Decomposing Modal Quantification

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Abstract

Providing a compositional interpretation procedure for discourses in which descriptions of complex dependencies between interrelated objects are incrementally built is a key challenge for natural language semantics. This paper focuses on the interactions between the entailment particle *therefore*, modalized conditionals and modal subordination. It shows that the dependencies between individuals and possibilities that emerge out of such interactions can receive a unified compositional account in a system couched in classical type logic that integrates and simplifies van den Berg's Dynamic Plural Logic and the classical Lewis-Kratzer analysis of modal quantification. The main proposal is that modal quantification is a composite notion, to be decomposed in terms of discourse reference to quantificational dependencies that is multiply constrained by the various components that make up a modal quantifier. The system captures the truth-conditional and anaphoric components of modal quantification in an even-handed way and, unlike previous accounts, makes the propositional contents contributed by modal constructions available for subsequent discourse reference.

1 Introduction

Providing a compositional interpretation procedure for sentences and discourses in which complex descriptions of dependencies between multiple, interrelated objects are incrementally built has proved to be a key challenge for formal theories of natural language interpretation. Consider, for example, the following discourse, part of the text of an LSAT logic puzzle.[1]

(1) [Preamble] An amusement park roller coaster includes five cars, numbered 1 through 5 from front to back. Each car accommodates up to two riders, seated side by side. Six people – Tom,

[1] Available online at http://www.west.net/~stewart/lwsample%5Bq%5D.htm.

> Gwen, Laurie, Mark, Paul and Jack – are riding the coaster at the same time. Laurie is sharing a car. Mark is not sharing a car and ...

The first sentence sets up the basic situation we are invited to consider, namely a roller coaster that has five cars. The text contains two quantificational expressions – the singular indefinite *a roller coaster* and the cardinal indefinite *five cars*, the first of which takes scope over the second.

The second sentence elaborates on these objects. The quantifier *each car* refers back to the five cars and further constrains the information we have about them: any one of these five cars can accommodate *up to two riders*. Yet again, the first quantifier (the distributive universal *each car*) takes scope over the second one (the modified cardinal indefinite *up to two riders*).

Against the background information provided by the first two sentences, the third sentence invites us to consider a more specific situation: six people are riding the coaster (which was introduced in the first sentence) at the same time. Ignoring the apposition that enumerates the names of the just mentioned six people, this sentence relates three sets of objects, contributed by the three nominal expressions in the sentence: six people, the anaphorically-retrieved roller coaster and the six periods of time that each person rides the coaster. The non-anaphoric definite *the same time* has narrow scope with respect to the indefinite *six people* and, thereby, requires these six periods of time to be identical.

The remaining sentences of the preamble provide additional information about how the six people and the five cars in our scenario are related. The preamble is then further elaborated upon by questions like (2a) through (2d) below.

(2)
  a. [Q1] Which of the following groups of riders could occupy the second car? (a) Laurie only. (b) ...
  b. [Q2] If Gwen is riding immediately behind Laurie's car and immediately ahead of Tom's car, all of the following must be true except: (a) Gwen is riding in the fourth car. (b) ...
  c. [Q3] Which one of the following statements cannot be true? ...
  d. [Q4] If Paul is riding in the second car, how many different combinations of riders are possible for the third car?

These questions invite us to consider alternative scenarios featuring the previously mentioned entities. Q1 above focuses on the second car and the hypothetical scenarios featuring different people (from our set of six) that ride in that car. Q2 invites us to consider a scenario in which three of the six people ride in consecutive cars etc. That is, our description relating a roller coaster, its five cars and its six riders (at a particular time) is enriched now by the fact that we consider various possibilities (i.e., hypothetical scenarios) and how these possibilities relate to the previously mentioned objects.

Moreover, the *propositional contents* of the sentences in the preamble must also be available in subsequent discourse. We need to be able to pool them together in a modal base (to use the terminology of Kratzer 1981) relative to which the modal verb *cannot* in Q3 above is interpreted. Furthermore, this discourse-contributed modal base can be subsequently updated with additional contents. For example, in Q4, the content of the conditional antecedent *if Paul is riding in the second car* is temporarily added and we are asked to identify certain possibilities compatible with the resulting modal base.

To successfully solve logic puzzles like this one, i.e., to correctly answer their questions, we need to have a precise understanding of the sets of objects and the relations between them that are incrementally described by the quantifiers, modals, pronouns etc. occurring in the text of the puzzle. That is, we need to be able to associate such natural-occurring texts with precisely-specified meanings, which, in turn, can form the basis for subsequent logical reasoning. As Lev (2007:10) observes, "whereas for humans the language understanding part of logic puzzles is trivial but the reasoning is difficult, for computers it is clearly the reverse."

The goal of this paper is to argue that we can make good progress on the first front, i.e., formally specifying precise meanings for discourses in which intricate descriptions of interrelated objects are incrementally constructed, if we generalize the classical Tarskian semantics for first-order logic in two ways.

First, we will take our contexts of evaluation to be modeled by a set of assignments G instead of a single assignment g. In the Montagovian tradition, the variable assignment is an essential part of the context of evaluation, storing the referents of anaphoric pronouns, past tense etc. Given that we want to elaborate on sets of referents and relations between them, we take our context of evaluation to consist of a set of assignments. In this way, we will be able to elaborate on the relations between the sets of values associated with variables over individuals x, y etc. and the set of possibilities associated with variables over possible worlds w etc. Such a set of assignments G can be represented as a matrix, exemplified in (3) below. The rows of the matrix represent variable assignments g_1, g_2, g_3 etc. The columns represent variables x, y, w etc. The objects in the cells of the matrix are values that assignments assign to variables: $car_1 = g_1(x)$, $car_2 = g_2(x)$, $rider_1 = g_1(y)$, $rider_2 = g_2(y)$,

$v_1 = g_1(w), v_2 = g_2(w)$ etc.

(3)

G	...	x	y	...	w	...
g_1	...	car_1	$rider_1$	...	v_1	...
g_2	...	car_2	$rider_2$	...	v_2	...
g_3	...	car_3	$rider_3$	...	v_3	...
...	...	...	...	...	...	...

or simply:

...	x	y	...	w	...
...	car_1	$rider_1$	...	v_1	...
...	car_2	$rider_2$	...	v_2	...
...	car_3	$rider_3$	...	v_3	...
...	...	...	...	...	...

The second generalization is going from a static semantics, where expressions are interpreted relative to a single context of evaluation G, to a dynamic semantics, where expressions are interpreted relative to a pair of contexts of evaluation $\langle G, H \rangle$ – see, e.g., Discourse Representation Theory (DRT; Kamp 1981, Kamp & Reyle 1993), File Change Semantics (FCS; Heim 1982) and Dynamic Predicate Logic (DPL; Groenendijk & Stokhof 1991).

The first member of the pair, i.e., G, is the input context relative to which natural language expressions are interpreted. This part is exactly as in static semantics. The second member of the pair, i.e., H, is the output context, which is the context that results after natural language expressions are interpreted. Interpreting natural language expressions relative to such pairs of contexts, i.e., as programs that incrementally update the discourse context, enables us to incrementally build the complex descriptions of interrelated objects needed for the interpretation of logic puzzles (among other things).

The present paper develops a formal semantics along these lines, building on van den Berg (1996) (see also Krifka 1996 and Nouwen 2003), and shows how discourses involving quantifiers, indefinites, modal verbs and pronouns can be compositionally interpreted, where composition is understood in the classical Fregean / Montagovian sense.

The focus is on modal expressions and their interactions with each other and with individual-level anaphora in discourse. The main proposal is that quantifiers over individuals and possible worlds should be decomposed into smaller, atomic components that manipulate contexts of evaluation in simple ways – and which together conspire to associate quantifiers, modal verbs, conditionals etc. with their intuitively correct truth conditions and anaphoric potential.

Thus, the intra- and cross-sentential interactions between quantification and anaphora in the modal and individual domains receive a unified compositional account in a system couched in classical type logic that integrates and simplifies van den Berg's Dynamic Plural Logic and the classical Lewis-Kratzer analysis of modal quantification. The system captures the truth-conditional and anaphoric components of modal quantification in an even-handed way and, unlike previous accounts, makes the propositional contents contributed by modal constructions available for subsequent discourse reference.

1.1 Relating Variables / Discourse Referents and Variable Assignments

We can think of variables, a.k.a. discourse referents (drefs), and variable assignments in two ways. Classical static and dynamic semantics takes variables to be basic entities and variable assignments to be composite objects, namely functions from variables to appropriate values. Taking variables to be the basic building blocks is pre-theoretically appealing: as Karttunen (1976) and Webber (1978) first argued, natural language interpretation involves an irreducible notion of discourse-level reference and the referents that are introduced, constrained and related to each other in discourse are distinct from the actual referents.

However, we want to make our discourse-interpretation procedure compositional at the sub-clausal level – and we can preserve the Montagovian solution to the problem of compositionality if we change our perspective on the relationship between variables and assignments and take assignments to be the basic building blocks and variables to be the composite, functional objects. The idea is to think of variables as projection functions over assignments (following Landman 1986): instead of a variable assignment g taking the variables x, y, w etc. as arguments and assigning them an individual or a possible world as values, we 'type-lift' variables and think of them as projection functions that take assignments / sequences of objects as arguments and return individuals or possible worlds as values.[2]

To reflect this change in perspective, we will use u_1, u_2, u, u' etc. to denote 'type-lifted' variables / drefs for individuals and p_1, p_2, p, p' etc. to denote 'type-lifted' variables / drefs for possible worlds. Also, we will use i, j, i_1, i_2, i', i'' etc. to denote variable assignments. The value of a dref u_1 at an assignment i is obtained by applying the function denoted by the dref to the atomic entity denoted by the assignment:

[2]For more discussion of these issues, see Groenendijk & Stokhof (1990), Chierchia (1995), Muskens (1996), Sternefeld (2001), Szabolcsi (2003) and Brasoveanu (2008) among others.

$u_1(i)$ – or $u_1 i$ for short. A suitable set of axioms ensures that the atomic entities i, j etc. behave as variable assignments; see Muskens (1996) and Brasoveanu (2008) among others for more discussion.

Viewed in this way, i.e., as functions from variable assignments to individuals, variables become parallel to Montagovian individual concepts – and open the way for defining a compositional dynamic system in the tradition of Montague semantics. For example, the individual $u_1 i$ is the individual that the dref u_1 denotes relative to the assignment i and the possible world $p_1 i$ is the world that the dref p denotes relative to i, much like the individual concept f_{chair} denoted by the definite description *the chair of the linguistics department* associates different individuals $f_{chair}(t)$, $f_{chair}(t')$ etc. with different times of evaluation t, t' etc.

The values, i.e., the objects in the cells of a matrix, remain the same, only this time they are the actual referents that drefs have relative to different variable assignments: $car_1 = u_1 i_1$, $car_2 = u_1 i_2$, $rider_1 = u_2 i_1$, $rider_2 = u_2 i_2$, $v_1 = p_1 i_1$, $v_2 = p_1 i_2$ etc. This is shown in (4) below, which is identical to (3) above except for the notational changes we just introduced.

(4)

I	...	u_1	u_2	...	p_1	...
i_1	...	car_1	$rider_1$	...	v_1	...
i_2	...	car_2	$rider_2$	...	v_2	...
i_3	...	car_3	$rider_3$	...	v_3	...
...	...	...	...	...	...	...

or simply:

...	u_1	u_2	...	p_1	...
...	car_1	$rider_1$	...	v_1	...
...	car_2	$rider_2$	...	v_2	...
...	car_3	$rider_3$	...	v_3	...
...	...	...	...	...	...

A dref stores a set of values relative to a set of assignments I – or, as we will call it following the dynamic semantics tradition, relative to a plural info state I. These sets of values are represented in the columns of matrices like (4) above. For example, the set of individuals $u_1 I := \{u_1 i : i \in I\}$ is the u_1 column of matrix I above and the set of possible worlds (i.e., the proposition) $p_1 I := \{p_1 i : i \in I\}$ is the p_1 column. Thus, columns associated with drefs for possible worlds encode propositional contents, i.e., classical truth conditions – and rows encode anaphoric information about values and about dependencies between values that determine these truth conditions.

Under this view, natural language sentences and discourses denote programs incrementally updating such plural info states. The semantic values of sentences and discourses are, therefore, binary relations between an input plural info state I – the info state that is updated – and an output plural info state J – the info state that is the result of the update.

Quantification over individuals and possible worlds is defined in terms of matrices instead of single assignments and the semantics of the non-quantificational part (lexical items like *car*, *ride* etc.) consists of rules for how to fill out a matrix.

This paper is dedicated to the detailed analysis of several discourses involving quantifiers, modals and anaphora that exemplify the ways in which these expressions interact to incrementally build complex descriptions of relations between sets of objects. These (mostly constructed) discourses will perforce be shorter and sketchier than the naturally-occurring logic puzzle texts. This will enable us to focus exclusively on the analysis of modals, quantifiers and pronouns. The remainder of this section introduces these discourses and briefly discusses them.

1.2 Quantificational and Modal Subordination

Quantifiers like *every convention* in (5a/6a) below (from Karttunen 1976) are typical examples of expressions that introduce – and elaborate on previously introduced – dependencies. In all examples, subscripts indicate the anaphors and superscripts their antecedents, following the notational convention in Barwise (1987).

(5) a. Harvey courts a^u woman at every convention.
b. She$_u$ is very pretty.

(6) a. Harvey courts a^u woman at every convention.
b. She$_u$ always comes to the banquet with him.
c. [The$_u$ woman is usually also very pretty.]

Consider, for example, the initial sentence (5a/6a) in the two discourses above. This sentence is ambiguous between two quantifier scopings: it "can mean that, at every convention, there is some woman that Harvey courts or that there is some woman that Harvey courts at every convention. [. . .] Harvey always courts the same [woman] [. . .] [or] it may be a different [woman] each time" (Karttunen 1976:377). The contrast between (5b) and (6b) is that the former allows only for the 'same woman' reading of sentence (5a/6a), while the latter is also compatible with the 'possibly different women' reading.

Discourse (5) raises the following question: how can we capture the fact that a singular pronoun in sentence (5b) can interact with and disambiguate quantifier scopings in sentence (5a)?

The fact that number morphology on the pronoun *she*$_u$ is crucial is shown by the discourse in (7) below, where the (preferred) relative scoping of *every convention* and *a*^{u} *woman* is the opposite of the one in discourse (5).

(7) a. Harvey courts a^u woman at every convention.
b. They$_u$ are very pretty.

We can see that it is indeed quantifier scopings that are disambiguated if we replace the indefinite *a*^{u} *woman* in (5a) with *exactly one*^{u} *woman*. This yields two truth-conditionally independent scopings: (*i*) *exactly one*^{u} *woman>>every convention*, which is true in situation in which Harvey courts more than one woman per convention as long as there is exactly one that he never fails to court, and (*ii*) *every convention>>exactly one*^{u} *woman*, where Harvey courts exactly one woman per convention, but the woman can be different from convention to convention.

Discourse (6) raises the following questions. First, why is it that adding an adverb of quantification, i.e., *always* / *usually*, preserves both readings of sentence (6a) and makes them available for the discourse as whole? Moreover, on the newly available reading of sentence (6a), i.e., the *every convention>>a*^{u} *woman* scoping, how can we capture the intuition that the singular pronoun *she*$_u$ and the adverb *always* in sentence (6b) elaborate on the quantificational dependency between conventions and women introduced in sentence (6a), i.e., how can we capture the intuition that we seem to have simultaneous anaphora to the two quantifier domains and the quantificational dependency between them?

Thus, quantifiers are not the only kind of expressions that can introduce and elaborate on dependencies. Pronouns, e.g., *she*$_u$ in (5b/6b), and indefinites, e.g., *a*^{u} *woman* in (5a/6a), can also do this and interact with the dependencies introduced by quantificational expressions.

The kind of interaction between quantifiers, indefinites and morphologically singular anaphora exemplified in discourses (5) and (6) above has become known under the label of quantificational subordination (see Heim 1990:139,(2)). Quantificational subordination phenomena suggest that the notion of generalized quantification involved in natural language interpretation should in fact be decomposed into at least two components:

(*i*) a static generalized quantifier component relating sets of individuals (as in Barwise & Cooper 1981 among many others)
(*ii*) one or more components operating over plural info states that regulate the dynamics of dependencies

Decomposing quantification along these lines enables us to account for the contrast between discourses (5) and (6) while preserving the Montagovian solution to the compositionality problem. The basic idea is that plural info states enable us to store both quantifier domains (in the columns of the matrix) and quantificational dependencies (in the rows of the matrix), pass them across sentential boundaries and further elaborate on them – for example, by letting a singular pronoun like *she* constrain the cardinality of a previously introduced quantifier domain.

This account of quantificational subordination generalizes to modal subordination. The resulting analysis of the modal-subordination discourse in (8) below (based on Roberts 1989) is point-for-point parallel to the analysis of the quantificational-subordination discourse in (6) above. We are, therefore, able to capture the anaphoric and quantificational parallels between the individual and modal domains argued for in Geurts (1995/1999), Frank (1996), van Rooy (1998), Stone (1999), Bittner (2001) and Schlenker (2005) among others (building on Partee 1973, 1984).

(8) a. A^u wolf might come in.
b. It_u would eat Harvey first.

1.3 Entailment Particles

In addition, we want to be able to analyze the more complex interactions between modal and individual-level anaphora exhibited by discourses like (9) below (attributed to Thomas Aquinas).

(9) a. [A] man cannot live without joy.
b. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures.

We will focus on only one of the meaning dimensions of this discourse, namely the entailment relation established by *therefore* between the modal premise (9a) and the modal conclusion (9b).

First, we want to capture the meaning of the entailment particle *therefore*, which relates the content of the premise (9a) and the content of the conclusion (9b) and requires the latter to be entailed by the former. We will take the content of a sentence to be truth-conditional in nature, i.e., to be the set of possible worlds in which the sentence is true, and entailment to be content inclusion, i.e., (9a) entails (9b) iff for any world w, if (9a) is true in w, so is (9b).[3]

[3]Modeling the entailment relation expressed by *therefore* as a truth-conditional relation, i.e., as requiring inclusion between two sets of possible worlds, cannot account for the fact that the discourse *π is an irrational number, therefore Fermat's last theorem is true* is not intuitively acceptable as a valid entailment and it cannot be accepted as a mathematical proof despite the fact that both sentences are necessary truths (i.e., they are true in every possible world). For expository simplicity, we will ignore hyper-

Second, we are interested in the meanings of (9a) and (9b). We will take meaning to be context-change potential, i.e., to encode both content (truth conditions) and anaphoric potential.

Thus, on one hand, we are interested in the contents of (9a) and (9b). They are both modal quantifications. Sentence (9a) involves a circumstantial modal base (to use the terminology introduced in Kratzer 1981) and says that, in view of the circumstances, i.e., given that God created man in a particular way, as long as a man is alive, he must find some thing or other pleasurable. Sentence (9b) involves the same modal base and elaborates on the preceding modal quantification: in view of the circumstances, if a man is alive and has no spiritual pleasure, he must have a carnal pleasure. Importantly, we need to make the contents of (9a) and (9b) accessible in discourse so that the entailment particle *therefore* can cross-sententially relate them.

On the other hand, we are interested in the anaphoric potential of (9a) and (9b), i.e., in the anaphoric connections between them. These connections are explicitly represented in discourse (10) below, which is intuitively equivalent to (9) albeit more awkwardly phrased.

(10) a. If a^{u_1} man is alive, he$_{u_1}$ must find something^{u_2} pleasurable / he$_{u_1}$ must have a^{u_2} pleasure.
b. Therefore, if he$_{u_1}$ doesn't have any^{u_3} spiritual pleasure, he$_{u_1}$ must have a^{u_4} carnal pleasure.

The indefinite *a*^{u_1} *man* in the antecedent of the conditional in (10a) introduces the dref u_1, which is anaphorically retrieved by the pronoun *he*$_{u_1}$ in the antecedent of the conditional in (10b). This is an instance of modal subordination, i.e., an instance of simultaneous modal and individual-level anaphora: the interpretation of the conditional in (10b) is such that we seem to covertly duplicate the antecedent of the conditional in (10a) – if *a man is alive and* he doesn't have any spiritual pleasure, he must have a carnal one.

We will henceforth discuss the more transparent discourse in (10) instead of the naturally-occurring one in (9). The challenge posed by (10) is that we need to compositionally assign meanings to the three discourse segments listed below – and we need to do that in such a way that we capture both the intuitively correct truth conditions of the whole discourse and the modal and individual-level anaphoric connections between the two conditionals and within each one of them.

(*i*) the modalized conditional in (10a), i.e., the premise
(*ii*) the modalized conditional in (10b), i.e., the conclusion

intensionality throughout the paper, but nothing in the proposed account prevents us from replacing sets of worlds with a suitably finer-grained notion of content.

(*iii*) the entailment particle *therefore*, which relates the premise and the conclusion

Plural info states enable us to analyze discourse (9/10) as a network of interrelated anaphoric connections – and the validity of the Aquinas argument will emerge as a consequence of the intertwined individual-level and modal anaphora.

The analysis brings further support to the idea that the dynamic turn in natural language semantics should *explicitly* preserve and incorporate the classical, static approach to meaning and reference. In fact, to analyze the Aquinas argument in (9/10), we use propositional drefs p_1, p_2 etc. to make classical, static propositional contents available for subsequent discourse reference – i.e., not merely available in the meta-language as part of the recursive definition of truth and satisfaction, but available in the representation / object language –, which in turn enables us to analyze the argument as relating drefs storing the propositional contents of its premise and conclusion.

The paper is structured as follows. Section 2 discusses the simpler case of quantification over individuals, its decomposition in a dynamic system based on plural info states and the resulting account of quantificational subordination. A good part of the material is not new, so the discussion will be fairly compressed (see Brasoveanu 2010 for more details). Working knowledge of Dynamic Predicate Logic (DPL, Groenendijk & Stokhof 1991) and Compositional DRT (CDRT, Muskens 1996) is probably a prerequisite for section 2 and the rest of the paper (for a review, see Brasoveanu 2007:ch.2,3 among many others). Section 3 tackles the more complex case of modal quantification and introduces the intensional dynamic system within which modal and individual-level quantification, as well as modal and quantificational subordination, receive a parallel decompositional analysis. Section 4 shows that the account of entailment particles and of the Aquinas discourse follows in this system. Finally, section 5 briefly compares this account with previous analyses of modal quantification and modal subordination.

2 Decomposing Quantification over Individuals

The goal of this section is to introduce the basic dynamic system that enables us to account for quantificational subordination. By simply adding possible-world drefs in the following section, we will be able to account for modal subordination in a way that explicitly captures the anaphoric and quantificational parallels between the individual and modal domains.

The knowledgeable reader eager to get to get to the heart of the matter, namely modal quantification and modal subordination, should feel free to skim this section and start reading in earnest only the next one. The less eager reader will find that I have repeatedly highlighted the parts of this section that will be more or less directly imported in the next section (and these parts make up the bulk of this section). Moreover, the dynamic perspective taken on individual-level quantification in this section is, in fact, a modal perspective:[4] variable assignments are conceptualized as atomic, basic entities, i.e., points / states / worlds in a modal model, and operations on variable assignments are ultimately conceptualized as modal operations over such points, in the spirit of van Benthem (1997) (see also Ben-Shalom 1996 and Marx & Venema 1997 among others).

Thus, in this section, we want to capture the fact that discourse (5) above allows for only one of the two quantifier scopings of sentence (5a/6a), while discourse (6) allows for both scopings.

Informally, the analysis proceeds as follows. First, sentence (5a) updates the discourse-initial info state ∅ – which stores no discourse information whatsoever – by introducing the dref u_1 that stores the (possibly singleton) set of women that Harvey courts at some convention or other and the dref u_2 that stores all the conventions. This update can happen in two ways, depending on whether the indefinite scopes over the universal quantifier or *vice versa*, as shown in (11) and (12).

(11) $\emptyset \xrightarrow{\text{a}^{u_1}\text{ woman (H. courts at every}^{u_2}\text{ convention)}}$

u_1	u_2	
$woman_1$	$conv_1$	$woman_1$ is courted at every convention
$woman_1$	$conv_2$	
$woman_1$	$conv_3$	

(12) $\emptyset \xrightarrow{\text{at every}^{u_2}\text{ convention (H. courts a}^{u_1}\text{ woman)}}$

u_1	u_2	
$woman_1$	$conv_1$	$woman_1$ is courted at $conv_1$
$woman_2$	$conv_2$	$woman_2$ is courted at $conv_2$
$woman_3$	$conv_3$	$woman_3$ is courted at $conv_3$

Irrespective of which quantifier scoping we choose for sentence (5a), the singular pronoun she_{u_1} in sentence (5b) constrains the set of u_1-

[4] As Paul Dekker graciously reminded me (p.c.).

women to be a singleton set. This is easily satisfied in (11), where the indefinite takes scope over the universal quantifier. In the case of (12), however, the singleton requirement contributed by the singular number morphology on the pronoun she_{u_1} makes a non-trivial contribution: it requires all the cells in the u_1-column to store the same entity, as shown in (13) below. Thus, irrespective of which quantifier scoping we choose for sentence (5a), the only available reading for discourse (5) as a whole is the wide-scope indefinite reading.

(13)

u_1	u_2
$woman_1$	$conv_1$
$woman_2$	$conv_2$
$woman_3$	$conv_3$

$\xRightarrow{\text{she}_{u_1} \text{ is very pretty}}$

u_1	u_2
$woman_{1/2/3}$	$conv_1$
$woman_{1/2/3}$	$conv_2$
$woman_{1/2/3}$	$conv_3$

$\{woman_1, woman_2, woman_3\}$
is a singleton, i.e.:
$woman_1 = woman_2 = woman_3$

The fact that discourse (6) – in contrast to (5) – is also compatible with the narrow-scope indefinite reading is due to the fact that the quantificational adverb $always_{u_2}$ in (6b) can take scope over the singular pronoun she_{u_1} and neutralize the effect that singular number morphology has on the cardinality of the previously introduced set of women.

This neutralization is a consequence of the discourse-level distributivity operator **dist** that quantificational expressions contribute. The **dist** operator distributes over plural info states in the sense that it requires the update in its scope to be interpreted relative to singleton subsets of the input plural info state, as shown in (14) below. So, the singleton requirement contributed by the singular pronoun she_{u_1} is interpreted relative to single rows and is trivially satisfied.

(14)

u_1	u_2
$woman_1$	$conv_1$
$woman_2$	$conv_2$
$woman_3$	$conv_3$

$\xRightarrow{\text{always}_{u_2}\ \mathbf{dist}_{u_2}(\text{she}_{u_1} \text{ comes to the banquet with him})}$

$$\left\{ \begin{array}{ll} \begin{array}{c|c} u_1 & u_2 \\ \hline \mathit{woman}_1 & \mathit{conv}_1 \end{array} & \{\mathit{woman}_1\} \text{ is a singleton} \\ \\ \begin{array}{c|c} u_1 & u_2 \\ \hline \mathit{woman}_2 & \mathit{conv}_2 \end{array} & \{\mathit{woman}_2\} \text{ is a singleton} \\ \\ \begin{array}{c|c} u_1 & u_2 \\ \hline \mathit{woman}_3 & \mathit{conv}_3 \end{array} & \{\mathit{woman}_3\} \text{ is a singleton} \end{array} \right\}$$

Note that we allow for models in which Harvey courts more than one woman at a convention. The singleton requirement contributed by singular number morphology on anaphoric pronouns requires uniqueness relative to the *local* plural info state and not globally, relative to the entire model – as Russellian (non-anaphoric) definite descriptions would. Moreover, we allow for models in which Harvey courts more than one woman at a convention and not every woman courted by Harvey comes to the banquet with him – we only require one of the women he courts at a convention to come to the banquet of that convention with him. We will further discuss this matter in due course, as well as the related issue of weak vs strong readings for singular donkey anaphora.

The analysis of the modal subordination discourse in (8) above will proceed in a way that is strictly parallel to the analysis of the quantificational subordination discourse in (6) – as shown by the sequence of updates depicted in (15) below. The only difference is that the modal verbs *might* and *would* quantify over possible worlds as opposed to individuals.

(15) $\emptyset \xRightarrow{\text{might}^{p}(\text{a}^{u} \text{ wolf come in}))}$

$$\begin{array}{c|c} p & u \\ \hline v_1 & \mathit{wolf}_1 \\ v_2 & \mathit{wolf}_2 \\ v_3 & \mathit{wolf}_3 \end{array} \quad \begin{array}{l} \\ \text{in world } v_1, \mathit{wolf}_1 \text{ comes in} \\ \text{in world } v_2, \mathit{wolf}_2 \text{ comes in} \\ \text{in world } v_3, \mathit{wolf}_3 \text{ comes in} \end{array}$$

$\xRightarrow{\text{would}_{p}\ \mathbf{dist}_{p}(\text{it}_{u} \text{ eat Harvey first})}$

$$\left\{ \begin{array}{ll} \begin{array}{|c|c|} \multicolumn{1}{c}{p} & \multicolumn{1}{c}{u} \\ \hline v_1 & wolf_1 \\ \hline \end{array} & \begin{array}{c} \{wolf_1\} \text{ is a singleton and} \\ wolf_1 \text{ eats Harvey first in } v_1 \end{array} \\ \\ \begin{array}{|c|c|} \multicolumn{1}{c}{p} & \multicolumn{1}{c}{u} \\ \hline v_2 & wolf_2 \\ \hline \end{array} & \begin{array}{c} \{wolf_2\} \text{ is a singleton and} \\ wolf_2 \text{ eats Harvey first in } v_2 \end{array} \\ \\ \begin{array}{|c|c|} \multicolumn{1}{c}{p} & \multicolumn{1}{c}{u} \\ \hline v_3 & wolf_3 \\ \hline \end{array} & \begin{array}{c} \{wolf_3\} \text{ is a singleton and} \\ wolf_3 \text{ eats Harvey first in } v_3 \end{array} \end{array} \right\}$$

2.1 The Basics: Plural Info States, DRSs, Conditions and Compositionality

The main formal innovation relative to classical DRT / FCS / DPL is that, just as in Dynamic Plural Logic (van den Berg 1996), information states I, J etc. are modeled as sets of variable assignments i, j etc. Plural info states enable us to encode discourse reference to both quantifier domains and quantificational dependencies and pass this anaphoric information across sentential boundaries, which is exactly what we need to account for the interpretation of discourses (5), (6) and (8).

More precisely, we need the following two ingredients. First, we need a suitable meaning for generalized determiners (over individuals) that will store two things in the output plural info state:

(*i*) the restrictor and nuclear scope sets of individuals that are introduced by the determiner
(*ii*) the quantificational dependencies between these sets and any other quantifiers / indefinites

For example, we store the sets of individuals and the dependencies between them introduced by the universal *every convention* in (6a) and the indefinite *a woman* in its nuclear scope. These sets and dependencies are available for subsequent anaphoric retrieval – e.g., *always* and *she* in (6b) are simultaneously anaphoric both to the two sets of conventions and women and to the dependency between these sets introduced in (6a).

Second, we need a suitable meaning for singular number morphology on pronouns like *she* above that requires the dref anaphorically retrieved by the pronoun to store a singleton set of individuals. Plural info states are, once again, crucial: they store and pass on structured sets (i.e., sets of values plus their associated dependencies / structure), so we can constrain their cardinality by subsequent, syntactically non-local anaphoric elements.

Modal verbs will be analyzed as the modal counterparts of generalized determiners and verbal moods (e.g., indicative) as the modal counterparts of pronouns.

To formalize these meanings for generalized determiners and singular anaphors, we work with a Dynamic Ty2 logic, i.e., with the Logic of Change in Muskens (1996) that reformulates dynamic semantics (Kamp 1981, Heim 1982, Groenendijk & Stokhof 1991) in Gallin's Ty2 (Gallin 1975).

We have three basic types: type t (truth values), type e (individuals; variables: $x, y, \dots$) and type s (variable assignments; variables: $i, j, \dots$). A suitable set of axioms ensures that the entities of type s behave as variable assignments (see Muskens 1996, Brasoveanu 2008 for more details).

A dref for individuals u is a function of type se from assignments i_s to individuals x_e (subscripts on terms indicate their type). Intuitively, the individual $u_{se}i_s$ is the individual that the assignment i assigns to the dref u. Dynamic info states $I, J, \dots$ are plural: they are sets of variable assignments, i.e., terms of type st. An individual dref u stores a set of individuals relative to a plural info state I: $u[I]$ is the image of the set of assignments I under the function u.

(16) $u[I] := \{u_{se}i_s : i \in I\}$

A sentence is interpreted as a Discourse Representation Structure (DRS), which is a binary relation of type $(st)((st)t)$ between an input state I_{st} and an output state J_{st}, as shown in (17).

(17) $[\mathbf{newdrefs} \mid \mathbf{conditions}] := \lambda I_{st}.\lambda J_{st}.\ I[\mathbf{newdrefs}]J \wedge \mathbf{conditions}J$

A DRS requires:

(i) the input info state I to differ from the output state J at most with respect to the **new drefs**

(ii) all the **conditions** to be satisfied relative to the output state J

The definition of dref introduction is given in (18) below. This definition is based on the familiar notion of dref introduction $i[u]j$ in DPL and CDRT, which relates single variable assignments i and j. Intuitively, the DPL / CDRT notion of dref introduction $i[u]j$ – a.k.a. random (re)assignment of value to a variable u – is interpreted as: the output assignment j differs from the input assignment i at most with respect to the value it assigns to u (see Groenendijk & Stokhof 1991, Muskens 1996 and Brasoveanu 2008 among others for more discussion and the exact definition of this notion). The binary relation $i[u]j$ is an equivalence relation over total variable assignments.

We need to generalize this binary relation $i[u]j$ between single assignments i and j to a binary relation $I[u]J$ between sets of assignments I and J – i.e., we need to generalize $i[u]j$ to a relation between plural info states. We do this cumulative-quantification style, as shown in (18): $I[u]J$ requires any input assignment $i \in I$ to have a $[u]$-successor assignment $j \in J$ and, *vice versa*, any output assignment $j \in J$ should have a $[u]$-predecessor assignment $i \in I$. The binary relation $I[u]J$ is an equivalence relation over sets of total variable assignments.

(18) $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

Multiple dref introduction is defined in terms of dynamic conjunction ";", which in turn is defined as DRS composition (i.e., binary relation composition), as shown in (19) below. Note the difference between dynamic conjunction and classical, static conjunction "$\wedge$": the former is an abbreviation, while the latter is part of the Dynamic Ty2 logic. An example is provided in (21).

(19) $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$

(20) $[u_1, \ldots, u_n] := [u_1]; \ldots ; [u_n]$

(21) $[u_1, u_2 \mid \text{WOMAN}\{u_1\}, \text{CONVENTION}\{u_2\}, \text{COURTED-AT}\{u_1, u_2\}]$
$:= \lambda I_{st}.\lambda J_{st}.\ I[u_1, u_2]J \wedge \text{WOMAN}\{u_1\}J$
$\wedge\ \text{CONVENTION}\{u_2\}J \wedge \text{COURTED-AT}\{u_1, u_2\}J$

DRSs of the form shown in (22) are tests. An example is provided in (23).

(22) $[\textbf{conditions}] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge \textbf{conditions}J$

(23) $[\text{COURTED-AT}\{u_1, u_2\}] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge \text{COURTED-AT}\{u_1, u_2\}J$

Note that DRSs like (21) above are simply a conjunction of an update introducing the new drefs followed by a test containing all the conditions, as shown below.

(24) $[u_1, u_2 \mid \text{WOMAN}\{u_1\}, \text{CONVENTION}\{u_2\}, \text{COURTED-AT}\{u_1, u_2\}] =$
$[u_1, u_2];\ [\text{WOMAN}\{u_1\}, \text{CONVENTION}\{u_2\}, \text{COURTED-AT}\{u_1, u_2\}]$

Conditions denote sets of info states, i.e., they are terms of type $(st)t$, and they are interpreted distributively relative to a plural info state. For example, $\text{COURTED-AT}\{u_1, u_2\}$ is a dynamic condition based on the static relation between individuals COURTED-AT of type $e(et)$ and a plural info state I is in the set denoted by this condition iff $\forall i_s \in I(\text{COURTED-AT}(u_1 i, u_2 i))$, as shown in (25).

(25) $\text{COURTED-AT}\{u_1, u_2\} :=$
$\lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(\text{COURTED-AT}(u_1 i, u_2 i))$
(prelim. version)

I	...	u_1	u_2	...
i_1	...	$\alpha_1\ (= u_1 i_1)$	$\beta_1\ (= u_2 i_1)$	...
		$\text{COURTED-AT}(\alpha_1, \beta_1)$		
i_2	...	$\alpha_2\ (= u_1 i_2)$	$\beta_2\ (= u_2 i_2)$	...
		$\text{COURTED-AT}(\alpha_2, \beta_2)$		
i_3	...	$\alpha_3\ (= u_1 i_3)$	$\beta_3\ (= u_2 i_3)$	...
		$\text{COURTED-AT}(\alpha_3, \beta_3)$		
...	...	...	...	...

Given the underlying type logic, Montague-style compositionality at sub-clausal level follows in the usual way. More precisely, the compositional aspect of interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) saturated expressions, i.e., names and sentences. Let us abbreviate them as **e** and **t**.

An extensional static logic, for example, identifies **e** with e and **t** with t. The translation of the English noun *woman* is of type **et**, i.e., et: *woman* $\rightsquigarrow \lambda x_e.\ \text{WOMAN}_{et}(x)$. The determiner *every* is of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, i.e., $(et)((et)t)$: *every* $\rightsquigarrow \lambda X_{et}.\lambda X'_{et}.\ \forall x_e(X(x) \rightarrow X'(x))$.

For our dynamic system based on plural info states, we only need to change the abbreviations for **e** and **t**. We let **t** abbreviate $(st)((st)t)$, i.e., a sentence is interpreted as a DRS, and we let **e** abbreviate se, i.e., a name is interpreted as a dref. The denotation of the noun *woman* is still of type **et**, as shown in (26) below. Moreover, the determiner *every* is still of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$ – and its dynamic interpretation will be discussed later in this section.

(26) *woman* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [\text{WOMAN}_{et}\{v\}]$

Later on, we will be able to intensionalize this extensional system by simply adding a basic type for possible worlds: we will build intensions by relativizing the corresponding extensions to possible-world drefs.

2.2 Indefinites and Pronouns

This subsection is dedicated to the analysis of the simple discourse in (27) below. The goal is to see the formal system in action and to show how indefinites, pronouns and basic patterns of cross-sentential anaphora are analyzed in this system.

(27) a. A^u wolf came in.

b. It_u ate $Harvey^{u'}$.

To model the fact that the discourse-initial info state does not contain any information, we introduce the dummy individual ★. This individual is the universal falsifier, i.e., any lexical relation that has ★ as one of its arguments, e.g., $\text{WOLF}(\bigstar)$ or $\text{EAT}(\bigstar, \alpha_1)$, is false.[5] The dummy assignment $i_\bigstar$ assigns the dummy individual ★ to every dref. The discourse-initial info state that contains no anaphoric-information is the plural info state containing only the dummy assignment $I_\bigstar = \{i_\bigstar\}$.

(28)

$I_\bigstar$	...	u_1	u_2	u_3	...
$i_\bigstar$	...	★	★	★	...

or simply:

...	u_1	u_2	u_3	...
...	★	★	★	...

The dummy info state $I_\bigstar$ enables us to capture the fact that using pronouns out-of-the-blue is infelicitous. Classical DRT / FCS captures this by using partial – instead of total – variable assignments. The dummy individual enables us to keep the underlying logic as simple / classical as possible: we work with total, not partial, assignments and we work with a total, two-valued logic – in contrast to the partial logic in van den Berg (1996).

Given the introduction of the universal falsifier ★, we need to interpret lexical relations (i.e., atomic conditions) distributively relative to the *non-dummy* sub-state of the input plural info state I, as shown below.

(29) $I_{u \neq \bigstar} := \{i_s \in I : ui \neq \bigstar\}$

(30) $\text{WOLF}\{u\} := \lambda I_{st}.\ I_{u \neq \bigstar} \neq \emptyset \wedge \forall i_s \in I_{u \neq \bigstar}(\text{WOLF}(ui))$

(31) $I_{u \neq \bigstar, u' \neq \bigstar} := \{i_s \in I : ui \neq \bigstar \wedge u'i \neq \bigstar\}$

(32) $\text{EAT}\{u, u'\} :=$
$\lambda I_{st}.\ I_{u \neq \bigstar, u' \neq \bigstar} \neq \emptyset \wedge \forall i_s \in I_{u \neq \bigstar, u' \neq \bigstar}(\text{EAT}(ui, u'i))$

(33) $I_{u_1 \neq \bigstar, \ldots, u_n \neq \bigstar} := \{i_s \in I : u_1 i \neq \bigstar \wedge \ldots \wedge u_n i \neq \bigstar\}$

[5] We ensure that any lexical relation R of arity n – i.e., of type $e^n t$, defined as in Muskens (1996:157-158): $e^0 t := t$ and $e^{m+1} t := e(e^m t)$ – yields falsity whenever ★ is one of its arguments by letting $R \subseteq (D_e^{\mathfrak{M}} \setminus \{\bigstar\})^n$.

(34) $R\{u_1, \ldots, u_n\} :=$
$\lambda I_{st}.\ I_{u_1 \neq \bigstar, \ldots, u_n \neq \bigstar} \neq \emptyset \wedge \forall i_s \in I_{u_1 \neq \bigstar, \ldots, u_n \neq \bigstar}(R(u_1 i, \ldots, u_n i))$

The translation of any sentence or discourse, hence also the translation of (27) above, will be a DRS D. This DRS is true relative to an input info state I, in particular, the dummy info state $I_\bigstar = \{i_\bigstar\}$, iff there is an output state J such that D relates I and J. In other words, a DRS D is true relative to I iff there is at least one way to successfully update I with D.

(35) A DRS D of type **t** is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

We capture cross-sentential anaphora between the indefinite a^u *wolf* and the pronoun it_u in (27) in very much the same way as classical DRT / FCS / DPL: the indefinite introduces a dref u that the pronoun later retrieves. The translations for the singular indefinite and the singular pronoun are provided in (36) and (37) below. The translations have the expected Montagovian form: the indefinite takes as arguments a restrictor property P and a nuclear scope property P', introduces a new dref u and predicates the two properties of this dref; the pronoun is the Montagovian type-lift of the anaphorically-retrieved dref u. In both translations, singular number morphology contributes a condition $\mathbf{sing}(u)$ that requires uniqueness of the non-dummy value of the dref u relative to the plural info state I; $|uI|$ denotes the cardinality of the set uI.[6] In contrast, plural pronouns do not require uniqueness, as shown in (40). Recall that drefs are assigned only atomic (i.e., semantically singular) entities as values; for a version of Dynamic Ty2 that countenances non-atomic individuals in addition to plural info states (i.e., both domain-level and discourse-level plurality), see Brasoveanu (2008).

(36) $a^u \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u \mid \mathbf{sing}(u)];\ P(u);\ P'(u)$

(37) $it_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{sing}(u)];\ P(u)$

(38) $uI := \{ui : i_s \in I_{u \neq \bigstar}\}$

(39) $\mathbf{sing}(u) := \lambda I_{st}.\ |uI| = 1$

(40) $they_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u \neq \emptyset];\ P(u)$

(41) $u \neq \emptyset := \lambda I_{st}.\ uI \neq \emptyset$

The proper name $Harvey^{u'}$ introduces a new dref u' and constrains it to pick out the individual denoted by the non-logical constant HARVEY (of type e) relative to any assignment $i \in I$.

(42) $Harvey^{u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u' \mid u' = \textsc{harvey}];\ P(u')$

[6] See Nouwen (2007) for a similar proposal in a closely related framework.

(43) $u' = \text{HARVEY} := \lambda I_{st}.\ u'I = \{\text{HARVEY}\}$

The two sentences of discourse (27) are compositionally translated as shown in (44) and (45) below.

(44) a. *wolf* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [\text{WOLF}\{v\}]$
b. *a^u wolf* $\rightsquigarrow \lambda P'_{\mathbf{et}}.\ [u \,|\, \mathbf{sing}(u)];\ [\text{WOLF}\{u\}];\ P'(u)$
c. *came in* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [\text{COME-IN}\{v\}]$
d. *a^u wolf came in* $\rightsquigarrow [u \,|\, \mathbf{sing}(u)];\ [\text{WOLF}\{u\}];\ [\text{COME-IN}\{u\}]$

(45) a. *ate* $\rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [\text{EAT}\{v, v'\}])$
b. *ate Harvey$^{u'}$* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [u' \,|\, u' = \text{HARVEY}];\ [\text{EAT}\{v, u'\}]$
c. *it$_u$ ate Harvey$^{u'}$* $\rightsquigarrow [\mathbf{sing}(u)];\ [u' \,|\, u' = \text{HARVEY}];\ [\text{EAT}\{u, u'\}]$

Conjoining the translations of the two sentences gives us the translation for the entire discourse in (27), provided in (46) below. The DRSs in (47) and (48) are equivalent ways of representing this discourse: (47) has the same format as classical DRT boxes (represented in a linearized way), while (48) is completely explicit about each atomic update contributed by discourse (27).

(46) $[u \,|\, \mathbf{sing}(u)];\ [\text{WOLF}\{u\}];\ [\text{COME-IN}\{u\}];$
$[\mathbf{sing}(u)];\ [u' \,|\, u' = \text{HARVEY}];\ [\text{EAT}\{u, u'\}]$

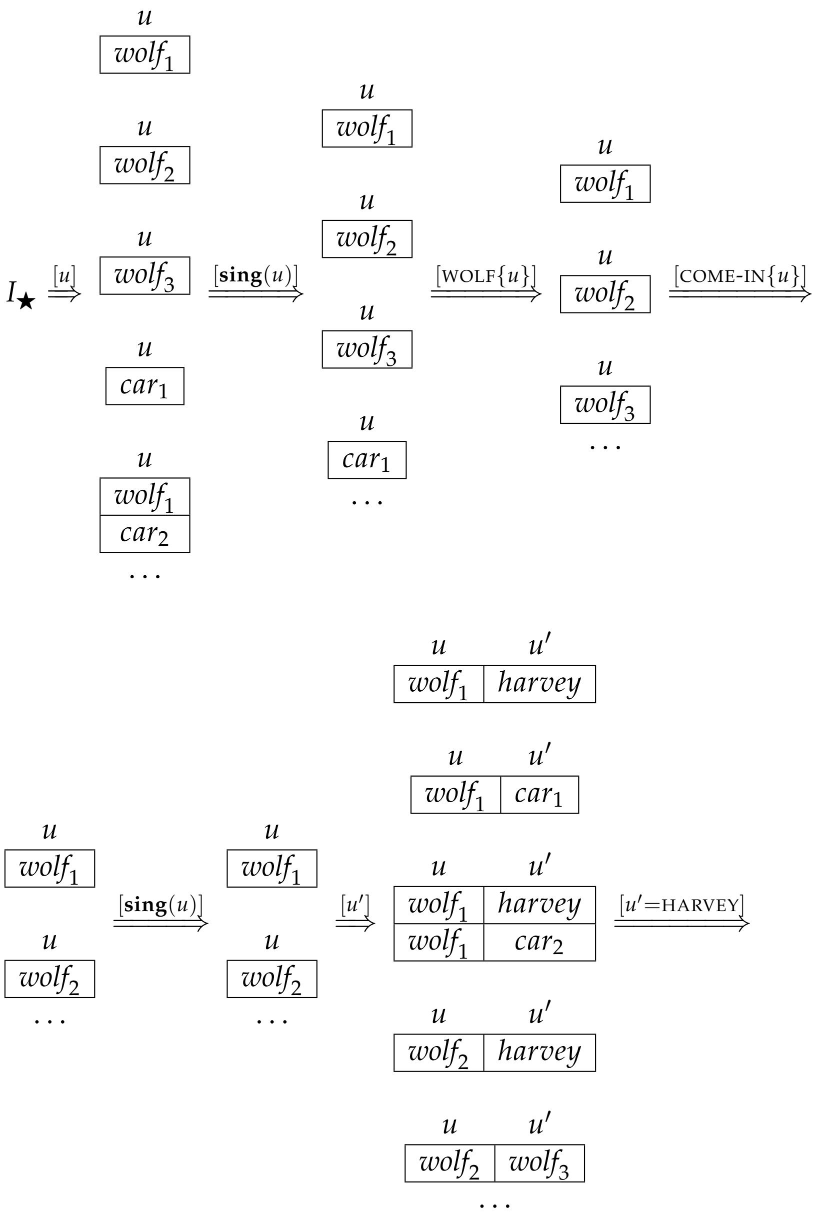
(47) $[u, u' \,|\, \mathbf{sing}(u), \text{WOLF}\{u\}, \text{COME-IN}\{u\}, u' = \text{HARVEY}, \text{EAT}\{u, u'\}]$

(48) $[u];\ [\mathbf{sing}(u)];\ [\text{WOLF}\{u\}];\ [\text{COME-IN}\{u\}];$
$[\mathbf{sing}(u)];\ [u'];\ [u' = \text{HARVEY}];\ [\text{EAT}\{u, u'\}]$

While the DRT-style representation in (47) is the most readable, the representation in (48) provides insight into the internal workings of the update procedure. This is graphically depicted in (49) below.

We start with the dummy info state $I_{\bigstar}$ that contains no anaphoric information. Then, we introduce a new dref $[u]$. The result is many plural info states, some containing only one row, some containing two rows etc. and assigning all possible individuals or combinations thereof to the newly introduced dref u. That is, we now have a graph with many paths. This is the result of the fact that our DRSs are relations between info states and not functions, i.e., they are non-deterministic updates. The test $[\mathbf{sing}(u)]$ eliminates some of the paths in the graph, namely all those paths that end in an info state assigning more than one entity to the dref u. The test $[\text{WOLF}\{u\}]$ eliminates further paths in the graph – namely all those that end in an info state where u is not assigned a wolf. The test $[\text{COME-IN}\{u\}]$ eliminates all the wolves that didn't come in.

The test [**sing**(u)] contributed by the pronoun is vacuously satisfied, so it doesn't eliminate any more paths in the graph. We now introduce another dref u' that extends the graph in many different ways. The subsequent test $[u' = \text{HARVEY}]$ prunes down the graph by eliminating all info states that don't assign the individual *harvey* to u'. Finally, the test $[\text{EAT}\{u, u'\}]$ keeps only the info states such that, for any row i in those info states, the individual ui ate the individual $u'i$.

(49) 

$$\begin{array}{c} \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array} \\[1em] \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_2} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array} \\ \ldots \end{array} \xrightarrow{[\text{EAT}\{u,u'\}]} \begin{array}{c} \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array} \\ \ldots \end{array}$$

The picture in (49) above might seem overwhelming at first. However, except for the fact that we allow plural info states (i.e., matrices with multiple rows), this is in no way different from the way interpretation proceeds in classical first-order logic or in classical DRT / FCS. Such graphs are implicit in their recursive definitions of truth and satisfaction. We will follow their lead and keep the graphs implicit, i.e., from now on, we will depict updates by choosing a single, typical path in the graph. For example, the update contributed by discourse (27) will be represented as shown in (50) below – or in abbreviated form, as shown in (51).

(50) $$I_\star \xrightarrow{[u]} \begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array} \xrightarrow{[\mathbf{sing}(u)]} \begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array} \xrightarrow{[\text{WOLF}\{u\}]} \begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array} \xrightarrow{[\text{COME-IN}\{u\}]}$$

$$\begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array} \xrightarrow{[\mathbf{sing}(u)]} \begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array} \xrightarrow{[u']} \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array} \xrightarrow{[u'=\text{HARVEY}]}$$

$$\begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array} \xrightarrow{[\text{EAT}\{u,u'\}]} \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array}$$

(51) $$I_\star \xrightarrow{[u|\mathbf{sing}(u),\text{WOLF}\{u\},\text{COME-IN}\{u\}]} \begin{array}{c} u \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} \\ \hline \end{array}$$

$$\xrightarrow{[u'|u'=\text{HARVEY},\text{EAT}\{u,u'\}]} \begin{array}{cc} u & u' \\ \hline \multicolumn{1}{|c|}{\mathit{wolf}_1} & \multicolumn{1}{c|}{\mathit{harvey}} \\ \hline \end{array}$$

The definition of truth in (35) above basically says that a DRS D is true if there is at least one path through the graph denoted by D. Again, this is just as in classical first-order logic or in classical DRT / FCS, except this is implicit in their definitions of truth and satisfaction.

We end this subsection with the definition of dynamic negation, provided in (52) below, which derives the intuitively correct truth conditions for examples like *Linus didn't bring an umbrella*, and with the translations for the anaphoric readings of singular and plural definite articles, which are parallel to the translations for singular and plural pronouns in (37) and (40) above.[7] Later on, we will analyze verbal moods as the modal counterparts of anaphoric pronouns.

[7] Semantically distinguishing between singular and plural definite articles is supported by the fact that other languages, e.g., Romance languages, have overt number morphology on definite articles.

(52) $\sim D := \lambda I_{st}.\ I \neq \emptyset \wedge \forall H_{st} \neq \emptyset (H \subseteq I \rightarrow \neg \exists K_{st}(DHK))$

(53) $the_{\mathbf{sg}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{sing}(u)];\ P(u);\ P'(u)$

(54) $the_{\mathbf{pl}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u \neq \emptyset];\ P(u);\ P'(u)$

2.3 Desiderata for Dynamic Generalized Quantification

A definition of dynamic generalized quantification, in both the individual and the modal domain, should satisfy several desiderata. We will discuss these desiderata here with respect to quantification over individuals, but the same considerations apply to quantification over possible worlds.

First, we want our notion of dynamic quantification to account for donkey anaphora, exemplified in (55) below.

(55) Every^{u} farmer who owns a$^{u'}$ donkey beats it$_{u'}$.

Second, we want to avoid the proportion problem that unselective quantification ("unselective" in the sense of Lewis 1975) runs into. The sentence in (56) below exemplifies this problem. Intuitively, (56) is false in a situation in which there are ten farmers, nine have a single donkey each and they do not beat it, while the tenth has twenty donkeys and he is busy beating them all. The unselective interpretation of the *most*-quantification, however, incorrectly predicts that the sentence is true in such a situation because more than half of the ⟨*farmer*, *donkey*⟩-pairs (twenty out of twenty-nine) are such that the farmer beats the donkey. Thus, dynamic generalized determiners should relate sets of individuals (of type *et*) and not sets of assignments (of type *st*).

(56) Most^{u} farmers who own a$^{u'}$ donkey beat it$_{u'}$.

Third, generalized quantification should be compatible with both strong and weak donkey readings. That is, we want to allow for the different interpretations associated with the donkey anaphora in (57) (Heim 1990) and (58) (Pelletier & Schubert 1989) below. The interpretation of (57) is: most slave-owners were such that, for *every* (strong reading) slave they owned, they also owned his offspring. The interpretation of (58) is: every dime-owner will put *some* (weak reading) dime of her / his in the meter.

(57) Most^{u} people that owned a$^{u'}$ slave also owned his$_{u'}$ offspring.

(58) Every^{u} person who has a$^{u'}$ dime will put it$_{u'}$ in the meter.

Finally, as the discourses in (5) and (6) above indicate, dynamic quantification should be defined in such a way that we make available the restrictor and nuclear scope sets of individuals for subsequent anaphoric take-up. In addition, the quantificational dependencies between different quantifiers / indefinites should also be anaphorically available.

More precisely, generalized quantification supports anaphora to two sets:

(*i*) the maximal set of individuals satisfying the restrictor update
(*ii*) the maximal set of individuals satisfying both the restrictor update and the nuclear scope update[8] (that is, we build conservativity into our representation of generalized quantification; this is needed for, e.g., donkey anaphora)

The discourse in (59) below exemplifies anaphora to nuclear scope sets. Sentence (59b) is interpreted as: the people that went to the beach are the students that left the party after 5 a.m. – which, in addition, formed a majority of the students at the party.

(59) a. Most$^{u'}$ students left the party after 5 a.m.
b. They$_{u'}$ went directly to the beach.

The discourses in (60) and (61) exemplify anaphora to restrictor sets. Using downward monotonic quantifiers like *no*^{u} *student* and *very few*^{u} *people* is important for this. Consider (60) first: any successful update with a *no*^{u}-quantification ensures that the nuclear scope set is empty (given that we build conservativity into our representation of generalized quantification) – and anaphora to it is therefore infelicitous. The only possible anaphora in (60) is restrictor-set anaphora.

(60) a. No^{u} student left the party later than 10 p.m.
b. They$_{u}$ had classes early in the morning.

Restrictor set anaphora is the only possible one in (61) too. This is because nuclear scope anaphora would yield a contradictory interpretation for (61b), namely: most of the people with a rich uncle that inherit his fortune don't inherit his fortune.

(61) a. Very few^{u} people with a rich uncle inherit his fortune.
b. Most of them$_{u}$ don't.

Given these four desiderata, we translate dynamic generalized determiners as shown in (62).

[8] We ignore anaphora to complement sets, i.e., sets obtained by taking the complement of the nuclear scope relative to the restrictor, e.g., *Few students were paying attention in class. They were tired.*; see Nouwen (2003) for arguments that complement-set anaphora is a pragmatic, not semantic, phenomenon (I am indebted to an anonymous reviewer for bringing this point to my attention).

(62) $det^{u,u'\sqsubseteq u} \rightsquigarrow$
$$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}({}_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}({}_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u,u'\}]$$

This translation is in the spirit of van den Berg (1996) (cf. van den Berg (1996:149,(4.1))).

Let us examine it in detail. First, a determiner $det^{u,u'\sqsubseteq u}$ introduces two drefs u and u': u is the restrictor dref and u' is the nuclear scope dref. Given the conservativity of natural language determiners, the nuclear scope dref is a subset of the restrictor dref: $u' \sqsubseteq u$.

Second, determiners relate a restrictor dynamic property $P_{\mathbf{et}}$ and a nuclear scope dynamic property $P'_{\mathbf{et}}$. When these dynamic properties are applied to their respective drefs, we obtain a restrictor DRS $P(u)$ of type **t** and a nuclear scope DRS $P'(u')$, also of type **t**.

The drefs u and u' and the properties P and P' are the basic building blocks of the three updates in (62). The first update, namely $\mathbf{max}^{u}({}_{\langle u\rangle}(P(u)))$, has three components: the operator $\mathbf{max}^{u}(\dots)$, the distributivity operator ${}_{\langle u\rangle}(\dots)$ and the DRS $P(u)$. This update ensures that u stores the maximal set of individuals, i.e., $\mathbf{max}^{u}(\dots)$, such that, when we take each u-individual separately, i.e., ${}_{\langle u\rangle}(\dots)$, this individual satisfies the restrictor dynamic property, i.e., $P(u)$. Once again, recall that the values assigned to the dref u are atomic, i.e., semantically singular, but plural info states store collections of such singular values.

The second update, namely $\mathbf{max}^{u'\sqsubseteq u}({}_{\langle u'\rangle}(P'(u')))$, ensures that the nuclear scope set u' is obtained in much the same way as the restrictor set u, except for the requirement that u' is the maximal structured subset of u, i.e., $\mathbf{max}^{u'\sqsubseteq u}(\dots)$.

Finally, the third update, namely $[\mathbf{DET}\{u,u'\}]$, is a test: we test that the restrictor set u and the nuclear scope set u' stand in the relation denoted by the corresponding static determiner **DET**.

To formally spell out the meaning for generalized determiners in (62) above, we need:

(*i*) two operators over plural info states, namely a maximization operator $\mathbf{max}^{u}(\dots)$ and a distributivity operator ${}_{\langle u\rangle}(\dots)$

(*ii*) a notion of structured inclusion $u' \sqsubseteq u$ that requires the subset to preserve the quantificational dependencies, i.e., the structure, associated with the individuals in the superset

The following subsections introduce dynamic quantification over individuals and the resulting analysis of quantificational subordination. The reader should take heart in the fact that, once this work is done, the analysis of modal quantification and modal subordination will follow by transferring all the developed notions from the domain of individuals to the domain of possible worlds.

2.4 Structured Inclusion, Maximization and Distributivity

We start with the notion of structured inclusion. Consider, for example, the discourse in (63) below, where u_1 stores the set of conventions and u_2 stores the set of corresponding women. Assume that, in (63a), *every*^{u_1} *convention* takes scope over *a*^{u_2} *woman* and the correlation between u_1-conventions and u_2-women is the one represented in (64) below. That is, the correlation / dependency between conventions and women is the binary relation $\{\langle\alpha_1,\beta_1\rangle, \langle\alpha_2,\beta_2\rangle, \langle\alpha_3,\beta_3\rangle, \langle\alpha_4,\beta_4\rangle\}$.

(63) a. Harvey courts a^{u_2} woman at every^{u_1} convention.

b. She$_{u_2}$ usually$^{u_3 \subseteq u_1}$ comes to the banquet with him.

(64) Two possible ways to introduce the subset dref u_3:

I	u_1	u_2	u_3 ($u_3 \subseteq u_1, u_3 \not\Subset u_1$)	u_3 ($u_3 \Subset u_1$)
i_1	α_1	β_1	α_1	α_1
i_2	α_2	β_2	α_3	α_2
i_3	α_3	β_3	α_1	$\bigstar$
i_4	α_4	β_4	α_2	α_4

(65) $u_3 \subseteq u_1 := \lambda I_{st}.\ u_3[I] \subseteq u_1[I]$

(66) $u_3 \Subset u_1 := \lambda I_{st}.\ \forall i_s \in I(u_3 i = u_1 i \vee u_3 i = \bigstar)$

Intuitively, the adverb *usually* in (63b) is anaphoric to the set of conventions introduced in (63a) – and sentence (63b) is interpreted as follows: at most conventions, the woman courted by Harvey *at that convention* comes to the banquet with him. Thus, we want to select a set that consists of a majority of conventions, i.e., we want to select a *most*-subset of the u_1-column in matrix (64) above. At the same time, we want to preserve the dependencies associated with the entities in this subset – which dependencies are encoded in the rows of the matrix.

The simplest notion of inclusion is the one defined in (65) above and symbolized by $\subseteq$ (the customary symbol). This is a notion of value-inclusion because it is concerned exclusively with sets of values. That is, it is concerned with the information stored in the columns of a matrix and completely disregards structure, i.e., the information stored in the rows of a matrix.

For example, the leftmost u_3 column in matrix (64) above satisfies the condition $u_3 \subseteq u_1$: the dref u_3 is a value-subset of the dref u_1 because $u_3 I = \{\alpha_1, \alpha_2, \alpha_3\} \subseteq u_1 I = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. We correctly store in u_3 most u_1-conventions (three out of four), but we fail to preserve the dependency between u_1-conventions and u_2-women established in (63a), i.e., the relation $\{\langle\alpha_1,\beta_1\rangle, \langle\alpha_2,\beta_2\rangle, \langle\alpha_3,\beta_3\rangle, \langle\alpha_4,\beta_4\rangle\}$: as far as u_3

and u_2 are concerned, α_1 is still correlated with β_1, but it is now also correlated with β_3, α_2 is now correlated with β_4 (not β_2) and α_3 with β_2 (not β_3). We therefore fail to derive the intuitively-correct interpretation for sentence (63b) – and for discourse (63) as a whole.

We obtain similarly incorrect results for donkey sentences like the one in (67) below. The restrictor of the quantification introduces a dependency between all the donkey-owning u_1-farmers and the u_2-donkeys that they own. The nuclear scope set u_3 needs to contain most u_1-farmers, but in such a way that the correlated u_2-donkeys remain the same. That is, the nuclear scope set contains a *most*-subset of donkey-owning farmers that beat *their respective donkey(s)*. The notion of value-only inclusion in (65) is, yet again, inadequate.

(67) Most$^{u_1, u_3 \subseteq u_1}$ farmers who own a^{u_2} donkey beat it$_{u_2}$.

So, to capture the intra-sentential and cross-sentential interaction between anaphora and quantification, we need the notion of *structured inclusion* defined in (66) above, whereby we go from a superset to a subset by discarding rows in the matrix. The subset is then guaranteed to contain *only* the dependencies associated with the superset (but not necessarily *all* dependencies – see below).

To formalize this, we follow van den Berg (1996) and use the dummy individual ★ as a tag for the cells in the matrix that should be discarded in order to obtain a structured subset u_3 of a superset u_1 – as shown by the rightmost u_3 column in (64) above. However, unlike van den Berg (1996), we will not take the dummy individual ★ to require making the underlying logic partial.

The notion of structured inclusion $\Subset$ in (66) above ensures that the subset inherits *only* the superset structure – but we also need it to inherit *all* the superset structure. We achieve this by means of the second conjunct in definition (68) below. This conjunct is needed to account for the strong donkey sentence in (57) above (among other things), which is interpreted as talking about *every* slave owned by any given person. That is, the nuclear scope set in (57), which is a *most*-subset of the restrictor set, needs to inherit *all* the superset structure: each slave owner in the nuclear scope set needs to be associated with *every* slave that s/he owned.

(68) $u' \sqsubseteq u := \lambda I_{st}.\ (u' \Subset u)I \wedge \forall i_s \in I(ui \in u'I \rightarrow ui = u'i)$

We turn now to the maximization and distributivity operators $\mathbf{max}^u$ and $\mathbf{dist}_u$, defined in the spirit of van den Berg (1996). Together, maximization and distributivity enable us to dynamize λ-abstraction over both values (i.e., quantifier domains) and structure (i.e., quantificational dependencies). That is, $\mathbf{max}^u$ and $\mathbf{dist}_u$ enable us to extract and store the restrictor and nuclear scope structured sets needed to define dynamic generalized quantification.

Consider the definition of $\mathbf{max}^u$ in (69) below first. The first conjunct $([u]; D)IJ$ introduces u as a new dref and makes sure that each u-individual stored in the output state J satisfies D. So, we ensure that u stores *only* individuals that satisfy D. The second conjunct enforces maximality: there is no output state K that stores u-individuals satisfying D and that is a strict superset of J. So, we ensure that u stores *all* individuals that satisfy D relative to J.

(69) $\mathbf{max}^u(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \neg\exists K_{st}(([u]; D)IK \wedge J_{u \neq \bigstar} \subsetneq K_{u \neq \bigstar})$

The definition of maximization is given in terms of local maxima, i.e., $\neg\exists K_{st}(([u]; D)IK \wedge J_{u \neq \bigstar} \subsetneq K_{u \neq \bigstar})$, and not in terms of a global supremum, i.e., $\forall K_{st}(([u]; D)IK \rightarrow K_{u \neq \bigstar} \subseteq J_{u \neq \bigstar})$, to allow for the fact that the DRS D could contain a singular indefinite, i.e., a non-deterministic update of the form $[u' \mid \mathbf{sing}(u')]$, that could in principle be satisfied by multiple single individuals.

Maximal structured subsets can now be defined as shown in (70).

(70) $\mathbf{max}^{u' \sqsubseteq u}(D) := \mathbf{max}^{u'}([u' \sqsubseteq u]; D)$

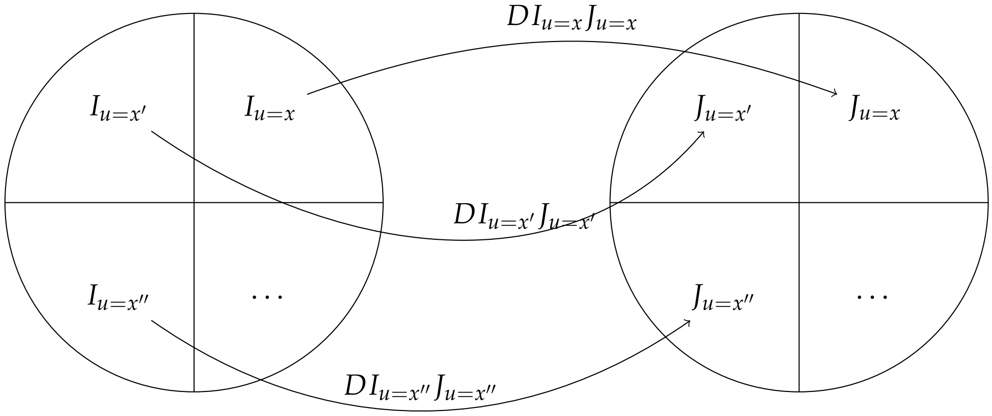
The definition of distributivity in (72) below – depicted in (73) – states that updating an info state I with a DRS D *distributively* over a dref u means:

(*i*) generating the u-partition of I, namely $\{I_{u=x} : x \in uI\}$
(a partition cell $I_{u=x}$ is defined as shown in (71) below)
(*ii*) updating each cell $I_{u=x}$ in the partition with the DRS D
(*iii*) taking the union of the resulting output info states

(71) $I_{u=x} := \{i \in I : ui = x\}$

(72) $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ uI = uJ \wedge \forall x_e \in uI(DI_{u=x}J_{u=x})$

(73) Updating the info state I with the DRS D distributively over the dref u:



The first conjunct in (72) is required to ensure that there is a bijection between the partition induced by the dref u over the input state I and the one induced over the output state J. Without this conjunct, we could introduce arbitrary new values for u in the output state J, i.e., arbitrary new partition cells.[9] The second conjunct in (72) is the one that actually defines the distributive update: the DRS D relates every partition cell in the input state I to the corresponding partition cell in the output state J, as shown in (73) above.

2.5 Dynamic Generalized Quantifiers

The translation for generalized determiners is provided in (77) below. The justification for the fact that we use the distributivity operators $_{\langle u \rangle}(\dots)$ and $_{\langle u' \rangle}(\dots)$ in the translation of generalized determiners has to do with the existential commitment customarily associated with new dref introduction.

(74) $_u(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\bigstar} = J_{u=\bigstar} \wedge uI \neq \emptyset \wedge \mathbf{dist}_u(D)I_{u\neq\bigstar}J_{u\neq\bigstar}$

(75) $_{\langle u \rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\bigstar} = J_{u=\bigstar} \wedge (uI = \emptyset \rightarrow I = J) \wedge$
$(uI \neq \emptyset \rightarrow \mathbf{dist}_u(D)I_{u\neq\bigstar}J_{u\neq\bigstar})$

(76) $\mathbf{DET}\{u, u'\} := \lambda I_{st}.\ \mathbf{DET}(uI, u'I)$,
where **DET** is a static determiner.

(77) $det^{u,u' \sqsubseteq u} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\langle u \rangle}(P(u)));\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

The existential commitment associated with new dref introduction is built into:

[9] Nouwen (2003:87) was the first to observe that we need to add the first conjunct in (72) to the original definition of distributivity in van den Berg (1996:145,(18)).

(i) the definition of lexical relations – see the conjunct $I_{u_1 \neq \bigstar, \ldots, u_n \neq \bigstar} \neq \emptyset$ in (34) above

(ii) the definition of the operator ${}_{u}(\ldots)$ – see the conjunct $uI \neq \emptyset$ in (74) above

We need these non-emptiness requirements because the pair of empty info states $\langle \emptyset_{st}, \emptyset_{st} \rangle$ is, on one hand, in the denotation of $[u]$ for any dref u (see definition (18) above) and, on the other hand, in the denotation of $\mathbf{dist}_u(D)$ for any dref u and DRS D (see definition (72) above).

Crucially, however, there is no existential commitment in the translation of $det^{u,u' \sqsubseteq u}$, which employs the distributivity operators ${}_{\langle u \rangle}(\ldots)$ and ${}_{\langle u' \rangle}(\ldots)$ defined in (75) above. The fact that we use these distributivity operators enables us to capture the meaning of both upward and downward monotonic quantifiers by means of the same translation.

The problem posed by downward monotonic quantifiers is that their nuclear scope set can or has to be empty. For example, after a successful update with a $no^{u,u' \sqsubseteq u}$-quantification, the nuclear scope set u' is necessarily empty, i.e., the dref u' always stores only the dummy individual $\bigstar$ relative to the output info state. This, in turn, means that no lexical relation in the nuclear scope DRS that has u' as an argument can be satisfied. The second conjunct $uI = \emptyset \rightarrow I = J$ in (75) resolves the conflict between the emptiness requirement enforced by a *no*-quantification and the non-emptiness requirement enforced by lexical relations.[10]

Another important feature of the translation in (77) above is the fact that it uses maximization operators to extract both the restrictor and the nuclear scope set of individuals. These **max** operators (and the nuclear scope one in particular) are essential for the derivation of the correct truth conditions associated generalized quantifiers – and downward monotonic quantifiers in particular.

The **max**-based definition of generalized quantification makes an independent – and correct – prediction: it predicts that anaphora to re-

[10]Even if definition (77) allows for empty restrictor and nuclear scope sets, we still capture the fact that subsequent anaphora to such empty sets is infelicitous (e.g., anaphora to the nuclear scope sets in (60) and (61) above) because pronouns contribute non-emptiness requirements – see the $\mathbf{sing}(u)$ condition contributed by *it* in (37) above and the $u \neq \emptyset$ condition contributed by *they* in (40).

Moreover, the fact that the second conjunct in (75) requires the identity of the input and output states I and J correctly predicts that anaphora to both empty restrictor / nuclear scope sets and indefinites in restrictor / nuclear scope DRSs associated with such empty sets is infelicitous. For example, the nuclear scope DRS of a successful $no^{u,u' \sqsubseteq u}$-quantification, i.e., $\mathbf{max}^{u' \sqsubseteq u}({}_{\langle u' \rangle}(P'(u')))$, will always be a test. Hence, we correctly predict that anaphora to any indefinites in the nuclear scope of a *no*-quantification is infelicitous, e.g., *Harvey courts a*$^{u''}$ *woman at no*$^{u,u' \sqsubseteq u}$ *convention. #She*$_{u''}$ *is very pretty / #They*$_{u''}$ *are very pretty* (on the narrow-scope indefinite reading).

strictor / nuclear scope sets is always anaphora to *maximal* sets, i.e., E-type anaphora.[11] Thus, the maximality of anaphora to quantifier sets is an automatic consequence of the fact that we independently need **max**-operators to formulate truth-conditionally correct dynamic meanings for quantifiers. This is one of the major results in van den Berg (1996), preserved here.

We end this subsection with the observation that maximization and distributivity enable us to give appropriate translations for non-anaphoric definites. Russellian definites are translated by combining the $\mathbf{max}^u$ operator and the $\mathbf{sing}(u)$ condition, as shown in (78) below. This is needed to interpret the DP *the banquet* in (6b) above. Link-style plural definites (under their distributive reading) are translated as shown in (79).

(78) $the^{\mathbf{sg}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(P(u));\ [\mathbf{sing}(u)];\ P'(u)$

(79) $the^{\mathbf{pl}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u({}_u(P(u)));\ [u \neq \emptyset];\ {}_u(P'(u))$

2.6 Quantificational Subordination

We can now analyze discourses (5) and (6). We start with the two quantifier scopings that are possible for the discourse-initial sentence (5a/6a). For simplicity, we assume that the two scopings are due to the two different lexical entries for the ditransitive verb *court-at* provided in (80) and (81) below: $court\text{-}at^1$ assigns the indefinite *a woman* wide scope relative to *every convention*, while $court\text{-}at^2$ assigns it narrow scope. This quantifier scoping mechanism is just a matter of presentational convenience; any other one (quantifier raising, Cooper storage, type-shifting etc.) would be equally suitable. The basic syntactic structure of sentence (5a/6a) is given in (82).

(80) $court\text{-}at^1 \rightsquigarrow$
$\lambda Q'_{(\mathbf{et})\mathbf{t}}.\lambda Q''_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q''(\lambda v''_{\mathbf{e}}.\ [\text{COURT-AT}\{v, v', v''\}]))$

(81) $court\text{-}at^2 \rightsquigarrow$
$\lambda Q'_{(\mathbf{et})\mathbf{t}}.\lambda Q''_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q''(\lambda v''_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ [\text{COURT-AT}\{v, v', v''\}]))$

(82) $Harvey^{u_1}$ [[$court\text{-}at^{1/2}$ [a^{u_4} *woman*]] [$every^{u_2, u_3 \sqsubseteq u_2}$ *convention*]]

We will assume that the restrictor set of the *every*-quantification is non-empty, so we can safely replace the distributivity operators ${}_{\langle u_2 \rangle}(\ldots)$ and ${}_{\langle u_3 \rangle}(\ldots)$ with the simpler distributivity operators ${}_{u_2}(\ldots)$ and ${}_{u_3}(\ldots)$. The representations of the two quantifier scopings for sentence (5a/6a)

[11] Recall the Evans examples *Few senators admire Kennedy and they are very junior* and *Harry bought some sheep. Bill vaccinated them* – in addition to (59), (60) and (61) above.

are provided in (85) and (86) below (redundant distributivity operators are omitted).[12]

(83) $every^{u_2, u_3 \sqsubseteq u_2} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_2}({}_{u_2}(P(u_2)));\ \mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}(P'(u_3)));$
$[\mathbf{EVERY}\{u_2, u_3\}]$

(84) $every^{u_2, u_3 \sqsubseteq u_2}$ *convention* $\rightsquigarrow$
$\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}]);\ \mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}(P'(u_3)));$
$[\mathbf{EVERY}\{u_2, u_3\}]$

(85) a^{u_4} *woman*$>>$$every^{u_2, u_3 \sqsubseteq u_2}$ *convention* $\rightsquigarrow$
$[u_1 \mid u_1 = \text{HARVEY}];\ [u_4 \mid \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}];$
$\mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}]);\ \mathbf{max}^{u_3 \sqsubseteq u_2}([\text{COURT-AT}\{u_1, u_4, u_3\}]);$
$[\mathbf{EVERY}\{u_2, u_3\}]$

(86) $every^{u_2, u_3 \sqsubseteq u_2}$ *convention*$>>$$a^{u_4}$ *woman* $\rightsquigarrow$
$[u_1 \mid u_1 = \text{HARVEY}];\ \mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}]);$
$\mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}([u_4 \mid \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]));$
$[\mathbf{EVERY}\{u_2, u_3\}]$

The representation in (85) updates the discourse-initial info state $I_\star$ as follows. First, we store Harvey in u_1 and one woman in u_4. Then, we store the set of all conventions in u_2 in a pointwise manner, i.e., one convention per row in the matrix. This is tantamount to associating Harvey and the u_4-woman with each and every convention in the resulting plural info state. The next update introduces u_3 and stores in it the set of all conventions at which Harvey courts the u_4-woman. Finally, we test that the set of u_2-conventions, i.e., all of them, and the set of u_3-conventions, i.e., the conventions where Harvery courts the u_4-woman, stand in the **EVERY** relation, i.e., we have that $u_2 I \subseteq u_3 I$. If this final test is satisfied, the update in (85) is true relative to the input state $I_\star$ – and this can happen iff there is a woman such that Harvey courts her at every convention, as depicted in (87) below.

(87) $I_\star \xrightarrow{[u_1 \mid u_1 = \text{HARVEY}]} \begin{array}{|c|}\hline u_1 \\ \hline harvey \\ \hline \end{array} \xrightarrow{[u_4 \mid \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}]}$

[12] I assume that the following constraint (possibly pragmatic in nature – e.g., manner / relevance based) is satisfied by every new dref introduction update: the dummy individual ★ is not assigned as a value for any newly introduced dref unless this is absolutely necessary, i.e., required for the satisfaction of subsequent updates.

For example, in (85), the dref u_1 will only store *harvey* in the output info state (and not *harvey* and the dummy individual ★), the dref u_4 will only store a woman in the output state and u_2 will only store conventions (all of them). Given the fact that u_3 is a structured subset of u_2, u_3 is the only dref that could conceivably store the dummy individual ★ in (part of) the output state – but even this is not possible in (85) because of the final condition $\mathbf{EVERY}\{u_2, u_3\}$.

u_1	u_4
harvey	*woman*$_1$

$\xrightarrow{\mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}])}$

u_1	u_4	u_2
harvey	*woman*$_1$	*conv*$_1$
harvey	*woman*$_1$	*conv*$_2$
harvey	*woman*$_1$	*conv*$_3$

$\xrightarrow{\mathbf{max}^{u_3 \sqsubseteq u_2}([\text{COURT-AT}\{u_1,u_4,u_3\}]);\ [\mathbf{EVERY}\{u_2,u_3\}]}$

u_1	u_4	u_2	u_3
harvey	*woman*$_1$	*conv*$_1$	*conv*$_1$
harvey	*woman*$_1$	*conv*$_2$	*conv*$_2$
harvey	*woman*$_1$	*conv*$_3$	*conv*$_3$

woman$_1$ is courted by *harvey* at every convention

The representation in (86) updates the discourse-initial info state $I_\bigstar$ as follows. First, we store Harvey in u_1. Then, we introduce the set of all conventions relative to the dref u_2. Then, we store in u_3 the set of conventions such that, when we take each convention one at a time, we can introduce one u_4-woman relative to it such that Harvery courts this woman at the convention under consideration. The distributive operator $_{u_3}(\dots)$ ensures that the u_4-women are introduced relative to one u_3-convention at a time and, at the end of this distributive update, they are collected together in the output info state in such a way that, for every row, the u_4 woman in that row was courted at the u_3 convention in that row. Importantly, the women may be different from convention to convention. Finally, we test that the set of u_2-conventions, i.e., all of them, and the set of u_3-conventions, i.e., the conventions where Harvery courts the corresponding u_4-woman, stand in the **EVERY** relation. If this final test is satisfied, the update in (86) is true relative to the input state $I_\bigstar$ – and this can happen iff every convention is such that Harvey courts some woman or other at that convention, as depicted in (88) below.

(88) $I_\bigstar \xrightarrow{[u_1 | u_1 = \text{HARVEY}]}$

u_1
harvey

$\xrightarrow{\mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}])}$

u_1	u_2
harvey	*conv*$_1$
harvey	*conv*$_2$
harvey	*conv*$_3$

$\xrightarrow{\mathbf{max}^{u_3 \sqsubseteq u_2}(_{u_3}(\dots))}$

$$\left\{ \begin{array}{l}
\begin{array}{ccc} u_1 & u_2 & u_3 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_1} & \multicolumn{1}{c|}{conv_1} \\ \hline \end{array} \xRightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]} \\
\begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_1} & \multicolumn{1}{c|}{conv_1} & \multicolumn{1}{c|}{woman_1} \\ \hline \end{array} \\
\\
\begin{array}{ccc} u_1 & u_2 & u_3 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_2} & \multicolumn{1}{c|}{conv_2} \\ \hline \end{array} \xRightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]} \\
\begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_2} & \multicolumn{1}{c|}{conv_2} & \multicolumn{1}{c|}{woman_2} \\ \hline \end{array} \\
\\
\begin{array}{ccc} u_1 & u_2 & u_3 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_3} & \multicolumn{1}{c|}{conv_3} \\ \hline \end{array} \xRightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]} \\
\begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \\ \hline \multicolumn{1}{|c|}{harvey} & \multicolumn{1}{c|}{conv_3} & \multicolumn{1}{c|}{conv_3} & \multicolumn{1}{c|}{woman_3} \\ \hline \end{array}
\end{array} \right\}$$

$$\xRightarrow{[\mathbf{EVERY}\{u_2, u_3\}]} \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{u_1} & \multicolumn{1}{c}{u_2} & \multicolumn{1}{c}{u_3} & \multicolumn{1}{c}{u_4} \\ \hline harvey & conv_1 & conv_1 & woman_1 \\ \hline harvey & conv_2 & conv_2 & woman_2 \\ \hline harvey & conv_3 & conv_3 & woman_3 \\ \hline \end{array}$$

$woman_1$ is courted by *harvey* at $conv_1$
$woman_2$ is courted by *harvey* at $conv_2$
$woman_3$ is courted by *harvey* at $conv_3$

We can now see how sentence (5b) – in particular, the singular morphology on the pronoun she_{u_4} – forces the wide-scope indefinite reading: the condition $\mathbf{sing}(u_4)$ in (89) below effectively conflates the two scopings by requiring the set of u_4-women obtained after updating with (85) or (86) to be a singleton. This requirement leaves the truth conditions derived on the basis of (85) untouched, but makes the truth conditions associated with (86) strictly stronger. This is because $\mathbf{sing}(u_4)$ requires the set of women $\{woman_1, woman_2, woman_3\}$ stored in the final output state in (88) above to be a singleton set, i.e., it requires that $woman_1 = woman_2 = woman_3$.

(89) she_{u_4} *is very pretty* $\rightsquigarrow [\mathbf{sing}(u_4), \text{VERY-PRETTY}\{u_4\}]$

In contrast, sentence (6b) contains the adverb of quantification $always^{u_5 \sqsubseteq u_3}$, which can take scope above or below the singular pronoun she_{u_4}. In the former case, the u_4-uniqueness requirement is weakened by being relativized to u_3-conventions. As shown in (90) below, we take the meaning of $always^{u_5 \sqsubseteq u_3}$ to be a universal quantification over an anaphorically-retrieved restrictor, which is none other than the nuclear scope dref introduced by the quantifier $every^{u_2, u_3 \sqsubseteq u_2}$ *convention* in the preceding sentence. The general format for the interpretation of quantificational expressions that anaphorically retrieve their restrictors is provided in (91).

(90) $always^{u_5 \sqsubseteq u_3} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u_5 \sqsubseteq u_3}({}_{u_5}(P(u_5)));\ [\mathbf{EVERY}\{u_3, u_5\}]$

(91) $det^{u' \sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u' \sqsubseteq u}({}_{\langle u' \rangle}(P(u')));\ [\mathbf{DET}\{u, u'\}]$

The definite description *the banquet* in (6b) is a Russellian definite description (see (78) above), which contributes existence and model-level uniqueness (relativized to conventions: there is a unique banquet per convention[13]). However, for simplicity, we will assume that sentence (6b) contributes a transitive predication of the form COME-WITH-HARVEY-TO-BANQUET-OF, abbreviated as COME, that relates women and conventions and that can be translated in two different ways corresponding to the two possible relative scopes of she_{u_4} and $always^{u_5 \sqsubseteq u_3}$, as shown in (92) and (93) below. That is, the scoping technique is the same as in (80) and (81) above. The translation in (92) gives the pronoun she_{u_4} wide scope relative to the adverb $always^{u_5 \sqsubseteq u_3}$, while the translation in (93) gives the pronoun narrow scope relative to the adverb.

(92) $come\text{-}to\text{-}banquet\text{-}of^1 \rightsquigarrow$
$\lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda Q'_{(\mathbf{et})\mathbf{t}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v_{\mathbf{e}}.\ [\text{COME}\{v', v\}]))$

(93) $come\text{-}to\text{-}banquet\text{-}of^2 \rightsquigarrow$
$\lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda Q'_{(\mathbf{et})\mathbf{t}}.\ Q(\lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ [\text{COME}\{v', v\}]))$

(94) $she_{u_4}\ [[always^{u_5 \sqsubseteq u_3}]\ come\text{-}to\text{-}banquet\text{-}of^{1/2}]$

The two translations for sentence (6b), obtained on the basis of the syntactic structure in (94) above, are provided in (95) and (96) below (redundant distributivity operators are omitted).

(95) $she_{u_4} >> always^{u_5 \sqsubseteq u_3} \rightsquigarrow$
$[\mathbf{sing}(u_4)];\ \mathbf{max}^{u_5 \sqsubseteq u_3}([\text{COME}\{u_4, u_5\}]);\ [\mathbf{EVERY}\{u_3, u_5\}]$

(96) $always^{u_5 \sqsubseteq u_3} >> she_{u_4} \rightsquigarrow$
$\mathbf{max}^{u_5 \sqsubseteq u_3}({}_{u_5}([\mathbf{sing}(u_4), \text{COME}\{u_4, u_5\}]));\ [\mathbf{EVERY}\{u_3, u_5\}]$

[13] The existence and uniqueness are contributed by the Russellian definite article, translated as: $the^{u_6} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_6}(P(u_6));\ [\mathbf{sing}(u_6)];\ P'(u_6)$. The relational noun *banquet* is anaphoric to u_3-conventions and is translated as: $banquet_{u_3} \rightsquigarrow \lambda v_{\mathbf{e}}.\ [banquet\{v, u_3\}]$ (this is the set of banquets v organized at convention u_3). Putting the two translations together, we obtain the following representation for our Russellian definite description: $the^{u_6}\ banquet_{u_3} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u_6}([\text{BANQUET}\{u_6, u_3\}]);\ [\mathbf{sing}(u_6)];\ P(u_6)$. The relativized uniqueness effect, i.e., the intuition that the banquet is unique per u_3-convention, is due to the fact that the definite description is in the scope of the adverb $always^{u_5 \sqsubseteq u_3}$ and, therefore, in the scope of the distributivity operator ${}_{u_5}(\dots)$ contributed by the adverb.

Thus, there are two possible representations for sentence (6a), i.e., (85) and (86), and two possible representations for sentence (6b), i.e., (95) and (96). Out of the four combinations, three end up effectively requiring the indefinite a^{u_4} *woman* to have wide scope relative to $every^{u_2, u_3 \sqsubseteq u_2}$ *convention*. The fourth combination (86+96), provided in (97) below, encodes the 'narrow-scope indefinite' reading that is intuitively available for discourse (6), but not for (5).

(97) $$[u_1 \mid u_1 = \text{HARVEY}];\ \mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}]);$$
$$\mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}([u_4 \mid \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]));$$
$$[\mathbf{EVERY}\{u_2, u_3\}];$$
$$\mathbf{max}^{u_5 \sqsubseteq u_3}({}_{u_5}([\mathbf{sing}(u_4), \text{COME}\{u_4, u_5\}]));\ [\mathbf{EVERY}\{u_3, u_5\}]$$

(98) $I_\bigstar \xrightarrow{[u_1 | u_1 = \text{HARVEY}]}$

u_1
harvey

$\xrightarrow{\mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}])}$

u_1	u_2
harvey	$conv_1$
harvey	$conv_2$
harvey	$conv_3$

$\xrightarrow{\mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}(\ldots))}$

$\Bigg\{$

u_1	u_2	u_3
harvey	$conv_1$	$conv_1$

$\xrightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]}$

u_1	u_2	u_3	u_4
harvey	$conv_1$	$conv_1$	$woman_1$

u_1	u_2	u_3
harvey	$conv_2$	$conv_2$

$\xrightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]}$

u_1	u_2	u_3	u_4
harvey	$conv_2$	$conv_2$	$woman_2$

u_1	u_2	u_3
harvey	$conv_3$	$conv_3$

$\xrightarrow{[u_4 | \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]}$

u_1	u_2	u_3	u_4
harvey	$conv_3$	$conv_3$	$woman_3$

$\Bigg\}$

$\xrightarrow{[\mathbf{EVERY}\{u_2, u_3\}]}$

u_1	u_2	u_3	u_4
harvey	$conv_1$	$conv_1$	$woman_1$
harvey	$conv_2$	$conv_2$	$woman_2$
harvey	$conv_3$	$conv_3$	$woman_3$

$\xrightarrow{\mathbf{max}^{u_5 \sqsubseteq u_3}({}_{u_5}(\ldots))}$

$$\left\{ \begin{array}{c} \text{state}_1 \xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]} \text{state}_1' \\ \text{state}_2 \xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]} \text{state}_2' \\ \text{state}_3 \xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]} \text{state}_3' \end{array} \right\}$$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_1$	$conv_1$	$woman_1$	$conv_1$

$\xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]}$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_1$	$conv_1$	$woman_1$	$conv_1$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_2$	$conv_2$	$woman_2$	$conv_2$

$\xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]}$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_2$	$conv_2$	$woman_2$	$conv_2$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_3$	$conv_3$	$woman_3$	$conv_3$

$\xrightarrow{[\mathbf{sing}(u_4),\text{COME}\{u_4,u_5\}]}$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_3$	$conv_3$	$woman_3$	$conv_3$

$\xrightarrow{[\mathbf{EVERY}\{u_3,u_5\}]}$

u_1	u_2	u_3	u_4	u_5
harvey	$conv_1$	$conv_1$	$woman_1$	$conv_1$
harvey	$conv_2$	$conv_2$	$woman_2$	$conv_2$
harvey	$conv_3$	$conv_3$	$woman_3$	$conv_3$

As depicted in (98) above, the 'narrow-scope indefinite' reading is available because the two occurrences of the condition $\mathbf{sing}(u_4)$ are in the scope of two distributivity operators over conventions ${}_{u_3}(\dots)$ and ${}_{u_5}(\dots)$.

Note that we allow for models in which Harvey courts more than one woman at a convention and, in addition, not every woman courted by Harvey comes to the banquet with him. We only require one of the women he courts at a convention to come to the banquet of that convention with him. That is, singular indefinites – and subsequent singular anaphora to them – have a weak reading. This issue is the main focus of the next subsection.

In sum, the present dynamic system – dubbed Plural Compositional DRT (PCDRT) – enables us to formulate in *classical type logic* a compositional account of the intra- and cross-sentential interactions between quantifiers, anaphora and number morphology exhibited by the quantificational subordination discourses in (5) and (6) above. The dynamics of plural info states and the static meaning for generalized determiners are integrated in an even-handed way, which enables us to provide an analysis of quantificational subordination as structured anaphora to quantifier domains.

2.7 Donkey Anaphora

Our current meanings for generalized determiners, indefinites and pronouns derive weak donkey readings. Consider again the typical weak donkey sentence in (58) above. Its PCDRT translation, which derives the intuitively-correct interpretation, is provided in (99) below.

(99) $\mathbf{max}^{u}({}_{u}([\text{PERSON}\{u\}];\ [u' \mid \mathbf{sing}(u'), \text{DIME}\{u'\}, \text{HAVE}\{u, u'\}]));$
$\mathbf{max}^{u'' \sqsubseteq u}({}_{u''}([\mathbf{sing}(u'), \text{PUT-IN-METER}\{u'', u'\}]));$
$[\mathbf{EVERY}\{u, u''\}]$

As the PCDRT system currently stands, it cannot derive the intuitively-correct interpretation for strong donkey sentences like (55) above. However, in the spirit of Dekker (1993) (see also Schwarzschild 1989), we can import the co-indexation mechanism in Heim (1982) and capture strong donkey readings. Moreover, we will preserve our solution to the proportion problem and the account will automatically generalize to mixed weak & strong donkey sentences of the kind discussed in Brasoveanu (2008).

The main proposal is as follows: we let the indefinites that intuitively receive a strong reading behave as open formulas, very much along the lines of classical DRT and FCS. Thus, strong indefinites are exactly like ordinary, weak indefinites except they do not introduce their own dref, which is instead introduced by the main generalized determiner of the donkey sentence. In other words, we allow generalized determiners to be *multiply* selective instead of singly selective.

Importantly, we do not run into the proportion problem because we have decomposed quantification and separated the static **DET** condition from the maximization and distributivity operators that regulate the dynamics of dependencies.

The resulting indexation of the paradigmatic example of strong donkey sentences is provided in (100) below. The universal determiner introduces its restrictor dref u, its nuclear scope dref u'' and the strong donkey dref u'. The indefinite is basically anaphoric to this dref (just as the donkey pronoun is) and its translation is identical to the one for singular anaphoric definites in (53) above.

(100) Every$^{u, u'' \sqsubseteq u, u'}$ farmer who owns a$_{u'}$ donkey beats it$_{u'}$.

This co-indexation mechanism is just a way to represent the contextual, pragmatically-determined coercion of the meaning of dynamic generalized determiners that Kanazawa (1994) argues for. Taking co-indexation to be the representation of this kind of coercion / quantifier domain manipulation correctly restricts the availability of strong readings to quantificational environments and predicts that the indefinite can be 'reinterpreted' as a definite only in this kind of configurations.

The translation for multiply selective generalized determiners is given in terms of multiply selective maximization and distributivity operators. These are a straightforward generalization of the singly selective operators we have already defined, as shown below.

(101) $\mathbf{max}^{u,u'}(D) := \lambda I_{st}.\lambda J_{st}.\ ([u,u'];D)IJ \wedge$
$\neg\exists K_{st}(([u,u'];D)IK \wedge J_{u\neq\bigstar,u'\neq\bigstar} \subsetneq K_{u\neq\bigstar,u'\neq\bigstar})$[14]

(102) $\mathbf{dist}_{u,u'}(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ \forall x \forall x'(I_{u=x,u'=x'} \neq \emptyset \leftrightarrow J_{u=x,u'=x'} \neq \emptyset) \wedge$
$\forall x \forall x'(I_{u=x,u'=x'} \neq \emptyset \rightarrow DI_{u=x,u'=x'}J_{u=x,u'=x'})$[15]

(103) $_{u,u'}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{u=\bigstar} = J_{u=\bigstar} \wedge I_{u'=\bigstar} = J_{u'=\bigstar}) \wedge$
$I_{u\neq\bigstar,u'\neq\bigstar} \neq \emptyset \wedge \mathbf{dist}_{u,u'}(D)I_{u\neq\bigstar,u'\neq\bigstar}J_{u\neq\bigstar,u'\neq\bigstar}$[16]

(104) $_{\langle u,u'\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{u=\bigstar} = J_{u=\bigstar} \wedge I_{u'=\bigstar} = J_{u'=\bigstar}) \wedge$
$(I_{u\neq\bigstar,u'\neq\bigstar} = \emptyset \rightarrow I = J) \wedge$
$(I_{u\neq\bigstar,u'\neq\bigstar} \neq \emptyset \rightarrow \mathbf{dist}_{u,u'}(D)I_{u\neq\bigstar,u'\neq\bigstar}J_{u\neq\bigstar,u'\neq\bigstar})$[17]

(105) $det^{u,u''\sqsubseteq u,u'} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u,u'}({}_{\langle u,u'\rangle}(P(u)));\ \mathbf{max}^{u''\sqsubseteq u}({}_{\langle u'',u'\rangle}(P'(u'')));$
$[\mathbf{DET}\{u,u''\}]$[18]

The strong donkey sentence in (100) above is translated as shown in (107) below. Just as before, we assume that the domain of the universal quantifier is non-empty, so we can use the simpler distributivity operators $_{u,u'}(\ldots)$ and $_{u'',u'}(\ldots)$.

(106) $every^{u,u''\sqsubseteq u,u'} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u,u'}({}_{u,u'}(P(u)));\ \mathbf{max}^{u''\sqsubseteq u}({}_{u'',u'}(P'(u'')));$
$[\mathbf{EVERY}\{u,u''\}]$

(107) $\mathbf{max}^{u,u'}({}_{u,u'}([\text{FARMER}\{u\}];\ [\mathbf{sing}(u'), \text{DONKEY}\{u'\}, \text{OWN}\{u,u'\}]));$
$\mathbf{max}^{u''\sqsubseteq u}({}_{u'',u'}([\mathbf{sing}(u'), \text{BEAT}\{u'',u'\}]));\ [\mathbf{EVERY}\{u,u''\}]$

The sequence of updates in (107) proceeds as follows. The restrictor update $\mathbf{max}^{u,u'}({}_{u,u'}(\ldots))$ stores under u and u' all the pairs of individuals

[14] $\mathbf{max}^{u_1,\ldots,u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ ([u_1,\ldots,u_n];D)IJ \wedge$
$\neg\exists K_{st}(([u_1,\ldots,u_n];D)IK \wedge J_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar} \subsetneq K_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar})$.

[15] $\mathbf{dist}_{u_1,\ldots,u_n}(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ \forall x_1 \ldots \forall x_n(I_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset \leftrightarrow J_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset) \wedge$
$\forall x_1 \ldots \forall x_n(I_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset \rightarrow DI_{u_1=x_1,\ldots,u_n=x_n}J_{u_1=x_1,\ldots,u_n=x_n})$.

[16] $_{u_1,\ldots,u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{u_1=\bigstar} = J_{u_1=\bigstar} \wedge \ldots \wedge I_{u_n=\bigstar} = J_{u_n=\bigstar}) \wedge$
$I_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar} \neq \emptyset \wedge \mathbf{dist}_{u_1,\ldots,u_n}(D)I_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar}J_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar}$.

[17] $_{\langle u_1,\ldots,u_n\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{u_1=\bigstar} = J_{u_1=\bigstar} \wedge \ldots \wedge I_{u_n=\bigstar} = J_{u_n=\bigstar}) \wedge$
$(I_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar} = \emptyset \rightarrow I = J) \wedge$
$(I_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar} \neq \emptyset \rightarrow \mathbf{dist}_{u_1,\ldots,u_n}(D)I_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar}J_{u_1\neq\bigstar,\ldots,u_n\neq\bigstar})$.

[18] $det^{u,u'\sqsubseteq u,u_1,\ldots,u_n} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u,u_1,\ldots,u_n}({}_{\langle u,u_1,\ldots,u_n\rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}({}_{\langle u',u_1,\ldots,u_n\rangle}(P'(u')));$
$[\mathbf{DET}\{u,u'\}]$.

such that, relative to any row i in the output info state, ui is a farmer and $u'i$ is a donkey that ui owns. Importantly, the $\mathbf{sing}(u')$ condition contributed by the singular indefinite $a_{u'}$ *donkey* is in the scope of the distributivity operator $_{u,u'}(\dots)$, which ensures that this singleton condition is vacuously satisfied. The nuclear scope update stores under u'' all the u-farmers that beat each and every one of their corresponding u'-donkeys. This maximal & distributive reading for the singular donkey pronoun $it_{u'}$ is due to the distributivity operator $_{u'',u'}(\dots)$, which instructs us to examine each *pair* consisting of a farmer and a donkey, i.e., each row i in the matrix, and check that the farmer $u''i$ in that pair beats the corresponding donkey $u'i$. Thus, the distributivity operator $_{u'',u'}(\dots)$ ensures the vacuous satisfaction of the second occurrence of the condition $\mathbf{sing}(u')$, which is contributed by the singular donkey pronoun. Finally, we check that the set of u-individuals, i.e., farmers that own at least one donkey, is included in the set of u''-individuals, i.e., farmers that own at least one donkey and beat every single donkey they own. This sequence of updates is depicted in (108) below.

(108) $I_\bigstar \xRightarrow{\mathbf{max}^{u,u'}(_{u,u'}(\dots))}$

u	u'
$farmer_1$	$donkey_1$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_1$	$donkey_1$

u	u'
$farmer_1$	$donkey_2$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_1$	$donkey_2$

u	u'
$farmer_2$	$donkey_3$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_2$	$donkey_3$

u	u'
$farmer_3$	$donkey_4$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_3$	$donkey_4$

u	u'
$farmer_3$	$donkey_5$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_3$	$donkey_5$

u	u'
$farmer_3$	$donkey_6$

$\xRightarrow{[\text{FARMER}\{u\}];[\mathbf{sing}(u'),\text{DONKEY}\{u'\},\text{OWN}\{u,u'\}]}$

u	u'
$farmer_3$	$donkey_6$

, i.e.,

u	u'
$farmer_1$	$donkey_1$
$farmer_1$	$donkey_2$
$farmer_2$	$donkey_3$
$farmer_3$	$donkey_4$
$farmer_3$	$donkey_5$
$farmer_3$	$donkey_6$

$\xRightarrow{\mathbf{max}^{u''\sqsubseteq u}(_{u'',u'}(\ldots))}$

$$
\left\{
\begin{array}{ll}
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_1 & donkey_1 & farmer_1 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_1 & donkey_1 & farmer_1 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow} \\
\\
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_1 & donkey_2 & farmer_1 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_1 & donkey_2 & farmer_1 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow} \\
\\
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_2 & donkey_3 & farmer_2 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_2 & donkey_3 & farmer_2 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow} \\
\\
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_4 & farmer_3 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_4 & farmer_3 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow} \\
\\
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_5 & farmer_3 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_5 & farmer_3 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow} \\
\\
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_6 & farmer_3 \\ \hline \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_3 & donkey_6 & farmer_3 \\ \hline \end{array} & \overset{[\mathbf{sing}(u'),\text{BEAT}\{u'',u'\}]}{\Longrightarrow}
\end{array}
\right\}
$$

$$
\overset{[\mathbf{EVERY}\{u,u''\}]}{\Longrightarrow}
\begin{array}{|c|c|c|} \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} & \multicolumn{1}{c}{u''} \\ \hline farmer_1 & donkey_1 & farmer_1 \\ \hline farmer_1 & donkey_2 & farmer_1 \\ \hline farmer_2 & donkey_3 & farmer_2 \\ \hline farmer_3 & donkey_4 & farmer_3 \\ \hline farmer_3 & donkey_5 & farmer_3 \\ \hline farmer_3 & donkey_6 & farmer_3 \\ \hline \end{array}
$$

In sum, the translation in (107) derives the intuitively-correct, strong reading for sentence (100) because the multiply selective distributivity operators ${}_{u,u'}(\dots)$ and ${}_{u'',u'}(\dots)$ neutralize the two occurrences of the $\mathbf{sing}(u')$ condition – which is now vacuously satisfied. Consequently, the maximization operator $\mathbf{max}^{u,u'}(\dots)$ in the restrictor stores in u' *all* the donkeys owned by each u-farmer – and not only one such donkey – and the nuclear scope retrieves all these donkeys one at a time.

We correctly predict that singular donkey anaphora can only have weak, i.e., existential & singular, or strong, i.e., maximal & distributive, readings – irrespective of which generalized determiner we co-index the indefinite with. This is because the co-indexation is cashed out in terms of multiply selective maximization and distributivity operators that are common to all determiners and that are completely separate from the static **DET** condition that is specific to each generalized determiner.

The present account of donkey anaphora generalizes to *mixed* weak & strong relative-clause donkey sentences like the one in (109) below, which are problematic for many static and dynamic accounts (see Brasoveanu 2008 for more discussion).

(109) Every$^{u_1,u_4 \sqsubseteq u_1,u_2}$ person who buys a$_{u_2}$ book on `amazon.com` and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

The most salient interpretation of sentence (109) is that, for *every* book (strong reading) that any credit-card owner buys on `amazon.com`, there is *some* credit card (weak reading) that s/he uses to pay for the book. In particular, the credit card can vary from book to book, e.g., I can use my MasterCard to buy set theory books and my Visa to buy detective novels, which means that even weak indefinites like *a*^{u_3} *credit card* can introduce non-singleton sets. For each buyer, the two sets of objects, i.e., all the books purchased on `amazon.com` and some of the credit cards that the buyer has, are correlated and the dependency between these sets – left implicit in the restrictor of the quantification – is specified in the nuclear scope: each book is correlated with the credit card that was used to pay for it. This paraphrase of the meaning of sentence (109) is formalized in classical (static) first-order logic as shown in (110) below.

(110) $\forall x \forall y (\text{PERSON}(x) \wedge \text{BOOK}(y) \wedge \text{BUY}(x,y) \wedge$
$\exists z (\text{CARD}(z) \wedge \text{HAVE}(x,z)) \rightarrow$
$\exists z' (\text{CARD}(z') \wedge \text{HAVE}(x,z') \wedge \text{USE-TO-PAY}(x,z',y)))$

The indefinite a_{u_2} *book* receives a strong reading, which in our account means that it is co-indexed with the determiner *every*$^{u_1,u_4 \sqsubseteq u_1,u_2}$. The resulting translation for this mixed-reading donkey sentence is provided in (112) below.

(111) $every^{u_1,u_4 \sqsubseteq u_1,u_2} \leadsto$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_1,u_2}({}_{u_1,u_2}(P(u_1)));\ \mathbf{max}^{u_4 \sqsubseteq u_1}({}_{u_4,u_2}(P'(u_4)));$
$[\mathbf{EVERY}\{u_1,u_4\}]$

(112) $\mathbf{max}^{u_1,u_2}({}_{u_1,u_2}([\text{PERSON}\{u_1\}];\ [\mathbf{sing}(u_2), \text{BOOK}\{u_2\}, \text{BUY}\{u_1,u_2\}];$
$[u_3 \mid \mathbf{sing}(u_3), \text{CARD}\{u_3\}, \text{HAVE}\{u_1,u_3\}]));$
$\mathbf{max}^{u_4 \sqsubseteq u_1}({}_{u_4,u_2}([\mathbf{sing}(u_2), \mathbf{sing}(u_3), \text{USE-TO-PAY}\{u_4,u_3,u_2\}]));$
$[\mathbf{EVERY}\{u_1,u_4\}]$

The update proceeds as follows. First, we store in u_1 all the people that buy books and have credit cards and in u_2 all the books that they buy. Then, relative to each u_1-person and each u_2-book, we (non-deterministically) store in u_3 one credit card that the person has. This is a consequence of the fact that the $\mathbf{sing}(u_3)$ condition is in the scope of the multiply selective distributivity operator ${}_{u_1,u_2}(\dots)$. The nuclear scope update stores in u_4 all the u_1 people such that, for each of the u_2-books they buy, they use the corresponding u_3-card to pay for the book. Finally, we test that the set of u_1-people is included in the set of u_4-people. The entire sequence of updates is true iff there is at least one way to successfully update the initial info state $I_\star$ with this sequence of updates, which is the case iff the first-order formula in (110) above can be satisfied.

3 Decomposing Modal Quantification

This section extends Plural Compositional DRT (PCDRT) with drefs for possible worlds, which enables us to decompose dynamic modal quantification and provide an analysis of modal subordination that is parallel to the analysis of quantificational subordination.

3.1 Intensional PCDRT

We extend Dynamic Ty2 (and PCDRT) with modal drefs by adding a new basic type $\mathbf{w}$ for possible worlds. The result is a Dynamic Ty3 logic with four basic types: t (truth values), e (individuals; variables: $x, x', \dots$), $\mathbf{w}$ (possible worlds; variables: $w, w', \dots$) and s (variable assignments; variables: $i, i', \dots, j, j', \dots$); D_t, D_e, $D_{\mathbf{w}}$ and D_s are non-empty and pairwise disjoint sets.

In the spirit of van Rooy (1998) and Stone (1999), we analyze modal anaphora by means of drefs for static modal objects. This enables us to explicitly capture the parallels between anaphora and quantification in the individual and modal domains. The resulting Intensional PCDRT system takes the research program in Muskens (1996), i.e., the unification of Montague semantics and DRT, one step further: it unifies in classical type logic the static Lewis (1973)-Kratzer (1981) analysis of modal quantification and the extensional Dynamic Plural Logic of van den Berg (1996).

Just as before, subscripts on terms indicate their types: $x_e, w_{\mathbf{w}}, i_s, \dots$. We also subscript lexical relations with their world variable – for example, $\text{SEE}_w(x, y)$ is intuitively interpreted as x sees y in world w. These notational conventions, meant to improve readability, are the reason for using boldfaced $\mathbf{w}$ for the type of possible worlds (while all the other basic types are italicized) – it distinguishes this type from the subscripted world variable w.

Just as a dref for individuals u is a function of type se from assignments i_s to individuals x_e, a dref for possible worlds p is a function of type $s\mathbf{w}$ from assignments i_s to possible worlds $w_{\mathbf{w}}$. Intuitively, the world $p_{s\mathbf{w}}i_s$ is the world that i assigns to the dref p.

A dref p stores a set of worlds, i.e., a proposition, with respect to an info state I, as shown in (113) below: $p[I]$ is the image of the set of assignments I under the function p.

(113) $p[I] := \{p_{s\mathbf{w}}i_s : i \in I\}$

Possible-world drefs have two uses:

(*i*) they store possible scenarios (in the sense of Stone 1999), e.g., the set of worlds introduced by the conditional antecedent in (10a), i.e., a possible scenario containing a man that is alive, which is further specified by the consequent of the conditional in (10a)

(*ii*) they store propositional contents, e.g., the content of the entire conditional in (10a), i.e., the content of the premise of the Aquinas argument (this is just a specific, restricted version of the previous use and, as such, can be derived from it)

In an intensional Fregean / Montagovian framework, the compositional aspect of interpretation is largely determined by the types for the extensions of the saturated expressions, i.e., names and sentences, plus the type that enables us to build intensions out of these extensions. Let us abbreviate them as **e**, **t** and **s**, respectively. A sentence is still interpreted as a DRS, i.e., as a relation between info states, hence $\mathbf{t} := (st)((st)t)$. A name is still interpreted as an individual dref, hence $\mathbf{e} := se$. Finally, $\mathbf{s} := s\mathbf{w}$, i.e., we use the type of possible-world drefs to build intensions.

The basic translations for some lexical items are provided in table (114) below. They are simply the intensional counterparts of their extensional PCDRT translations. We will henceforth use the following notational conventions:

- $u, u', \ldots$ are drefs (i.e., constants) of type **e** and $v, v', \ldots$ are variables of type **e**
- $p, p', \ldots$ are drefs (i.e., constants) of type **s** and $q, q', \ldots$ are variables of type **s**
- $\mathcal{P}, \mathcal{P}', \ldots$ are variables over dynamic propositions; their type is **st**
- $P, P', \ldots$ are variables over dynamic intensional properties of individuals; their type is $\mathbf{e}(\mathbf{st})$
- $Q, Q', \ldots$ are variables over dynamic intensional quantifiers; their type is $(\mathbf{e}(\mathbf{st}))(\mathbf{st})$

(114)	Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$ $\mathbf{s} := s\mathbf{w}$
	alive	$\lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [\text{ALIVE}_q\{v\}]$ where ALIVE is of type $e(\mathbf{w}t)$	$\mathbf{e}(\mathbf{st})$
	have	$\lambda Q_{(\mathbf{e}(\mathbf{st}))(\mathbf{st})}.\lambda v_{\mathbf{e}}.$ $Q(\lambda v'_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [\text{HAVE}_q\{v, v'\}])$ where HAVE is of type $e(e(\mathbf{w}t))$	$((\mathbf{e}(\mathbf{st}))(\mathbf{st}))(\mathbf{e}(\mathbf{st}))$
	man	$\lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [\text{MAN}_q\{v\}]$ where MAN is of type $e(\mathbf{w}t)$	$\mathbf{e}(\mathbf{st})$
	he$_u$	$\lambda P_{\mathbf{e}(\mathbf{st})}.\lambda q_{\mathbf{s}}.\ [\mathbf{sing}_q(u)];\ P(u)(q)$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
	Harvey^u	$\lambda P_{\mathbf{e}(\mathbf{st})}.\lambda q_{\mathbf{s}}.$ $[u \mid u = \text{HARVEY}];\ P(u)(q)$	$(\mathbf{e}(\mathbf{st}))(\mathbf{st})$
	a^u	$\lambda P_{\mathbf{e}(\mathbf{st})}.\lambda P'_{\mathbf{e}(\mathbf{st})}.\lambda q_{\mathbf{s}}.$ $[u \mid \mathbf{sing}_q(u)];\ P(u)(q);\ P'(u)(q)$	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
	det$^{u,u'\sqsubseteq u}$	$\lambda P_{\mathbf{e}(\mathbf{st})}.\lambda P'_{\mathbf{e}(\mathbf{st})}.\lambda q_{\mathbf{s}}.$ $\mathbf{max}^u(_{\langle u\rangle}(P(u)(q)));$ $\mathbf{max}^{u'\sqsubseteq u}(_{\langle u'\rangle}(P'(u')(q)));$ $[\mathbf{DET}_q\{u, u'\}]$	$(\mathbf{e}(\mathbf{st}))((\mathbf{e}(\mathbf{st}))(\mathbf{st}))$
	not	$\lambda \mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ [\sim \mathcal{P}(q)]$	$(\mathbf{st})(\mathbf{st})$

The intensional singleton requirement $\mathbf{sing}_p(u)$, the intensional condition $\mathbf{DET}_p\{u, u'\}$ and intensional lexical relations are defined below. Just as we have a dummy individual $\bigstar_e$, we will assume that there is a dummy world $\bigstar_{\mathbf{w}}$ relative to which all lexical relations are false, i.e., any n-ary relation of the form $R_w(x_1, \ldots, x_n)$ is false if w is $\bigstar$. We can think of the dummy world $\bigstar_{\mathbf{w}}$ as the world where no individual exists. Lexical relations are false in $\bigstar_{\mathbf{w}}$ because a relation between individuals obtains at a world only if the individuals exist in that world.

(115) $I_{p\neq\bigstar} := \{i \in I : pi \neq \bigstar\}$

(116) $pI := \{pi : i \in I_{p\neq\bigstar}\}$

(117) $I_{p=w} := \{i \in I : pi = w\}$

(118) $\mathbf{sing}_p(u) := \lambda I_{st}.\ pI \neq \emptyset \wedge \forall w \in pI(\mathbf{sing}(u)I_{p=w})$

(119) $\mathbf{DET}_p\{u, u'\} := \lambda I_{st}.\ pI \neq \emptyset \wedge \forall w \in pI(\mathbf{DET}\{u, u'\}I_{p=w})$

(120) $I_{p\neq\bigstar,u_1\neq\bigstar,\ldots,u_n\neq\bigstar} :=$
$\{i_s \in I : pi \neq \bigstar \wedge u_1 i \neq \bigstar \wedge \ldots \wedge u_n i \neq \bigstar\}$

(121) $R_p\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{p\neq\bigstar,u_1\neq\bigstar,\ldots,u_n\neq\bigstar} \neq \emptyset \wedge$
$\forall i_s \in I_{p\neq\bigstar,u_1\neq\bigstar,\ldots,u_n\neq\bigstar}(R_{pi}(u_1 i, \ldots, u_n i))$

The definition of intensional atomic conditions in (121) above relies on static lexical relations $R_w(x_1, \ldots, x_n)$ of the expected intensional type

$e^n(\mathbf{w}t)$. For any type τ, $e^n\tau$ is defined as the smallest set of types such that $e^0\tau := \tau$ and $e^{m+1}\tau := e(e^m\tau)$.

The notions of new dref introduction, structured inclusion, maximization and distributivity for possible-world drefs are parallel to the corresponding notions for individual-level drefs.

(122) $[p] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[p]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[p]j))$

(123) $p' \Subset p := \lambda I_{st}.\ \forall i_s \in I(p'i = pi \vee p'i = \bigstar)$

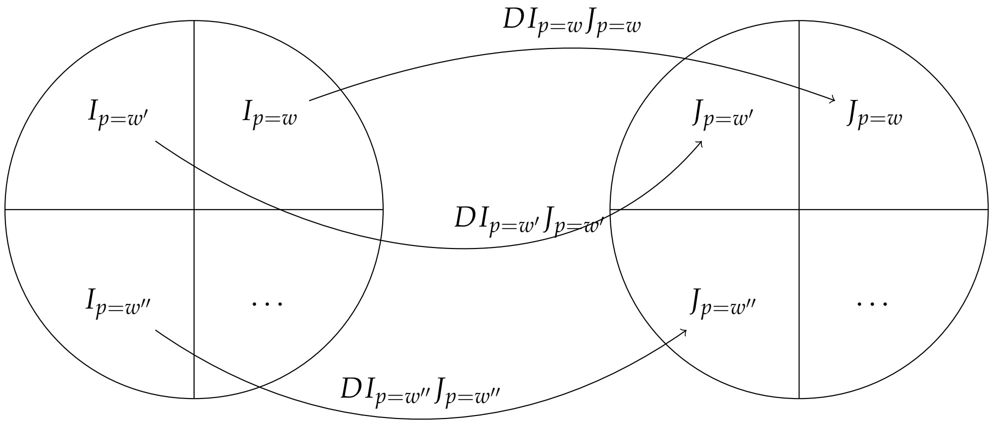
(124) $p' \sqsubseteq p := \lambda I_{st}.\ (p' \Subset p)I \wedge \forall i_s \in I(pi \in p'I \rightarrow pi = p'i)$

(125) $\mathbf{max}^p(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ ([p]; D)IJ \wedge \neg\exists K_{st}(([p]; D)IK \wedge J_{p \neq \bigstar} \subsetneq K_{p \neq \bigstar})$

(126) $\mathbf{max}^{p' \sqsubseteq p}(D) := \mathbf{max}^{p'}([p' \sqsubseteq p]; D)$

(127) $\mathbf{dist}_p(D) := \lambda I_{st}.\lambda J_{st}.\ pI = pJ \wedge \forall w_{\mathbf{w}} \in pI(DI_{p=w}J_{p=w})$

(128) Updating the info state I with the DRS D distributively over the dref p:



(129) ${}_p(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\bigstar} = J_{p=\bigstar} \wedge pI \neq \emptyset \wedge \mathbf{dist}_p(D)I_{p \neq \bigstar}J_{p \neq \bigstar}$

(130) ${}_{\langle p \rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{p=\bigstar} = J_{p=\bigstar} \wedge (pI = \emptyset \rightarrow I = J) \wedge$
$(pI \neq \emptyset \rightarrow \mathbf{dist}_p(D)I_{p \neq \bigstar}J_{p \neq \bigstar})$

We define sentential negation as a test (see *not* $\rightsquigarrow \lambda\mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ [\sim \mathcal{P}(q)]$ in table (114) above) only for simplicity. We could have easily defined it as introducing a maximal possible-world dref p storing all and only the worlds satisfying the 'sentence radical' in the scope of negation. This is needed to account for examples of modal subordination like (131) below, which can be easily analyzed within the framework developed in this section.

(131) a. Linus^u does not^p have a$^{u'}$ car.
b. He$_u$ would$_p$ have nowhere to park it$_{u'}$.

3.2 Indicative Sentences

We will analyze the discourse in (132) below all over again to see the intensional system in action. The two sentences of this discourse are compositionally translated in Intensional PCDRT as shown in (133) and (134) below.

(132) a. A^{u} wolf came in.

b. It_{u} ate $\text{Harvey}^{u'}$.

(133) a. *wolf* $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [\text{WOLF}_q\{v\}]$

b. a^{u} *wolf* $\rightsquigarrow \lambda P'_{\mathbf{e(st)}}.\lambda q_{\mathbf{s}}.\ [u \mid \mathbf{sing}_q(u)];\ [\text{WOLF}_q\{u\}];\ P'(u)(q)$

c. a^{u} *wolf came in* $\rightsquigarrow$
$\lambda q_{\mathbf{s}}.\ [u \mid \mathbf{sing}_q(u)];\ [\text{WOLF}_q\{u\}];\ [\text{COME-IN}_q\{u\}]$

(134) a. *ate* $\rightsquigarrow \lambda Q_{(\mathbf{e(st)})(\mathbf{st})}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [\text{EAT}_q\{v, v'\}])$

b. *ate* $\textit{Harvey}^{u'} \rightsquigarrow \lambda v_{\mathbf{e}}.\lambda q_{\mathbf{s}}.\ [u' \mid u' = \text{HARVEY}];\ [\text{EAT}_q\{v, u'\}]$

c. $\textit{it}_{u}$ *ate* $\textit{Harvey}^{u'} \rightsquigarrow$
$\lambda q_{\mathbf{s}}.\ [\mathbf{sing}_q(u)];\ [u' \mid u' = \text{HARVEY}];\ [\text{EAT}_q\{u, u'\}]$

The translations of the two sentences, more precisely, of the two 'sentence radicals', are two dynamic propositions of type **st**. We assume that each of the two sentences has an indicative mood morpheme (morphologically realized as part of their past tense morphology), the meaning of which is provided in (135) below. The indicative mood takes the dynamic proposition $\mathcal{P}_{\mathbf{st}}$ denoted by the remainder of the sentence (i.e., by the 'sentence radical' in its scope) and applies it to the designated dref for the actual world p^*. We capture the deictic nature of indicative morphology by the fact that its translation is parallel to the translation of singular pronouns (see (37) above).[19] The single world that p^* stores relative the input state I is (for the purposes of the current discourse / conversation) the actual world w^*.

[19]This analysis of indicative mood is simply a proof-of-concept intended to show that verbal moods can be fruitfully analyzed in parallel to pronouns. The analysis needs to be further developed to account for (among other things) the contrast between indicative and subjunctive in English – and, in richer mood systems, for the contrast between indicative and various kinds of non-indicative moods. The hope is that the resulting analysis of mood systems can be usefully compared to analyses of pronominal systems that realize various contrasts, e.g., proximal vs distal demonstratives, indexical vs anaphoric personal pronouns, overt vs covert pro-forms, past vs present tense etc. For more discussion, see Stone (1997) and Bittner (2001, 2007) among others.

(135) $\mathbf{ind}_{p^*} \rightsquigarrow \lambda \mathcal{P}_{\mathbf{st}}.\ [\mathbf{sing}(p^*)];\ \mathcal{P}(p^*)$[20]

(136) $\mathbf{sing}(p) := \lambda I_{st}.\ |pI| = 1$

When we apply the two indicative morphemes to the two 'sentence radicals' of our discourse, as shown in (137) below, we obtain the final DRS translations for the two sentences. Dynamically conjoining these DRSs gives us the translation for the entire discourse, provided in (138) below. We can rewrite this in the more familiar DRT format as shown in (139).

(137) $\mathbf{ind}_{p^*}(a^u\ wolf\ came\ in).\ \mathbf{ind}_{p^*}(it_u\ ate\ Harvey^{u'})$

(138) $[\mathbf{sing}(p^*)];\ [u \,|\, \mathbf{sing}_{p^*}(u)];\ [\text{WOLF}_{p^*}\{u\}];\ [\text{COME-IN}_{p^*}\{u\}];$
$[\mathbf{sing}(p^*)];\ [\mathbf{sing}_{p^*}(u)];\ [u' \,|\, u' = \text{HARVEY}];\ [\text{EAT}_{p^*}\{u, u'\}]$

(139) $[u, u' \,|\, \mathbf{sing}(p^*), \mathbf{sing}_{p^*}(u), \text{WOLF}_{p^*}\{u\}, \text{COME-IN}_{p^*}\{u\},$
$u' = \text{HARVEY}, \text{EAT}_{p^*}\{u, u'\}]$

Given that our default discourse-initial state $I_\bigstar$ assigns the dummy world $\bigstar$ to all world-drefs, including p^*, we follow Bittner (2007) and assume that a startup update precedes all discourses. This startup update is just the "commonplace" update of Stalnaker (1978) and "include[s] any information which the speaker assumes his audience can infer from the performance of the speech act" (Stalnaker 1978). Thus, the startup update introduces the dref p^* (among other things) and constrains it to store the world w^* in which the speech act is performed, as shown in (140) below. The two anaphoric indicative moods in (137) above can now be successfully interpreted and the sequence of updates in (138) can now be depicted as shown in (142) below.

(140) $[p^* \,|\, p^* = w^*]$

(141) $p^* = w^* := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(p^* i = w^*)$

(142)
$$I_\bigstar \xrightarrow{[p^*|p^*=w^*]} \begin{array}{c} p^* \\ \boxed{w^*} \end{array} \xrightarrow{[\mathbf{sing}(p^*)]} \begin{array}{c} p^* \\ \boxed{w^*} \end{array}$$
$$\xrightarrow{[u|\mathbf{sing}_{p^*}(u), \text{WOLF}_{p^*}\{u\}, \text{COME-IN}_{p^*}\{u\}]} \begin{array}{|c|c|} \multicolumn{1}{c}{p^*} & \multicolumn{1}{c}{u} \\ \hline w^* & wolf_1 \\ \hline \end{array} \xrightarrow{[\mathbf{sing}(p^*)]} \begin{array}{|c|c|} \multicolumn{1}{c}{p^*} & \multicolumn{1}{c}{u} \\ \hline w^* & wolf_1 \\ \hline \end{array}$$
$$\xrightarrow{[\mathbf{sing}_{p^*}(u)]} \begin{array}{|c|c|} \multicolumn{1}{c}{p^*} & \multicolumn{1}{c}{u} \\ \hline w^* & wolf_1 \\ \hline \end{array} \xrightarrow{[u'|u'=\text{HARVEY}, \text{EAT}_{p^*}\{u,u'\}]} \begin{array}{|c|c|c|} \multicolumn{1}{c}{p^*} & \multicolumn{1}{c}{u} & \multicolumn{1}{c}{u'} \\ \hline w^* & wolf_1 & harvey \\ \hline \end{array}$$

[20] We could take the dref p^* to store the current Context Set (Stalnaker 1978). Since p^* stores exactly one world relative to a plural info state (by virtue of $\mathbf{sing}(p^*)$), we could think of the Context Set as the set $p^*I_1 \cup p^*I_2 \cup p^*I_3 \cup \ldots$, whose elements are all the worlds that p^* stores relative to the plural info states $I_1, I_2, I_3, \ldots$ that are still live options at any given point in discourse.

3.3 Dynamic Modal Quantifiers

Following Kratzer (1981), we analyze modal verbs as quantifiers over possible worlds that are contextually parametrized. The contextually-provided parameters are the modal base β and the ordering source ω, represented as indices on modal verbs, as shown in (143) and (144) below.

(143) A wolf might$_{\beta,\omega}$ come in.

(144) If a wolf comes in, it might$_{\beta,\omega}$ eat Harvey.

Our dynamic notion of modal quantification is parallel to the notion of dynamic quantification over individuals proposed in the previous section. The translation for modal verbs in (145) below is parallel to the translation for determiners that anaphorically retrieve their restrictor in (91) above. The translation for modalized conditionals in (146) below, i.e., for *if*-clauses that modify a matrix clause containing a modal verb, is parallel to the translation for generalized determiners in (77) above. The *if*-clause is a way to overtly provide the restrictor for the modal quantifier. Note that the type of the translations in (145) and (146) are exactly the types we would expect in an intensional Montagovian framework, i.e., $(\mathbf{st})(\mathbf{st})$ and $\mathbf{st}((\mathbf{st})(\mathbf{st}))$, respectively.

(145) $modal^{p'\sqsubseteq p}_{\beta,\omega} \rightsquigarrow$
$\lambda\mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathcal{P}(p')));\ [\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}]$

(146) $if^{p}{+}modal^{p'\sqsubseteq p}_{\beta,\omega} \rightsquigarrow$
$\lambda\mathcal{P}_{\mathbf{st}}.\lambda\mathcal{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{\langle p\rangle}(\mathcal{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathcal{P}'(p')));$
$[\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}]$

Just as generalized determiners relate two dynamic properties P and P' of type **et**, modal verbs relate two dynamic propositions $\mathcal{P}$ and $\mathcal{P}'$ of type **st**. These dynamic propositions are used to extract a maximal restrictor set of worlds and a maximal nuclear scope set of worlds, which are stored in the drefs p and p'. These drefs are then related by a modal condition **MODAL** that is relativized to a modal base β and an ordering source ω. This condition contributes the static modal force that is specific to each modal quantifier.

Thus, the modal condition $\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}$ is parallel to the $\mathbf{DET}\{u,u'\}$ condition. The only difference is that the modal condition brings in the extra contextual parameters β and ω – and this is where the static analysis of modal quantification in Lewis (1973)-Kratzer (1981) is incorporated into our dynamic notion of modal quantification. Both β and ω are drefs of type $s(\mathbf{w}t)$, i.e., they are drefs for sets of worlds, and they store a set of sets of worlds, i.e., a set of propositions, relative to a plural info state I, as shown in (147) below. The dummy value for drefs like β and ω is the singleton set $\{\bigstar\}$ whose sole member is the dummy world, as shown in (148).

(147) a. $\beta I := \{\beta i : i \in I_{\beta \neq \{\bigstar\}}\}$
b. $\omega I := \{\omega i : i \in I_{\omega \neq \{\bigstar\}}\}$

(148) a. $I_{\beta \neq \{\bigstar\}} := \{i \in I : \beta i \neq \{\bigstar\}\}$
b. $I_{\omega \neq \{\bigstar\}} := \{i \in I : \omega i \neq \{\bigstar\}\}$

The sets of propositions βI and ωI are none other than Kratzer's static conversational backgrounds. That is, they are the set of propositions $\mathcal{B}$ of type $(\mathbf{w}t)t$ that is the contextually-provided modal base, and the set of propositions $\mathcal{O}$ of type $(\mathbf{w}t)t$ that is the contextually-provided ordering source – as shown in (149) below. Encoding conversational backgrounds by means of drefs directly captures their context-sensitive nature. Moreover, encoding them by means of drefs for propositions enables us to capture the fact that the propositional contents of sentences in logic puzzles like (1) above can be assembled together to form such conversational-background drefs.

(149) a. $\mathcal{B} = \beta I = \{\beta i : i \in I_{\beta \neq \{\bigstar\}}\}$
b. $\mathcal{O} = \omega I = \{\omega i : i \in I_{\omega \neq \{\bigstar\}}\}$

The ordering source $\mathcal{O}$ induces a strict partial order $<_{\mathcal{O}}$ over the set of all possible worlds, as defined in (150). We will assume that all the strict partial orders of the form $<_{\mathcal{O}}$ satisfy the Generalized Limit Assumption in (152) below – so, the **Ideal** function in (153) is well-defined. This function extracts the subset of $\mathcal{O}$-ideal worlds from the set of worlds W.

(150) $w <_{\mathcal{O}} w' := \{W_{\mathbf{w}t} \in \mathcal{O} : w' \in W\} \subsetneq \{W_{\mathbf{w}t} \in \mathcal{O} : w \in W\}$

(151) $w \leq_{\mathcal{O}} w' := w <_{\mathcal{O}} w' \vee w = w'$[21]

(152) Generalized limit assumption: for any proposition $W_{\mathbf{w}t}$ and ordering source $\mathcal{O}_{(\mathbf{w}t)t}$,
$\forall w \in W(\exists w' \in W(w' \leq_{\mathcal{O}} w \wedge \neg\exists w'' \in W(w'' <_{\mathcal{O}} w')))$

(153) The **Ideal** function: for any proposition $W_{\mathbf{w}t}$ and ordering source $\mathcal{O}_{(\mathbf{w}t)t}$,
$\mathbf{Ideal}_{\mathcal{O}}(W) := \{w \in W : \neg\exists w' \in W(w' <_{\mathcal{O}} w)\}$

[21] Alternatively: $w \leq_{\mathcal{O}} w' := \{W_{\mathbf{w}t} \in \mathcal{O} : w' \in W\} \subseteq \{W_{\mathbf{w}t} \in \mathcal{O} : w \in W\}$.

The static modal relations **NEC** and **POS** can now be defined in the familiar way, as shown in (154) and (155) below. The intersection of the restrictor set W and the (itself intersected) modal base $\mathcal{B}$, i.e., $\bigcap\mathcal{B} \cap W$, provides the set of worlds from which we select $\mathcal{O}$-ideal worlds. Necessity requires W' to contain all the ideal worlds, while possibility requires W' to contain only some of the ideal worlds.

(154) $\mathbf{NEC}_{\mathcal{B},\mathcal{O}}(W, W') := \mathbf{Ideal}_{\mathcal{O}}(\bigcap\mathcal{B} \cap W) \subseteq W'$

(155) $\mathbf{POS}_{\mathcal{B},\mathcal{O}}(W, W') := \mathbf{Ideal}_{\mathcal{O}}(\bigcap\mathcal{B} \cap W) \cap W' \neq \emptyset$

The modal conditions contributed by dynamic modal quantifiers can now be defined as shown in (156) below, exemplified for necessity and possibility modals in (157) and (158). Note that the third conjunct in (157) and the third conjunct in (158) are exactly the static modal relations **NEC** and **POS** defined in (154) and (155) above.

(156) $\mathbf{MODAL}_{p,\beta,\omega}\{p', p''\} :=$
$\lambda I_{st}.\ I_{p=\bigstar} = \emptyset \wedge \mathbf{sing}(p)I \wedge \mathbf{MODAL}_{\beta I,\omega I}(p'I, p''I)$

(157) $\mathbf{NEC}_{p,\beta,\omega}\{p', p''\} :=$
$\lambda I_{st}.\ I_{p=\bigstar} = \emptyset \wedge \mathbf{sing}(p)I \wedge \mathbf{NEC}_{\beta I,\omega I}(p'I, p''I)$

(158) $\mathbf{POS}_{p,\beta,\omega}\{p', p''\} :=$
$\lambda I_{st}.\ I_{p=\bigstar} = \emptyset \wedge \mathbf{sing}(p)I \wedge \mathbf{POS}_{\beta I,\omega I}(p'I, p''I)$

The world-dref p provides the world relative to which the entire modal quantification is evaluated. This is the reason for the first two conjuncts $I_{p=\bigstar} = \emptyset$ and $\mathbf{sing}(p)I$, which require the dref p to store only one world relative to info state I. That is, they ensure that the dref p is either the actual-world dref p^* or is in the scope of a distributivity operator $_p(\dots)$, which means that the entire modal condition is in the scope of such an operator.

We therefore ensure that the modal base βI and the ordering source ωI are the modal base and ordering source associated with only one world – namely the world of evaluation pI. To put it differently, we make use of plural info states to simplify the types of conversational-background drefs. The type of conversational backgrounds in Kratzer (1981) is $\mathbf{w}((\mathbf{w}t)t)$, i.e., a function f that associates with each world w, taken as the world of evaluation, a set of propositions $f(w)$, which is the conversational background f at the world of evaluation w. We could in principle have drefs for such functions as our conversational-background drefs; their type would be $s(\mathbf{w}((\mathbf{w}t)t))$. But we are able to work with conversational-background drefs of the lower type $s(\mathbf{w}t)$ because we let plural info states encode the dependency between the world of evaluation and its contextually-associated set of propositions.

Finally, the sets of worlds $p'I$ and $p''I$ in (156/157/158) above store the restrictor and nuclear scope sets of the modal quantifier. By virtue of our plural info states, these sets are also guaranteed to be the restrictor and the nuclear scope sets of worlds corresponding to world of evaluation pI.

3.4 A Parallel Account of Modal and Quantificational Subordination

We can now give a compositional account of modal subordination examples like the one in (8) above, repeated in (159) below, that is completely parallel to the analysis proposed for the quantificational subordination example in (6) above, repeated in (160).

(159) a. A^u wolf might come in.
b. It$_u$ would eat Harvey first.

(160) a. Harvey courts a^u woman at every convention.
b. She$_u$ always comes to the banquet with him.
c. [The$_u$ woman is usually also very pretty.]

Under its most salient interpretation, discourse (159) says that, for all the speaker knows, it is a possible that a wolf comes in. Moreover, for *any* such epistemic possibility of a wolf coming in, the wolf eats Harvey first. The modal subordination discourse in (159) is parallel to the discourse in (160) because the interactions between the indefinite *a*^{u} *wolf* and the modal *might*, on one hand, and the singular pronoun *it*$_u$ and the modal *would*, on the other hand, are parallel to the interactions between *a*^{u} *woman* and *every convention* and between *she*$_u$ and *always*.

Discourse-initial modals like *might* in (159a) have to anaphorically retrieve their restrictor. However, no world drefs are available for anaphoric take-up if the discourse is interpreted relative to the dummy info state $I_\bigstar$. We will assume that a world dref storing the set of all possible worlds is always available in discourse for anaphoric retrieval. Formally, we introduce a dref p storing the set of all possible worlds as part of our startup update, which will be of the form shown in (161) below: first, we set the dref p^* to the actual world w^*, then we make the set of all possible worlds salient in discourse by storing it in the dref p.

(161) $[p^* \mid p^* = w^*];\ \mathbf{max}^p([p \Subset p])$[22]

[22]Instead of $\mathbf{max}^p([p \Subset p])$, we could equally well use $\mathbf{max}^p([p = p])$, where $p = p' := \lambda I_{st}.\ \forall i_s \in I(pi = p'i)$.

We want to capture the *de dicto* reading of sentence (159a) in which the indefinite a^u *wolf* has narrow scope relative to the modal *might* – and the fact that sentence (159b) preserves and elaborates on this *de dicto* reading.

The translations for the two modal verbs *might* (in (159a)) and *would* (in (159b)) are provided in (162) and (163) below. Just like the adverb of quantification *always* in sentence (160b) above, these modal verbs anaphorically retrieve their restrictor drefs: *might* is anaphoric to the set of all possible worlds p that is by default available as part of the startup update and *would* is anaphoric to the nuclear scope dref p' introduced by *might*.[23]

(162) $might^{p' \sqsubseteq p}_{\beta,\omega} \rightsquigarrow$
$\lambda \mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p' \sqsubseteq p}({}_{p'}(\mathcal{P}(p')));\ [\mathbf{POS}_{q,\beta,\omega}\{p, p'\}]$

(163) $would^{p'' \sqsubseteq p'}_{\beta,\omega} \rightsquigarrow$
$\lambda \mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'' \sqsubseteq p'}({}_{p''}(\mathcal{P}(p'')));\ [\mathbf{NEC}_{q,\beta,\omega}\{p', p''\}]$

Note that the distributivity operators ${}_{\langle p \rangle}(\dots)$ and ${}_{\langle p' \rangle}(\dots)$ in the translations of *might* and *would* have been replaced *salva veritate* by the simpler operators ${}_{p}(\dots)$ and ${}_{p'}(\dots)$. This substitution is made possible by the fact that the modal relation **POS** contributed by *might* has a built-in existential commitment: by the very definition of **POS**, there must be a non-empty restrictor set of worlds p and a non-empty nuclear scope set of worlds p'. The same applies to the meaning of anaphoric *would*: since the restrictor set of *would* (i.e., p') is non-empty, its nuclear scope set p'' must also be non-empty given that **NEC** is parametrized by the same modal base β and ordering source ω as **POS**.

The contextually-supplied modal base β (of type $s(\mathbf{w}t)$) for both *might* and *would* is epistemic: it associates with each q-world the set of

[23] Thus, we build the meaning for run-of-the-mill quantifiers like *every*, *most* etc. out of three relatively independent parts: the first two parts are the two updates introducing the restrictor and nuclear scope sets (these two updates regulate the dynamics of dependencies) and the final, static part contributes the truth conditions associated with generalized quantifiers. This is not intended as a claim about how we actually process quantifiers. The motivation for this way of building the meaning of quantifiers is that it provides a general pattern that can be easily modified to obtain meanings for various other quantificational expressions. For example, if we omit the first update (the one that introduces the restrictor set), we obtain the meaning for adverbs of quantification like *always*, *usually* etc. and modal verbs like *might*, *should* etc., which anaphorically retrieve their restrictor. We might also be able to analyze floated *each* along similar lines – e.g., we could let it anaphorically retrieve its restrictor set and simply distribute over it, i.e., we would omit both the first and the third update and let the second update consist of an anaphoric distributivity operator only.

propositions that the speaker believes in that q-world. The contextually-supplied ordering source ω is empty, which means that it does not contribute anything to the meaning of the two modal quantifications. Once again, this is information that is not explicitly introduced in discourse, but is part of the background knowledge that the participants in the conversation are assumed to have. Since we are not concerned here with the exact nature of this common-background knowledge and how it comes about, we will just take it to also be part of the startup update, as shown in (164) below. Furthermore, we place conditions like $\mathbf{epistemic}\{p^*, \beta\}$ or $\mathbf{empty}\{\omega\}$ on the β and ω drefs. Strictly speaking, these conditions are just placeholders for the common-ground knowledge that they allude to. They are supposed to be intuitively interpreted as: β is an epistemic base relative to the evaluation world p^* and ω is an empty ordering source – but we will not attempt here to explicitly formalize them. One way to define emptiness for conversational-background drefs is tentatively suggested in (165).

(164) $[p^* \mid p^* = w^*]$; $\mathbf{max}^p([p \Subset p])$;
$[\beta, \omega \mid \mathbf{epistemic}\{p^*, \beta\}, \mathbf{empty}\{\omega\}]$

(165) a. $\mathbf{empty}\{\omega\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(\omega i = \{\bigstar\})$
b. $\mathbf{empty}\{\beta\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(\beta i = \{\bigstar\})$

Since the restrictor set for *might* in (162) above consists of all possible worlds, the modal verb ends up quantifying over the set of worlds compatible with the epistemic modal base β, i.e., over the set of worlds $\bigcap \beta I$, where I is the info state obtained after the startup update in (164) above. This is intuitively correct: when uttered out of the blue, sentence (159a) is interpreted as asserting that, for all the speaker knows, it is possible that a wolf comes in. The follow-up sentence in (159b) further elaborates on the wolf-coming-in possibilities epistemically entertained by the speaker. This is represented in (166) below by indexing both *might* and *would* with the same epistemic modal base β and by letting *would* be anaphoric to the world-dref p' introduced by *might*. The empty ordering source ω does not make any truth-conditional contribution because $\mathbf{Ideal}_{\mathcal{O}}(W) = W$ if $\mathcal{O}$ is empty, so we will henceforth omit it in our representations.

(166) $\mathbf{ind}_{p^*}(might_{\beta}^{p' \sqsubseteq p}(a^u\ wolf\ come\ in))$.
$\mathbf{ind}_{p^*}(would_{\beta}^{p'' \sqsubseteq p'}(it_u\ eat\ Harvey^{u'}\ first))$

The compositionally-derived translation for our modal subordination discourse is provided in (167).

(167) $[\mathbf{sing}(p^*)];$
$\mathbf{max}^{p' \sqsubseteq p}(_{p'}([u \mid \mathbf{sing}_{p'}(u), \text{WOLF}_{p'}\{u\}, \text{COME-IN}_{p'}\{u\}]));$
$[\mathbf{POS}_{p^*,\beta}\{p, p'\}];$
$[\mathbf{sing}(p^*)];$
$\mathbf{max}^{p'' \sqsubseteq p'}(_{p''}([\mathbf{sing}_{p''}(u)];\ [u' \mid u' = \text{HARVEY}, \text{EAT}_{p''}\{u, u'\}]));$
$[\mathbf{NEC}_{p^*,\beta}\{p', p''\}]$

Given the info state obtained after the startup update, (167) instruct us to do the following. First, we introduce the dref p' and store in it the maximal subset of p-worlds in which a wolf u comes in. Since p is the set of all possible worlds, p' will store the set of all worlds in which a wolf u comes in. Then, we test that the nuclear scope p' is a β-based epistemic possibility relative to the restrictor p. Formally, we check that there is at least one world w which is simultaneously a p-world and a β-world and which, in addition, is also a p'-world. That is, we check that there is an epistemic possibility of a wolf coming in. Then, we introduce the dref p'' and store in it all the p'-worlds in which the corresponding u-wolf eats Harvey first. Finally, we test that the nuclear scope p'' is a β-based epistemic necessity relative to the restrictor p'. Formally, we check if any world w that is both a p'-world and a β-world is also a p''-world. That is, we check if any epistemic possibility of a wolf coming in is such that the wolf featuring in that epistemic possibility eats Harvey first.

Note that the singular indefinite a^u *wolf* and the singular pronoun it_u anaphoric to it receive a weak reading. That is, we allow for epistemic possibilities in which several wolves come in and we require one of those wolves (but not necessarily all of them) to eat Harvey first. Thus, given any epistemic possibility of a wolf coming in, the wolf is unique relative to the *local* plural info state – but not globally, relative to the entire epistemic possibility under consideration. The issue of weak vs strong readings in modal environments is further discussed in the next subsection, in preparation for our analysis of the Aquinas discourse, which involves an instance of modal subordination that clearly has a strong reading.

The sequence of updates in (167) is depicted in (168) below.

(168) $I_\bigstar \xrightarrow{[p^* \mid p^* = w^*];\mathbf{max}^p([p \Subset p]);[\beta \mid \mathbf{epistemic}\{p^*, \beta\}]}$

p^*	p	β
w^*	w^*	$\{w^*, w_1, w_2\}$
w^*	w_1	$\{w^*, w_1, w_2\}$
w^*	w_2	$\{w^*, w_1, w_2\}$
w^*	w_3	$\{w^*, w_1, w_2\}$

$\xrightarrow{[\mathbf{sing}(p^*)]}$

p^*	p	β
w^*	w^*	$\{w^*, w_1, w_2\}$
w^*	w_1	$\{w^*, w_1, w_2\}$
w^*	w_2	$\{w^*, w_1, w_2\}$
w^*	w_3	$\{w^*, w_1, w_2\}$

$\xrightarrow{\mathbf{max}^{p' \sqsubseteq p}(_{p'}(\ldots))}$

$\left\{\right.$

p^*	p	β	p'
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*

$\xrightarrow{[u|\mathbf{sing}_{p'}(u), \text{WOLF}_{p'}\{u\}, \text{COME-IN}_{p'}\{u\}]}$

p^*	p	β	p'	u
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$

p^*	p	β	p'
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1

$\xrightarrow{[u|\mathbf{sing}_{p'}(u), \text{WOLF}_{p'}\{u\}, \text{COME-IN}_{p'}\{u\}]}$

p^*	p	β	p'	u
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$

p^*	p	β	p'
w^*	w_3	$\{w^*, w_1, w_2\}$	w_3

$\xrightarrow{[u|\mathbf{sing}_{p'}(u), \text{WOLF}_{p'}\{u\}, \text{COME-IN}_{p'}\{u\}]}$

p^*	p	β	p'	u
w^*	w_3	$\{w^*, w_1, w_2\}$	w_3	$wolf_3$

$\left.\right\}$

$\xrightarrow{[\mathbf{POS}_{p^*,\beta}\{p, p'\}]}$

p^*	p	β	p'	u
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$
w^*	w_2	$\{w^*, w_1, w_2\}$	★	★
w^*	w_3	$\{w^*, w_1, w_2\}$	w_3	$wolf_3$

$\xrightarrow{[\mathbf{sing}(p^*)]}$

p^*	p	β	p'	u
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$
w^*	w_2	$\{w^*, w_1, w_2\}$	★	★
w^*	w_3	$\{w^*, w_1, w_2\}$	w_3	$wolf_3$

$\xrightarrow{\mathbf{max}^{p'' \sqsubseteq p'}(_{p''}(\ldots))}$

p^*	p	β	p'	u	p''
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$	w^*

$\xrightarrow{[\mathbf{sing}_{p''}(u)];[u' \mid u'=\text{HARVEY},\text{EAT}_{p''}\{u,u'\}]}$

p^*	p	β	p'	u	p''	u'
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$	w^*	$harvey$

p^*	p	β	p'	u	p''
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$	w_1

$\xrightarrow{[\mathbf{sing}_{p''}(u)];[u' \mid u'=\text{HARVEY},\text{EAT}_{p''}\{u,u'\}]}$

p^*	p	β	p'	u	p''	u'
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$	w_1	$harvey$

$\xrightarrow{[\mathbf{NEC}_{p^*,\beta}\{p',p''\}]}$

p^*	p	β	p'	u	p''	u'
w^*	w^*	$\{w^*, w_1, w_2\}$	w^*	$wolf_1$	w^*	$harvey$
w^*	w_1	$\{w^*, w_1, w_2\}$	w_1	$wolf_2$	w_1	$harvey$
w^*	w_2	$\{w^*, w_1, w_2\}$	★	★	★	★
w^*	w_3	$\{w^*, w_1, w_2\}$	w_3	$wolf_3$	★	★

Thus, the representation in (167) above captures the intuitively correct truth conditions for the modal subordination discourse in (159).[24] Moreover, as desired, the representation in (167) is parallel to the corresponding representation of the quantificational subordination discourse in (97) above, repeated in (169) below for ease of reference.

[24] The logical form and the compositionally-obtained translation for the negation-based modal subordination discourse in (131) above are provided in (3) and (4) below. The crucial component is the translation for negation in (1) that introduces a maximal possible-world dref p.

(1) $not^p \rightsquigarrow \lambda\mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^p(_{\langle p\rangle}(\mathcal{P}(p)));\ [\mathbf{sing}(q), q \neq p]$

(2) $p \neq p' := \lambda I_{st}.\ \forall i_s \in I(pi \neq p'i)$

(3) $\mathbf{ind}_{p^*}(not^p(Linus^u\ have\ a^{u'}\ car))$.
$\mathbf{ind}_{p^*}(would_\beta^{p'\sqsubseteq p}(he_u\ have\text{-}nowhere\text{-}to\text{-}park\ it_{u'}))$.

(4) $[\mathbf{sing}(p^*)];$
$\mathbf{max}^p(_{\langle p\rangle}([u, u' \mid u = \text{LINUS}, \mathbf{sing}_p(u'), \text{CAR}_p\{u'\}, \text{HAVE}_p\{u, u'\}]));$
$[\mathbf{sing}(p^*), p^* \neq p];$
$[\mathbf{sing}(p^*)];$
$\mathbf{max}^{p'\sqsubseteq p}(_{\langle p'\rangle}([\mathbf{sing}_{p'}(u), \mathbf{sing}_{p'}(u'), \text{HAVE-NOWHERE-TO-PARK}_{p'}\{u, u'\}]));$
$[\mathbf{NEC}_{p^*,\beta}\{p, p'\}]$

(169) $[u_1 \mid u_1 = \text{HARVEY}]$; $\mathbf{max}^{u_2}([\text{CONVENTION}\{u_2\}])$;
$\mathbf{max}^{u_3 \sqsubseteq u_2}({}_{u_3}([u_4 \mid \mathbf{sing}(u_4), \text{WOMAN}\{u_4\}, \text{COURT-AT}\{u_1, u_4, u_3\}]))$;
$[\mathbf{EVERY}\{u_2, u_3\}]$;
$\mathbf{max}^{u_5 \sqsubseteq u_3}({}_{u_5}([\mathbf{sing}(u_4), \text{COME}\{u_4, u_5\}]))$; $[\mathbf{EVERY}\{u_3, u_5\}]$

Importantly, given our analysis of modal verbs as dynamic quantifiers, we automatically predict that anaphora to nuclear scope sets of worlds is always maximal. This is exactly what we need to account for the standard case of modal subordination in (159) above, the most salient interpretation of which is that: there is a possibility of a wolf coming in and, for *any* such epistemic possibility, the wolf eats Harvey first. That is, the modal *would* is anaphoric to all the epistemically-accessible worlds in which a wolf comes in, not only to some of them. This is parallel to the maximality associated with (E-type) anaphora to quantifiers over individuals.

We will end this subsection with a brief discussion of an important difference between quantificational and modal subordination. Consider the infelicitous example in (170) below (from Stone 1999; see also references therein).

(170) a. John might$^{p' \sqsubseteq p}$ be eating a^u cheesesteak.
b. #It$_u$ is$_{p^*}$ very greasy.

(171) $u\,\mathbf{in}\,p := \lambda I_{st}.\ I_{u \neq \bigstar, p \neq \bigstar} \neq \emptyset \wedge \forall i_s \in I_{u \neq \bigstar, p \neq \bigstar}(ui\,\mathbf{in}\,pi)$,
where **in** is a constant of type $e(\mathbf{w}t)$

Intuitively, this discourse is infelicitous because the hypothetical cheesesteak is anaphorically retrieved in the actual world – and the property of being very greasy cannot hold in the actual world of an entity that does not actually exist.

Note that the discourse (170) is felicitous if the indefinite *a^u cheesesteak* has a *de re*, wide-scope reading, but we will ignore this less salient reading for the moment and focus on the more salient *de dicto*, narrow-scope reading of the indefinite that yields infelicity.

As Stone (1999:21) suggests, we can derive the infelicity of this example if we associate every pronoun with a presupposition of existence relative to a modal dref – in this case, the modal dref p^* associated with the property of being very greasy that takes the pronoun as an argument. This presupposition is of the form given in (171) above (Stone's actual implementation is different).

The proposal is that the pronoun *it$_u$* in (170b) contributes such a presupposition of existence relative to the actual world dref p^*, i.e., $u\,\mathbf{in}\,p^*$. This presupposition, however, is not satisfied because the indefinite *a^u cheesesteak* in (170a) has a *de dicto* reading (i.e., it has narrow scope relative to the modal verb) and introduces the u-individual only relative to the epistemically-accessible p'-worlds contributed by $might^{p'}$, i.e., we can only satisfy u **in** p'.

In sum, the discourse in (170) is infelicitous because the most salient reading of sentence (170a) is the *de dicto* one, while sentence (170b) requires a *de re* reading to satisfy the existence presupposition contributed by the pronoun it_u. In contrast, the discourse in (172) below is felicitous because the *de re* reading of sentence (172a) is salient enough. The very same presuppositional mechanism that accounts for the infelicity of (170) enables us to account for the fact that the only available reading for discourse (172) as a whole is the *de re* one.

(172) a. A^u wolf $\text{might}_\beta^{p' \sqsubseteq p}$ come into our cabin tonight.
b. Linus saw it_u last night circling dangerously close to the cabin.

This difference between quantificational subordination and modal subordination also accounts for the fact that, in the former case, we can anaphorically retrieve a narrow scope indefinite by means of a plural pronoun, while in the latter case, we cannot, as shown by the contrast between (173) and (174) below.[25] This is because, in the case of quantificational subordination, the plural pronoun, e.g., $they_u$ in (173b) below, can have 'wide scope' and retrieve *all* the objects brought to salience by the narrow-scope indefinite a^u *woman* in (173b). That is, in the case of quantificational subordination, we can collect all the quantificationally-subordinated entities and elaborate on all of them in subsequent discourse.

In contrast, sentence (174b) is infelicitous because a modally-subordinated dref, i.e., a dref introduced by a singular indefinite that has narrow scope relative to a modal, cannot be subsequently retrieved by a 'wide-scope' pronoun, be it singular (as in (170b) above) or plural (as in (174b)). This is because the existence presupposition of the form u **in** p associated with the 'wide-scope' pronoun cannot be satisfied by the modally-subordinated, narrow-scope antecedent.

(173) a. Harvey courts a^u woman at every convention.
b. They_u are (always) very pretty.

(174) a. A^u wolf might come in.
b. #Linus saw them_u last night circling dangerously close to the cabin.
c. #They_u would eat Harvey first.

[25] I am indebted to an anonymous reviewer for bringing this point to my attention.

Finally, we can account for the infelicity of sentence (174c), which features a '*de dicto*' plural pronoun anaphoric to a *de dicto* singular indefinite, by means of the 'Maximize Presupposition' principle usually invoked in such cases (see, for example, Sauerland 2003 and references therein). That is, the **sing** presupposition contributed by singular pronouns (which, for simplicity, we have treated as part of the at-issue / asserted meaning) is stronger than the non-emptiness presupposition contributed by plural pronouns (which we also took to be part of the at-issue / asserted meaning) – and, *ceteris paribus*, the item with the stronger presupposition should be used if that presupposition can be satisfied.

The above observations about plural anaphora are fairly tentative and should be taken *cum grano salis*. We cannot do justice here to the variety of issues raised by plural anaphora and its interactions with individual-level and modal quantification. Our main goal is to provide a framework in which we can precisely formulate the relevant distinctions and that enables us to formalize such issues in the *lingua franca* of classical many-sorted type logic. This goal is more basic – and more modest – than the empirical investigation of the range of possible anaphoric relations and their (differing) interactions with quantificational elements in the individual and modal domains.

To conclude, anaphora and quantification in the individual and modal domains are analyzed in a systematically parallel way, from the types for drefs to the format for the translations of quantificational and anaphoric expressions. The fact that this is an empirically and theoretically desirable goal has been repeatedly observed in the literature – see Geurts (1995/1999), Stone (1997, 1999), Frank (1996), Bittner (2001) and Schlenker (2005) among others, extending the parallel between the individual and temporal domains argued for in Partee (1973, 1984). The differences in anaphoric accessibility between quantificational and modal subordination arise as a consequence of independently-motivated constraints on the way in which we can establish dependencies that simultaneously involve possibilities (i.e., hypothetical scenarios) and individuals featuring in them.

3.5 Modalized Conditionals and Donkey Anaphora

We analyze conditional antecedents as plural Russellian definite descriptions (see (79) above) in the modal domain – with the addition that such plural definites in the modal domain are always distributive, unlike their possibly collective counterparts in the individual domain. This analysis builds on the proposals in Stone (1999), Bittner (2001) and Schlenker (2004).

As already indicated in (146) above, we analyze *if* as a dynamic λ-abstractor over possible worlds, i.e., as a morpheme that extracts the content of a dynamic proposition $\mathcal{P}_{\mathbf{st}}$ and stores it in a newly introduced dref p, as shown in (175) below. In a modalized conditional, this dref p provides the restrictor set of worlds anaphorically retrieved by the modal verb in the matrix clause.

(175) $if^{p} \rightsquigarrow \lambda\mathcal{P}_{\mathbf{st}}.\ \mathbf{max}^{p}({}_{p}(\mathcal{P}(p)))$

Thus, the proposal goes against the received wisdom (which is not independently or cross-linguistically motivated; see, e.g., Bittner 2001) that the update in (146) above comes about by simply ignoring the semantic contribution of the overt item *if* and taking the adjoined *if*-clause to provide the restrictor for the generalized quantifier denoted by the modal verb in the matrix clause.

Rather, we propose that the adjoined *if*-clause introduces a topical dref that the matrix clause comments on. Except for the requirement that the matrix clause has to comment on the topical dref (or drefs, as the case may be; see the discussion of strong donkey conditionals below) contributed by the *if*-clause, there is no other constraint on how the meanings of the two clauses are combined. That is, we will use the default mode of dynamic meaning combination: dynamic conjunction.

Alternatively, we can stay closer to the received wisdom about conditionals and take the adjoined *if*-clause as a whole to be an intersective modifier of type **st** that modifies, i.e., is conjoined with, the modalized matrix clause as a whole, itself of type **st**. The relevant meaning for *if* is provided in (176) below and the corresponding compositional interpretation procedure for modalized conditionals is schematized in (177). The antecedent denotes a dynamic proposition, the consequent also denotes a dynamic proposition and the two propositions are intersected / conjoined; see the appendix of Brasoveanu (2008) for the required notions of generalized dynamic conjunction and generalized sequencing.

(176) $if^{p} \rightsquigarrow \lambda\mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathcal{P}(p)))$

(177) a. $if^{p}(subordinate\ clause) \rightsquigarrow \lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathcal{P}_{subord}(p)))$

b. $modal^{p'\sqsubseteq p}_{\beta,\omega}(matrix\ clause) \rightsquigarrow$
$\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathcal{P}_{matrix}(p')));\ [\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}]$

c. $[\ [if^{p}(subordinate\ clause)]\ [modal^{p'\sqsubseteq p}_{\beta,\omega}(matrix\ clause)]\ ]$
$\rightsquigarrow \lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathcal{P}_{subord}(p))) \sqcap \lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathcal{P}_{matrix}(p')));$
$[\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}]$
$\rightsquigarrow \lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{p}(\mathcal{P}_{subord}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathcal{P}_{matrix}(p')));$
$[\mathbf{MODAL}_{q,\beta,\omega}\{p,p'\}]$

Consider, for example, the modalized conditional in (178) below. The LF of this conditional is schematically represented in (179). The compositionally-obtained translations of the two clauses *if...* and *should...* are dynamically conjoined and the result is provided in (180). For simplicity, we omit the (redundant) distributivity operator contributed by *if*^{p} and use the simpler distributivity operator $_{p'}(\ldots)$ instead of $_{\langle p' \rangle}(\ldots)$.

(178) If^p it's raining, Linus^u should$^{p' \sqsubseteq p}_{\beta,\omega}$ bring an$^{u'}$ umbrella.

(179) $\mathbf{ind}_{p^*}([\ [\mathit{if}^p(\mathit{it's\ raining})]$
$[\mathit{should}^{p' \sqsubseteq p}_{\beta,\omega}(\mathit{Linus}^u\ \mathit{bring\ an}^{u'}\ \mathit{umbrella})]\ ])$

(180) $[\mathbf{sing}(p^*)];\ \mathbf{max}^p([\text{RAINING}_p]);$
$\mathbf{max}^{p' \sqsubseteq p}(_{p'}([u \mid u = \text{LINUS}];$
$[u' \mid \mathbf{sing}_{p'}(u'), \text{UMBRELLA}_{p'}\{u'\}, \text{BRING}_{p'}\{u, u'\}]));$
$[\mathbf{NEC}_{p^*,\beta,\omega}\{p, p'\}]$

The update in (180) proceeds as follows. First, we store in p the set of all worlds in which it is raining. Then, we store in p' all the p-worlds in which Linus brings an umbrella. Finally, we test whether the p'-worlds include all the deontically-ideal p-worlds – where 'deontically ideal' is determined in terms of the actual world p^* and the contextually-provided modal base β and deontic ordering source ω. This is depicted in (181) below.

(181) $I_\bigstar \xRightarrow{[p^*|p^*=w^*];[\beta,\omega|\ldots]}$

p^*	β	ω
w^*	W_1	V_1
w^*	W_2	V_2
w^*	W_3	V_3

$\xRightarrow{[\mathbf{sing}(p^*)]}$

p^*	β	ω
w^*	W_1	V_1
w^*	W_2	V_2
w^*	W_3	V_3

$\xRightarrow{\mathbf{max}^p([\text{RAINING}_p])}$

p^*	β	ω	p
w^*	W_1	V_1	w^*
w^*	W_2	V_2	w_1
w^*	W_3	V_3	w_2

$\xRightarrow{\mathbf{max}^{p' \sqsubseteq p}(_{p'}(\ldots))}$

p^*	β	ω	p	p'
w^*	W_2	V_2	w_1	w_1

$$\xrightarrow{[u \mid u=\text{LINUS}];[u' \mid \mathbf{sing}_{p'}(u'),\text{UMBRELLA}_{p'}\{u'\},\text{BRING}_{p'}\{u,u'\}]}$$

p^*	β	ω	p	p'	u	u'
w^*	W_2	V_2	w_1	w_1	*linus*	*umbrella*$_1$

p^*	β	ω	p	p'
w^*	W_3	V_3	w_2	w_2

$$\xrightarrow{[u \mid u=\text{LINUS}];[u' \mid \mathbf{sing}_{p'}(u'),\text{UMBRELLA}_{p'}\{u'\},\text{BRING}_{p'}\{u,u'\}]}$$

p^*	β	ω	p	p'	u	u'
w^*	W_3	V_3	w_2	w_2	*linus*	*umbrella*$_2$

$$\xrightarrow{[\mathbf{NEC}_{p^*,\beta,\omega}\{p,p'\}]}$$

p^*	β	ω	p	p'	u	u'
w^*	W_1	V_1	w^*	$\star$	$\star$	$\star$
w^*	W_2	V_2	w_1	w_1	*linus*	*umbrella*$_1$
w^*	W_3	V_3	w_2	w_2	*linus*	*umbrella*$_2$

This analysis immediately generalizes to modalized conditionals that contain instances of donkey anaphora. Consider, for example, the conditional in (182) below, based on an example in Partee (1984). The most salient reading for the donkey anaphora is the weak one: we consider all the worlds in which Linus has at least one credit card and store them in p; then, we check that all the p-worlds that are deontically-ideal (relative to the contextually-provided modal base β and ordering source ω) are such that Linus uses one of his credit cards instead of cash. The compositionally-obtained Intensional PCDRT representation is provided in (183). This account of weak donkey conditionals is parallel to the account of weak relative-clause donkey sentences in (99) above.

(182) If^p Linus^u has a$^{u'}$ credit card, he$_u$ should$^{p' \sqsubseteq p}_{\beta,\omega}$ use it$_{u'}$ here instead of cash.

(183) $[\mathbf{sing}(p^*)]$; $\mathbf{max}^p({}_p([u \mid u = \text{LINUS}];$
$[u' \mid \mathbf{sing}_p(u'), \text{CARD}_p\{u'\}, \text{HAVE}_p\{u,u'\}]))$;
$\mathbf{max}^{p' \sqsubseteq p}({}_{p'}([\mathbf{sing}_{p'}(u), \mathbf{sing}_{p'}(u'), \text{USE}_{p'}\{u,u'\}]))$;
$[\mathbf{NEC}_{p^*,\beta,\omega}\{p,p'\}]$

(184) $I_\star \xrightarrow{[p^* \mid p^*=w^*];[\beta,\omega \mid \ldots]}$

p^*	β	ω
w^*	W_1	V_1
w^*	W_2	V_2
w^*	W_3	V_3

$\xrightarrow{[\mathbf{sing}(p^*)]}$

p^*	β	ω
w^*	W_1	V_1
w^*	W_2	V_2
w^*	W_3	V_3

$\xrightarrow{\mathbf{max}^p({}_p(\dots))}$

p^*	β	ω	p
w^*	W_1	V_1	w^*

$\xrightarrow{[u \mid u{=}\text{LINUS}];[u' \mid \mathbf{sing}_p(u'),\text{CARD}_p\{u'\},\text{HAVE}_p\{u,u'\}]}$

p^*	β	ω	p	u	u'
w^*	W_1	V_1	w^*	*linus*	$card_1$

p^*	β	ω	p
w^*	W_2	V_2	w_1

$\xrightarrow{[u \mid u{=}\text{LINUS}];[u' \mid \mathbf{sing}_p(u'),\text{CARD}_p\{u'\},\text{HAVE}_p\{u,u'\}]}$

p^*	β	ω	p	u	u'
w^*	W_2	V_2	w_1	*linus*	$card_2$

p^*	β	ω	p
w^*	W_3	V_3	w_2

$\xrightarrow{[u \mid u{=}\text{LINUS}];[u' \mid \mathbf{sing}_p(u'),\text{CARD}_p\{u'\},\text{HAVE}_p\{u,u'\}]}$

p^*	β	ω	p	u	u'
w^*	W_3	V_3	w_2	*linus*	$card_3$

, i.e.,

p^*	β	ω	p	u	u'
w^*	W_1	V_1	w^*	*linus*	$card_1$
w^*	W_2	V_2	w_1	*linus*	$card_2$
w^*	W_3	V_3	w_2	*linus*	$card_3$

$\xrightarrow{\mathbf{max}^{p' \sqsubseteq p}({}_{p'}(\dots))}$

p^*	β	ω	p	u	u'	p'
w^*	W_1	V_1	w^*	*linus*	$card_1$	w^*

$\xrightarrow{[\mathbf{sing}_{p'}(u),\mathbf{sing}_{p'}(u'),\text{USE}_{p'}\{u,u'\}]}$

p^*	β	ω	p	u	u'	p'
w^*	W_1	V_1	w^*	*linus*	$card_1$	w^*

p^*	β	ω	p	u	u'	p'
w^*	W_3	V_3	w_2	*linus*	$card_3$	w_2

$\xrightarrow{[\mathbf{sing}_{p'}(u),\mathbf{sing}_{p'}(u'),\text{USE}_{p'}\{u,u'\}]}$

p^*	β	ω	p	u	u'	p'
w^*	W_3	V_3	w_2	*linus*	$card_3$	w_2

$\xrightarrow{[\mathbf{NEC}_{p^*,\beta,\omega}\{p,p'\}]}$

p^*	β	ω	p	u	u'	p'
w^*	W_1	V_1	w^*	*linus*	$card_1$	w^*
w^*	W_2	V_2	w_1	*linus*	$card_2$	★
w^*	W_3	V_3	w_2	*linus*	$card_3$	w_2

We also account for the strong donkey conditional in (185) below, based on an example in Kratzer (1981), in a way that is parallel to the analysis of strong relative-clause donkey sentences in (100) and (107) above. The representation in (186) derives the intuitively-correct interpretation: p stores all the worlds in which a murder happens and u stores all the corresponding murders; then, we check that each and every u-murder is investigated in all the deontically-ideal p-worlds.

(185) If^{p,u} a$_u$ murder happens, it$_u$ must$^{p' \sqsubseteq p,u}_{\beta,\omega}$ be investigated.

(186) $[\mathbf{sing}(p^*)];$
$\mathbf{max}^{p,u}({}_{p,u}([\mathbf{sing}_p(u), \text{MURDER}_p\{u\}, \text{HAPPEN}_p\{u\}]));$
$\mathbf{max}^{p' \sqsubseteq p}({}_{p',u}([\mathbf{sing}_{p'}(u), \text{INVESTIGATED}_{p'}\{u\}]));$
$[\mathbf{NEC}_{p^*,\beta,\omega}\{p, p'\}]$

(187) $$I_\bigstar \xrightarrow{[p^*|p^*=w^*];[\beta,\omega|\ldots]} \begin{array}{c|c|c} p^* & \beta & \omega \\ \hline w^* & W_1 & V_1 \\ w^* & W_2 & V_2 \\ w^* & W_3 & V_3 \end{array}$$

$$\xrightarrow{[\mathbf{sing}(p^*)]} \begin{array}{c|c|c} p^* & \beta & \omega \\ \hline w^* & W_1 & V_1 \\ w^* & W_2 & V_2 \\ w^* & W_3 & V_3 \end{array} \xrightarrow{\mathbf{max}^{p,u}({}_{p,u}(\ldots))}$$

$\Big\{$

p^*	β	ω	p	u
w^*	W_1	V_1	w^*	$murder_1$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_1	V_1	w^*	$murder_1$

p^*	β	ω	p	u
w^*	W_1	V_1	w^*	$murder_2$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_1	V_1	w^*	$murder_2$

p^*	β	ω	p	u
w^*	W_2	V_2	w_1	$murder_3$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_2	V_2	w_1	$murder_3$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_4$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_4$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_5$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_5$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_6$

$\xRightarrow{[\mathbf{sing}_p(u),\text{MURDER}_p\{u\},\text{HAPPEN}_p\{u\}]}$

p^*	β	ω	p	u
w^*	W_3	V_3	w_2	$murder_6$

$\Big\}$, i.e.,

p^*	β	ω	p	u
w^*	W_1	V_1	w^*	$murder_1$
w^*	W_1	V_1	w^*	$murder_2$
w^*	W_2	V_2	w_1	$murder_3$
w^*	W_3	V_3	w_2	$murder_4$
w^*	W_3	V_3	w_2	$murder_5$
w^*	W_3	V_3	w_2	$murder_6$

$\xRightarrow{\mathbf{max}^{p' \sqsubseteq p}(_{p',u}(\dots))}$

$\left\{ \right.$

p^*	β	ω	p	u	p'
w^*	W_2	V_2	w_1	$murder_3$	w_1

$\xRightarrow{[\mathbf{sing}_{p'}(u), \text{INVESTIGATED}_{p'}\{u\}]}$

p^*	β	ω	p	u	p'
w^*	W_2	V_2	w_1	$murder_3$	w_1

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_4$	w_2

$\xRightarrow{[\mathbf{sing}_{p'}(u), \text{INVESTIGATED}_{p'}\{u\}]}$

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_4$	w_2

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_5$	w_2

$\xRightarrow{[\mathbf{sing}_{p'}(u), \text{INVESTIGATED}_{p'}\{u\}]}$

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_5$	w_2

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_6$	w_2

$\xRightarrow{[\mathbf{sing}_{p'}(u), \text{INVESTIGATED}_{p'}\{u\}]}$

p^*	β	ω	p	u	p'
w^*	W_3	V_3	w_2	$murder_6$	w_2

$\left. \right\}$

$\xRightarrow{[\mathbf{NEC}_{p^*,\beta,\omega}\{p,p'\}]}$

p^*	β	ω	p	u	p'
w^*	W_1	V_1	w^*	$murder_1$	$\bigstar$
w^*	W_1	V_1	w^*	$murder_2$	$\bigstar$
w^*	W_2	V_2	w_1	$murder_3$	w_1
w^*	W_3	V_3	w_2	$murder_4$	w_2
w^*	W_3	V_3	w_2	$murder_5$	w_2
w^*	W_3	V_3	w_2	$murder_6$	w_2

The relevant definitions for the maximization and distributivity operators are provided below – they are strictly parallel to the ones needed for strong relative-clause donkey sentences.

(188) $\mathbf{max}^{p,u}(D) := \lambda I_{st}.\lambda J_{st}.\ ([p,u];D)IJ \wedge$
$\neg\exists K_{st}(([p,u];D)IK \wedge J_{p\neq\bigstar,u\neq\bigstar} \subsetneq K_{p\neq\bigstar,u\neq\bigstar})$

(189) $\mathbf{dist}_{p,u}(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ \forall w \forall x(I_{p=w,u=x} \neq \emptyset \leftrightarrow J_{p=w,u=x} \neq \emptyset) \wedge$
$\forall w \forall x(I_{p=w,u=x} \neq \emptyset \rightarrow DI_{p=w,u=x}J_{p=w,u=x})$

(190) ${}_{p,u}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{p=\bigstar} = J_{p=\bigstar} \wedge I_{u=\bigstar} = J_{u=\bigstar}) \wedge$
$I_{p\neq\bigstar,u\neq\bigstar} \neq \emptyset \wedge \mathbf{dist}_{p,u}(D)I_{p\neq\bigstar,u\neq\bigstar}J_{p\neq\bigstar,u\neq\bigstar}$

(191) ${}_{\langle p,u\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ (I_{p=\bigstar} = J_{p=\bigstar} \wedge I_{u=\bigstar} = J_{u=\bigstar}) \wedge$
$(I_{p\neq\bigstar,u\neq\bigstar} = \emptyset \rightarrow I = J) \wedge$
$(I_{p\neq\bigstar,u\neq\bigstar} \neq \emptyset \rightarrow \mathbf{dist}_{p,u}(D)I_{p\neq\bigstar,u\neq\bigstar}J_{p\neq\bigstar,u\neq\bigstar})$

The account also generalizes to mixed weak & strong donkey conditionals, first discussed in Dekker (1993). An example and its analysis are provided in (192) and (193) below.

(192) If^{p,u} a$_u$ driver has a$^{u'}$ dime, she$_u$ should$^{p' \sqsubseteq p,u}_{\beta,\omega}$ put it$_{u'}$ in the meter.

(193) $[\mathbf{sing}(p^*)]$; $\mathbf{max}^{p,u}({}_{p,u}([\mathbf{sing}_p(u), \text{DRIVER}_p\{u\}];$
$[u' \mid \mathbf{sing}_p(u'), \text{DIME}_p\{u'\}, \text{HAVE}_p\{u, u'\}]))$;
$\mathbf{max}^{p' \sqsubseteq p}({}_{p',u}([\mathbf{sing}_{p'}(u), \mathbf{sing}_{p'}(u'), \text{PUT-IN-METER}_{p'}\{u, u'\}]))$;
$[\mathbf{NEC}_{p^*,\beta,\omega}\{p, p'\}]$

4 Entailment as Modal Anaphora

This section shows that the intensional dynamic system we have argued for enables us to analyze entailment particles like *therefore* and discourses like (9/10) above that involve multiple layers of modal quantification and modal subordination across these layers.

Schematically, we analyze discourse (9/10) as shown in (194) below.

(194)
a. PREMISE^{p_1}
   i. if^{p_2,u_1}: a$_{u_1}$ man is alive
   ii. must$^{p_3 \sqsubseteq p_2,u_1}_{\beta,\omega}$: he$_{u_1}$ has a^{u_2} pleasure
b. THEREFORE$^{p_4 \sqsubseteq p_1}_{\beta^*,\omega^*}$
   i. if$^{p_5 \Subset p_2,u_1}$: not(he$_{u_1}$ has a^{u_3} spiritual pleasure)
   ii. must$^{p_6 \sqsubseteq p_5,u_1}_{\beta,\omega}$: he$_{u_1}$ has a^{u_4} carnal pleasure

The representation in (194) is a network of anaphoric connections. Consider the conditional in (194a) first. We introduce a new dref p_1 that stores the content of the whole conditional – that is, p_1 is the premise of the Aquinas argument as a whole. Then, the morpheme *if*^{p_2,u_1} in (194a-i) introduces a new dref p_2 that stores the content of the antecedent. In addition, the indefinite *a man* receives a strong donkey reading, i.e., given our mechanism of co-indexation, we introduce a new dref u_1 that stores all the men that are alive in each of the p_2-worlds.

Then, the modal verb *must* in (194a-ii) introduces a new dref p_3 that contains all the p_2-worlds in which the corresponding u_1-men have a pleasure. Finally, we test that the set of ideal p_2-worlds – ideal relative to a circumstantial modal base β and an empty ordering source ω – is included in the set of p_3-worlds.

We take *therefore* to contribute a necessity modal relation, just as the modal verb *must*. The particle *therefore* is anaphoric to dref p_1, i.e., the premise, and introduces a new dref p_4 that stores the set of p_1-worlds that also satisfy the conclusion of the Aquinas argument, i.e., the conditional (194b). Then, like any necessity modal, *therefore* requires the set of p_1-worlds that are ideal relative to the empty modal base β^* and the empty ordering source ω^* to be included in the set of p_4-worlds. The modal base β^* and the ordering source ω^* are empty because *therefore* is interpreted as logical consequence. Importantly, since β^* and ω^* are empty, the particle *therefore* effectively requires the set of p_1-worlds to be identical to the set of p_4-worlds.

Thus, we analyze the Aquinas discourse as a modal quantification that relates two embedded modal quantifications.

We finally turn to the second modal quantification, i.e., the conditional in (194b) above, which is modally subordinated to the previous conditional in (194a). Unlike all the conditional antecedents that we have considered up until now, the antecedent in (194b-i) is anaphoric to the dref p_2 introduced by the antecedent in (194a-i). The dref p_5 is a structured subset of p_2, symbolized as $p_5 \Subset p_2$. We need structured inclusion because we want p_5 to preserve the structure associated with the p_2-worlds, i.e., we want p_5 to preserve the quantificational dependency between p_2-worlds and the u_1-men that are alive in them. Since the dref u_1 received a strong reading in (194a-i), we preserve this strong reading in the modally subordinated antecedent in (194b-i), hence the indexation of the particle *if*$^{p_5 \Subset p_2, u_1}$.

Nothing special needs to be said about the modalized consequent in (194b-ii); its indexation and interpretation are parallel to (194a-ii).

The remainder of this section examines the particle *therefore* in more detail, then turns to a discussion of anaphoric conditionals and concludes with the fully explicit formal analysis of the Aquinas discourse.

4.1 *Therefore* as a Necessity Modal

We indicated that the entailment particle *therefore* relates static *propositional contents* and not dynamic meanings. We can see this by examining the discourses in (195) and (196) below: in both cases, the content (i.e., truth conditions) of the premise(s) and the content of the conclusion stand in an inclusion relation, while their meanings (i.e., context-change potentials) do not.

(195) a. Every^{u_1} man saw a^{u_2} woman.
b. Therefore, they$_{u_1}$ noticed them$_{u_2}$.

(196) a. A^{u_1} wolf might^{p_1} enter the cabin.
b. It$_{u_1}$ would$_{p_1}$ see John^{u_2}.
c. Therefore, it$_{u_1}$ would$_{p_1}$ notice him$_{u_2}$.

Further support for the idea that *therefore* relates contents is provided by the fact that the felicity of *therefore*-discourses is context-dependent, as shown by the discourse in (197) below: entailment obtains if (197) is uttered on a Thursday in a discussion about John, but not otherwise. The context-sensitivity of such discourses is expected if *therefore* relates contents – since contents are determined in a context-dependent way, while meanings are not.

(197) a. He$_{John}$ came back three days ago$_{Thursday}$.
b. Therefore, John came back on a Monday.

Moreover, we suggested that the entailment particle *therefore* should be analyzed as a necessity modal relation. One argument for such an analysis is that it predicts we can interpret *therefore* in different ways, i.e., relative to different modal bases and ordering sources.

For example, *therefore* expresses logical necessity in the Aquinas discourse, as well as in the three discourses in (195), (196) and (197) above. But it can also express causal consequence, as in (198) below, metaphysical necessity, as in (199), a form of practical inference, as in (200), and various other kinds of necessity / consequence, as in (201), (202) and (203).

(198) Reviewers are usually people who would have been poets, historians, biographers, etc., if they could; they have tried their talents at one or the other, and have failed; *therefore* they turn critics. (Samuel Taylor Coleridge, *Lectures on Shakespeare and Milton*)

(199) I blog; *therefore*, I am. (various hits on `www.google.com`)

(200) We cannot put the face of a person on a stamp unless said person is deceased. My suggestion, *therefore*, is that you drop dead. (attributed to J. Edward Day; letter, never mailed, to a petitioner who wanted himself portrayed on a postage stamp)

(201) In view of the head and neck symptoms of pneumomediastinum and cervical emphysema during labor – which include dyspnea, cough, sore throat, pain on swallowing, and dysphagia – otolaryngologists might be consulted and should *therefore* be aware of these conditions in order to recognize and treat them. (COCA – Corpus of Contemporary American English, `www.americancorpus.org`)

(202) There has been a general fear that juvenile delinquency might lead to adult criminality, and *therefore*, must always be curtailed before contaminating the society. (COCA)

(203) To say things might go back the way they were, *therefore* we should not do anything, is bad thinking. (COCA)

Another argument for analyzing *therefore* as a necessity modal is that it enables us to capture the intuitive equivalence between *therefore* discourses like (204) below and modalized conditionals like (205): they are equivalent provided we add the premise *A man saw a woman* to (205).

(204) A man saw a woman. Therefore, he noticed her.

(205) If a man saw a woman, he obviously / necessarily noticed her.

The translation for the particle *therefore* as it is interpreted in the Aquinas discourse is provided in (206) below.

(206) $therefore^{p_4 \sqsubseteq p_1}_{\beta^*,\omega^*} \rightsquigarrow$
$\lambda \mathcal{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p_4 \sqsubseteq p_1}({}_{p_4}(\mathcal{P}(p_4)));\ [\mathbf{NEC}_{q,\beta^*,\omega^*}\{p_1, p_4\}]$

(207) $[\mathbf{empty}\{\beta^*\}, \mathbf{empty}\{\omega^*\}]$

Since *therefore* expresses logical consequence, both its modal base β^* and its ordering source ω^* are empty, as shown in (207) above. This effectively requires p_4 to be identical to p_1 with respect to both values and structure. Formally, given the output state J obtained after the update $\mathbf{max}^{p_4 \sqsubseteq p_1}(\dots)$ in (206) above, the **NEC** condition effectively tests if $p_1 j = p_4 j$ for any assignment $j \in J$. Consequently, p_1 can be freely substituted for p_4 and we can simplify the translation of *therefore* as shown in (208) below. This translation clearly exhibits the anaphoric nature of the particle *therefore*.

(208) $therefore_{p_1} \rightsquigarrow \lambda \mathcal{P}_{\mathbf{st}}.\ {}_{p_1}(\mathcal{P}(p_1))$

The anaphoric nature of the entailment particle *therefore*, which needs a dref p_1 as its restrictor, triggers the insertion of a covert content-extraction operator $PREMISE^{p_1}$ that scopes over the entire modalized conditional in (194a) and stores the content of this conditional in p_1.

(209) $PREMISE^{p_1} \rightsquigarrow \lambda \mathcal{P}_{\mathbf{st}}.\ \mathbf{max}^{p_1}({}_{p_1}(\mathcal{P}(p_1)))$

We assume the existence of such covert operators only for presentational convenience. A more plausible analysis would be to automatically extract propositional contents and introduce new drefs to store these contents as part of the Common Ground (CG) update process (see Stalnaker 1978). While the present dynamic system is well-suited to formalize CG update and its interaction with quantification and anaphora, this issue is largely orthogonal to our present concerns, so we will ignore it.[26]

[26] Note that the Common Ground is a set of propositions incrementally updated in discourse by the addition of new propositions – and is different from the Context Set, which is a set of worlds, namely the intersection of all the propositions in the Common Ground. In the present system, we can take the dref p^* of type $s\mathbf{w}$ to store the Context Set. The Common Ground could be formalized by means of a modal base dref β^* of type $s(\mathbf{w}t)$.

4.2 Anaphoric Conditionals

The antecedent (194b-i) of the second conditional in the Aquinas discourse is anaphoric, i.e., modally subordinated, to the antecedent (194a-i) of the first conditional. We capture this by requiring the dref p_5 to be a maximal structured subset of p_2, as shown in (210) below.

(210) $if^{p_5 \Subset p_2} \rightsquigarrow \lambda\mathcal{P}_{\mathbf{st}}.\ \mathbf{max}^{p_5 \Subset p_2}({}_{p_5}(\mathcal{P}(p_5)))$

(211) $\mathbf{max}^{p' \Subset p}(D) := \mathbf{max}^{p'}([p' \Subset p]; D)$

This analysis seems to postulate two translations for the complementizer *if*: a non-anaphoric one in (194a-i) and an anaphoric one in (194b-i). But we can easily unify them by taking discourse-initial occurrences of *if* to be anaphoric the set of all possible worlds introduced as part of our startup update (see (161) above).

The fact that we use the weaker notion of structured inclusion $\Subset$ in the above translation for anaphoric *if* instead of the stronger one $\sqsubseteq$ requires some justification. Consider the two notions of anaphoric maximization $\mathbf{max}^{p' \sqsubseteq p}(D)$ and $\mathbf{max}^{p' \Subset p}(D)$ more closely.

The former ($\mathbf{max}^{p' \sqsubseteq p}(D)$) is more restrictive, i.e., stronger with respect to values: p' stores all and only the p-worlds such that all their associated dependencies satisfy the DRS D. The latter ($\mathbf{max}^{p' \Subset p}(D)$) is less restrictive, but this makes it stronger with respect to structure / dependencies: p' doesn't store only the p-worlds such that all their associated dependencies satisfy the DRS D; p' stores both these worlds and the p-worlds such that only some of their associated dependencies satisfy D. That is, p' collects as many p-dependencies that satisfy D as possible.

Consider again the modally-subordinated conditional in (194b). It is interpreted as: if a man is alive and he doesn't have any spiritual pleasure, he must have a carnal pleasure. That is, we quantify over all the p_2-worlds in which there is at least one u_1-man without spiritual pleasures and, simultaneously, over all such u_1-men. That is, we do not quantify only over the worlds in which *all* men are without spiritual pleasures. We quantify over both these worlds and the worlds in which there is at least one man without spiritual pleasures.

Formally, we need to quantify over the maximal subset of p_2-worlds such that at least some of their associated u_1-dependencies satisfy the antecedent (194b-i) – and, at the same time, over the maximal subset of these u_1-dependencies that satisfy (194b-i). This is what the structure-maximization operator $\mathbf{max}^{p' \Subset p}(D)$ achieves – as opposed to the operator $\mathbf{max}^{p' \sqsubseteq p}(D)$.

So, the antecedent of the conditional in (194b-i) stores in p_5 all the p_2-worlds where some u_1-man is alive and without spiritual pleasures and associates with each of these worlds all the corresponding u_1-men that have no spiritual pleasures. The consequent in (194b-ii) above can now receive its intuitively-correct interpretation, i.e., it requires us to check that each one of the u_1-men without spiritual pleasures has a carnal pleasure in the corresponding p_5-world.

Thus, anaphoric conditionals bring into sharper focus the main idea behind our account of modal subordination: modal subordination is just quantifier domain restriction via structured modal anaphora, i.e., simultaneous anaphora both to previously introduced sets of values, i.e., worlds and individuals, and to the previously established dependencies between these sets.

In general, structure-maximization operators seem to be needed in downward-entailing contexts that contain anaphoric items. Consider, for example, the relative-clause donkey sentences with downward-monotonic quantifiers in (212) (from Rooth 1987) and (213) below.

(212) No parent with a son still in high school has ever lent him the car on a weeknight.

(213) Few parents with a son still in high school have ever lent him the car on a weeknight.

Intuitively, (212) is falsified by any parent who has a son in high school and who has lent him the car on a weeknight, even if the parent has another son who never got the car (a similar observation can be made with respect to sentence (213)).

We analyze (212) as a strong (not weak, as the received wisdom has it!) donkey sentence whose main determiner *no* contributes a structure-maximization operator over its nuclear scope, as shown in (214) and (215) below.

(214) No$^{u,u''\Subset u,u'}$ parent with a$_{u'}$ son still in high school has ever lent him$_{u'}$ the car on a weeknight.

(215) $\mathbf{max}^{u,u'}(_{\langle u,u'\rangle}([\text{PARENT}\{u\}];\ [\mathbf{sing}(u'), \text{SON}\{u'\}, \text{WITH}\{u,u'\}]));$
$\mathbf{max}^{u''\Subset u}(_{\langle u'',u'\rangle}([\mathbf{sing}(u'), \text{LEND-CAR}\{u'',u'\}]));\ [\mathbf{NO}\{u,u''\}]$

(216) $\mathbf{max}^{u'\Subset u}(D) := \mathbf{max}^{u'}([u' \Subset u]; D)$

The first update in (215), i.e., the restrictor of the *no*-quantification, stores in u all the parents with at least one son and in u' all their corresponding sons. The second update, i.e., the nuclear scope of the *no*-quantification, stores in u'' all the u-parents that lent their car to *at least one* of their corresponding u'-sons. Finally, the $\mathbf{NO}\{u,u''\}$ condition effectively requires u'' to store the empty set – that is, there are no parents who lent their car to any one of their sons.

4.3 The Aquinas Discourse

The translation of the entire Aquinas discourse is provided in (217) below. Given the definition of truth in (35) above, we assign the intuitively correct truth conditions to this discourse. That is, according to the translation in (217), the argument made by Aquinas goes through. The premise establishes that the set of ideal worlds among the p_2-worlds is such that any man u_1 has at least one pleasure. The conclusion follows because, in all the ideal p_2-worlds, pleasures are spiritual or carnal (just as in the actual world w^*) and any man has at least one pleasure. Hence, if a man u_1 has no spiritual pleasure, he must have at least one carnal pleasure.

(217) *If*^{p_2,u_1} *a*$_{u_1}$ *man is alive, he*$_{u_1}$ *must*$^{p_3 \sqsubseteq p_2,u_1}_{\beta,\omega}$ *have a*^{u_2} *pleasure. Therefore*$_{p_1}$*, if*$^{p_5 \Subset p_2,u_1}$ *he*$_{u_1}$ *doesn't have any*^{u_3} *spiritual pleasure, he*$_{u_1}$ *must*$^{p_6 \sqsubseteq p_5,u_1}_{\beta,\omega}$ *have a*^{u_4} *carnal pleasure.* $\rightsquigarrow$

$$\begin{aligned}
&[\mathbf{sing}(p^*), \mathbf{circumstantial}\{p^*, \beta\}, \mathbf{empty}\{\omega\}];\\
&\mathbf{max}^{p_1}({}_{p_1}(\mathbf{max}^{p_2,u_1}({}_{p_2,u_1}([\mathbf{sing}_{p_2}(u_1), \text{MAN}_{p_2}\{u_1\}, \text{ALIVE}_{p_2}\{u_1\}]));\\
&\mathbf{max}^{p_3 \sqsubseteq p_2}({}_{p_3,u_1}([\mathbf{sing}_{p_3}(u_1)];\\
&[u_2 \mid \mathbf{sing}_{p_3}(u_2), \text{PLEASURE}_{p_3}\{u_2\}, \text{HAVE}_{p_3}\{u_1, u_2\}]));\\
&[\mathbf{NEC}_{p_1,\beta,\omega}\{p_2, p_3\}]));\\
&{}_{p_1}(\mathbf{max}^{p_5 \Subset p_2}({}_{p_5,u_1}([\sim [\mathbf{sing}_{p_5}(u_1)];\ [u_3 \mid \mathbf{sing}_{p_5}(u_3),\\
&\text{SPIRITUAL}_{p_5}\{u_3\}, \text{PLEASURE}_{p_5}\{u_3\}, \text{HAVE}_{p_5}\{u_1, u_3\}]]));\\
&\mathbf{max}^{p_6 \sqsubseteq p_5}({}_{p_6,u_1}([\mathbf{sing}_{p_6}(u_1)];\ [u_4 \mid \mathbf{sing}_{p_6}(u_4),\\
&\text{CARNAL}_{p_6}\{u_4\}, \text{PLEASURE}_{p_6}\{u_4\}, \text{HAVE}_{p_6}\{u_1, u_4\}]));\\
&[\mathbf{NEC}_{p_1,\beta,\omega}\{p_5, p_6\}])
\end{aligned}$$

Once again, we follow Kratzer (1981) and take the modal base and ordering source drefs β and ω (relative to which the premise and the conclusion are interpreted) to be contextually supplied and suitably constrained. In particular, the condition $\mathbf{circumstantial}\{p^*, \beta\}$ in (217) above constrains the modal base β to share with the actual world p^* the same (contextually-relevant) set of circumstances. The proposition in (218) below is one such shared circumstance.

(218) $\{w : \forall x_e(\text{PLEASURE}_w(x) \rightarrow (\text{SPIRITUAL}_w(x) \vee \text{CARNAL}_w(x)))\}$

5 Comparison with Previous Approaches

The main goal of the present paper was to explore how far we can get in the analysis of modal and individual-level quantification and anaphora

– and their intra- and cross-sentential interactions – with just two generalizations of the classical Tarskian semantics for first-order logic: going dynamic in the style of Dynamic Predicate Logic (Groenendijk & Stokhof 1991), i.e., taking denotations to be pairs of an input and an output context of evaluation, and going plural in the style of Dynamic Plural Logic (van den Berg 1996), i.e., taking contexts of evaluation to consist of sets of assignments instead of single assignments. We preserved as much of classical Tarskian semantics as possible, e.g., we worked with total assignments (formally better behaved than partial assignments, stacks, referent systems etc.), so that we can identify the extensions that are crucial to our analysis.

Following Muskens (1996) (among many others), we also preserved the Montagovian solution to the problem of compositionality (at subclausal level), since there seemed to be no compelling reason to believe that the two generalizations of classical Tarskian semantics will require any change in this respect. The only essential addition to Muskens (1996) was the introduction of maximization and distributivity operators (in the spirit of van den Berg 1996), needed to define a dynamic version of λ-abstraction.

The resulting Intensional PCDRT system reformulates van den Berg's Dynamic Plural Logic in classical type logic, simplifies it in the process and unifies it with the static Lewis (1973)-Kratzer (1981) analysis of modal quantification in an even-handed way. An important consequence is that we are able to associate modal quantifications with *contents*, i.e., the propositions these quantifications express in a particular context, and are able to account for the fact that the entailment particle *therefore* in the Aquinas discourse and modal verbs in logic puzzles can relate such contents.

The approach we took was one of four types of dynamic approaches to modal subordination explored in the previous literature. These types of approaches differ with respect to the way in which they encode the quantificational dependencies between possible scenarios, e.g., the epistemic possibilities of a wolf coming in, and the individuals that feature in these scenarios, e.g., the wolves in these epistemic possibilities that come in.

The first class consists of accommodation accounts – exemplified primarily by Roberts (1987, 1989, 1996), but see also the more recent approach in Geurts (2009) –, where there are no modal drefs of any kind and the associations between possible scenarios and the individuals that feature in them is captured at the level of logical form, i.e., by accommodating / copying the DRSs that introduce the relevant individual-level drefs into the restrictor or nuclear scope DRSs of another modal operator. Most of the work in these approaches is done by such DRS-copying rules, for which a systematic, suitably-constrained theory is still pending (see Geurts 2009 for a recent attempt to work towards the foundations of such a theory).

The naturally-occurring logic-puzzle texts exemplified at the beginning of the paper show that it is really important to provide a formally explicit and detailed theory of DRS copying / accommodation. The reason is that these logic puzzles *can be solved* – and, to solve them, we need to be able to build precise semantic representations for discourses in which complex relations between sets of objects are incrementally introduced and described. Simply invoking some underspecified form of DRS copying / accommodation will not provide the highly specific and detailed representations that support the logical reasoning required to solve these puzzles.

The second class includes analyses like the ones proposed in Kibble (1994, 1995), Geurts (1995/1999), Frank (1996), Frank & Kamp (1997), van Rooy (2001) and Asher & McCready (2007), which take modal quantifiers to relate dynamically-valued drefs, i.e., in the simplest case, drefs for information states – where, following Heim (1982), an information state is basically represented as a set of $\langle world, assignment\rangle$-pairs.

In these approaches, the dependency between possibilities and individuals is encoded in the drefs for information states: every $\langle world, assignment\rangle$-pair is such that the assignment stores the individual-level drefs that have been introduced with respect to that world. These approaches to modal subordination are parallel to the parametrized-sum-individuals approaches to donkey anaphora and quantificational subordination in Rooth (1987) and Krifka (1996). The only difference is that, instead of summing atomic individuals, each of which is parametrized with a variable assignment, these approaches 'sum' possible worlds that are parametrized with assignments.

The third class consists of encapsulated quantification accounts, e.g., Stone (1997, 1999) and Bittner (2001, 2007), where modal quantifiers relate drefs for static objects. Modal drefs in such accounts are of type $s(\mathbf{w}(\mathbf{w}t))$, i.e., they are drefs for accessibility relations, and individual-level drefs are of type $s(\mathbf{w}e)$, i.e., they are drefs for individual concepts.

The quantificational dependency between possibilities and individuals is encoded in the complex static objects that these drefs have as values. For example, in a sentence like *A wolf might come in*, the modal *might* introduces a dref of type $s(\mathbf{w}(\mathbf{w}t))$ which, with respect to a given assignment i_s, stores a relation of type $\mathbf{w}(\mathbf{w}t)$ that maps (the current candidates for) the actual world to the set of epistemically-accessible worlds in which a wolf comes in. The indefinite *a wolf* introduces a dref of type $s(\mathbf{w}e)$ which, relative to an assignment i_s, stores a function mapping every epistemically-accessible world w in which a wolf comes in to that particular wolf.

Building on proposals in van den Berg (1996) and van Rooy (1998), we used of a fourth way of capturing quantificational dependencies between possibilities and individuals, namely plural information states. Just as in encapsulated quantification accounts, drefs for possibilities have static objects as values – in particular, they are of type $s\mathbf{w}$, storing a possible world w relative to each assignment i. Drefs for individuals have the usual type se. But, unlike in encapsulated quantification accounts, the quantificational dependencies between possibilities and individuals are stored in the plural info states that are incrementally updated in discourse and not in the static objects that the modal and individual-level drefs have as values. Our dynamic modal relations are relativized to a world-dref (p^* by default), so we can associate modal quantifications with their propositional contents and thereby account for the Aquinas discourse.

Thus, the differences between the present account and previous approaches stem from the choices made with respect to three dimensions of variation:

(*i*) the use of modal drefs that have static vs dynamic objects as values
(*ii*) encoding quantificational dependencies by means of functions / relations vs plural info states
(*iii*) being able to associate propositional contents with dynamic modal quantifiers

The many dynamic systems instantiating these four types of approaches and combinations thereof are rather complex (given the empirical complexity of the target phenomena) and one can fairly easily imagine a variety of more-or-less natural extensions for most of them.[27] Consequently, a comparison between these systems with the aim of dismissing some of them as empirically or theoretically inadequate is ultimately misguided – given the currently available empirical evidence and the current level of theory development.

This section will therefore be confined to the discussion of several kinds of phenomena that make our choices with respect to (*i*), (*ii*) and (*iii*) above at least *prima facie* plausible.

We begin with point (*iii*). The Lewis-Kratzer static semantics for modal quantification automatically associates propositional contents with modal quantifiers. This enables us to understand why modal statements can be related by particles like *therefore*, i.e., can be (informa-

[27] I am indebted to Paul Dekker and an anonymous reviewer for really driving this point home.

tively) said to entail one another, in the same way that non-modal statements can be said to entail one another. In contrast, most dynamic analyses of modal quantification, including van Rooy (1998) and Stone (1999), are formulated in such a way that they automatically fuse modal statements with the background information (i.e., with the input context of evaluation) and do not extract their contents and keep track of these contents as separate discourse entities. While this is not a necessary feature of dynamic systems, it is a natural consequence of the view that meaning is context / information update.[28] Consequently, these dynamic systems do not capture the meaning of items like *therefore* that relate such contents. One can imagine ways of 'lifting' the dynamic meanings of sentences in a *therefore*-discourse from the context and extract their static contents – since all dynamic systems on the market associate sentences with truth conditions (in the meta-language, however, not in the representation / object language!) – but this would be an artificial way to proceed. The way in which Intensional PCDRT integrates dynamic plural logic and the Lewis-Kratzer semantics for modal quantification delivers this automatically.

Turning now to point (*ii*), an argument in favor of decomposed quantification is that encapsulated-quantification approaches (which, in a broad sense, include approaches that make use of choice and / or Skolem functions to account for donkey anaphora and quantificational subordination) do not store the quantificational dependencies introduced in discourse in the database that is meant to store discourse-related information, i.e., in the information states. They instead store quantificational dependencies in the meaning of lexical items, be they the indefinite-like items that introduce new drefs or the pronoun-like items that retrieve them.

The point, which van den Berg (1996) already makes with respect to plural anaphora, can be more easily clarified if we consider the quantificational-subordination examples in (219) and (220) below. The argument based on modal subordination would be parallel to this.

(219) a. Every^{u} man loves a$^{u'}$ woman.
b. They$_{u}$ bring them$_{u'}$ flowers to prove this.

(220) a. Every^{u} boy made a$^{u'}$ paper flower and gave it$_{u'}$ to a$^{u''}$ girl.
b. They$_{u''}$ thanked them$_{u}$ for the$_{u'}$ very nice gifts.

Consider (219) (from van den Berg 1996) first. Sentence (219a) establishes a twofold dependency between men and the women that they

[28] I am indebted to Paul Dekker and an anonymous reviewer for suggesting this line of argumentation.

love. Then, sentence (219b) further elaborates on this dependency. Encapsulated-quantification approaches have to make use of functions from individuals to individuals of type *ee* – or relations between individuals of type $e(et)$ – to capture the intuition that sentence (219b) elaborates on the dependency introduced in sentence (219a). That is, either the quantifiers $every^u$ *man* and $a^{u'}$ *woman* or the pronouns $they_u$ and $them_{u'}$ – or both – have to have such functions / relations as part of their semantic values.

Now consider discourse (220). Sentence (220a) establishes a threefold dependency between boys, flowers and girls and sentence (220b) further elaborates on this dependency. In this case, encapsulated-quantification approaches need to make use of functions and / or relations that are more complex than the ones needed for discourse (219). Therefore, the semantic values assigned to quantifiers and / or pronouns will have to be more complex in the case of (220), despite the fact that the very same lexical items are used.

That is, quantifiers and / or pronouns denote functions / relations of different arities depending on the discourse context, i.e., depending on how many simultaneous anaphoric connections are established in a particular discourse. And these functions / relations become a lot more complex as soon as we start to explicitly represent anaphora to and quantification over possible worlds, times, locations, eventualities, degrees etc. In sum, the argument against encapsulated-quantification approaches is the following: since the arity of the functions / relations denoted by pronouns and / or quantifiers is determined by the discourse context, we should encode this context dependency in the info state (the purpose of which is to store precisely this kind of discourse information) and not in the meaning of the lexical items themselves.

There are two, more specific features of our analysis of modal quantification that distinguish it from the encapsulated approach in Stone (1999) – and they should be mentioned because Stone (1999) has been an obvious source of inspiration for the present account. First, Stone (1999) treats modal bases and ordering sources as static objects (see the definitions for necessity and possibility in Stone 1999:27,(47)). In contrast, we introduce drefs for modal bases and ordering sources, thus providing a dynamic treatment for all the context-dependent components of modal quantification that Kratzer (1981) argues for.

Second, we employ maximal *unparametrized* restrictor and nuclear scope sets in the definition of modal quantification. In contrast, Stone (1999) introduces restrictor and nuclear scope sets for modal quantifiers by means an **if**-update, with a Lewis-style similarity ordering source built into it (see Stone 1999:17,(34)). To see that the built-in parametrization is too restrictive, consider the deontic conditional in (221) below (based on Kratzer 1981). Intuitively, (221) does not involve a similarity ordering source because the conditional simply states that, according to the law, the deontically ideal worlds among the set of *all* worlds where there is a murder are such that the murderer goes to jail. The deontic quantification is not restricted to the set of worlds where there is a murder and which are as similar as possible to the actual world, since many of the facts in the actual world are orthogonal to the legal requirement specified by (221).

(221) If there is a murder, the murderer must go to jail.

Another argument for maximal unparametrized nuclear scope sets is provided by the discourse in (222) below (from Roberts 1996).

(222) a. You should buy a^u lottery ticket and put it_u in a safe place.
b. [You're a person with good luck.]
c. It_u might be worth millions.

Sentence (222c) elaborates on *any* possible scenario in which you buy a lottery ticket (and put it in a safe place). Crucially, sentence (222c) does not elaborate only on deontically-ideal scenarios of this sort, as it would be the case if the nuclear-scope set of worlds introduced by sentence (222a) were parametrized by the modal base and ordering source contributed by the modal verb *should*.

We finally turn to point (*i*). The availability of both weak and strong donkey readings in modalized conditionals seems to favor Intensional PCDRT over systems with dynamically-valued modal drefs or accommodation-based accounts. Accommodation-based approaches like Roberts (1987, 1989) account only for strong donkey readings, a feature they inherit from the underlying classical DRT framework. More interestingly, approaches that use drefs for information states also account only for strong readings.

For example, the definitions in Frank (1996:98,(36)) and Geurts (1995/1999:154,(43b)) update a set F of $\langle world, assignment\rangle$-pairs with a DRS K (the denotation of which is a binary relation between $\langle world, assignment\rangle$-pairs) by taking the image of the set F under the relation denoted by K. That is, the output set G of $\langle world, assignment\rangle$-pairs obtained after updating F with K is the set $G = \{\langle w', g'\rangle : \exists \langle w, g\rangle \in F(\langle w, g\rangle\, K\, \langle w', g'\rangle)\}$. This kind of update predicts that, after we interpret the antecedent of the conditional in (182) above, for example, the output set of $\langle world, assignment\rangle$-pairs will contain all the credit cards that Linus has, which in turn incorrectly predicts that the conditional in (182) requires Linus to use all his credit cards.

Various repairs can be imagined, which would presumably try to uncouple the part of the update that targets worlds, which still needs to be maximal / 'strong', and the part of the update that targets variable assignments, which should optionally be non-maximal / 'weak'. The question that arises is to what extent such repairs would move these dynamic systems towards the Intensional PCDRT end of the spectrum.

We end this section with the observation that weak and strong donkey readings also pose problems for the account in van Rooy (1998) and for Stone (1999). The system in van Rooy (1998) derives only strong readings because indefinites are analyzed as quantifiers, so they always have a maximal / strong reading.

In contrast, Stone (1999) derives only weak readings because indefinites introduce drefs for individual concepts (they are functions of type $s(\mathbf{w}e)$), hence, for each possible world, the dref will store exactly one individual. Such drefs are, basically, drefs for choice functions: given a world w, the individual concept will choose a particular entity that is a credit card that Linus has in w. Thus, Stone (1999) can account for the weak-donkey conditional in (182), but not for the strong-donkey conditional in (185), where the indefinite in the antecedent needs to introduce all the murders that happen in any given world w.

An easy repair would be to introduce drefs for properties, i.e., drefs of type $s(\mathbf{w}(et))$, which, relative to a given world w, would store the set of all murders in w. However, this strategy fails to predict the correct interpretation for more complex examples involving multiple instances of strong donkey anaphora, e.g., (223) below. If u simply stores all farmers that own a donkey in w and u' stores all donkeys owned by a farmer in w, there is no way to keep track of the *own*-relation in the consequent of the conditional: every farmer should feed the donkey(s) that he owns, not other donkeys owned by some other farmer.

(223) If a^{u} farmer owns a$^{u'}$ donkey, he$_{u}$ should feed it$_{u'}$ properly.

6 Conclusion

The paper introduced a variety of phenomena involving intra-sentential and cross-sentential quantificational dependencies between individuals and possibilities and argued that they can receive a unified compositional account if their analysis is seen as part of a general project of investigating the fine structure and dynamics of quantifier-internal and quantifier-external contexts of evaluation.

The main proposal is that modal quantification is a composite notion, to be analyzed in terms of discourse reference to quantificational dependencies that is multiply constrained by the various components that make up a modal quantifier. In particular, modal and individual-level quantification should be decomposed in such a way that the 'counting' / 'quantifying' component specific to each quantifier is separated from the general dynamics of quantifier-dependency interpretation.

We purposefully left open the question of whether the decomposition of quantifiers is at the object level (i.e., at the level of logical form) or at the level of the meta-language or translation language (i.e., quantifiers come with a complex interpretation rule, but they are not complex expressions at the level of logical form). Generalized quantifiers over individuals were decomposed at the meta-level, while modalized conditionals were decomposed at the level of logical form. In general, whether a quantificational item / construction – in English or any other language – should be decomposed at the object level or at the meta-level is an empirical issue, as it depends on the morpho-syntax of that particular item / construction (among other things).

The idea that the Montagovian solution to the problem of compositionality and the underlying logic of Montague semantics are compatible with dynamic semantics informs the entire paper. The resulting dynamic system, couched in classical type logic, systematically captures the anaphoric and quantificational parallels between the individual and modal domains.

The truth-conditional and anaphoric components of modal quantification are captured in an even-handed way and, unlike previous accounts, we make the propositional contents contributed by modal constructions available for subsequent discourse reference, which enables us to analyze discourses like (9/10) that crucially involve structured anaphora to such propositional contents.

The main goal of the paper was to provide a framework in which issues and distinctions pertaining to the interactions between singular / plural anaphora and individual-level / modal quantification can be precisely formulated and explicitly formalized in the *lingua franca* of classical many-sorted type logic. The hope is that an empirical program that systematically investigates the range of possible anaphoric relations and their differing interactions with various quantificational elements can be built on this foundation.

Acknowledgments

I am grateful to Daniel Altshuler, Pranav Anand, Johan van Benthem, Sam Cumming, Donka Farkas, Tim Fernando, Hans Kamp, Sarah Murray, Jessica Rett, Maribel Romero, Robert van Rooy, Philippe Schlenker, Katrin Schulz, Magda Schwager, Roger Schwarzschild, Will Starr, Henk Zeevat and Ede Zimmermann for comments and discussion. I am particularly grateful to Maria Bittner, Paul Dekker and Matthew Stone for their continued support and interest in this project and their extensive comments and advice. Finally, I want to thank three *Journal of Semantics* anonymous reviewers and Anna Szabolcsi

for their detailed comments on an earlier draft of this paper and the impetus to improve it in many respects. The usual disclaimers apply.

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