Sentence-Internal Readings of *Same* / *Different* as Quantifier-Internal Anaphora

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1 Deictic and Sentence-Internal Readings of *Same/Different*

- **Goal**: provide a unified account of the deictic/sentence-external and sentence-internal readings of *same* / *different*

- these readings have been known to exist at least since Carlson (1987), but no unified account has been proposed (see Barker (2007) and Matushansky (2007) for recent discussions) despite the fact that, in language after language, the same lexical item is used for both readings

(1) Deictic / sentence-external readings:
a. Mary recited *The Raven.*
b. Then, every boy recited a different poem.
(different from *The Raven*).

- the interpretation of *different* in (1b) is sentence external in the sense that it is anaphoric to a discourse referent (dref) introduced in the previous sentence (1a)

- in (1), *different* relates two drefs and requires their values, i.e., the actual entities, to be distinct

(2) Sentence-internal readings:
Every boy recited a different poem.
(for any two boys a and b, a's poem is different from b's poem)

- the sentence-internal reading in (2) seems to relate values of only one dref, introduced by the narrow-scope indefinite *a poem*

- these values, i.e., the recited poems, co-vary with the values of the dref introduced by the universal quantifier *every boy* – and *different* requires the poems to be distinct relative to distinct boys

- **Generalization** (Carlson 1987): sentence-internal readings are licensed in English by distributive quantifiers, e.g., *every boy* in (2), or by distributively interpreted pluralities

- compare the following felicitous example (plural & distributive) ...

(3) The boys recited different poems. (Carlson 1987)

- ... and the following infelicitous examples (singular and plural & collective, respectively)

(4) #Mary recited a different poem.
(no sentence-internal readings with singulars)

(5) #The boys gathered around different fires.
(no sentence-internal readings with collective plurals)[1]

- we focus on sentence-external readings and sentence-internal readings under morphologically singular, semantically distributive quantifiers like *every boy*, since these are the readings that are cross-linguistically realized by the same lexical item

Main proposal:

- distributive quantification temporarily makes available two drefs within its nuclear scope, the values of which are required by sentence-internal uses of *same* / *different* to be identical / distinct ...

- ... much as their deictic uses require the values of two drefs to be identical / distinct

[1] The sentence-internal reading is available if *the boys* denotes a set of groups of boys – and each group gathered around a different fire. Such group-level distributivity is basically the same as individual-level distributivity, modulo the fact that it licenses collective predicates like *gather*. This reading will not be discussed in the paper.

General background project – decomposing quantification:

- *same* and *different* provide further support for the idea that natural language quantification is a composite notion ...
- ... to be analyzed in terms of discourse reference to dependencies that is multiply constrained by the various components that make up a quantifier

2 Sentence-External Readings as Cross-Sentential Anaphora

- deictic / sentence-external readings are just an instance of cross-sentential anaphora, of the same kind as the typical discourse ...

(6) **a.** A^{u_0} man came in. **b.** He$_{u_0}$ sat down.

- this discourse is straightforwardly analyzed in DRT (Kamp 1981, Kamp & Reyle 1993) / FCS (Heim 1982) / DPL (Groenendijk & Stokhof 1991)
- the indefinite in sentence (6a) introduces a dref u_0 – symbolized by the superscript on the indefinite article
- this dref is then retrieved by the pronoun in (6b) – symbolized by the subscript on the anaphoric pronoun
- discourse (6) as a whole is represented by the following two Discourse Representation Structures (DRSs), a.k.a. (linearized) boxes

(7) $[u_0 \mid man\{u_0\}, come\_in\{u_0\}]$; $[sit\_down\{u_0\}]$

- DRSs are pairs of the form [**new drefs** | **conditions**], the first member of which consists of the newly introduced drefs, while the second member consists of the conditions that the previously introduced drefs have to satisfy
- the first DRS in (7) is contributed by sentence (6a)
- we introduce a new dref u_0 and require its value to be a man that came in
- the second DRS, contributed by sentence (6b), does not introduce any new drefs (the first member of the pair is empty, so we omit it)
- it just further constrains the previously introduced dref u_0 to store an individual that sat down
- the two DRSs are dynamically conjoined, symbolized as ";"
- dynamic conjunction ensures that the anaphoric information contributed by the first DRS (i.e., the fact that u_0 stores a man that came in) is available to the second DRS

The analysis of deictic / sentence-external readings follows the same general format.

(8) **a.** Mary^{u_0} recited *The Raven*^{u_1}.
b. Then, every^{u_2} boy recited a^{u_3} different$_{u_1,u_3}$ poem.

- the proper name *The Raven* in (8a) introduces a new dref u_1 storing the poem *The Raven*
- this dref is subsequently retrieved by the adjective *different* in (8b)
- *different* constrains the value of the anaphorically retrieved dref u_1 in two ways
  - **first**, it requires u_1 to satisfy the conditions contributed by the nominal phrase following *different* – in this case, it requires u_1 to be a poem
  - to see this, replace the indefinite *a poem* in (8b) with the indefinite *a different passage of Scripture* – this yields an infelicitous discourse
  - this requirement is a presupposition, as shown by the standard S-tests for presupposition projection, e.g., the question *Did every boy recite a different passage of Scripture?* is also infelicitous in the context of sentence (8a)
  - **secondly**, *different* requires the value of the anaphorically retrieved dref u_1 to be distinct from the value of the dref contributed by the indefinite article that precedes *different* – in this case, u_3
  - this requirement is part of the asserted / at-issue content, as the S-tests also show, e.g., consider *different* under negation: *Mary recited* The Raven*, as she promised, but Linus didn't recite a different poem, despite what he promised* – the poem Linus recited is not distinct from *The Raven*

3 Sentence-Internal Readings as Quantifier-Internal Anaphora

Proposal:

- sentence-internal readings of *same* / *different* are parallel to the sentence-external ones in that they also involve anaphora and relate two drefs, requiring their values to be identical (for *same*) or distinct (for *different*)

- distributive quantifiers like *every*^{u_0} *boy* introduce a distributive operator $\mathbf{dist}_{u_0}$ relative to which the nuclear scope of the quantifier is evaluated, as shown in (9) below

- the $\mathbf{dist}_{u_0}$ operator checks in a *distributive, pointwise* manner whether the restrictor set of the quantifier (stored in the dref u_0) satisfies the nuclear scope of the quantification

(9) Every^{u_0} boy $\quad \mathbf{dist}_{u_0}$(recited a^{u_1} different$^{+2}_{u_1}$ poem).

(10)
$$\emptyset \xRightarrow{\text{Every}^{u_0}\text{ boy}} \begin{array}{|c|}\multicolumn{1}{c}{u_0}\\ \hline boy_1\\ \hline boy_2\\ \hline boy_3\\ \hline\end{array} \xRightarrow{\mathbf{dist}_{u_0}(\text{recited a}^{u_1}\text{different}^{+2}_{u_1}\text{poem})}$$

$$\left\{\begin{array}{l} \begin{array}{|c|c|}\multicolumn{1}{c}{u_0} & \multicolumn{1}{c}{u_1}\\ \hline boy_1 & poem_1\\ \hline\end{array} * \begin{array}{|c|c|}\multicolumn{1}{c}{u_0} & \multicolumn{1}{c}{u_1}\\ \hline boy_2 & poem_2\\ \hline\end{array} \ \& \ poem_1 \neq poem_2 \\ \begin{array}{|c|c|}\multicolumn{1}{c}{u_0} & \multicolumn{1}{c}{u_1}\\ \hline boy_1 & poem_1\\ \hline\end{array} * \begin{array}{|c|c|}\multicolumn{1}{c}{u_0} & \multicolumn{1}{c}{u_1}\\ \hline boy_3 & poem_3\\ \hline\end{array} \ \& \ poem_1 \neq poem_3 \\ \qquad\qquad\qquad \text{etc.} \end{array}\right\}$$

$$\xRightarrow{\text{summing all the updates}} \begin{array}{|c|c|l}\multicolumn{1}{c}{u_0} & \multicolumn{1}{c}{u_1} & \\ \hline boy_1 & poem_1 & boy_1 \text{ chose } poem_1\\ \hline boy_2 & poem_2 & boy_2 \text{ chose } poem_2\\ \hline boy_3 & poem_3 & boy_3 \text{ chose } poem_3\\ \hline\end{array}$$

This pointwise, distributive update proceeds as shown in (10) above:

- the quantifier *every boy*^{u_0} introduces a new dref u_0 that stores the restrictor set of the quantifier (i.e., the set of boys)

- then, we temporarily introduce two new drefs, each storing one and only one boy in the restrictor set u_0; the two boys stored by the two drefs must be distinct

- then, we predicate the nuclear scope of the quantification of each temporary dref and simultaneously make all the necessary updates ('simultaneously' means something like 'simultaneous recursion' here) – in particular, we associate each of the two boys under consideration with their corresponding u_1-poems

- the adjective *different*$^{+2}_{u_1}$ is interpreted *in situ*, i.e., within the indefinite a^{u_1} ... *poem*, and it is anaphoric to the dref u_1 introduced by the indefinite

- *different*$^{+2}_{u_1}$ tests that, for the two u_0-boys that we are currently considering, their corresponding u_1-poems are distinct (*same* would check that their corresponding u_1-poems are identical)

- the superscript +2 on *different* is the one that tells us where to look for the poems: they are stored by the drefs u_1 and u_{1+2} (i.e., u_3). This is a consequence of the fact that the $*$ operator in (10) above concatenates 'boy-poem' sequences

- the superscript on sentence-internal *different* is not arbitrary: it reflects how many drefs have been introduced prior to the occurrence of sentence-internal *different*; in our case, the superscript is +2 because we have previously introduced the two drefs u_0 and u_1

- the superscript is basically the length of the sequence of individuals relative to which *different* is interpreted – more precisely, the length of the initial sub-sequence up to and including the dref that is introduced by the indefinite DP that *different* is a part of; however, a more systematic theory of anaphora 'indexation' in stack-based PCDRT is a project I leave for future research (as Bittner (2007) argues, such a theory can and should be provided in stack-based dynamic systems)

- finally, we repeat this procedure for any two distinct individuals stored in u_0 (i.e., any two individuals in the restrictor set) and, then, we sum together all the updates thus obtained

The procedural flavor of the above informal description is largely just an expository device. The actual definition of the **dist** operator directly encodes the non-procedural, guiding intuition that ...

- sentence-internal readings of *same* / *different* provide a window into the internal structure of distributive quantification

- distributivity does not merely involve selecting one individual at a time from the restrictor set and checking that the nuclear scope holds of this individual, but ...

- distributivity involves selecting *pairs* of distinct individuals and *simultaneously* evaluating the nuclear scope relative to each individual

- this is why *same* / *different* are licensed only in the nuclear scope of distributive quantifiers or distributively interpreted pluralities (as Carlson (1987) observes): the very process of distributively evaluating the nuclear scope temporarily constructs the same kind of contexts that license anaphoric, sentence-external readings

- in a nutshell, the analysis is just this: sentence-internal readings are quantifier-internal / distributivity-internal anaphora

Items like $\texttt{other}_{u_n}$ can have only sentence-external readings because they do not have the additional meaning component that is symbolized here as a superscript on $\texttt{different}^{m}_{u_n}$. This additional, 'superscripted' meaning component allows for both sentence-internal and sentence-external readings as follows:

- for sentence-internal readings, m is a positive integer and the analysis proceeds as shown above
- for sentence-external readings, m is a negative integer such that $-n \leq m$ (this ensures that $0 \leq n+m$, so u_{n+m} is indexed with a positive integer) – in this case, the dref u_{n+m} is in fact one of the drefs introduced before u_n and it functions very much like the dref u_n functions for sentence-external only $\texttt{other}_{u_n}$

Thus, the main difference between lexical items that allow only for sentence-external readings and lexical items that allow for both of them is that the latter kind have an extra superscript m, which can be a positive or a negative integer and which is added to the index of the dref u_n introduced by the indefinite article (hence the dref u_{n+m}). The superscript m is the one that enables lexical items like $\texttt{different}$ to take advantage of the particular environment created by distributive quantifiers, i.e., to be 'bound' and have sentence-internal readings. Since $\texttt{other}_{u_n}$ is not lexically specified as having this extra superscripted parameter, it can have only sentence-external readings.

4 Formalization: Stacks and Plural Information States

This section discusses the formalization of the two main features of the analysis, namely:

- interpreting expressions relative to *sets* of variable assignments and not single assignments (the assignments are the rows storing boys and poems in (10) above; **dist** operators distribute over such sets of assignments)
- making multiple drefs simultaneously available by *concatenating* variable assignments (this is what happens when we simultaneously consider two boys and their poems in the scope **dist** operators)

These two features are formalized by plural information states and stacks, respectively.

- plural info states enable us to store the restrictor sets of quantifiers like *every boy*u_0 and pass them on to the **dist** operators that license sentence-internal readings in the nuclear scope of such distributive quantifiers
- using stacks and not partial / total variable assignments enables us to define a notion of stack concatenation, symbolized as $*$, that is crucial for simultaneously making available two drefs in the scope of **dist** operators, e.g., in (10), we are able to require *poem*$_1$ and *poem*$_2$ to be distinct only if both of them are simultaneously available in the same stack

4.1 Stacks

- we work with stacks / sequences of individuals instead of total or partial variable assignments (following Bittner (2001, 2007), Nouwen (2003, 2007) and references therein)
- the main motivation for using stacks is that, when we introduce new drefs, we never override old drefs and, therefore, never lose previously introduced anaphoric information: we always add information to a stack and we do this in an orderly manner, based on the particular position in the stack that the update targets
- one consequence of this fact for our analysis is that we can easily define a notion of stack concatenation, which is crucial for the definition of the **dist** operators we need
- we represent the empty positions in a stack i by storing the dummy individual # in that position

0	1	...	$n-1$	n	$n+1$	...
α_0	α_1	...	α_{n-1}	#	#	...

- the dummy individual # makes any lexical relation false, i.e., # is the universal falsifier[2]
- the length of a stack i, abbreviated $\mathbf{lng}(i)$, is provided by the 'leftmost' position in which the stack stores an individual different from the universal falsifier # – to which we need to add 1, because the first position in the stack is the 0-th position.

(11) **Abbreviation – stack length**[3]

$$\mathbf{lng}(i) := \begin{cases} 1 + \iota n.\ (i)_n \neq \# \wedge \forall n' > n((i)_{n'} = \#) & \text{if } \exists n((i)_n \neq \# \wedge \forall n' > n((i)_{n'} = \#)) \\ 0 & \text{if } \forall n((i)_n = \#) \\ \# & \text{otherwise} \end{cases}$$

[2]We ensure that any lexical relation R of arity n, i.e. of type $e^n t$, defined recursively as in Muskens (1996: 157-158): $e^0 t := t$ and $e^{m+1}t := e(e^m t)$, yields falsity whenever # is one of its arguments by letting $R \subseteq (D_e^{\mathfrak{M}} \setminus \{\#\})^n$.

[3]The "otherwise" case covers stacks of infinite length, for example, the stack storing the universal falsifier # at all odd-number positions $1, 3, 5, \ldots$ and individuals different from # at the other positions.

- here's an example of a stack of length 4 – that is, $\mathbf{lng}(i) = 4$; the cells storing the universal falsifier # are simply omitted

0	1	2	3
α	β	γ	δ

u_0	u_1	u_2	u_3
α	β	γ	δ

- the positions in a stack can be indicated by either natural numbers or – as we will do from now on – drefs that have natural numbers as indices
- the indices on drefs are essential: they indicate the stack position where the value of the dref is stored

4.2 Plural Information States

- just as in Dynamic Plural Logic (van den Berg 1996), information states I, J etc. are modeled as *sets* of stacks i, j etc.
- such *plural* info states can be represented as matrices with stacks (sequences) as rows, as shown below.

Info State I	u_0	u_1	u_2	...
i_1	α_1 (i.e., $u_0 i_1$)	β_1 (i.e., $u_1 i_1$)	γ_1 (i.e., $u_2 i_1$)	...
i_2	α_2 (i.e., $u_0 i_2$)	β_2 (i.e., $u_1 i_2$)	γ_2 (i.e., $u_2 i_2$)	...
i_3	α_3 (i.e., $u_0 i_3$)	β_3 (i.e., $u_1 i_3$)	γ_3 (i.e., $u_2 i_3$)	...
...	...	...	...	...

Quantifier domains (sets) are stored columnwise: $\{\alpha_1, \alpha_2, \dots\}$, $\{\beta_1, \beta_2, \dots\}$ etc.	**Quantifier dependencies** (relations) are stored rowwise: $\{\langle\alpha_1, \beta_1\rangle, \langle\alpha_2, \beta_2\rangle, \dots\}$, $\{\langle\alpha_1, \beta_1, \gamma_1\rangle, \langle\alpha_2, \beta_2, \gamma_2\rangle, \dots\}$ etc.

- plural info states enable us to encode discourse reference to both quantifier domains, i.e. *values*, and quantificational dependencies, i.e. *structure*
- the values are the sets of objects that are stored in the columns of the matrix, e.g., the dref u_0 stores a set of individuals $\{\alpha_1, \alpha_2, \alpha_3, \dots\}$ relative to a plural info state because u_0 is assigned an individual by each stack/row
- the structure is encoded in the rows of the matrix: for each stack/row i_1, i_2 etc. in the info state, the individual assigned to the dref u_0 (for example) by that stack is structurally correlated with the individual assigned to the dref u_1 (and/or u_2, and/or u_3 etc.) by the same stack
- from now on, we will use simpler representations for plural info states – we will only indicate the drefs and the stored individuals (omitting the universal falsifier), as exemplified below

u_0	u_1	u_2	...
α_1	β_1	γ_1	...
α_2	β_2	γ_2	...
α_3	β_3	γ_3	...
...	...	...	...

4.3 Concatenating Stacks and Plural Info States

(12) **Abbreviation – projection functions over stacks**
$(i)_n$ is the individual stored at position n (a.k.a. u_n) in stack i.

(13) **Abbreviation – stack update**
$i[n]j$ (a.k.a. $i[u_n]j$) $:= \forall m < n((j)_m = (i)_m) \wedge \forall m > n((j)_m = (i)_{m-1})$
(j is the stack obtained by shifting all the i-individuals at positions greater than or equal to n by one position and introducing a new random individual at position n)

(14) **Abbreviation – concatenating stacks and individuals** (based on Bittner 2007, Nouwen 2007)
$i * x := \iota j.\ i[\mathbf{lng}(i)]j \wedge (j)_{\mathbf{lng}(i)} = x$
($i * x$ is the stack obtained by appending the individual x at the end of stack i)

(15) **Abbreviation – concatenating stacks** (Nouwen 2007)
$i * j := (i * (j)_0) * ... * (j)_{\mathbf{lng}(j)-1}$
($i * j$ is obtained by appending the first individual in stack j, namely $(j)_0$, at the end of stack i, then appending the second individual in j at the end of the resulting stack etc.)

(16) **Abbreviation – concatenating plural info states** (Nouwen 2007)
$I * J := \{i * j : i \in I \wedge j \in J\}$

For example, within the scope of the $\mathbf{dist}_{u_0}$ operator in (10) above, we concatenate two stacks of length 2 to obtain a stack of length 4:

u_0	u_1
boy_1	$poem_1$

$*$

u_0	u_1
boy_2	$poem_2$

$=$

u_0	u_1	u_2	u_3
boy_1	$poem_1$	boy_2	$poem_2$

We will also concatenate plural info states, for example:

u_0	u_1
boy_1	$poem_1$
boy_2	$poem_2$
boy_3	$poem_3$

$*$

u_0	u_1
boy_2	$poem_2$

$=$

u_0	u_1	u_2	u_3
boy_1	$poem_1$	boy_2	$poem_2$
boy_2	$poem_2$	boy_2	$poem_2$
boy_3	$poem_3$	boy_2	$poem_2$

4.4 Independent Motivation for Plural Info States and Stacks

Both plural info states and stacks are independently motivated.

Independent Motivation for Plural Info States

- Brasoveanu (2007) argues that we need a semantics based on plural info states to account for quantificational subordination (among other things)
- consider the example of quantificational subordination in (17) (from Karttunen 1976)

(17) **a.** Harvey courts a^{u_0} woman at every^{u_1} convention.
b. She$_{u_0}$ always$_{u_1}$ comes to the banquet with him.
[**c.** The$_{u_0}$ woman is usually$_{u_1}$ also very pretty.]

- one of the interpretations of discourse (17) is that Harvey courts a different woman at every convention and, at each convention, the woman courted by Harvey at that convention comes with him to the banquet of the convention
- the singular pronoun *she*$_{u_0}$ and the adverb *always*$_{u_1}$ in sentence (17b) elaborate on the quantificational dependency between conventions and women introduced in sentence (17a)

Plural info states enable us to give a semantics for sentence (17a) that, as a result of the very process of interpreting sentence (17a):

- introduces two quantifier domains (the conventions and the women) and a quantificational dependency between them (the 'being courted by Harvey' relation)
- stores the quantifier domains and quantificational dependency in a plural info state
- passes on this info state to sentence (17b), which further elaborates on it

Thus, we need plural info states not only for the quantifier-internal dynamics that licenses the sentence-internal readings if *same* / *different*, but also for the quantifier external dynamics involved in quantificational subordination.

Independent Motivation for Stacks

- the example of cross-sentential anaphora to quantifier domains in (18) below (based on an example in Nouwen 2007) provides similarly independent motivation for the use of stacks and stack-concatenation operations

(18) **a.** Every^{u_0} boy chose a^{u_1} poem.
b. Then, they$_{u_0}$ each$_{u_0}$ recited it$_{u_1}$ / them$^{+2}_{u_1}$.

In sentence (18b), we can refer back to the narrow-scope indefinite a^{u_1} *poem*:

- with the singular pronoun *it*$_{u_1}$, in which case (18b) says that each boy recited the poem he chose – that is, we elaborate on the quantificational dependency between boys and poems introduced in sentence (18a)
- with the plural pronoun *them*$_{u_1}$, in which case (18b) says that each boy recited all the poems under consideration

That is, in the scope of the distributor each$_{u_0}$ in sentence (18b), we need to have access to both the dependency between boys and poems and the entire set of poems under consideration.

Nouwen (2007) proposes to give a semantics for *each*$_{u_0}$ in terms of stack concatenation to account for the availability of both distributive / dependent and collective / independent anaphora in its scope.

(19) **Abbreviation** – the empty stack
$i_{\#} := \iota i.\ \mathbf{lng}(i) = 0$

The update contributed by sentence (18a) relates an input and an output plural info state:

- input state: the singleton set containing the empty stack – this is the initial info state that stores no anaphoric information
- output state: a set of stacks that stores all the boys in its first column and their corresponding poems in the second column (the boy-poem dependency is stored stack-wise)

(20) $\{i_{\#}\}$ $\xRightarrow{\text{Every}^{u_0}\text{ boy chose a}^{u_1}\text{ poem}}$

u_0	u_1	
boy$_1$	*poem*$_1$	*boy*$_1$ chose *poem*$_1$
boy$_2$	*poem*$_2$	*boy*$_2$ chose *poem*$_2$
boy$_3$	*poem*$_3$	*boy*$_3$ chose *poem*$_3$

The update contributed by sentence (18b), in particular, by the distributor *each*$_{u_0}$, further updates the output info state of the previous sentence by:

- temporarily introducing each boy, one at a time, and his corresponding poem
- concatenating the boy and the poem currently under consideration with the input stack

- checking that the update in the scope of $each_{u_0}$ holds relative to the resulting stacks of length 4, which can license both distributive / dependent anaphora (i.e., the singular pronoun) and collective / independent anaphora (i.e., the plural pronoun)

- the plural pronoun $them^{+2}_{u_1}$ is marked as independent / collective by its superscript +2; this superscript indicates that the pronoun retrieves not the single u_1 poem currently under consideration, but all the poems, which are stored two positions to the right of u_1, i.e., by dref $u_{1+2} = u_3$

- just as in the case of sentence-internal *different*, the superscript on independent pronouns is not arbitrary: it depends on how many drefs have been previously introduced; in our case, the superscript is +2 because we introduced the two drefs u_0 and u_1 prior to the occurrence of the independent pronoun *them*

(21)

u_0	u_1
boy_1	$poem_1$
boy_2	$poem_2$
boy_3	$poem_3$

$\xRightarrow{\text{They}_{u_0}\ \text{each}_{u_0}\ (\text{recited it}_{u_1}/\text{them}^{+2}_{u_1})}$

u_0	u_1	u_2	u_3
boy_1	$poem_1$	boy_1	$poem_1$
boy_1	$poem_1$	boy_2	$poem_2$
boy_1	$poem_1$	boy_3	$poem_3$

u_0	u_1	u_2	u_3
boy_2	$poem_2$	boy_1	$poem_1$
boy_2	$poem_2$	boy_2	$poem_2$
boy_2	$poem_2$	boy_3	$poem_3$

u_0	u_1	u_2	u_3
boy_3	$poem_3$	boy_1	$poem_1$
boy_3	$poem_3$	boy_2	$poem_2$
boy_3	$poem_3$	boy_3	$poem_3$

The cross-sentential availability of multiple drefs in (18) is made possible by the fact that the distributor *each* temporarily introduces new drefs by:

- selecting a subset of stacks from a particular plural info state

- appending this subset of stacks to another set of stacks

We use the same stack-concatenation technique to define the quantifier-internal distributive operator that we need to unify sentence-internal and sentence-external readings of *same* / *different*.

Appendix 1. Three Uses of *Different*, Cross-linguistically

Bulgarian:

(22) **a.** *Meri izrecitira* Garvanăt.
Mary recited *Raven.the*
'Mary recited *The Raven.*'

b. *Sled tova, vsjako momče izrecitira (po) (edno) različno/drugo stihotvorenie.*
After that, every boy recited (DIST) (one) different poem
'Then, every boy recited a different poem.'

(23) *Vsjako momče izrecitira (edno) različno stihotvorenie.*
Every boy recited (one) different poem
'Every boy recited a different poem.'[4]

(24) *Momčetata izrecitiraha različni stihotvorenija.*
Boys.the recited different.pl poems
'The boys recited different poems.'[5]

French (see also Laca & Tasmowski (2003)):

(25) **a.** *Marie a récité* Le Corbeau.
Maria HAS recited *The Raven*
'Mary recited *The Raven.*'

b. *Puis, chaque garçon a récité un autre poème / un poème différent.*
Then, every boy HAS recited an other poem / a poem different
'Then, every boy recited a different poem.'

(26) *Chaque garçon a récité un poème différent.*
Every boy HAS recited a poem different
'Every boy recited a different poem.'

[4]The following structure is also possible: *Vsjako momče izrecitira po edno različno stihotvorenie* (Every boy recited DIST one different poem).

[5]The following structure is also possible: *Momčetata izrecitiraha po edno različno stihotvorenije* (Boys.the recited DIST one different poem).

(27) *Les garçons ont récité des poèmes différents.*
The.pl boys HAVE recited DE.pl poems different.pl
'The boys recited different poems.'

German (see also Beck (2000)):

(28) ***a.*** *Maria sagte* Der Rabe *auf.*
Maria said *The Raven* PART
'Mary recited *The Raven.*'

b. *Dann sagte jeder Junge ein anderes Gedicht auf.*
Then said every boy an other poem PART
'Then, every boy recited a different poem.'

(29) *Jeder Junge sagte ein anderes Gedicht auf.*
Every boy said an other poem PART
'Every boy recited a different poem.'[6]

(30) *Die Jungen sagten verschiedene Gedichte auf.*
The boys said different poems PART
'The boys recited different poems.'[7]

Greek:

(31) ***a.*** *I Maria apingile* To Koraki
The Mary recited *The Raven*
'Mary recited *The Raven.*'

b. *Meta kathe aghori / ta aghoria apingil-e/an ena dhiaforetiko piima.*
Then every boy / the boys recited-3sg/pl one different poem
'Then, every boy / the boys recited a different poem.'

(32) *Kathe aghori apingile apo ena dhiaforetiko piima.*
Every boy recited DIST(lit.:from) one different poem
'Every boy recited a different poem.'

[6]The following structure is also possible: *Jeder Junge sagte ein eigenes Gedicht auf* (Every boy said an own poem PART).
[7]The following structure is also possible: *Die Jungen sagten unterschiedliche Gedichte auf.* (The boys said different poems PART).

(33) *Ta aghoria apingilan dhiaforetika piimata.*
The boys recited.pl different.pl poems
'The boys recited different poems.'

Hebrew:

(34) ***a.*** *meri diklema et ha-orev*
Mary recited.3.sg.fem Acc DEF-raven
'Mary recited *The Raven.*'

b. *ve-az kol yeled diklem šir axer*
and-then every boy recited-3.sg.masc poem not-the-same
'Then, every boy recited a different poem.'

(35) *kol yeled diklem šir axer*
every boy recited-3.sg.masc poem not-the-same
'Every boy recited a different poem.'

(36) *ha-y(e)ladim diklemu širim šonim*
DET-boys recited-3.pl poems different
'The boys recited different poems.'

Hindi:

(37) ***a.*** *Mary-ne* The Raven *recite kii*
Mary-Erg *The.Raven*.fem recite do.pfv.fem
'Mary recited *The Raven.*'

b. *phir har laRke-ne ek alag kavita recite kii*
then every boy-Erg a different poem.fem recite do.pfv.fem
'Then, every boy recited a different poem.'

(38) *har laRke-ne ek alag kavita recite kii*
every boy-Erg a different poem.fem recite do.pfv.fem
'Every boy recited a different poem.'

(39) *aRkoN-ne alag alag kavitaaeN recite kiiN*
boys-Erg different different poems.fem recite do.pfv.fem.pl
'The boys recited different poems.'

Hungarian:

(40) ***a.*** *Mari el-szavalta* A Hollo-*t.*
Mari away-recite *The Raven*-Acc
'Mary recited *The Raven.*'

b. *Aztan minden fiu el-szavalt egy mas verset.*
Then every boy away-recite an other poem.Acc
'Then, every boy recited a different poem.'

(41) *Minden fiu mas-mas verset szavalt el.*
Every boy other-other poem.Acc recite away
'Every boy recited a different poem.'

(42) *A fiuk mas-mas verseket szavaltak el.*
The boys other-other poem.pl.Acc recite away
'The boys recited different poems.'

Romanian:

(43) ***a.*** *Maria a recitat* Corbul.
Mary HAS recited *Raven.the*
'Mary recited *The Raven.*'

b. *Apoi, fiecare băiat a recitat un alt poem.*
Then, every boy HAS recited a different poem
'Then, every boy recited a different poem.'

(44) *Fiecare băiat a recitat cîte un alt poem.*
Every boy HAS recited CÎTE a different poem.
'Every boy recited a different poem.'

(45) *Băieţii au recitat poeme diferite.*
Boys.the HAVE recited poems different.pl
'The boys recited different poems.'

Russian (see also Matushansky (2007)):

(46) ***a.*** *Mary pro-chita-la* Voron
Mary pfv-read-pst.3s.fem *Raven*
'Mary recited *The Raven.*'

b. *Potom kazhdyj mal'chik pro-chita-l drugoe stixotvorenie.*
Afterwards every boy pfv-read-pst.3s different poem
'Then, every boy recited a different poem.'

(47) *Kazhdyj mal'chik pro-chita-l svoje stixotvorenie.*
Every boy pfv-read-pst.3s own poem
'Every boy recited a different poem.'[8]

(48) *Mal'chiki pro-chita-li raznye stixotvorenija.*
Boys pfv-read-pst.3pl different poems
'The boys recited different poems.'

Spanish:

(49) ***a.*** *María recitó* El Cuervo
Mary recite.pst.3s *The Raven*
'Mary recited *The Raven.*'

b. *Después de eso, cada chico recitó un poema distinto/diferente*
After DE that, each boy recite.pst.3s a poem distinct/different
'Then, every boy recited a different poem.'

(50) *Cada chico recitó un poema distinto/diferente.*
Each boy recite.pst.3s a poem distinct.masc.sg/different.masc.pl
'Every boy recited a different poem.'

(51) *Los chicos recitaron poemas distintos/diferentes*
The boys recited poems distinct.masc.pl/different.masc.pl
'The boys recited different poems.'

[8] The following structure is also possible: *Kazhdyj mal'chik prochital po stixotvoreniju* (every boy read DIST poem.Dat).

Appendix 2. Stack-based Plural Compositional DRT (PCDRT)

Stack-based Dynamic Ty2

We work with a Dynamic Ty2 logic, i.e., basically, with the Logic of Change in Muskens (1996), which reformulates dynamic semantics (Kamp 1981, Heim 1982) in Gallin's Ty2 (Gallin 1975). We have three basic Types: (i) e (individuals, including the set of natural numbers $\mathbb{N}$) – variables: $x, y, ...$; constants: $linus, gabby, ...$; variables over natural numbers: $m, n, ...$, (ii) t (truth values) – $\mathbb{T}, \mathbb{F}$; (iii) s (stacks) – variables: $i, j, ...$ Four axioms ensure that the entities of type s behave as stacks.

(52) **Ax1** (stack identity in terms of projection functions):
$\forall i_s \forall i'_s(\forall n((i)_n = (i')_n) \rightarrow i = i')$
Ax2 (stacks have finite length): $\forall i_s(\exists n(\mathbf{lng}(i) = n))$[9]
Ax3 (the empty stack exists): $\exists i_s(\mathbf{lng}(i) = 0)$
Ax4 (enough stacks): $\forall i_s \forall n \forall x_e(x \neq \# \rightarrow \exists j(i[n]j \wedge (j)_n = x))$

Stack-based PCDRT

Discourse referents (drefs) u_0, u_1 etc. of type se are just projection functions over stacks. Conditions are sets of info states, i.e., sets of sets of stacks (terms of type $(st)t$). DRSs are binary relations between info states / sets of stacks (i.e., terms of type $(st)((st)t)$).

(53) $u_n := \lambda i_s.\ (i)_n$, e.g., $u_0 := \lambda i.\ (i)_0$, $u_1 := \lambda i.\ (i)_1$ etc.

(54) $i[u_n]j := \forall m < n((j)_m = (i)_m) \wedge \forall m > n((j)_m = (i)_{m-1})$[10]

(55) $I[u_n]J := \forall i_s \in I(\exists j_s \in J(i[u_n]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u_n]j))$

(56) $I_{u_{m_1} \neq \#, ..., u_{m_n} \neq \#} := \{i_s \in I : u_{m_1}i \neq \# \wedge \ldots \wedge u_{m_n}i \neq \#\}$

(57) $R\{u_{m_1}, ..., u_{m_n}\} := \lambda I_{st}.\ I_{u_{m_1} \neq \#, ..., u_{m_n} \neq \#} \neq \emptyset \wedge \forall i_s \in I_{u_{m_1} \neq \#, ..., u_{m_n} \neq \#}(R(u_{m_1}i, \ldots, u_{m_n}i))$
(lexical relations, for any n-ary relation R of type $e^n t$, where $e^0 t := t$ and $e^{n+1}t := e(e^n t))$

(58) $I_{u_n = x} := \{i_s \in I : u_n i = x\}$

(59) $I_{u_n \neq x} := \{i_s \in I : u_n i \neq x\}$

(60) $u_n I := \{u_n i : i_s \in I_{u_n \neq \#}\}$

(61) $u_n = x := \lambda I_{st}.\ u_n I = \{x\}$ (identity between drefs and individuals – needed for proper names)

(62) $u_n = u_m := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u_n i = u_m i)$ (identity between drefs)

(63) Atomic DRSs: $[C] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge CJ$

(64) Tests: $[C_1, ..., C_m] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge C_1 J \wedge\ ...\ \wedge C_m J$

(65) Dynamic conjunction: $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$

(66) Multiple dref introduction: $[u_{m_1}, ..., u_{m_n}] := [u_{m_1}];\ ...\ ; [u_{m_n}]$

(67) DRSs: $[u_{m_1}, ..., u_{m_n} \mid C_1, ..., C_m] := [u_{m_1}, ..., u_{m_n}]; [C_1, ..., C_m]$

(68) Truth: a DRS D of type $\mathbf{t}$ is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

Maximization and Distributivity

(69) $\mathbf{max}^{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ ([u_n]; D)IJ \wedge \forall K_{st}(([u_n]; D)IK \rightarrow u_n K \subseteq u_n J)$

(70) $\mathbf{each}_{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ u_n I = u_n J \wedge I_{u_n = \#} = J_{u_n = \#} \wedge \forall x_e \in u_n I(D(I_{u_n = x} * I)(J_{u_n = x} * I))$
(based on Nouwen 2007)

(71) $\mathbf{dist}_{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ u_n I = u_n J \wedge I_{u_n = \#} = J_{u_n = \#} \wedge (|u_n I| = 1 \rightarrow DI_{u_n \neq \#}J_{u_n \neq \#}) \wedge \forall x_e \in u_n I \forall x'_e \in u_n I(x \neq x' \rightarrow D(I_{u_n = x} * J_{u_n = x'})(J_{u_n = x} * J_{u_n = x'}))$

Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics become available. In more detail, the compositional aspect of interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences. Abbreviate them as $\mathbf{e}$ and $\mathbf{t}$. An extensional static logic identifies $\mathbf{e}$ with e and $\mathbf{t}$ with t. The translation of the English noun ***boy*** is of type $\mathbf{et}$, i.e. et: ***boy*** $\rightsquigarrow \lambda x_e.\ boy_{et}(x)$. The generalized determiner ***every*** is of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, i.e. $(et)((et)t)$: ***every*** $\rightsquigarrow \lambda S_{et}.\lambda S'_{et}.\ \forall x_e(S(x) \rightarrow S'(x))$. PCDRT assigns the following dynamic types to the 'meta-types' $\mathbf{e}$ and $\mathbf{t}$: $\mathbf{t}$ abbreviates $(st)((st)t)$, i.e. a sentence is interpreted as a DRS, and $\mathbf{e}$ abbreviates se, i.e. a name is interpreted as a dref. The denotation of the noun ***boy*** is still of type $\mathbf{et}$, the determiner ***every*** is still of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$ etc.

Basic Translations

(72) ***poem*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [poem_{et}\{v\}]$, i.e. ***boy*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I = J \wedge poem_{et}\{v\}J$

[9] This is equivalent to $\forall i_s(\mathbf{lng}(i) \neq \#)$.

[10] Or we can use the stronger version: $i[u_n]j := \forall m < n((j)_m = (i)_m \wedge (i)_m \neq \#) \wedge \forall m > n((j)_m = (i)_{m-1})$.

(73) $\texttt{recite} \rightsquigarrow \lambda Q_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.[recite\{v, v'\}])$

(74) $\texttt{each} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{each}_v(P(v))$

(75) $\texttt{every}^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \max^{u_n}(P(u_n));\ \mathbf{dist}_{u_n}(P'(u_n))$

(76) $\mathbf{singleton}\{u_n\} := \lambda I_{st}.\ |u_n I| = 1$

(77) $\texttt{a}^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u_n \,|\, \mathbf{singleton}\{u_n\}];\ P(u_n);\ P'(u_n)$

(78) $\texttt{it}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{singleton}\{u_n\}];\ P(u_n)$

(79) independent pronouns: $\texttt{it}^{+m}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{singleton}\{u_{n+m}\}];\ P(u_{n+m})$

(80) $u_n \neq \emptyset := \lambda I_{st}.\ u_n I \neq \emptyset$

(81) $\texttt{they}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \neq \emptyset];\ P(u_n)$

(82) independent pronouns: $\texttt{they}^{+m}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_{n+m} \neq \emptyset];\ P(u_{n+m})$

(83) $\texttt{Linus}^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \mid u_n = linus];\ P(u_n)$ (where $linus_e$ is an individual constant of type e)

(84) $\mathbf{disjoint}\{u_n, u_{n'}\} := \lambda I_{st}.\ I \neq \emptyset \wedge u_n I \cap u_{n'} I = \emptyset$

(85) $\texttt{other}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \underline{P(u_n)};\ [\mathbf{disjoint}\{u_n, v\}];\ P(v)$ (presuppositions are underlined)

(86) $\texttt{different}^{m}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \underline{P(u_{n+m})};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}];\ P(v)$, where u_n has to be the dref introduced by the indefinite article immediately preceding $\texttt{different}$

(87) $\mathbf{identical}\{u_n, u_{n'}\} := \lambda I_{st}.\ I \neq \emptyset \wedge u_n I = u_{n'} I$

(88) $\texttt{same}^{m}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \underline{P(u_{n+m})};\ [\mathbf{identical}\{u_{n+m}, u_n\}];\ P(v)$, where u_n has to be the dref introduced by the definite article immediately preceding $\texttt{different}$

Sample Derivations

(89) $\texttt{other}_{u_{n'}}\ \texttt{poem} \rightsquigarrow \lambda v_{\mathbf{e}}.\ \underline{[poem\{u_{n'}\}]};\ [\mathbf{disjoint}\{u_{n'}, v\}];\ [poem\{v\}]$

(90) $\texttt{an}^{u_n}\,\texttt{other}_{u_{n'}}\ \texttt{poem} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ [u_n \,|\, \mathbf{singleton}\{u_n\}];\ \underline{[poem\{u_{n'}\}]};$
$[\mathbf{disjoint}\{u_{n'}, u_n\}];\ [poem\{u_n\}];\ P'(u_n)$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \underline{[poem\{u_{n'}\}]};$
$[u_n \,|\, \mathbf{singleton}\{u_n\}, \mathbf{disjoint}\{u_{n'}, u_n\}, poem\{u_n\}];\ P'(u_n)$

(91) $\texttt{different}^{m}_{u_n}\ \texttt{poem}$
$\rightsquigarrow \lambda v_{\mathbf{e}}.\ \underline{[poem\{u_{n+m}\}]};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}];\ [poem\{v\}]$

(92) $\texttt{a}^{u_n}\ \texttt{different}^{m}_{u_n}\ \texttt{poem} \rightsquigarrow \lambda P'_{\mathbf{et}}.\ [u_n \,|\, \mathbf{singleton}\{u_n\}];\ \underline{[poem\{u_{n+m}\}]};$
$[\mathbf{disjoint}\{u_{n+m}, u_n\}];\ [poem\{u_n\}];\ P'(u_n)$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \underline{[poem\{u_{n+m}\}]};$
$[u_n \,|\, \mathbf{singleton}\{u_n\}, \mathbf{disjoint}\{u_{n+m}, u_n\}, poem\{u_n\}];\ P'(u_n)$

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