{max}^P(_{\langle p \rangle}(\mathbb{P}(p)));\ [p' \mid p' \sqsubseteq p, \mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}];\ _{\langle p' \rangle}(\mathbb{P}'(p'))$$

The definition in (72) is formally simpler than the one in (68) because it has only one maximization operator. But the additional complexity of (68) is both theoretically and empirically motivated.

The theoretical advantage of the new definition in (68) over the previous definition in (72) is that the new definition systematically and explicitly captures the parallel between modal quantification and individual-level quantification as analyzed in chapter **6**. For convenience, I repeat the definition of individual-level quantification in (73) below. The reader can easily check that it is point-for-point parallel to the definition in (68) above.

73. $det^{u,u' \sqsubseteq u} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}.\ \mathbf{max}^u(_{\langle u \rangle}(P(u)));\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

Empirically, the new definition is better than the previous one in at least two respects. As we will see, these two empirical advantages are a direct consequence of the parallel between the dynamic definition of modal quantification and its individual-level counterpart in (73) above. Let us examine them in turn.

First, the new definition generalizes to downward monotonic modal quantifiers (i.e. to modal determiners / modal relations that are downward monotonic in their right argument) like *impossible*, *improbable*, *unlikely* etc. To see this, note that, just as the individual-level quantification in (74) below is incompatible with (75) (i.e. *Few men left* entails that *It is not the case that most men left*), the modal quantification in (76) is incompatible with (77) (i.e. *Given the available evidence, it is improbable / unlikely that it will rain* entails that *It is not the case that, given the available evidence, it is probable / likely that it will rain*).

74. Few men left.
75. Most men left.
76. Given the available evidence, it is improbable / unlikely that it will rain.

77. Given the available evidence, it is probable / likely that it will rain.

This shows that, when computing the meaning of updates containing (individual-level or modal) determiners that are downward monotonic in their right argument, we need to have access to the maximal nuclear scope set (of individuals or possible worlds). To see this, consider the definition of dynamic individual-level quantification in (78) below, which does not introduce the maximal nuclear scope set and which is parallel to the old definition of modal quantification in (72) above.

78. **A definition of individual-level quantification that fails for determiners that are downward monotonic in their right argument:**

$$det^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\textbf{et}}.\lambda P'_{\textbf{et}}.\ \textbf{max}^u(\langle u \rangle(P(u)));\ [u' \mid u' \sqsubseteq u,\ \textbf{DET}\{u,\ u'\}];\ \langle u \rangle(P'(u'))$$

The definition in (78) captures the meaning of upward monotonic quantifiers, e.g. *Most men left* is correctly interpreted as: introduce the maximal set *u* of individuals that satisfies the restrictor dynamic property, i.e. the maximal set of men; then, nondeterministically introduce some subset *u'* of the restrictor set *u* that is a *most*-subset (i.e. it is more than half of the restrictor set). If there is at least one such non-deterministically introduced subset *u'* that also satisfies the nuclear scope dynamic property, then the *most*-quantification is successful.

However, the definition in (78) fails to capture the meaning of downward monotonic quantifiers, e.g. *Few men left* is incorrectly interpreted as: introduce the maximal set *u* of individuals that satisfies the restrictor dynamic property, i.e. the maximal set of men; then, nondeterministically introduce some subset *u'* of the restrictor set *u* that is a *few*-subset (i.e. it is less than half of the restrictor set, possibly the empty set). If there is at least one such non-deterministically introduced subset *u'* that also satisfies the nuclear scope dynamic property (let us assume that the empty set vacuously satisfies any property), then the *few*-quantification is successful.

This meaning for *few* fails to capture the fact that *Few men left* is incompatible with *Most men left* because, even if we are successful in introducing a *few*-subset that satisfies the nuclear scope property, it can still be the case that a *most*-subset, for example, also satisfies that property, i.e. a successful update with *Few men left* does not rule out a

successful update with *Most men left* (this is a direct consequence of the proposition relating witness sets and quantifier monotonicity in Barwise & Coopeer 1981: 104[24]; for a closely related discussion, see fn. **15** in section **3.3** of chapter **6**).

And, just as the definition of individual-level quantification in (78) above fails to account for the fact that *Few men left* is incompatible with *Most men left*, the parallel definition of modal quantification in (72) above fails (*mutatis mutandis*) to account for the fact that *Given the available evidence, it is improbable that it will rain* is incompatible with *Given the available evidence, it is probable that it will rain*.

The second advantage of the new definition of dynamic modal quantification over the previous one is that we predict without any additional stipulation that anaphora to the nuclear scope set of a modal quantification is always maximal – which is exactly what we need to account for the standard case of modal subordination in (53) above (i.e. *A^u wolf might come in. It_u would attack Harvey first*) and, also, for the more complex example involving interactions between *therefore* and modal subordination in (9) above (i.e. *A^u wolf might enter the cabin. It_u would see John^{u'}. Therefore, it_u would notice him_{u'}*).

In more detail: recall that, under its most salient interpretation, discourse (53) is interpretating as asserting that: (**i**) for all the speaker knows, it is a possible that a wolf comes in, and, in addition, (**ii**) for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first. That is, the modal *would* is anaphoric to all the epistemically accessible worlds in which a wolf comes in and not only to some of them.

However, according to the old definition, the modal verb *might* introduces only some (and not necessarily all) the epistemically accessible worlds in which a wolf comes in. Consequently, we would need an additional stipulation to the effect that, at least in discourse (53), *might* introduces the maximal set of epistemically accessible worlds satisfying the nuclear scope.

I can think of two ways of justifying the additional maximality stipulation associated with anaphora to *might* in discourse (53), namely: (**i**) modal anaphora is

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[24] Page references to Partee & Portner (2002).

parallel to donkey anaphora and, in discourse (53), we have an instance of strong donkey-like modal anaphora and (**ii**) modal anaphora is parallel to plural anaphora and plural anaphora is always maximal.

However, as we will presently see, these two justifications do not hold under scrutiny. In contrast, the fact that the novel definition of modal quantification in (68) introduces the maximal unparametrized nuclear scope set of worlds is independently motivated by the need to capture the meaning of downward monotonic modal quantifiers.

Moreover, this *explanation* for the maximality of modal anaphora – i.e. the fact that the maximality of modal anaphora (analyzed as structured anaphora to quantifier domains) emerges as a consequence of independently justified meanings for dynamic generalized quantifiers – is parallel to the explanation provided in section **3.3** of chapter **6** above for the maximality of E-type anaphora in the invidual domain (recall the Evans examples *Few^u congressmen admire Kennedy and they$_u$ are very junior* and *Harry bought some^u sheep. Bill vaccinated them$_u$*).

Let us examine the first suggestion above, namely the idea that modal anaphora is, in general, parallel to donkey anaphora (and not parallel to E-type anaphora to quantifier domains) and that discourse (53) is basically an instance of strong donkey anaphora in the modal domain.

This hypothesis derives the intuitively correct truth-conditions for discourse (53) since the modal *might* in (53a) has a strong donkey reading and, therefore, introduces the *maximal* set of epistemically accessible possible worlds in which a wolf comes in (see the PCDRT analysis of weak / strong donkey ambiguities in chapter **5**). The modal anaphor *would* in (53b) will then retrieve this maximal set of worlds and further elaborate on them, much like the anaphor *it$_u$* in *Every farmer who owns a$^{str:u}$ donkey beats it$_u$* retrieves all the donkeys owned by any particular farmer.

The problem with this hypothesis is that we expect to find instances of modal anaphora that have weak donkey-like readings – and I am not aware of any examples of this kind. All the examples of cross-sentential modal anaphora to *might* of the same form as discourse (53) above seem to require maximality – and the same maximality

requirement seem to be obligatory in cases in which *might* occurs embedded in conditional structures. Consider, for example, the conditional in (79) below, where the (putatively donkey-like) modal *might* occurs in the antecedent of a conditional and the purpose infinitival clause *to kill it* in the consequent is (presumably) anaphoric to the epistemically accessible possible worlds introduced by *might*.

79. If you think a rat might come in, you should bring some poison to kill it. [25]
80. If you think a rat might come in, then you should bring some poison so that:

    if a rat does come in, you'll have a way to kill it / #you might have a way to kill it.

As the intuitively correct paraphrase in (80) above shows, the modal anaphora does not have a weak reading: the infinitival clause is anaphoric to all the worlds in which a rat comes in and not only to some of the (epistemically accessible) worlds in which a rat comes in.

The second suggestion made above is that modal anaphora is parallel to plural anaphora and, given that plural anaphora is always maximal, this explains why modal anaphora to *might* is always maximal. Much like the previous hypothesis, this one also derives the intuitively correct truth-conditions for discourse (53). But it ultimately faces the same problems as the "modal anaphora as donkey anaphora" idea – and this is because plural anaphora is in fact not always maximal / strong.

Plural donkey anaphora to *some* does indeed seem to always be maximal / strong, both in cross-sentential cases (the Evans example: *Harry bought some^u sheep. Bill vaccinated them$_u$[26]*) and in the case of intra-sentential plural donkey anaphora – see for example (81) below.

81. Every person who has some^u dimes will put them$_u$ in the meter.

However, the maximality effect in all these cases seems to be due to the determiner *some*, because plural anaphora to cardinal indefinites can very well be non-maximal /

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[25] This example incorporates several modifications suggested to me by Jessica Rett (p.c.).

[26] Based on Evans (1980): 217, (8) (page references are to Evans 1985).

weak, as shown by the cross-sentential example and the donkey sentence in (82) and (83) below.

82. Harry bought two^u sheep and Bill vaccinated them$_u$.

But Bill didn't vaccinate all the sheep that Harry bought on the same occasion / But Bill didn't vaccinate the three other sheep that Harry bought on the same occasion.

83. Every person who has two^u dimes will put them$_u$ in the meter.

Thus, the idea that modal anaphora is parallel to individual-level plural anaphora is problematic for the same reason as the "modal anaphora as donkey anaphora" hypothesis, because there seem to be no non-maximal instances of modal anaphora to *might* – which is exactly what we would expect under the "modal anaphora as anaphora to quantifier domains" view pursued throughout this section.

6.3. Conditional Antecedents vs. Modal Bases

As (68) indicates, I take modal generalized determiners to have a composite, conditional-like structure. The observation that antecedents of conditionals contribute to the restrictor, i.e. the modal base, of a modal quantification goes back at least to Kratzer (1981). A typical example (which, incidentally, provides an argument for ordering sources over an above modal bases) is given in (84) below.

84. If^p there is a$^{\mathbf{str}:u}$ murder, the$_u$ murderer must$_{\mu,\omega}^{p' \sqsubseteq p}$ go to jail.

The modalized conditional in (84) is interpreted as a modal quantification relativized to a contextually provided empty modal base μ and a contextually provided deontic ordering source ω (e.g. *in view of the law in the actual world*).

The antecedent of the conditional contributes the set p of all worlds where there is some murder or other. The modalized conditional is true if the consequent of the conditional is satisfied in all the deontically ideal worlds among the p-worlds intersected with the modal base worlds; since, in this case, the modal base is empty (hence it is

vacuously satisfied in any possible world), the restrictor of the quantification is just the set of *p*-worlds.

However, despite the fact that antecedents and modal bases should be lumped together in the evaluation of a modal quantification (as Kratzer 1981 has it), they should in fact be distinguished for anaphoric purposes: as example (84) shows, we can have donkey anaphora between the definite *the$_u$ murderer* in the consequent and the indefinite *a^u murder* in the antecedent. This is the reason for the systematic distinction between the conditional antecedent (i.e. the restrictor *stricto sensu*) and the modal base in the definition of dynamic modal quantification in (68) above.

The necessity to distinguish between conditional antecedents and modal bases is further supported by the discourses in (85) and (86) below (based on examples (7) and (10) in Stone 1999: 4-5), where we have instances of cross-sentential (modally subordinated) anaphora to dref's introduced in antecedents of conditionals[27].

85. **a.** If a^u wolf came in, John could escape (from it$_u$). **b.** It$_u$ might eat Mary though.

86. **a.** If a^u wolf came in, John could not legally kill it$_u$. **b.** But he still would have to.

6.4. Anaphoric Modal Quantifiers

Finally, just as quantifiers like *always* in (54b) above anaphorically retrieves its restrictor (more exactly: it is anaphoric to the nuclear scope dref introduced by the determiner *every* in (54a)), the modal quantifier *would* in (53b) anaphorically retrieves its restrictor – and, in a parallel way, *would* in (53b) is anaphoric to the nuclear scope dref introduced by the modal verb *might* in (53a). The general format for the translation of such anaphoric modal quantifiers is provided in (87) below.

87. $modal_{\mu,\omega}p^{p'\sqsubseteq p} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}(\mathbb{P}(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

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[27] Unlike the Aquinas discourse in (1/2) above, where the modally subordinated pronoun is located in the restrictor of the modal quantification, the modally subordinated pronoun in discourses (85) and (86) is located in the nuclear scope.

This concludes our brief survey of the version of Intensional PCDRT (IP-CDRT) that builds on the extended PCDRT system introduced in chapter **6**[28].

6.5. Subordination across Domains

We finally turn to the IP-CDRT analysis of the modal subordination discourse in (53). As desired, this analysis is the exact modal counterpart of the analysis of quantificational subordination in section 4 of chapter **6**.

Under its most salient interpretation, discourse (53) asserts that, for all the speaker knows, it is a possible that a wolf comes in and that, for *any* such epistemic possibility of a wolf coming in, the wolf attacks Harvey first. Thus, we are interested in the "narrow-scope indefinite" reading of discourse (53), wherein the indefinite *a wolf* in (53a) has narrow scope relative to the modal *might* and sentence (53b) preserves and elaborates on this *de dicto* reading.

The meanings for the two modal quantifiers *might* in (53a) and *would* in (53b) are provided in (88) and (89) below. Given that the modal relation **POS** contributed by *might* has a built-in existential commitment, i.e. there must be a non-empty restrictor set of worlds p of a non-empty nuclear scope set of worlds p' (see the definition in (67) above), we can simplify the meaning of *might* by replacing the operators $_{\langle p \rangle}(\ldots)$ and $_{\langle p' \rangle}(\ldots)$ with $_p(\ldots)$ and $_{p'}(\ldots)$. The same applies to the meaning of anaphoric *would* because, according to definition (66) above, if the restrictor set of *would* (i.e. p') is non-empty, then so must be its nuclear scope set p'' (given that *would* is parametrized by the same modal base as *might*).

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[28] For completeness, I provide below the revised intensional meanings for dynamic properties, generalized determiners, indefinite articles, pronouns and proper names.

(**i**) *girl* $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [girl_q\{v\}]$

(**ii**) $\mathbf{det}^{u, u' \sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^u(_{\langle u \rangle}(P(u)(q)));\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')(q)));\ [\mathbf{DET}\{u, u'\}]$

(**iii**) $a^{\mathsf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u];\ _u(P(u)(q));\ _u(P'(u)(q))$

(**iv**) $a^{\mathsf{str}:u} \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^u(_u(P(u)(q));\ _u(P'(u)(q)))$

(**v**) $he_u \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\mathbf{unique}\{u\}];\ _u(P(u)(q))$

(**vi**) *Harvey*$^u \rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u \mid u \in Harvey];\ _u(P(u)(q)),$ where $Harvey := \lambda i_s.\ harvey_e.$

88. $might_{\mu,\omega}{}^{p,p'\sqsubseteq p} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^p({}_p(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{p'}(\mathbb{P}'(p')));\ [\mathbf{POS}_{q,\mu,\omega}\{p,p'\}]$

89. $would_{\mu,\omega,p'}{}^{p''\sqsubseteq p'} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p''\sqsubseteq p'}({}_{p''}(\mathbb{P}(p'')));\ [\mathbf{NEC}_{q,\mu,\omega}\{p',p''\}]$

The contextually supplied modal base μ (of type $s(\mathbf{w}t)$) for both *might* and *would* is *epistemic*, e.g. it associates with each q-world the set of propositions that the speaker believes in that q-world. The contextually supplied ordering source ω is empty[29], which means that it does not contribute anything to the meaning of the two modal quantifications – and we will henceforth ignore it.

Given that the *might* quantification is discourse initial, we have to accommodate a restrictor proposition $\mathbb{P}_{\mathbf{st}}$ – and a natural choice is the trivial dynamic proposition $\lambda q_{\mathbf{s}}.$ $[q \in q]$. This ensures that the restrictor dref p introduced by *might* stores the set of all possible worlds (since the restrictor DRS is $\mathbf{max}^p({}_p([p \in p]))$), which in turn entails that we quantify over each and every world compatible with the epistemic modal base μ – and this is intuitively correct: when uttered out of the blue, sentence (53a) is interpreted as asserting that, for all the speaker knows, it is possible that a wolf comes in.

Finally, both modal quantifications in (53a) and (53b) are interpreted relative to the actual world, since the epistemic modal base μ for both quantifications is in fact the set propositions believed by the actual speaker in the actual world. I will capture this means of an indicative mood morpheme taking scope over the modal verbs. Thus, I will assume that sentences (53a) and (53b) have the logical forms provided in (90) and (91) below; the logical forms are followed by their compositionally derived IP-CDRT translations[30].

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[29] Emptiness can be required by a condition of the form $\mathbf{empty}\{\omega\} := \lambda I_{st}.\ \forall i_s \in I(\omega i = \{\#\})$, i.e., throughout the plural info state I, we assign to the dref ω of type $s(\mathbf{w}t)$ the dummy object of type $\mathbf{w}t$, which is the singleton set of the dummy world $\{\#_{\mathbf{w}}\}$.

[30] I employ the notational abbreviations and equivalences introduced in chapter **6** above (see appendix **0** of chapter **6** for the entire list), in particular:

(i) ${}_p(C) := \lambda I_{st}.\ I_{p\neq\#} \neq \emptyset \wedge \forall w \in pI_{p\neq\#}(CI_{p=w})$,   where C is a condition (of type $(st)t$)

(ii) ${}_p(\alpha_1, ..., \alpha_n) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#} = J_{p=\#} \wedge I_{p\neq\#}[\alpha_1, ..., \alpha_n]J_{p\neq\#}$,
      where $p \notin \{\alpha_1, ..., \alpha_n\}$ and $[\alpha_1, ..., \alpha_n] := [\alpha_1]; ...; [\alpha_n]$

(iii) ${}_p([C_1, ..., C_m]) = [{}_p(C_1), ..., {}_p(C_m)]$

90. **ind**$_{p^*}$(*might*$_\mu^{p,p'\sqsubseteq p}$ ($\lambda q_\mathbf{s}.[q \in q]$) ($a^{\mathbf{wk}:u}$ *wolf come in*))

 [**unique**$\{p^*\}$]; $_{p^*}$(**max**^{p}($_p$([$p \in p$])); **max**$^{p'\sqsubseteq p}$([$_{p'}(u) \mid _p(wolf_{p'}\{u\}), _p(come\_in_{p'}\{u\})$]));

   [**POS**$_{p^*,\mu}\{p, p'\}$])

91. **ind**$_{p^*}$(*would*$_{\mu,p'}^{p''\sqsubseteq p'}$ (*it*$_u$ *attack Harvey first*))

 [**unique**$\{p^*\}$]; $_{p^*}$(**max**$^{p''\sqsubseteq p'}$([$_{p''}$(**unique**$\{u\}$), $_{p''}(attack_{p''}\{u, Harvey\})$]));

   [**NEC**$_{p^*,\mu}\{p', p''\}$])

Intuitively, the DRS in (90) instructs us to update the default info state $\{i_\#\}$ as follows. First, since the entire modal quantification is relativized to the actual world and the epistemic modal base provided by the speaker's beliefs, we need to introduce the actual world dref p^* and the epistemic modal base dref μ. These updates are default start-up updates for any discourse whatsoever, i.e. they are what Stalnaker (1978) refers to as "commonplace" updates that "will include any information which the speaker assumes his audience can infer from the performance of the [assertion] speech act" (Stalnaker 1978; see also the related discussion about deictic pronouns in section **3.7** of chapter **6** above). Thus, I will assume that the DRS in (90) is in fact preceded by the start-up update in (92) below.

92. [$p^*, \mu \mid p^*{=}w^*$, **epistemic**$_{p^*}\{\mu\}$],

   where $p^*{=}w^* := \lambda I_{st}. \, p^*I_{p^*\neq\#}{=}\{w^*\}$

We are now able to test that the dref p^* contains a unique non-dummy world (in particular, the actual world w^*), as the first update in (90) instructs us to do.

Then, we introduce the dref p relative to the actual world dref p^* and store in it the set of all possible worlds (given that the condition $p \in p$ is vacuously satisfied). The next update instructs us to introduce the dref p' and store in it the maximal subset of p-worlds

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(iv) $_p$([$\alpha_1, \ldots, \alpha_n \mid C_1, \ldots, C_m$]) = [$_p(\alpha_1, \ldots, \alpha_n) \mid _p(C_1), \ldots, _p(C_m)$]).

that contain a wolf *u* that comes in. Given that *p* is the set of all possible worlds, *p'* will in fact store the set of all worlds that contain a wolf *u* that comes in.

We finally test that the nuclear scope *p'* is a μ-epistemic possibility relative to the restrictor *p*, which basically means that there is at least one possible world *w* which is both a *p*-world and a μ-world and which, in addition, is also a *p'*-world. In other words, the DRS in (90) is true iff there is an epistemic possibility of a wolf coming in.

The DRS in (91) instructs us to update the info state that we have obtained after processing (90) as follows. First, we test again that the dref *p** contains a unique non-dummy world, which we know is true (we have performed the same test in (90)). Then, we introduce the nuclear scope dref *p''*, which stores the maximal subset of the *p'*-worlds relative to which we have introduced a unique *u*-wolf and in which said wolf attacks Harvey first.

We finally test that the nuclear scope *p''* is a μ-epistemic necessity relative to the restrictor *p'*, which basically means that any possible world *w* which is both a *p'*-world and a μ-world is also a *p''*-world. In other words, the DRS in (91) is true iff *any* epistemically accessible possible world in which a wolf comes in is such that the wolf attacks Harvey first.

Thus, the IP-CDRT representation in (90) + (91) captures the intuitively correct truth-conditions for the modal subordination discourse in (53) above. Moreover, as desired, the representation in (90) + (91) is parallel to the corresponding PCDRT representation that captures the "narrow-scope indefinite" reading of the quantificational subordination discourse in (54) above. For convenience, I repeat this representation in (93) below (see **4.3** section of chapter **6** for more discusssion).

93. $\mathbf{max}^{u_1}([\textit{convention}\{u_1\}]); [_{u_1}(u_2)]; [_{u_1}(\textit{girl}\{u_2\}), _{u_1}(\textit{court\_at}\{\textit{Harvey}, u_2, u_1\})];$

  $[_{u_1}(\mathbf{unique}\{u_2\}), _{u_1}(\textit{come\_to\_banquet\_of}\{u_2, u_1\})]$

The differences between the two representations are only an artifact of the fact that, in the analysis provided in (93), we have conflated the restrictor and nuclear scope dref's for both the determiner *every* in (54a) and the anaphoric adverb *always* in (54b). This

simplification is, however, not possible for the modal representation in (90) + (91) because, unlike individual-level quantification, modal quantification is parametrized by a non-empty modal base and the restrictor and nuclear scope dref's of *might* and *would* cannot be conflated. The conflation is possible only of both the modal base and the ordering source are empty, as it was the case for the entailment particle *therefore* analyzed in section **4.3** of the present chapter and whose translation was simplified in much the same way as the translations of *every* and *always* in chapter **6**.

Thus, anaphora and quantification in the individual and modal domains are analyzed in a systematically parallel way in IP-CDRT, from the types of the dref's to the general format of the meanings associated with quantificational and anaphoric expressions. The fact that this formal feature is empirically and theoretically desirable has been repeatedly observed in the literature – see Stone (1997, 1999), Frank (1996), Geurts (1999), Bittner (2001), Schlenker (2005) among others, extending the parallel between the individual and temporal domains argued for in Partee (1973, 1984).

IP-CDRT – which builds on and unifies Muskens (1996), van den Berg (1996a) and Stone (1999) – is, to my knowledge, the first dynamic system that systematically captures the anaphoric and quantificational parallels between the individual and modal domains while, at the same time, keeping the underlying logic classical and preserving the Montagovian approach to compositionality.

6.6. *De Re* Readings

Consider the discourses in (94) and (95) below. In both cases, the only intuitively available reading for the indefinite a^u *wolf* is a *de re* reading, that is, the anaphoric pronoun it_u in the indicative sentences (94b)/(95b) rules out the "narrow-scope indefinite" reading $might_\mu^{p,p'\sqsubseteq p} >> a^u$ *wolf* for sentence (94a/95a).

94. **a.** $A^{\mathbf{wk}:u}$ wolf $might_\mu^{p,p'\sqsubseteq p}$ come in.

   b. It_u escaped yesterday from the zoo.[31]

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[31] Or: *A wolf might come in. It's the wolf that escaped yesterday from the zoo.*

95. **a.** A$^{\textbf{wk:}u}$ wolf might$_{\mu}^{p,p'\sqsubseteq p}$ come in.

    b. John saw it$_u$ yesterday night standing dangerously close to the cabin.

Discourses (94) and (95) are parallel to the first discourse from Karttunen (1976) analyzed in chapter **6**, repeated in (96) below. Much as in (94) and (95) above, the anaphoric pronoun *she$_u$* in sentence (96b) rules out the "narrow-scope indefinite" reading *every$^{u'}$ convention>>a$^{\textbf{wk:}u}$ girl* for sentence (96a).

96. **a.** Harvey courts a$^{\textbf{wk:}u}$ girl at every$^{u'}$ convention.

    b. She$_u$ is very pretty.

According to the analysis in section **4.2** of chapter **6** above, this is a consequence of the fact that the two readings of sentence (96a) are effectively conflated by the condition **unique**$\{u\}$ condition contributed by the singular number morphology on the pronoun *she$_u$*. The PCDRT representations of the entire discourse in (96), derived on the basis of the two (conflated) quantifier scopings of (96a), are repeated in (97) and (98) below.

97. *a$^{\textbf{wk:}u}$ girl>>every$^{u'}$ convention*:

    $[u \mid girl\{u\}]; {}_u(\textbf{max}^{u'}([convention\{u'\}])); [{}_u(court\_at\{Harvey, u, u'\})];$

    $[\textbf{unique}\{u\}, very\_pretty\{u\}]$

98. *every$^{u'}$ convention>>a$^{\textbf{wk:}u}$ girl*:

    $\textbf{max}^{u'}([convention\{u'\}]); [{}_{u'}(u) \mid {}_{u'}(girl\{u\}), {}_{u'}(court\_at\{Harvey, u, u'\})];$

    $[\textbf{unique}\{u\}, very\_pretty\{u\}]$

This analysis, however, does not generalize to the modal case – and for a simple reason. The *de re* reading of the modal discourses in (94) and (95) above requires the common noun *wolf* to be interpreted relative to the dref for the actual world *p** over and above the fact that the indefinite *a^u wolf* in (94a/95a) brings to salience a single individual. The **unique**$\{u\}$ condition contributed by the pronoun *it$_u$* in (94b)/(95b) can constrain only the cardinality of the set of individuals introduced by the indefinite *a^u wolf* – but it cannot require them to be wolves in the actual world, i.e. to satisfy the condition *wolf$_{p*}\{u\}$*.

To see the problem more clearly, consider the two IP-CDRT representations of discourse (94) above provided in (99) and (100) below. For simplicity, I ignore the

unique$\{p^*\}$ condition and the $_{p^*}(\dots)$ operator contributed by the indicative mood **ind**$_{p^*}$ (as the reader can check, nothing crucial rests on this assumption).

99. $a^u\ wolf \gg might_\mu^{p,p'\sqsubseteq p}$, i.e. **ind**$_{p^*}($ $[a^{\mathbf{wk}:u}\ wolf]^\vee\ might_\mu^{p,p'\sqsubseteq p}$ $(\lambda q_{\mathbf{s}}.[q\in q])$ $(t_v\ come\ in)$):

$[u\ |\ wolf_{p^*}\{u\}];\ _u(\mathbf{max}^p(_p([p\in p]));\ \mathbf{max}^{p'\sqsubseteq p}([come\_in_{p'}\{u\}]);\ [\mathbf{POS}_{p^*,\mu}\{p,\ p'\}]);$

$[\mathbf{unique}\{u\},\ escape\_from\_zoo_{p^*}\{u\}]$

100. $might_\mu^{p,p'\sqsubseteq p} \gg a^u\ wolf$, i.e. **ind**$_{p^*}($ $might_\mu^{p,p'\sqsubseteq p}$ $(\lambda q_{\mathbf{s}}.[q\in q])$ $(a^{\mathbf{wk}:u}\ wolf\ come\ in)$):

$\mathbf{max}^p(_p([p\in p]));\ \mathbf{max}^{p'\sqsubseteq p}([_p(u)\ |\ _p(wolf_{p'}\{u\}),\ _p(come\_in_{p'}\{u\})]);\ [\mathbf{POS}_{p^*,\mu}\{p,\ p'\}];$

$[\mathbf{unique}\{u\},\ escape\_from\_zoo_{p^*}\{u\}]$

The representation in (99) provides the intuitively available *de re* reading: there is a u-individual that is a wolf in the actual p^*-world and there are some p'-worlds in which the u-individual comes in and that are μ-epistemic possibilites relative to the actual p^*-world. Note that we do not require the u-individual to be a wolf in these p'-worlds, but we can assume that, in all the relevant μ-accessible p'-worlds, the u-individual is a wolf because, on the one hand, it is a wolf in the actual p^*-world and, on the other hand, the μ-accessible worlds are also relativized to the actual p^*-world [32].

It is the representation in (100) that is problematic. Intuitively, the *de dicto* reading $might_\mu^{p,p'\sqsubseteq p} \gg a^u\ wolf$ should be ruled out, but the IP-CDRT representation in (100) incorrectly predicts that discourse (94) could have the following unavailable *de dicto* reading: there are some p'-worlds that are μ-epistemic possibilites relative to the actual p^*-world and there is this unique u-individual that is a wolf in each of the p'-worlds and that comes in in each of the p'-worlds. Moreover, the u-individual under consideration is such that it escaped from the zoo in the actual p^*-world. Note, in particular, that the u-individual can be a mouse or a giraffe in the actual world – and not necessarily a wolf.

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[32] Note that a similar reasoning can be used to account in IP-CDRT for the discourse in (i) below, due to Stone (1999): 8, (18), and which, as Stone (1999): 8-10 shows, poses significant problems for most alternative dynamic approaches to modal quantification (including Geurts 1995/1999, Frank 1996 and Frank & Kamp 1997).

(i) **a.** A^u wolf might walk in. **b.** We would be safe because John has a$^{u'}$ gun. **c.** He would use it$_{u'}$ to shoot it$_u$.

Thus, we observe that IP-CDRT over-generates with respect to modal subordination discourses like (94) above because it allows for an intuitively unavailable *de dicto* reading. However, the over-generation is not due to a peculiarity of the IP-CDRT system, but to the fact that the scopal interaction between a modal and an individual-level quantifier is more complex than the interaction between two individual-level quantifiers. In turn, this complexity is a consequence of the fact that the lexical relations contributed by a DP are always relativized to a modal dref and can, therefore, interact with a modal quantifier in a way that is independent from the interaction between that modal quantifier and the determiner heading the DP under consideration.

I will now briefly suggest a solution to this problem, following a proposal in Stone (1999). Stone (1999): 21 derives the infelicity of the example in (101) below by associating a presupposition of existence relative to a particular modal dref with every pronoun. This presupposition is of the form given in (102) below (Stone's actual implementation is different, but the basic proposal is the same as the one in (102), which is formulated in IP-CDRT terms).

101. **a.** John might$^{p'}$ be eating a^u cheesesteak. **b.** #It$_u$ is$_{p*}$ very greasy.
     (Stone 1999: 21, (40))

102. *u* **exists in** $p := \lambda I_{st}.\ I_{u\neq\#,p\neq\#}\neq\varnothing \wedge \forall i_s \in I_{u\neq\#,p\neq\#}(ui$ **exists in** $pi)$,
     where ***exists in*** is a constant of type $e(\mathbf{w}t)$[33].

   Abbreviation: **in** := **exists in**,
       i.e. we omit '**exists**', e.g. *u* **in** *p*, *x* **in** *w* etc.

The basic proposal in Stone (1999) (various technical details are, again, different) is that the pronoun *it$_u$* in (101b) contributes such a presupposition of existence relative to the actual world dref *p**, i.e. *u* **in** *p**. This presupposition, however, is not satisfied because the indefinite *a^u cheesesteak* in (101a) receives a narrow scope, *de dicto* reading and introduces the *u*-individual only relative to the epistemically accessible *p'*-worlds

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[33] This particular format for 'relativizing' the domain of individuals to possible worlds is due to Muskens (1995b). I use it in IP-CDRT only for its formal simplicity – and without any particular commitment to the possibilist (as opposed to the actualist) approach to quantified modal logic.

contributed by *might$^{p'}$*, i.e. *u* **in** *p'*. Consequently, the *u*-individual exists in the *p'*-epistemically accessible worlds, but not necessarily in the actual *p**-world, which makes the discourse in (101) infelicitous.

The discourse in (101) is infelicitous because the most salient reading for sentence (101a) is the *de dicto*, "narrow-scope indefinite" one, while sentence (101b) requires a *de re*, "wide-scope indefinite" reading to satisfy the existence presupposition contributed by the pronoun *it$_u$*. In contrast, the discourse in (94) above is felicitous because the *de re*, "wide-scope indefinite" reading for sentence (94a) is salient enough – but the same presuppositional mechanism that accounts the infelicity of (101) enables us to account for the fact that the only available reading for discourse (94) as a whole is the *de re* one.

More precisely, I propose to revise the IP-CDRT translations for indefinite articles and pronouns as shown in (103), (104) and (105) below. The new translations are identical to the ones proposed above (see fn. **28** in section **6.4** above) except for the addition of Stone-style existence conditions of the form *u* **in** *p*. The presuppositional status of such conditions when contributed by pronouns is indicated by underlining. A simplified version of the translation for pronouns – which is good enough for our current purposes – is provided in (106).

103. *a*$^{\textbf{wk}:u}$ $\rightsquigarrow$ $\lambda P_{\textbf{e(st)}}.\lambda P'_{\textbf{e(st)}}.\lambda q_{\textbf{s}}.$ $[u \mid u$ **in** $q];$ $_u(P(u)(q));$ $_u(P'(u)(q))$

104. *a*$^{\textbf{str}:u}$ $\rightsquigarrow$ $\lambda P_{\textbf{e(st)}}.\lambda P'_{\textbf{e(st)}}.\lambda q_{\textbf{s}}.$ $\textbf{max}^u([u$ **in** $q];$ $_u(P(u)(q));$ $_u(P'(u)(q)))$

105. *it$_{u,p}$* $\rightsquigarrow$ $\lambda P_{\textbf{e(st)}}.\lambda q_{\textbf{s}}.$ $[\underline{u \textbf{ in } p}, \underline{q \subseteq p}];$ $[\textbf{unique}\{u\}];$ $_u(P(u)(q))$ [34]

106. *it$_u$* $\rightsquigarrow$ $\lambda P_{\textbf{e(st)}}.\lambda q_{\textbf{s}}.$ $[\underline{u \textbf{ in } q}];$ $[\textbf{unique}\{u\}];$ $_u(P(u)(q))$

For concreteness, I will assume that the presuppositional conditions *$\underline{u}$* **$\underline{in}$** *$\underline{q}$* contributed by pronouns have to be satisfied as such in discourse, i.e. a condition of the

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[34] Alternatively (or: in addition), we can associate every lexical relation $R_q\{v_1, ..., v_n\}$ with a family of existence presuppositions of the form given in (i) below. For our current purposes, the simplified form in (ii) is sufficient. Just as before, underlining indicates presuppositional status.

(i) $\lambda v_n...\lambda v_1.\lambda q.$ $[\underline{v_1 \textbf{ in } p_1}, ..., \underline{v_n \textbf{ in } p_n}, \underline{q \subseteq p_1}, ..., \underline{q \subseteq p_n}];$ $[R_q\{v_1, ..., u_n\}]$

(ii) $\lambda v_n...\lambda v_1.\lambda q.$ $[\underline{v_1 \textbf{ in } q}, ..., \underline{v_n \textbf{ in } q}];$ $[R_q\{v_1, ..., u_n\}].$

form u **in** q has to be available (and 'accessible') in the representation of the previous discourse[35]. The revised (compositionally derived) IP-CDRT representations of discourse (94) are provided in (107) and (108) below. The presupposition $\underline{u\ \textbf{in}\ p*}$ contributed by the pronoun it_u in (94b) is satisfied in the *de re* representation in (107), but not in the *de dicto* representation in (108) – hence, we correctly predict that the only available reading for discourse (94) as a whole is the *de re* one.

107. $a^u\ wolf>>might_\mu{}^{p,p'\sqsubseteq p}$, i.e. $\textbf{ind}_{p*}(\ [a^{\textbf{wk}:u}\ wolf]^v\ might_\mu{}^{p,p'\sqsubseteq p}\ (\lambda q_\textbf{s}.[q\in q])\ (t_v\ come\ in)\ )$:

   $[u\ |\ u\ \textbf{in}\ p*,\ wolf_{p*}\{u\}]$;

   $_u(\textbf{max}^p(_p([p\in p])));\ \textbf{max}^{p'\sqsubseteq p}([come\_in_{p'}\{u\}]);\ [\textbf{POS}_{p*,\mu}\{p,p'\}])$;

   $[\underline{u\ \textbf{in}\ p*}];\ [\textbf{unique}\{u\},\ escape\_from\_zoo_{p*}\{u\}]$

108. $might_\mu{}^{p,p'\sqsubseteq p}>>a^u\ wolf$, i.e. $\textbf{ind}_{p*}(\ might_\mu{}^{p,p'\sqsubseteq p}\ (\lambda q_\textbf{s}.[q\in q])\ (a^{\textbf{wk}:u}\ wolf\ come\ in)\ )$:

   $\textbf{max}^p(_p([p\in p]));\ \textbf{max}^{p'\sqsubseteq p}([_{p'}(u)\ |\ u\ \textbf{in}\ p',\ {}_{p'}(wolf_{p'}\{u\}),\ {}_{p'}(come\_in_{p'}\{u\})])$;

   $[\textbf{POS}_{p*,\mu}\{p,p'\}]$;

   $[\underline{u\ \textbf{in}\ p*}];\ [\textbf{unique}\{u\},\ escape\_from\_zoo_{p*}\{u\}]$

7. Comparison with Alternative Approaches

Summarizing various points made throughout the present chapter (chapter **7**) and the previous two (chapters **5** and **6**), Intensional PCDRT differs from most previous dynamic approaches in at least three respects. The first difference is conceptual: PCDRT captures the idea that reference to structure is as important as reference to value and that the two should be treated in parallel (contra van den Berg 1996a, Krifka 1996b and Nouwen 2003 among others).

The second difference is empirical: the motivation for plural information states is provided by singular and intra-sentential donkey anaphora, in contrast to the previous

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[35] This is more in line with the binding / presupposition-as-anaphora theory of presupposition (van der Sandt 1992, Geurts 1995/1999, Kamp 2001 among others) rather than with the satisfaction theory (Karttunen 1974, Heim 1983b, 1992 among others), but I expect the solution to also be compatible with (some form of) the satisfaction theory. See Krahmer (1998), Geurts (1995/1999) and Beaver (2001) for comparative evaluations of the two theories.

literature (see van den Berg 1996a, Krifka 1996b and Nouwen 2003) which relies on plural and cross-sentential anaphora.

Finally, from a formal point of view, Intensional PCDRT accomplishes two non-trivial goals for the first time.

On the one hand, it is not obvious how to recast van den Berg's Dynamic Plural Logic in classical type logic, given that, among other things, the former logic is partial and it conflates discourse-level plurality (i.e. the use of plural information states) and domain-level plurality (i.e. non-atomic individuals) (see chapter **8** below for more discussion about this distinction).

On the other hand, previous dynamic reformulations of the analysis of modal quantification in Lewis (1973) / Kratzer (1981), e.g. the ones in Geurts (1995/1999), Frank (1996) and Stone (1999), are not satisfactory insofar as they fail to associate modal quantifications with *contents* (i.e. the propositions such quantifications express in a particular context) and cannot account for the fact that the entailment particle *therefore* can relate such contents as, for example, in the Aquinas discourse analyzed in the present chapter (see section **7.2** below for more details).

In general, the previous dynamic approaches to modal subordination fall into three broad categories based on the way in which they encode the quantificational dependencies between possible scenarios (e.g. the epistemically accessible possibilities of a wolf coming in) and the individuals that feature in these scenarios (e.g. whichever wolf enters in a particular epistemically accessible possibility):

- accommodation accounts, e.g. Roberts (1987, 1989, 1995, 1996), where there are no modal dref's of any kind and the associations between possible scenarios and the individuals that feature in them is captured at the level of logical form, i.e. by accommodating / copying the DRS's that introduce the relevant individual-level dref's into the restrictor or nuclear scope DRS's of another modal operator;

- analyses like the ones proposed in Kibble (1994, 1995), Portner (1994), Geurts (1995/1999), Frank (1996), Frank & Kamp (1997) and van Rooy (2001), which take modal quantifiers to relate dynamically-valued dref's, i.e. (in the simplest case)

dref's for information states, where, following Heim (1983b), an information state is basically represented as a set of <world, variable assignment> pairs; in these approaches, the dependency between possibilities and individuals is encoded in the dref's for information states: every <world, assignment> pair is such that the assignment stores the individual-level dref's that have been introduced with respect to that world; these approaches to modal subordination are parallel to the "parametrized sum individuals" approaches to donkey anaphora and quantificational subordination in Rooth (1987) and Krifka (1996b): instead of summing atomic individuals that are each parametrized with a variable assignment, these approaches 'sum' possible worlds that are each parametrized with a variable assignment;

- encapsulated quantification accounts, e.g. Stone (1997, 1999) and Bittner (2001, 2006), where modal quantifiers relate dref's for *static* objects (unlike Geurts 1995/1999, Frank 1996 and van Rooy 2001), namely dref's for accessibility relations. Thus, modal dref's in such accounts are of type $s(\mathbf{w}(\mathbf{w}t))$ and individual-level dref's are of type $s(\mathbf{w}e)$, i.e. they are dref's for individual concepts. The quantificational dependency between possibilities and individuals is encoded in the complex static objects that these dref's have as values. For example, in a sentence like *A wolf might come in*, the modal *might* introduces a dref of type $s(\mathbf{w}(\mathbf{w}t))$ which, with respect to a given 'assignment' i_s, stores a function of type $\mathbf{w}(\mathbf{w}t)$ that maps (the current candidates for) the actual world to the set of epistemically accessible worlds in which a wolf comes in; at the same time, the indefinite *a wolf* introduces a dref of type $s(\mathbf{w}e)$ which, relative to a given 'assignment' i_s, stores a function mapping every epistemically accessible world w in which a wolf comes in to the wolf that comes in in w.

Intensional PCDRT (IP-CDRT) makes use of a fourth way of capturing the quantificational dependencies between possibilities and individuals, namely plural information states. Just as in encapsulated quantification accounts, the IP-CDRT dref's for possibilities have static objects as values – in particular, they are of type $s\mathbf{w}$, storing a possible world w relative to each 'assignment' i. The dref's for individuals have the usual type se. But, unlike in encapsulated quantification accounts, the quantificational dependencies between possibilities and individuals are stored in the plural info states that

are incrementally updated in discourse and not in the static objects that the modal and individual-level dref's have as values.

For example, in a sentence like *A wolf might come in*, the modal *might* introduces a dref p of type $s\mathbf{w}$ which, with respect to a plural info state I_{st}, stores the set of worlds $pI :=$ $\{pi: i\in I\}$ in which a wolf comes in. The indefinite *a wolf* introduces a dref u of type se which, with respect to each world w in which a wolf comes in, stores the wolf or wolves that come in in w. That is, for every world w, the sub-state $I_{p=w} := \{i\in I: pi=w\}$ (which stores only the world w relative to p) stores the corresponding wolf or wolves relative to u, i.e. the set of wolves associated with w is $uI_{p=w} := \{ui: i\in I_{p=w}\}$. Thus, the dependency between worlds and wolves is stored in the plural info state I_{st} in a pointwise manner: for each $i_s\in I$, the wolf ui comes in in world pi.

The subset of the p-worlds that are epistemically accessible from the actual world $w*$ are also accessed via the the quantificational dependencies stored in the plural info state I_{st}. First, we have that the dref for the actual world $p*$ stores only the actual world $w*$ relative to the entire plural info state I_{st}, i.e. we have that $p*I=\{w*\}$ – consequently, the plural info state I is the same as $I_{p*=w*}$. Second, following the proposal in Kratzer (1981), IP-CDRT assumes that an epistemic modal base μ is contextually supplied: μ is a dref of type $s(\mathbf{w}t)^{36}$ and the dref μ stores a set of propositions $\mu I_{p*=w*} := \{\mu i: i\in I_{p*=w*}\}$ relative to the current plural info state $I_{p*=w*}$, hence relative to the actual world $w*$.

The differences between IP-CDRT and previous approaches stem from the two main features of its account of modal subordination: (**i**) the use of modal dref's that have static objects as values; (**ii**) the use of plural info states to encode quantificational dependencies.

7.1. Statically vs. Dynamically Valued Modal Dref's

The first feature, namely using modal dref's with static objects as values, is shared with encapsulated quantification accounts. Using modal dref's with static objects as

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[36] Note the simplification in type relative to the modal bases in Kratzer (1981), which have type $\mathbf{w}((\mathbf{w}t)t)$.

values has several advantages relative to the first two categories of approaches, i.e. accommodation approaches and approaches that use dref's with information states as values. Stone (1999) (see pp. 5-11 in particular) provides a lucid review of these two categories of approaches and a persuasive argument for using modal dref's with static objects as values, which I will not iterate here. I will only summarize the two main arguments – the first one is empirical, while the second is more theoretical in nature.

Empirically, the first two categories of approaches to modal subordination have difficulties accounting for discourses that involve multiple inter-related possible scenarios like the one in (109) below.

109. **a.** A^u wolf might walk in.

b. We would be safe because John has a$^{u'}$ gun.

c. He would use it$_{u'}$ to shoot it$_u$.

(Stone 1999: 8, (18))

As Stone (1999) puts it:

> "[The discourse in (109)] describes two situations: an actual present situation, in which John has a gun; and a possible future development of that situation, in which a wolf walks in. The last sentence of [(109)] illustrates that the speaker may refer both to the possible wolf and to John's gun in a description of that possible future. […] In previous dynamic approaches, scenarios are interpreted as sets of DYNAMIC objects, in which possible worlds are paired with assignments that indicate what entities are available for reference there. (Entities are introduced into a sequence of evolving SCENARIOS rather than into evolving representations of the DISCOURSE.)
> Because scenario referents explicitly inventory available referents, we can only refer to a gun in a scenario in which a gun has been explicitly added. This is incompatible with the pattern of reference in [(109)]. First, the discourse describes a possible elaboration of what we know, where a wolf comes in (and we are safe). Then the discourse evokes a further elaboration of our information which includes a gun. Although this elaboration describes reality, it nevertheless leaves the original hypothetical scenario unchanged. There is therefore no gun to refer to in the wolf-scenario."
> (Stone 1999: 8-9)

For more details, see Stone (1999): 9-11.

In contrast, encapsulated quantification approaches and IP-CDRT (see in particular the account of *de re* readings in section **6.6** above) can account for such discourses because they model possible scenarios as ordinary static objects and can relate multiple scenarios and the individuals featured in them in very much the same way as classical DRT / FCS / DPL approaches introduce and relate multiple individual dref's.

The theoretical argument in Stone (1999) against the first two kinds of approaches to modal subordination is that they fail to capture the anaphoric and quantificational parallels between the individual and modal domains argued for in Stone (1997, 1999), Bittner (2001, 2006) and Schlenker (2003, 2005b) among others. In contrast (as shown by the parallel analysis of quantificational and modal subordination in section **6** above), the theoretical architecture of IP-CDRT enables us to give a point-for-point parallel account of anaphora and quantification in the individual and modal domains, from the types assigned to the modal and individual dref's to the translations compositionally associated with anaphoric and quantificational expressions.

7.2. Plural Info States vs. Encapsulated Quantification

Let us turn now to the second feature of the IP-CDRT account of modal subordination, namely the use of plural info states to capture quantificational dependencies. This is the feature that distinguishes IP-CDRT from encapsulated quantification accounts (e.g. Stone 1997, 1999 and Bittner 2001, 2006).

There is one argument that seems to recommend the use of plural info states to encode quantificational dependencies as opposed to the use of encapsulated quantification: encapsulated quantification approaches (which, in a broad sense, include approaches that make use of choice functions and / or Skolem functions to account for donkey anaphora and quantificational subordination – see section **6** of chapter **5** above) do not store quantificational dependencies introduced in discourse in the database that is meant to store discourse-related information, i.e. in the information states, but in the meaning of the lexical items, be they the indefinite-like items that introduce new dref's or the pronoun-like items that retrieve them.

The point (already made in van den Berg 1994, 1996a with respect to individual-level plural anaphora) can be more easily clarified if we consider the quantificational subordination examples in (110) and (111) below. The modal subordination based argument is similar.

110. **a.** Every^u man loves a$^{u'}$ woman.

   b. They$_u$ bring them$_{u'}$ flowers to prove this.

   (van den Berg 1996a: 168, (16))

111. **a.** Every^u boy bought a$^{u'}$ flower and gave it$_{u'}$ to a$^{u''}$ girl.

   b. They$_{u''}$ thanked them$_u$ for the$_{u'}$ very nice gifts.

Consider (110) first. Sentence (110a) establishes a twofold dependency between men and the women that they love and sentence (110b) further elaborates on this dependency. Encapsulated quantification approaches have to make use of functions from individuals to individuals of type *ee* (or relations between individuals of type *e*(*et*)) to capture the intuition that sentence (110b) elaborates on the dependency introduced in sentence (110a). That is, either the quantifiers (*every^u man* and *a$^{u'}$ woman*) or the pronouns (*they$_u$* and *them$_{u'}$*) – or both – have to have such functions as (part of) their semantic value.

Now consider discourse (111). Sentence (111a) establishes a threefold dependency between boys, flowers and girls and sentence (111b) further elaborates on this dependency. In this case, encapsulated quantification approaches need to make use of functions and / or relations that are more complex than the ones needed for discourse (110). Therefore, the semantic values assigned to quantifiers and / or pronouns will have to be more complex in the case of (111), despite the fact that the very same lexical items are used.

That is, quantifiers and / or pronouns denote functions / relations of different arities depending on the discourse context, i.e. depending on how many simultaneous anaphoric connections are established in a particular discourse. And these functions / relations become a lot more complex as soon as we start to explicitly represent anaphora to and quantification over possible worlds, times, locations, eventualities, degrees etc.

Summarizing, the (mostly theoretical) argument for plural info states as opposed to encapsulated quantification approaches is the following: since the arity of the functions / relations denoted by pronouns and / or quantifiers is determined by the discourse context, we should encode this context dependency in the info state (the purpose of which is to store precisely this kind of discourse information) and not in the representation of the lexical items themselves.

Turning now to more empirical considerations, IP-CDRT and encapsulated quantification approaches seem to have a similar empirical coverage as far as the English phenomena considered in this chapter are concerned (although see the observations in sections **7.4** and **7.5** below). However, only future research will decide if IP-CDRT based approaches can also scale up to account for typologically different languages (e.g. Kalaallisut), which have been successfully analyzed in an encapsulated quantification dynamic framework (see for example Bittner 2006).

Note, however, that the two frameworks are not incompatible, since IP-CDRT can also make use of dref's that have more complex modal objects as values, e.g. the dref's for modal bases and ordering sources used in this chapter. But, even in such cases, the use of plural info states enables us to simplify the types of such dref's – much like the types of modal and individual-level dref's in Stone (1999) are simplified in IP-CDRT: we only need to use dref's for possible worlds of type $s\mathbf{w}$ in IP-CDRT as opposed to the dref's for accessibility relations of type $s(\mathbf{w}(\mathbf{w}t))$ in Stone (1999); also, we only need to use dref's for individuals of type *se* in IP-CDRT instead of the dref's for individual concepts of type $s(\mathbf{w}e)$ used in Stone (1999).

I will conclude this section with three more observations about the differences between IP-CDRT and the encapsulated quantification system in Stone (1999).

First, for simplicity, Stone (1999) treats modal bases and ordering sources as static objects (see the definitions for necessity and possibility in Stone 1999: 27, (47)). IP-CDRT introduces dref's for modal bases and ordering sources, thus providing a dynamic treatment for all the contextually dependent components of modal quantification argued for in Kratzer (1981).

Second, IP-CDRT employs maximal *unparametrized* restrictor and nuclear scope sets in the definition of modal quantification, in contrast to Stone (1999), who introduces restrictor and nuclear scope sets for modal quantifiers by means an *if*-update with a Lewis-style similarity ordering source built in (see Stone 1999: 17, (34)). To see that the built-in parametrization is too restrictive, consider the deontic conditional in (112) below (based on Kratzer 1981): (112) does not seem to involve a similarity ordering source because the conditional simply states that, according to the law, the deontically ideal worlds among the set of worlds where there is a murder are such that the murderer goes to jail. The deontic quantification is not restricted to the set of worlds where there is a murder and which are as similar as possible to the actual world since many of the facts in the actual world are orthogonal to the legal requirement specified by (112).

112. If there is a^u murder, the$_u$ murderer must go to jail.

Finally, in contrast to the IP-CDRT definitions, the definitions of necessity and possibility in Stone 1999: 27, (47) do not associate contents with modal quantifications, so they cannot account for the *therefore* discourses in (1/2) and (9) above, in which *therefore* relates contents and not meanings (i.e. context-change potentials); for more discussion, see section **2.2** of the present chapter.

7.3.   Conjunctions under Modals

Roberts (1996) (see also Roberts 1995) presents the following challenge for dynamic / anaphoric accounts of modal subordination. Consider the two discourses in (113) and (114) below (example (19) in Roberts 1996: 224 and example (3) in Roberts 1996: 216 respectively).

113. **a.** You should buy a^u lottery ticket and put it$_u$ in a safe place.

[**b.** You're a person with good luck.]

c. It$_u$ might be worth millions.

114. **a.** You should buy a^u lottery ticket and put it$_u$ in a safe place.

b. #It$_u$'s worth a million dollars.

Note that the *might* modal quantification in (113c) is restricted by the content of the first conjunct below the modal *should* in (113a), i.e. it is interpreted as asserting that, given that you're a generally lucky person, *if you buy a lottery ticket*, it might be worth millions. Crucially, (113c) is not restricted by the content of both conjuncts in (113a) or by the set of deontically ideal worlds contributed by *should*.

The challenge for dynamic approaches is to show that they do not under-generate, i.e. that they can account for the felicitous discourse in (113), and that they do not over-generate, i.e. that they can account for the infelicitous discourse in (114). In this section, I will briefly sketch how IP-CDRT can account for the first, felicitous example and derive the infelicity of the second. In the process, we will see that example (113) provides another empirical argument for the explicit introduction of contents in discourse.

The discourse in (113) is analyzed like the Aquinas discourse in (1/2) above, i.e. in terms of structured anaphora to propositions. The only component we need to add is a translation for *and* that introduces and relates the contents of its conjuncts, much like the analysis of conjunction in classical modal logic. A suitable translation is provided in (115) below, which, just as the translation for modal quantifiers in section **6.1** above, relies on structured inclusion to capture the anaphoric connections between the first and the second conjunct in (113a) above. Also, note that the conjunction *and* relates two maximal *unparametrized* sets of possible worlds – again, just like the definition of modal quantification in section **6.1** above.

115. $and^{p,p' \sqsubseteq p} \rightsquigarrow \lambda \mathbb{P}_{st}.\lambda \mathbb{P}'_{st}.\lambda q_s.\ \mathbf{max}^p(_p(\mathbb{P}(p)));\ \mathbf{max}^{p' \sqsubseteq p}(_{p'}(\mathbb{P}'(p')));\ [q \sqsubseteq p']$

The translation for the modal *should* in (113a) is provided in (116) below; it is the expected one, modulo the fact that we omit the distributivity operator $_{p'}(\dots)$ over the nuclear scope update[37].

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[37] This is needed to ensure that the structural dependencies introduced within the two conjuncts are properly inherited by the nuclear scope dref *p'* – and it can be seen as the limit case (no distributivity operator at all) of the variability with respect to nuclear scope distributivity operators argued for in section **6.2** of chapter **6** above.

116. *if^p + should$_{\mu,\omega}$$^{p'\sqsubseteq p}$* ⇝

$$\lambda\mathbb{P}_{\textbf{st}}.\lambda\mathbb{P}'_{\textbf{st}}.\lambda q_{\textbf{s}}. \textbf{max}^p(_p(\mathbb{P}(p))); \textbf{max}^{p'\sqsubseteq p}(\mathbb{P}'(p')); [\textbf{NEC}_{q,\mu,\omega}\{p, p'\}]$$

In (116), μ is an epistemic modal base, ω is a deontic ordering source, the antecedent $\mathbb{P}$ is accommodated as $\lambda q_{\textbf{s}}. [q{\in}q]$ (due to the fact that (113a) is discourse initial – see section **6.5** above for more discussion) and the consequent $\mathbb{P}'$ is the conjunction *you buy a lottery ticket and you put it in a safe place*, i.e. the dynamic proposition in (117) below. The final representation of (113a) has the form given in (118) below, which can be simplified as shown in (119), i.e. by omitting the dref p_2.

117. $\lambda q_{\textbf{s}}. \textbf{max}^{P_1}(_{p_1}(\mathbb{P}_1(p_1))); \textbf{max}^{P_2,\hat{\delta}\,P_1}(_{p_2}(\mathbb{P}_2(p_2))); [q{\sqsubseteq}p_2],$

    where $\mathbb{P}_1$ is "you buy a^u lottery ticket" and $\mathbb{P}_2$ is "you put it$_u$ in a safe place".

118. $\textbf{max}^p([p{\in}p]); \textbf{max}^{p'\sqsubseteq p}(\textbf{max}^{P_1}(_{p_1}([u \mid lottery\_ticket_{p_1}\{u\}, you\_buy_{p_1}\{u\}]));$

$$\textbf{max}^{P_2,\hat{\delta}\,P_1}(_{p_2}([you\_put\_in\_safe\_place_{p_2}\{u\}])); [p'{\sqsubseteq}p_2]);$$

$[\textbf{NEC}_{p*,\mu,\omega}\{p, p'\}],$

    where $p*$ is the dref for the actual world.

119. $\textbf{max}^p([p{\in}p]); \textbf{max}^{P_1,\hat{\delta}\,P}(_{p_1}([u \mid lottery\_ticket_{p_1}\{u\}, you\_buy_{p_1}\{u\}]));$

$$\textbf{max}^{p'\hat{\delta}\,P_1}(_{p'}([you\_put\_in\_safe\_place_{p'}\{u\}])); [\textbf{NEC}_{p*,\mu,\omega}\{p, p'\}]$$

Informally, the update in (119) instructs us to do the following operations on the default input info state $\{i_\#\}$. First, given that the modal verb is contextually dependent (much like deictic pronouns), we need to accommodate an update that introduces the dref for the actual world $p*$, the epistemic modal base μ and the deontic ordering source ω. Then, we process the first update in (119), namely $\textbf{max}^p([p{\in}p])$, which instructs us to add a p column to the input info state and store in it the set of all possible worlds.

The next update instructs us to add a p_1 column and store in it all the p worlds in which you buy a lottery ticket; also, we add a u column and store in it the lottery ticket(s) that you buy in each corresponding p_1-world. Then, we add a p' column and store in it all the p_1-worlds in which you put in a safe place the corresponding u-lottery ticket(s).

Finally, we test that all the ω-deontically ideal worlds among the μ-epistemically accessible p-worlds are included in p'. That is, since p stores the set of all possible worlds, we simply test that all the ω-deontically ideal worlds among the μ-epistemically accessible worlds are such that you buy a lottery ticket and put it in a safe place.

Crucially, at the end of the update contributed by sentence (113a), we have access to the set of p_1-worlds satisfying the first conjunct below the modal *should*, i.e. we have access to all the worlds in which you buy a lottery ticket. We will therefore be able to interpret sentence (113c) in the usual way, i.e. as simultaneously anaphoric to the modal dref p_1 and the individual-level dref u. Thus, IP-CDRT is able to capture all the structured anaphoric connections established in discourse (113) and derive the intuitively correct truth-conditions associated with it.

The IP-CDRT account of the infelicitous discourse in (114) is basically the same as the account of the infelicitous discourse in (101) above (see section **6.6** of the present chapter).

7.4. Weak / Strong Ambiguities under Modals

Donkey anaphora in modalized conditionals exhibits weak / strong ambiguities just as it does in (extensional) relative-clause donkey sentences. In particular, the conditional in (2a) above, repeated in (120) below, provides an instance of strong donkey anaphora, while the conditional in (121) below, due to Partee (1984), provides an instance of weak donkey anaphora.

120. If a^u man is alive, he$_u$ must find something pleasurable.
121. If you have a^u credit card, you should use it$_u$ here instead of cash.
      (Partee (1984): 280, fn. 12)

Given the analysis of the weak / strong ambiguity in chapter **5** above, it should be clear that IP-CDRT can account for both examples: the indefinite *a^u man* in (120) receives a strong reading (see section **4.1** of the present chapter), while the indefinite *a^u credit card* in (121) receives a weak reading. The intuitively correct truth-conditions for both discourses are derived in the usual way.

Weak / strong donkey ambiguities pose problems for all three categories of alternative approaches mentioned above. Accommodation-based approaches like Roberts (1987, 1989) can account only for strong donkey readings – a feature they inherit from the underlying classical DRT framework.

Approaches that use dref's for information states can also account only for strong readings. For example, the definitions of info state dref update in Frank (1996): 98, (36) and Geurts (1905/1999): 154, (43b) update a set F of <world, assignment> pairs with a DRS K (the denotation of which is a binary relation between <world, assignment> pairs) by taking the image of the set F under the relation denoted by K. That is, the output set G of <world, assignment> pairs obtained after updating F with K is the set $G = \{<w', g'>: \exists<w, g> \in F(<w, g>K<w', g'>)\}$. This kind of update predicts that, by the time we have processed the antecedent of the conditional in (121), the output set of <world, assignment> pairs will contain all the credit cards that you have, which in turn predicts that the conditional in (121) counter-intuitively requires you to use all your credit cards.

Finally, the encapsulated quantification approach in Stone (1999) can account only for weak donkey readings because indefinites introduce dref's for individual concepts (they are functions of type $s(\mathbf{w}e)$), hence, for each possible world, the dref will store exactly one individual. Such dref's are, basically, dref's for choice functions: given a world w, the individual concept will choose a particular entity that is a credit card you have in that world.

Thus, Stone (1999) can account for the weak reading conditional in (121) as follows: the indefinite in the antecedent (arbitrarily) chooses a credit card relative to each world w in which you have a non-empty set of credit cards; the consequent elaborates on this by requiring all the deontically ideal worlds w to be such that you use the corresponding card instead of cash.

By the same token, Stone (1999) cannot account for the strong reading conditional in (120), where the indefinite in the antecedent needs to introduce *all* the men that are alive in any given world w. An easy fix that would enable Stone (1999) to account for the strong donkey conditional in (120) would be to introduce dref's for properties, i.e. dref's

of type $s(\mathbf{w}(et))$ which, relative to a given world w, would store the set of all men that are alive in w. However, for the reasons mentioned in section **1** of chapter **5** above, this strategy would fail for more complex examples involving multiple strong indefinites.

7.5. Uniqueness Effects under Modals

Modal subordination discourses exhibit the same kind of uniqueness effects (and variability thereof) as quantificational subordination discourses. Consider again examples (2) (*If a^u man is alive, he$_u$ must find something pleasurable. Therefore, if he$_u$ doesn't have any spiritual pleasure, he must have a carnal pleasure*), (53) (*A^u wolf might come in. It$_u$ would attack Harvey first*), and (86) (*If a^u wolf came in, John could not legally kill it$_u$. But he still would have to*) above.

Discourse (53) seems to exhibit relativized uniqueness effects: it is (preferably) understood as talking about epistemic possibilities featuring a unique wolf coming in. In contrast, discourses (2) and (86) do not exhibit any uniqueness effects: (2) is not talking only about worlds / possibilities in which exactly one man is alive and (86) is interpreted as asserting that, if he wants to obey the law, John cannot kill any wolf or wolves that come in and, in addition, if he wants to survive, John has to kill any wolf or wolves that come in – neither the law, nor John's survival instinct are particularly geared towards possible scenarios in which a unique wolf comes in.

IP-CDRT can capture the relativized uniqueness effects associated with discourse (53) in much the same way as it captures the relativized uniqueness effects associated with quantificational subordination (see section **6.1** of chapter **6**). That is, if we use a strong / maximized indefinite article, the (relativized) uniqueness emerges from the interaction between the $\mathbf{max}^u$ operator contributed by the strong indefinite *a*$^{\mathbf{str}:u}$ *wolf* and the $\mathbf{unique}\{u\}$ condition contributed by the singular pronoun *it$_u$*.

The lack of uniqueness effects associated with discourses (2) and (86) can be captured in the same way as the lack of uniqueness effects associated with donkey anaphora (see section **6.2** of chapter **6** for details), i.e. by means of suitable distributivity operators that neutralize / vacuously satisfy the $\mathbf{unique}$ conditions contributed by singular pronouns.

Thus, IP-CDRT can capture the wavering nature of the uniqueness implications associated with modal subordination in much the same way as it captures the wavering nature of the uniqueness effects associated with quantificational subordination and donkey anaphora.

It is not obvious to me how the alternative approaches mentioned above can capture the behavior of uniqueness effects in modal subordination discourses – so, I will leave this issue as a topic for future investigation and discussion.

Appendix

A1. Intensional PCDRT: Definitions and Translations

122. **New Dref's, Structured Inclusion, Maximization and Distributivity.**

    a. $[p] := \lambda I_{st} J_{st}. \ \forall i_s \in I(\exists j_s \in J(i[p]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[p]j)))$

    b. $p' \Subset p := \lambda I_{st}. \ \forall i_s \in I(p'i = pi \vee p'i = \#)$

    c. $p' \sqsubseteq p := \lambda I_{st}. \ (p' \Subset p)I \wedge \forall i_s \in I(pi \in p'I_{p' \neq \#} \rightarrow pi = p'i)$

    d. $\mathbf{max}^p(D) := \lambda I_{st}.\lambda J_{st}. \ ([p]; D)IJ \wedge \forall K_{st}(([p]; D)IK \rightarrow pK_{p \neq \#} \subseteq pJ_{p \neq \#})$

    e. $\mathbf{max}^{p' \sqsubseteq p}(D) := \mathbf{max}^{p'}([p' \sqsubseteq p]; D)$

    f. $\mathbf{max}^{p \Subset p'}(D) := \lambda I_{st}.\lambda J_{st}. \ \exists H([p \mid p \Subset p']IH \wedge DHJ \wedge$

$$\forall K_{st}([p \mid p \Subset p']IK \wedge \exists L_{st}(DKL) \rightarrow K_{p \neq \#} \subseteq H_{p \neq \#})) \ ^{38}$$

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[38] This operator, more precisely $\mathbf{max}^{u' \Subset u}$, is independently required to analyze the example in (i) below within the revised PCDRT system of chapter **6**. This example can be easily analyzed within the system of chapter **5** (the only difference is that, to obtain the intuitively correct truth-conditions, we need the indefinite *a son* to be weak, not strong), but (i) poses problems for the revised definition of generalized quantification in chapter **6**, repeated in (ii) below for convenience. The problem is that (i) is falsified by any parent who has a son in high school and who has lent him the car on a weeknight even if said parent has another son who never got the car. This problem is posed by any determiner that is downward monotonic in his right argument, e.g. *Few parents with a son still in high school lend him the car on weekends* is intuitively falsified if most parents are such that they have a son in high school and they lent him the car on a weeknight even if, at the same time, all parents have at least one son who never got the car.

(i) No$^{u,u' \Subset u}$ parent with a$^{\mathbf{str}:u''}$ son still in high school has ever lent him$_{u''}$ the car on a weeknight.
   (Rooth 1987: 256, (48))

(ii) $det^{u,u' \sqsubseteq u} \rightsquigarrow \lambda P_{et}.\lambda P'_{et}. \ \mathbf{max}^u(_{\langle u \rangle}(P(u))); \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u'))); [\mathbf{DET}\{u, u'\}]$

g. $\mathbf{dist}_p(D) := \lambda I_{st}.\lambda J_{st}.\ \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \leftrightarrow J_{p=w}\neq\emptyset) \wedge \forall w_{\mathbf{w}}(I_{p=w}\neq\emptyset \rightarrow DI_{p=w}J_{p=w})$,

     i.e. $\mathbf{dist}_p(D) := \lambda I_{st}J_{st}.\ pI=pJ \wedge \forall w_{\mathbf{w}}\in pI(DI_{p=w}J_{p=w})$

h. $_p(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}\neq\emptyset \wedge \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#}$

i. $_{\langle p\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge (I_{p\neq\#}=\emptyset \rightarrow I=J) \wedge (I_{p\neq\#}\neq\emptyset \rightarrow \mathbf{dist}_p(D)I_{p\neq\#}J_{p\neq\#})$

j. $_p(C) := \lambda I_{st}.\ I_{p\neq\#}\neq\emptyset \wedge \forall w\in pI_{p\neq\#}(CI_{p=w})$,   where C is a condition (of type $(st)t$)

k. $_p(\alpha_1, \ldots, \alpha_n) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\#}=J_{p=\#} \wedge I_{p\neq\#}[\alpha_1, \ldots, \alpha_n]J_{p\neq\#}$,

     where $p\notin\{\alpha_1, ..., \alpha_n\}$ and $[\alpha_1, ..., \alpha_n] := [\alpha_1]; ...; [\alpha_n]$

The definition in (ii) is problematic for the following reason. First, note that, if the indefinite *a son* is weak, we obtain intuitively incorrect truth-conditions for (i) because, if the indefinite introduces only the u''-son who never got the car relative to the corresponding u-parent, the $\mathbf{NO}\{u, u'\}$ condition is verified and we incorrectly predict that (i) is true in such a situation. Second, note that, if the indefinite *a son* is strong, i.e. we introduce both the u''-son that got the car and the u''-son that didn't get it with respect to the corresponding u-parent, then the $\mathbf{max}^{u'\in u}$ operator used to extract the nuclear scope will discard this parent, i.e. this u-parent will not be stored in u', because it is not the case that this u-parent lends the car to all the corresponding u''-sons. Hence, yet again, the $\mathbf{NO}\{u, u'\}$ condition is verified and we incorrectly predict that (i) is true in such a situation.

However, using the $\mathbf{max}^{u'\in u}$ operator to provide the alternative translation in (iii) below for (certain occurrences of) determiners that are downward monotonic in their right argument yields the intuitively correct truth-conditions for example (i) if the indefinite *a son* is strong. The reason is that the $\mathbf{max}^{u'\in u}$ update will retain any u-parent that lent the car to at least one son – and the the $\mathbf{NO}\{u, u'\}$ condition (or the $\mathbf{FEW}\{u, u'\}$ condition etc.) will not be verified anymore.

(iii) $det^{u,u'\in u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\in u}(_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

Given that the $\mathbf{max}^{p'\in p}$ operator is associated, in the modal domain, with conditional antecedents, which are also downward monotonic, a fairly general procedure for translating individual-level and modal determiners seems to emerge: the right upward monotone determiners $det\!\uparrow$ (*every*, *most* etc.) should receive the $det^{\alpha,\alpha'\in\alpha}$ type of translation in (ii), while the right downward determiners $det\!\downarrow$ (*no*, *few* etc.) should receive the $det^{\alpha,\alpha'\in\alpha}$ type of translation in (iii). Also, if the restrictor of a determiner is anaphoric to another dref α'', then, for left upward determiners $\uparrow\!det$, they should be translated as $det^{\alpha\in\alpha'',\alpha'\in\alpha}$ (if they are right upward monotone) or $det^{\alpha\in\alpha'',\alpha'\in\alpha}$ (if they are right downward monotone). If the determiners are left downward monotone, i.e. $\downarrow\!det$ (*every*, *if* etc.) and their restrictor is anaphoric to a dref α'', they should be translated as $det^{\alpha\in\alpha'',\alpha'\in\alpha}$ (if they are right upward monotone) or $det^{\alpha\in\alpha'',\alpha'\in\alpha}$ (if they are right downward monotone). For instance, the *if+must* determiner in the second conditional (i.e. the conclusion) of the Aquinas argument in (1/2) receives the $det^{p\in p'',p'\in p}$ translation in (iv) below.

(iv) $det^{p\in p'',p'\in p} \rightsquigarrow \lambda \mathbb{P}_{\mathbf{st}}.\lambda \mathbb{P}'_{\mathbf{st}}.\ \mathbf{max}^{p\in p''}(_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\in p}(_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{DET}\{p, p'\}]$.

I leave the investigation of this suggestion – as well as the problem posed by the translation of non-monotonic determiners (e.g. *exactly n*) – for future research.

123. **Distributivity-based Equivalences.**

    a. $_p([C_1, \ldots, C_m]) = [_p(C_1), \ldots, _p(C_m)]$

    b. $_p([\alpha_1, \ldots, \alpha_n \mid C_1, \ldots, C_m]) = [_p(\alpha_1, \ldots, \alpha_n) \mid _p(C_1), \ldots, _p(C_m)])$

124. **Atomic Conditions.**

    a. $R_p\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{p \neq \#, u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset \wedge$

$$\forall i_s \in I_{p \neq \#, u_1 \neq \#, \ldots, u_n \neq \#}(R_{pi}(u_1 i, \ldots, u_n i))$$

    b. unique$\{p\} := \lambda I_{st}.\ I_{p \neq \#} \neq \emptyset \wedge \forall i_s, i'_s \in I_{p \neq \#}(pi = pi')$

    c. MODAL$_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#} = \emptyset \wedge \mathbf{unique}\{q\}I \wedge$

$$\mathbf{MODAL}_{\mu I \neq \{\#\}, \omega I \neq \{\#\}}\{pI_{p \neq \#}, p'I_{p' \neq \#}\},$$

        where μ (modal based dref) and ω (ordering source dref) are of type $s(\mathbf{w}t)$

    d. NEC$_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#} = \emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{NEC}_{\mu I \neq \{\#\}, \omega I \neq \{\#\}}\{pI_{p \neq \#}, p'I_{p' \neq \#}\},$

        where $\mathbf{NEC}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M) \cap W_1) \subseteq W_2,$

            where W_1 and W_2 are of type $\mathbf{w}t$

            and M (modal base) and O (ordering source) are of type $(\mathbf{w}t)t$

    e. POS$_{q,\mu,\omega}\{p, p'\} := \lambda I_{st}.\ I_{q=\#} = \emptyset \wedge \mathbf{unique}\{q\}I \wedge \mathbf{POS}_{\mu I \neq \{\#\}, \omega I \neq \{\#\}}\{pI_{p \neq \#}, p'I_{p' \neq \#}\},$

        where $\mathbf{POS}_{M,O}(W_1, W_2) := \mathbf{Ideal}_O((\cap M) \cap W_1) \cap W_2 \neq \emptyset,$

            where W_1 and W_2 are of type $\mathbf{w}t$

            and M (modal base) and O (ordering source) are of type $(\mathbf{w}t)t$

    f. Generalized Limit Assumption.

        For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\forall w \in W \exists w' \in W((w' <_O w \vee w' = w) \wedge \neg \exists w'' \in W(w'' <_O w'))$$

    g. The Ideal function.

        For any proposition $W_{\mathbf{w}t}$ and ordering source $O_{(\mathbf{w}t)t}$:

$$\mathbf{Ideal}_O(W) := \{w \in W: \neg \exists w' \in W(w' <_O w)\}$$

125. **Translations.**

    a. *if*^{p} + *modal*$_{\mu,\omega}{}^{p' \sqsubseteq p}$ ⤳

$$\lambda \mathbb{P}_{\mathbf{st}}.\lambda \mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^p(_{\langle p \rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p' \sqsubseteq p}(_{\langle p \rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$$

    b. *modal*$_{\mu,\omega p}{}^{p' \sqsubseteq p}$ ⤳ $\lambda \mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p' \sqsubseteq p}(_{\langle p \rangle}(\mathbb{P}(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p, p'\}]$

    c. *if*$^{p \sqsubseteq p''}$ + *modal*$_{\mu,\omega}{}^{p' \sqsubseteq p}$ ⤳

$$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p\in p''}(_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{\langle p\,\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p,\,p'\}]$$

d. indicative$_{p*}$ $\rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ [\mathbf{unique}\{p*\}];\ _{p*}(\mathbb{P}(p*)),$

        where $p*$ is the dref for the actual world

e. girl $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [girl_q\{v\}]$

f. det$^{u,u'\sqsubseteq u}$ $\rightsquigarrow$

        $\lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^u(_{\langle u\rangle}(P(u)(q)));\ \mathbf{max}^{u'\sqsubseteq u}(_{\langle u\,\rangle}(P'(u')(q)));\ [\mathbf{DET}\{u,\,u'\}]$

g. $a^{\mathbf{wk}:u}$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u];\ _u(P(u)(q));\ _u(P'(u)(q))$

h. $a^{\mathbf{str}:u}$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^u(_u(P(u)(q));\ _u(P'(u)(q)))$

i. he$_u$ $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [\mathbf{unique}\{u\}];\ _u(P(u)(q))$

j. Harvey^u $\rightsquigarrow \lambda P_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u\mid u\in Harvey];\ _u(P(u)(q)),$

        where $Harvey := \lambda i_s.\ harvey_e$

k. might$_{\mu,\omega}^{\ p,p'\sqsubseteq p}$ $\rightsquigarrow$

        $\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^p(_p(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}(_{p'}(\mathbb{P}'(p')));\ [\mathbf{POS}_{q,\mu,\omega}\{p,\,p'\}]$

l. would$_{\mu,\omega,p'}^{\ p''\sqsubseteq p'}$ $\rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p''\sqsubseteq p'}(_{p''}(\mathbb{P}(p'')));\ [\mathbf{NEC}_{q,\mu,\omega}\{p',\,p''\}]$

Chapter 8. Conclusion

This chapter contains a summary of the dissertation and briefly outlines two future extensions of the present work.

Summary

Handling the semantic connections established and elaborated upon in extended discourse represents a key challenge for understanding the notion of meaning involved in natural language interpretation. Devising a precise compositional interpretation procedure is particularly difficult for discourses involving complex descriptions of multiple related objects (individuals, events, times, propositions etc.), as for example, the discourses in (1), (2) and (3) below.

1. Every person who buys a^u computer and has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_u$.
2. **a.** Harvey^u courts a$^{u'}$ girl at every$^{u''}$ convention.
   b. She$_{u'}$ always$_{u''}$ comes to the banquet with him$_u$.
   (Karttunen 1976)
3. **a.** If^p a^u man is alive, he$_u$ must$_p$ find something pleasurable.
   b. Therefore, if$_p{}^{p'}$ he$_u$ doesn't have any spiritual pleasure, he$_u$ must$_{p'}$ have a carnal pleasure.
   (based on Thomas Aquinas)

The main achievement of this dissertation is the introduction of a representation language couched in classical type logic in which we can compositionally translate natural language discourses like (1), (2) and (3) above and capture their truth-conditions and the intricate anaphoric dependencies established in them.

The dissertation argues that discourse reference involves two equally important components with essentially the same interpretive dynamics, namely reference to *values*, i.e. (non-singleton) sets of objects (individuals and possible worlds), and reference to *structure*, i.e. the correlation / dependency between such sets, which is introduced and incrementally elaborated upon in discourse.

To define and investigate structured discourse reference, a new dynamic system couched in classical (many-sorted) type logic is introduced which extends Compositional DRT (Muskens 1996) with plural information states, i.e. information states are modeled as sets of variable assignments (following van den Berg 1996), which can be can be represented as matrices with assignments (sequences) as rows – as shown in the table in (4) below. A plural info state encodes both values (the columns of the matrix store sets of objects) and structure (each row of the matrix encodes a correlation / dependency between the objects stored in it).

4. Info State I	...	u	u'	p	p'	...
i_1	...	x_1 (i.e. ui_1)	y_1 (i.e. $u'i_1$)	w_1 (i.e. pi_1)	v_1 (i.e. $p'i_1$)	...
i_2	...	x_2 (i.e. ui_2)	y_2 (i.e. $u'i_2$)	w_2 (i.e. pi_2)	v_2 (i.e. $p'i_2$)	...
i_3	...	x_3 (i.e. ui_3)	y_3 (i.e. $u'i_3$)	w_3 (i.e. pi_3)	v_3 (i.e. $p'i_3$)	...
...	...	...	...	...	...	...

Values (sets of individuals or worlds): $\{x_1, x_2, x_3, ...\}$, $\{w_1, w_2, w_3, ...\}$ etc.   **Structure (relations between individuals and / or worlds):** $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, ...\}$, $\{<x_1, y_1, w_1>, <x_2, y_2, w_2>, <x_3, y_3, w_3>, ...\}$, $\{<w_1, v_1>, <w_2, v_2>, <w_3, v_3>, ...\}$ etc.

In Plural Compositional DRT (PCDRT), sentences denote relations between an input and an output plural info state. Indefinites and conditional antecedents non-deterministically introduce both values and structure, i.e. they introduce structured sets of individuals and possible worlds respectively; pronouns, verbal moods and modal verbs are anaphoric to such structured sets. Quantification over individuals and over possible worlds is defined in terms of matrices instead of single assignments and the semantics of the non-quantificational part becomes rules for how to fill out a matrix.

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

PCDRT enables us to account for a variety of phenomena, including: (**i**) mixed reading (weak & strong) relative-clause donkey sentences (chapter **5**), instantiated by example (1) above, (**ii**) quantificational subordination (chapter **6**), exemplified by discourse (2), and (**iii**) the complex interactions between entailment particles (i.e.

therefore), modal anaphora and modal subordination exhibited by discourse (3) above (chapter **7**).

In more detail, example (1) is a mixed reading (weak & strong) relative-clause donkey sentence which is interpreted as follows: for any person that is a computer buyer and credit card owner, for *every* (strong) computer s/he buys, s/he uses *some* (weak) credit card of her/his to pay for the computer. In particular, note that the weak indefinite $a^{u'}$ *credit card* co-varies with, i.e. is dependent on, the strong indefinite a^u *computer* (I can buy my Dell desktop with a MasterCard and my Toshiba laptop with a Visa) despite the fact that the two indefinites are syntactically trapped in their respective VP-conjuncts. The notion of plural info state employed in PCDRT enables us to capture this kind of non-local structured anaphoric dependencies (**i**) across VP-conjuncts and (**ii**) across clauses, i.e. between the two indefinites in the restrictor of the quantification in (1) and the two pronouns in the nuclear scope.

The PCDRT account successfully generalizes to the mixed reading DP-conjunction donkey sentences in (5) and (6) below, where the same pronoun is intuitively interpreted as having two distinct indefinites as antecedents – and the two indefinites have different readings (one is weak and the other is strong).

5. (Today's newspaper claims that, based on the most recent statistics:)
    Every^u company who hired a$^{\mathbf{str:}u'}$ Moldavian man, but no$^{u''}$ company who hired a$^{\mathbf{wk:}u'}$ Transylvanian man promoted him$_{u'}$ within two weeks of hiring.

6. (Imagine a Sunday fair where people come to sell their young puppies before they get too old and where the entrance fee is one dollar. The fair has two strict rules: all the puppies need to be checked for fleas at the gate and, at the same time, the one dollar bills also need to be checked for authenticity because of the many faux-monnayeurs in the area. So:)
    Everyone^u who has a$^{\mathbf{str:}u'}$ puppy and everyone$^{u''}$ who has a$^{\mathbf{wk:}u'}$ dollar brings it$_{u'}$ to the gate to be checked.

The above mixed reading DP-conjunction donkey sentences pose problems for the family of D-/E-type approaches to donkey anaphora because such approaches locate the

weak / strong donkey ambiguity at the level of the donkey pronouns. However, there is only one pronoun in both (5) and (6) above – and two distinct donkey readings associated with it. The PCDRT account, which locates the ambiguity at the level of the donkey indefinites, seems more plausible.

Furthermore, the PCDRT account predicts that the same indefinite cannot be interpreted as strong with respect to one pronoun (or any other kind of anaphor, e.g. a definite) and weak with respect to another pronoun – and this prediction seems to be borne out. By the same token, D-/E-type approaches predict the exact opposite: according to them, the same indefinite should be able to be interpreted as strong with respect to one pronoun and as weak with respect to another – which seems to be an incorrect prediction.

Discourse (2) is an instance of quantificational subordination. Crucially, its interpretation contrasts with the interpretation of discourse (7) below, whose first sentence is identical to (2a) above. Sentence (2a/7a) is ambiguous between two quantifier scopings: Harvey courts the same girl vs. a possibly different girl at every convention. Discourse (7) as a whole allows only for the "same girl" reading, while discourse (2) is compatible with both readings.

7. **a.** Harvey^u courts a$^{u'}$ girl at every$^{u''}$ convention. **b.** She$_{u'}$ is very pretty.
    (Karttunen 1976)

The non-local, cross-sentential interaction between quantifier scope and anaphora, in particular the fact that a *singular* pronoun in the second sentence can disambiguate between the two readings of the first sentence, can be captured in PCDRT because plural information states enable us to store both quantifier domains (i.e. values) and quantificational dependencies (i.e. structure), pass them across sentential boundaries and further elaborate on them, e.g. by letting a pronoun constrain the cardinality of a previously introduced quantifier domain.

The contrast between the two Karttunen examples is derived by giving a suitable dynamic reformulation of the independently motivated static meanings for generalized quantifiers and singular number morphology. In the process, we see how generalized

quantifiers enter anaphoric connections as a matter of course, usually functioning simultaneously as both indefinites and pronouns.

Finally, adding (discourse referents for) possible worlds to PCDRT enables us to account for discourse (3) above, which is a more explicit version of the naturally occurring discourse in (8) below.

8. [A] man cannot live without joy. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures.

(Thomas Aquinas)

Discourse (3) exhibits complex interactions between entailment particles (i.e. *therefore*), modal anaphora and modal subordination: on the one hand, *therefore* relates the propositional contents (formalized as sets of possible worlds) contributed by the premise (3a) and the conclusion (3b) and tests that they stand in an entailment relation; on the other hand, the premise and the conclusion themselves are modal quantifications and, consequently, relate a restrictor and a nuclear scope set of possible worlds.

Moreover, the propositional contents of the two modalized conditionals in (3a) and (3b) can be determined only if we are able to capture: (**i**) the donkey anaphoric connection between the indefinite a^u *man* in the antecedent of (3a) and the pronoun he_u consequent of (3a) and (**ii**) the fact that the antecedent of the conditional in (3b) is modally subordinationed to the antecedent of (3a), i.e. (3b) is interpreted as if the antecedent of (3a) is covertly repeated, i.e. as *if a man is alive and he doesn't have any spiritual pleasure, he must have a carnal pleasure*.

The discourse is analyzed in PCDRT as a network of structured anaphoric connections and the meaning (and validity) of the Aquinas argument emerges as a consequence of the intertwined individual-level and modal anaphora. Moreover, modal subordination is basically analyzed as quantifier domain restriction via structured modal anaphora; that is, the antecedent of (3b) is simultaneously anaphoric to the set of worlds and the set of individuals introduced by the the antecedent of (3a) and, also, to the quantificational dependency established between these two sets.

The dissertation is located at the intersection of two major research programs in semantics that have gained substantial momentum in the last fifteen years: (**i**) the development of theories and formal systems that unify different semantic frameworks and (**ii**) the investigation of the semantic parallels between the individual, temporal and modal domains. As the dissertation shows, one of the outcomes of bringing together these two research programs is a novel compositional account of non-local (modal and individual-level) quantificational dependencies as *anaphora to structure*.

The unification of different semantics frameworks, in particular Montague semantics, situation semantics and dynamic semantics (see Janssen 1986, Groenendijk & Stokhof 1990 and Muskens 1995a, 1995b, 1996 among others) enables us to incorporate the generally complementary strengths of these different frameworks and allows for an easy cross-framework comparison of alternative analyses of the same phenomenon.

Building on the Compositional DRT (CDRT) of Muskens (1996), chapters **2** through **4** of the dissertation incrementally develop a formal system couched in classical type logic which unifies dynamic semantics, in particular its account of basic kinds of cross-sentential / cross-clausal anaphora, and Montague semantics, in particular its compositional interpretation procedure and its account of generalized quantification.

The resulting CDRT+GQ system can compositionally account for a variety of phenomena, including cross-sentential anaphora, bound-variable anaphora, quantifier scope ambiguities and a fairly diverse range of relative-clause and conditional donkey sentences. Moreover, the analysis of donkey anaphora avoids the proportion problem and can account for simple instances of weak / strong donkey ambiguities. But CDRT+GQ cannot account for the three phenomena instantiated in (1), (2) and (3) above, i.e. mixed reading (weak & strong) relative-clause donkey sentences, quantificational subordination and the interaction between quantifier scope and number morphology on cross-sentential anaphora and modal anaphora, modal subordination and their interaction with entailment particles.

Plural Compositional DRT (PCDRT) pushes the framework unification program further and unifies in classical type logic the compositional analysis of selective

generalized quantification in Montague semantics, its account of quantifier scope ambiguities and singular number morphology with Dynamic Plural Logic (van den Berg 1994, 1996a, b). A novel, compositional account of mixed reading relative-clause donkey sentences (chapter **5**) and an account of quantificational subordination and its interaction with singular anaphora (chapter **6**) are some of the immediate benefits of this unification.

The introduction of (dref's for) possible worlds enables us to further extend PCDRT and unify it with the static Lewis (1973) / Kratzer (1981) analysis of modal quantification. The resulting Intensional PCDRT (IP-CDRT) system enables us to capture structured modal anaphora and modal subordination (chapter **7**).

The account brings further support to the idea that the dynamic turn in natural language semantics does not require us to abandon the classical approach to meaning and reference: I show that the classical notion of truth-conditional *content* (as opposed to *meaning*, which I take to be context-change potential) can be recovered within IP-CDRT and this enables us to analyze the entailment particle *therefore* as involving structured discourse reference to (propositional) contents, contributed by the premise(s) and the conclusion of an argument.

At the same time, Intensional PCDRT (IP-CDRT) pushes further the second research program, namely the investigation of anaphoric and quantificational parallels across domains.

The anaphoric (and quantificational) parallels between the individual and temporal domains have been noticed at least since Partee (1973, 1984) and they have been extended to the modal domain by Stone (1997, 1999) and, subsequently, by Bittner (2001, 2006) and Schlenker (2003, 2005b) among others.

IP-CDRT extends this research program and brings further support to the conjecture that our semantic competence is domain neutral by providing a point-for-point parallel account of quantificational and modal subordination. For example, the quantificational subordination discourse in (2) above is analyzed in the same way as the modal subordination discourse in (9) below; in particular, the interaction between *a^u girl-every*

convention and *she$_u$-always* in (2) is captured in the same way as the interaction between *a^u wolf-might* and *it$_u$-would* in (9).

9. A^u wolf might^p come in. It$_u$ would$_p$ attack Harvey first.
   (based on Roberts 1989)

IP-CDRT – which builds on and unifies Muskens (1996), van den Berg (1996a) and Stone (1999) – is, to my knowledge, the first dynamic system that systematically captures the anaphoric and quantificational parallels between the individual and modal domains (from the types of the discourse referents to the form that the translations of anaphoric and quantificational expressions have) while, at the same time, keeping the underlying logic classical and preserving the Montagovian approach to compositionality.

PCDRT differs from most previous dynamic approaches in at least three respects. The first difference is conceptual: PCDRT captures the idea that discourse reference to structure is as important as discourse reference to value and that the two have the same dynamics and should therefore be treated in parallel (contra van den Berg 1996a among others).

The second difference is empirical: the motivation for plural information states is provided by *singular* and *intra-sentential* donkey anaphora, in contrast to the previous literature which relies on *plural* and *cross-sentential* anaphora (see van den Berg 1996a, Krifka 1996b and Nouwen 2003 among others).

Finally, from a formal point of view, PCDRT accomplishes two non-trivial goals for the first time. On the one hand, it is not obvious how to recast van den Berg's Dynamic Plural Logic in classical type logic, given that, among other things, the former logic is partial and it conflates discourse-level plurality, i.e. plural information states, and domain-level plurality, i.e. non-atomic individuals (for more on this distinction, see the discussion of plural anaphora and quantification below).

On the other hand, previous dynamic analyses of modal quantification in the spirit of Lewis (1973) / Kratzer (1981), e.g. the ones in Geurts (1995/1999), Frank (1996) and Stone (1999), are not completely satisfactory insofar as they fail to associate modal quantifications with the propositional contents that they express (in a particular context)

and they fail to explicitly introduce these contents in discourse. Consequently, within these approaches, we cannot account for the fact that the entailment particle *therefore* relates such contents (across sentences), as shown, for example, by the Aquinas discourse in (3) above.

Two Extensions

The mostly foundational research pursued in this dissertation can be extended in various directions. I will outline here only two of them, namely:

- a cross-linguistic analysis of the interpretation and distribution of verbal moods when they occur under (particular kinds of) attitude verbs and in (particular kinds of) conditional structures;
- extending PCDRT with an account of plural anaphora and quantification.

De Se Attitudes and the Romanian Subjunctive B Mood

Intensional PCDRT seems to provide a suitable framework for a cross-linguistic investigation of aspect / tense / mood systems. I will illustrate the kind of issues that arise by briefly examining the interpretation and distribution of the subjunctive B mood in Romanian.

Romanian is the most widely spoken Romance language in the Balkan Sprachbund. Its distinctive position in the Indo-European spectrum has provided Romanian with a rich verbal morphology system, including two subjunctive (i.e. non-indicative finite) moods. The moods' distribution in intensional contexts is clearly interpretation-driven and the fine-grained distinctions drawn between different kinds of attitude reports and conditional structures suggest the existence of previously unnoticed semantic universals.

We will focus on the interpretation of the Romanian subjunctive B mood when embedded under attitude verbs like *crede* (believe), as shown in example (10) below. The main idea of the analysis is that subjunctive B is temporally and propositionally *de se* – thus extending the parallel between pronouns, tenses and moods to *de se* readings.

10. Maria crede   că   **ar**   fi   în   pericol.

    Mary believe.ind.pres.3s that **subjB**.3s be   in   danger.

    Mary believes that she is in danger.

Thus, the contrast between indicative and subjunctive B in Romanian is parallel to the contrast between overt pronouns (e.g. *John hopes that **he** will win*) and null PRO (e.g. *John hopes to win*) in the individual domain: as Chierchia (1989) and Schlenker (2003) observe, overt pronouns are compatible with both the *de se* and non-*de se readings*, while null PRO allows only for a *de se* reading. In particular, subjunctive B is parallel to PRO, in that it requires a temporally and propositionally *de se* reading, while indicative can, but does not have to receive such a reading.

Temporal *de se* means that the reported belief of being in danger is temporally located at the *internal now* of the believer, e.g., in (10) above, at the time at which Mary (correctly or not) thinks she entertains the belief that she is in danger. Propositional *de se* means that the believer has an attitude towards a 'self-referential' kind of content similar to the self-referential experience contents proposed by Searle (1983). For example, the content of my visual experience of seeing a yellow station wagon is that: (a) there is a yellow station wagon there and (b) the fact that there is a yellow station wagon there is causing *this* very visual experience. This 'self-referentiality' is the expression of the common sense intuition that having an experience or an attitude is assuming a particular point of view / perspective on the content of the experience or of the attitude.

Intuitively, a belief report with subjunctive B mood is propositionally *de se* insofar it *explicitly* encodes in the believed content this *perspectival* component inherent in any attitude; the form of such a report is basically *I believe that: p and p is what I believe*. Its redundancy is crucial in deriving two surprising empirical generalizations: on the one hand, in a report of the form *x believes that not p*, subjunctive B always takes wide scope with respect to embedded negation[1]; on the other hand, unlike the indicative mood, subjunctive B is incompatible with the adverb *probabil* (probably) in reports of the form *x believes that probably p*.

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[1] See Brasoveanu (2006a) for the clarification of what "wide scope" means in this particular context.

The interpretation of Romanian subjunctive B motivates a new analysis of attitude reports in terms of *centered propositions* as opposed to centered worlds (as in Lewis 1979a, Creswell & von Stechow 1982, Abusch 1997 among others). Moreover, these centered propositions have an essentially dynamic behavior: in a report of the form *x believes that p*, they are contributed by the matrix clause '*x believes...*' and then anaphorically retrieved and elaborated on by the embedded clause '...*that p*'.

The analysis of *de se* and *de re* belief in Lewis (1979a) involves three ingredients:

- *centered worlds*: the believed content is not a proposition, i.e. a set of worlds (as the standard analysis would have it[2]), but a property, or, equivalently, a set of centered worlds[3]. A centered world is a pair $<w, x^{self}>$, where w is a world and x^{self}, the center of world w, is the individual that Neo takes himself to be in w, i.e. the belief-internal 'self';

- *self ascription*: the verb believe is interpreted as a relation between an individual and a set of centered worlds (and not as a relation between an individual and a proposition). That is, we replace the function $\mathbf{dox}_{w*,x*}$ that returns a set of worlds (the set of $x*$'s doxastic alternatives to world $w*$) with a function $\mathbf{self\_ascribe}_{w*,x*}$, which returns a set of centered worlds $<w, x^{self}>$. Crucially, given the "two god" argument in Lewis (1979a), we might have two distinct self-ascribed pairs $<w, x^{self}>$ and $<w, y^{self}>$ that contain the same world w but different individuals x^{self} and y^{self};

- *acquaintance relations*: the reported belief is about an individual with whom the belief-internal 'self' is acquainted in a particular way. In the *de se* case, the acquaintance relation is the most intimate relation the belief-internal 'self' can have with any individual whatsoever, namely the identity relation.

The analysis of temporal *de se / de re* is parallel to the analysis of individual *de se / de re*. Following Abusch (1997), we only extend centered worlds with a variable for time: Heimson is self-ascribing in world $w*$ at time $t*$ a set of centered worlds that are now represented as triples $<w, x^{self}, t^{now}>$, where x^{self} is the individual that Heimson takes

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[2] See for example Hintikka (1969).

[3] See for example Creswell & von Stechow (1982) for more discussion.

himself to be in w and t^{now} is the time that Heimson takes his internal 'now' to be in w. Moreover, we will also have acquaintance relations relative to time intervals.

The incompatibility between subjunctive B and *probabil* in reports of the form *x believes that probably q* suggests that centered worlds should be generalized to centered propositions, i.e. to triples of the form $<p, f^{self}, g^{now}>$, where:

* p is a set of possible worlds (of type $\mathbf{w}t$), i.e. the set of x's doxastic alternatives;
* f^{self} is a relation[4] between worlds and individuals (of type $\mathbf{w}(et)$) that specifies, for each doxastic alternative $w \in p$, what individual(s) x takes himself to be in w;
* g^{now} is a relation between worlds and time-intervals (of type $\mathbf{w}(\tau t)$[5]).

The basic idea of the centered-propositions analysis is that, in a belief report of the form *x believes + embedded clause*, the matrix clause *x believes* sets up the context for the interpretation of the embedded clause by contributing a centered proposition relative to which the embedded clause is interpreted. The matrix clause basically introduces a centered proposition discourse referent (more exactly, three suitably related discourse referents – for p, f^{self} and g^{now}), which is (are) anaphorically accessed by the embedded clause.

The incompatibility between subjunctive B and *probabil* is a consequence of the fact that subjunctive B is anaphoric to the set of doxastic alternatives p and requires the proposition q expressed by the embedded clause to be true in every doxasatic alternative w in p, while *probabil* implicates that that there must be at least one world w in p where q is false (see Brasoveanu 2006a for more discussion).

This analysis is independently motivated by the fact that a subsequent matrix clause with a subjunctive B mood can also be interpreted relative to the same centered proposition (in fact: it has to be interpreted in this way) – as shown by (11) below. The

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[4] f^{self} and g^{now} are relations between worlds and individuals / times and not functions from worlds to individuals / times because of the "two god" argument in Lewis (1979a).

[5] Where τ is whatever type we decide to assign to temporal intervals, e.g. it might be a basic type or a characteristic function of convex sets of temporal instants etc.

subjunctive B sentence in (11b) has to be interpreted as a further elaboration on Mary's beliefs[6] and cannot be interpreted as stating that John has beautiful eyes in the actual world.

11. **a.** Maria   crede   că   Ion   **ar**   fi   chipeş.

    Mary   believes   that   John   **subjB**.3s   be   handsome.

    Mary believes that John is handsome.

    b. Ar   avea   ochi   frumoşi.

    subjB.3s   have   eyes   beautiful.

    [She believes that] he has beautiful eyes.

The fact that plural information states are basically designed to store and pass on information about quantificational dependencies between multiple objects makes IP-CDRT an ideal framework for the formalization of *de se* reports in terms of anaphora to centered propositions.

Basically, a verb like *believe* would introduce three discourse referents: p (a modal dref of type $s\mathbf{w}$), u^{self} (an individual-level dref of type se) and χ^{now} (a temporal dref of type $s\tau$). The correlation between worlds, individuals and times and anaphora to it is just another instance of discourse reference and anaphora to structure and, as expected, it will be store in a plural info state I_{st}. That is, instead of having to build the quantificational dependencies into complex functions (see the triples $<p, f^{self}, g^{now}>$ above), the dependencies emerge as a consequence of the independently motivated account of structured discourse reference in IP-CDRT: for each 'assignment' i_s in info state I, $u^{self}i$ is the individual that Heimson takes himself to be in world pi and $\chi^{now}i$ is the time that Heimson takes his internal 'now' to be in world pi.

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[6] We can even have modal subordination, as shown in (i) below.

(i) **a.** Maria crede că **ar** fi vampiri în LA.

   Mary believes that there are (**subjB**) vampires in LA.

   b. Ar intra noaptea în case şi **ar** ataca oamenii în somn.

   [She believes that] they break (**subjB**) into houses at night and attack (**subjB**) people in their sleep.

Given that IP-CDRT is couched in type logic, we preserve the static, compositional analysis of attitude reports while, at the same time, being able to account for the possibility of structured cross-sentential anaphora to centered propositions (see (11) above) and for the interpretation and distribution of the Romanian subjunctive B mood.

The analysis of subjunctive B sketched above raises at least the following three questions, which I leave for future research:

- how is subjunctive B located within the mood system of Romanian, in particular, how does its interpretation and distribution differ from indicative and subjunctive A (analyzed in Farkas 1985, 1992)?

- what are the similarities and differences between the Romanian subjunctive A and B moods and the non-indicative moods of other Indo-European and non-Indo-European languages, e.g. the French subjunctive investigated Schlenker (2005a) (among others), the German reportive subjunctive analyzed in Fabricius-Hansen & Saebø (2004), the English subjunctive (see Frank 1996, Stone 1997, Condoravdi 2001, Ippolito 2003 among others) or the Kalaallisut dependent moods analyzed in Bittner (2006)?

- can we successfully generalize IP-CDRT to capture the entire verbal mood system in Romanian and to accommodate a broader range of aspect / tense / mood systems attested in other languages?

Plural Anaphora and Quantification

Given that the main arguments for plural information states are based on morphologically singular anaphora and not on plural anaphora (as in the previous dynamic literature), the following question arises: what is the relationship between plural information states and the pluralities involved in morphologically plural anaphora?

My answer to this question is that the two notions of plurality are distinct, which goes against the seemingly received wisdom in the literature (see van den Berg 1996a, Krifka 1996b and Nouwen 2003 among others). Morphologically plural anaphora involves *domain-level plural reference*, i.e. non-atomic individuals of the kind countenanced in Link (1983) among many others. In contrast, plural information states

formalize a notion of *discourse-level plural reference* (more precisely: a notion of plural discourse reference), which encodes discourse reference to quantificational dependencies established and elaborated upon in discourse between (non-singleton) sets of objects, be they atomic and / or non-atomic individuals.

This systematic distinction and the ensuing extension of PCDRT with non-atomic individuals (see Brasoveanu 2006c) enable us to provide a unified account of several phenomena.

First, we can account for the fact that singular donkey anaphora can involve non-singleton sets of atomic individuals while being incompatible with collective predicates, as shown in (12) below.

12. #Every farmer who owns a^u donkey gathers it$_u$ around the fire at night.
    (based on Kanazawa 2001)

Second, we can capture the intuitive parallel between multiple singular and plural donkey anaphora exhibited by the examples in (13) and (14) below (see chapter **5** for the PCDRT analysis of (13)).

13. Every boy who bought a^u gift for a$^{u'}$ girl in his class asked her$_{u'}$ deskmate to wrap it$_u$.

14. Every parent who gives a^u balloon to two$^{u'}$ boys expects them$_{u'}$ to end up fighting (each other) for it$_u$.
    (based on an example due to Maria Bittner, p.c.)

The parallel between singular and plural donkey anaphora also covers weak donkey readings – see (15) and (16) below – and 'sage plant' example – see (17) and (18) below.

15. Every person who has a^u dime will put it$_u$ in the meter.
    (Pelletier & Schubert 1989)

16. Every person who has two^u dimes will put them$_u$ in the meter.

17. Everybody who bought a^u sage plant here bought eight others along with it$_u$.
    (Heim 1982/1988)

18. Everybody who bought two^u sage plants here bought seven others along with them$_u$.

The novel distinction between plural reference and plural discourse reference as well as the (partly novel) empirical observations above hardly begin to explore three important issues.

First, what are the necessary ontological (i.e. domain-level) commitments for an adequate treatment of plurality in natural language?

Second, what is the relationship between the instances of anaphora in the examples above that are both morphologically and semantically plural and morphologically singular anaphora that (usually) is semantically plural of the kind instantiated by quantificational subordination and telescoping discourses (see (19) and (20) below) – which were analyzed in chapter **6** above?

19. Every chess set comes with a^u spare pawn. It$_u$ is taped to the top of the box.
    (Sells 1984, 1985)
20. Each^u candidate for the space mission meets all our requirements. He$_u$ has a PhD in Astrophysics and extensive prior flight experience.
    (Roberts 1987)

Finally, is there any cross-linguistic variation in the morphological realization of semantically plural anaphora and quantification and, if so, what are the parameters of variation and what is their significance for the current theories of domain-level and discourse-level plurality?

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