Modified Numerals as Post-suppositions

I. The Problem and the Main Proposal The goal of this paper is to provide a compositional account of cumulative readings with non-increasing modified numerals (aka van Benthem's puzzle) – exemplified in (1) below – in terms of cardinality post-suppositions. We discuss only *exactly n* modified numerals, but the account generalizes to other non-increasing numerals (e.g., *at most n*).

1. Exactly three^x boys saw exactly five^y movies.

Figure 1

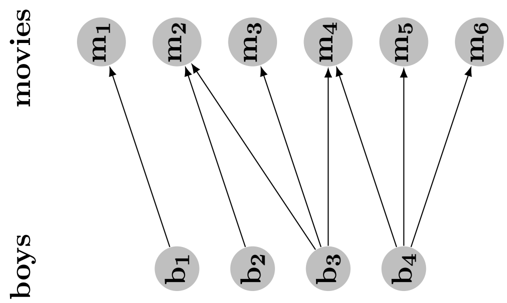
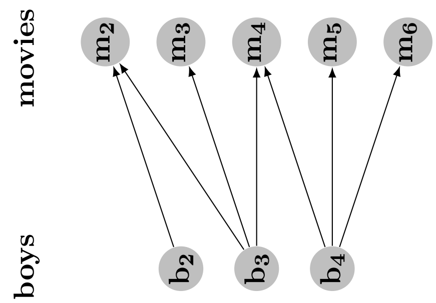


Figure 2



The reading of sentence (1) that we want to capture is the cumulative one, namely: consider the maximal number of boys that saw a movie and the maximal number of movies seen by a boy; there are three such boys and five such movies. Importantly (as Krifka 1999, Landman 2000 and Ferreira 2007 observe), this reading is different from: the maximal number of boys that saw exactly five movies is three. The situations depicted in the figures above distinguish between these two readings. Figure 1 is exactly like Figure 2, except for the addition of boy $\mathbf{b_1}$, movie $\mathbf{m_1}$ and the arrow between them symbolizing the seeing relation. The cumulative reading is intuitively false in Figure 1 (4 boys and 6 movies) and true in Figure 2. The second reading, however, is true in both cases.

The main proposal is that modified numerals make two kinds of contributions to the meaning of sentences like (1). Their asserted / at-issue contribution is a maximization operator that introduces the maximal set of entities that satisfies their restrictor and nuclear scope. The second contribution is a post-supposition, i.e., a cardinality constraint (e.g., exactly three) that needs to be satisfied relative to the *context that results after* the at-issue meaning is evaluated. For our current purposes, contexts are sets of variable assignments relative to which quantificational expressions are interpreted – and which are updated as a result of the interpretation of such expressions. That is, we work with a simplified version of Dynamic Plural Logic (DPlL, van den Berg 1996).

The fact that the cardinality constraints are post-suppositional also enables us to account for the fact that scalar implicatures are not normally associated with modified numerals, as opposed to unmodified numerals. The reason is that scalar implicatures are basically pragmatically contributed post-suppositions that are supposed to *non-redundantly* constrain the cardinality of the sets they target. Non-redundancy, however, cannot be satisfied in the case of *exactly n* modified numerals. And, given a suitable formulation of the cardinality constraints contributed by *at least n* and *at most n*, we can predict for them exactly the kind of implicatures that they trigger – see Geurts & Nouwen (2007) and Büring (2008) for recent discussions of the implicatures associated with *at least n*.

The main difference between the present account and Krifka (1999) is conceptual: we take modified numerals to constrain *quantificational* – and not focus – alternatives, where a quantificational alternative is one of the assignments satisfying a quantificational expression. Thus, we reconceptualize DPlL as the logic of quantificational alternatives in natural language interpretation.

II. Outline of the Formal Account We work with the usual models for FOL $\mathfrak{M} = \langle \mathfrak{D}, \mathfrak{I} \rangle$, where $\mathfrak{D}$ is the domain of individuals and $\mathfrak{I}$ is the basic interpretation function such that $\mathfrak{I}(R) \subseteq \mathfrak{D}^n$ for any n-ary relation R. An $\mathfrak{M}$-assignment g is a total function from the set of variables $\mathcal{V}$ to $\mathfrak{D}$. The essence of quantification in FOL is pointwise / variablewise manipulation of variable assignments, abbreviated as $h[x]g$ (h differs from g at most with respect to the value it assigns to x). We generalize this to sets of assignments $H[x]G$ cumulative-quantification style, as in (2) below. Clauses (5)-(10) are simplified versions of the corresponding ones in DPlL. Atomic formulas are tests, interpreted as in (5). Cardinality constraints on the values of variables are also tests – see (6). Dynamic conjunction and random assignment are interpreted DRT/FCS/DPL-style – see (7) and (8). Restricted existential quantification $\exists x[x = n \wedge \phi]\ (\psi)$ (intuitively, n ϕ-individuals are ψ), which corresponds to English unmodified numerals, is defined in (9). The maximization operator in (10) is just a dynamic form of λ-abstraction: any output set of assignments H stores in x the maximal set of values satisfying ϕ.

2. $H[x]G := \begin{cases} \text{for all } h \in H, \text{ there is a } g \in G \text{ such that } h[x]g \\ \text{for all } g \in G, \text{ there is a } h \in H \text{ such that } h[x]g \end{cases}$

3. $G(x) := \{g(x) : g \in G\}$ **4.** $|G(x)|$ is the cardinality of the set of individuals $G(x)$.

5. $[\![R(x_1, \ldots, x_n)]\!]^{\langle G,H\rangle} = \mathbb{T}$ iff $G = H$ and $\langle h(x_1), \ldots, h(x_n)\rangle \in \mathfrak{I}(R)$, for all $h \in H$.

6. $[\![x = n]\!]^{\langle G,H\rangle} = \mathbb{T}$ iff $G = H$ and $|H(x)| = n$.

7. $[\![\phi \wedge \psi]\!]^{\langle G,H\rangle} = \mathbb{T}$ iff there is a K s.t. $[\![\phi]\!]^{\langle G,K\rangle} = \mathbb{T}$ and $[\![\psi]\!]^{\langle K,H\rangle} = \mathbb{T}$.

8. Random assignment: $[\![[x]]\!]^{\langle G,H\rangle} = \mathbb{T}$ iff $H[x]G$. **9.** $\exists x[x = n \wedge \phi]\ (\psi) := [x] \wedge x = n \wedge \phi \wedge \psi$

10. $[\![\mathbf{M}x(\phi)]\!]^{\langle G,H\rangle} = \mathbb{T}$ iff $[\![[x] \wedge \phi]\!]^{\langle G,H\rangle} = \mathbb{T}$ and there is no H' s.t. $H \subsetneq H'$ and $[\![[x] \wedge \phi]\!]^{\langle G,H'\rangle} = \mathbb{T}$.

All this is more or less standard. The novelty is the addition of post-suppositions, formalized as tests (formulas that don't update their input set of assignments) that are introduced at certain points in the interpretation and that are passed on from info state to info state. That is, an info state is a set of assignments G indexed with a set of tests ζ, represented as $G[\zeta]$. All the operators above are interpreted in the same way, except that, if the input info state G is indexed with a set of tests ζ, this set is past on to the output state H. Thus, the interpretation function is not simply $[\![\cdot]\!]^{\langle G,H\rangle}$, but $[\![\cdot]\!]^{\langle G[\zeta],H[\zeta']\rangle}$, where ζ and ζ' are sets of tests – and $\zeta = \zeta'$ for all the operators above.

We mark a formula ϕ as a post-supposition by superscripting it. A post-suppositional formula does not update the input set of assignments – it is simply added to the set of tests associated with the output info state, as shown in (11) below. These tests are post-suppositional in the sense that they are required to be true relative to the final output info state. This is formalized by means of the definition of truth in (12) below, which treats the formulas $\psi_1, \ldots, \psi_m$ as post-suppositions because they are tests performed on the final output set of assignments H.

11. $[\![^{\phi}]\!]^{\langle G[\zeta],H[\zeta']\rangle} = \mathbb{T}$ iff $G = H$, ϕ is a test and $\zeta' = \zeta \cup \{\phi\}$.

12. Truth: a formula ϕ is true relative to an input info state $G[\emptyset]$, where $\emptyset$ is the empty set of tests, iff there is an output set of assignments H and a (possibly empty) set of tests $\{\psi_1, \ldots, \psi_m\}$ such that $[\![\phi]\!]^{\langle G[\emptyset],H[\{\psi_1,\ldots,\psi_m\}]\rangle} = \mathbb{T}$ and $[\![\psi_1 \wedge \ldots \wedge \psi_m]\!]^{\langle H[\emptyset],H[\emptyset]\rangle} = \mathbb{T}$.

Restricted *exactly* n quantification $\exists^{x=n}[\phi]\ (\psi)$ is defined in terms of maximization over the restrictor formula ϕ and the nuclear scope formula ψ, plus the cardinality post-supposition $x = n$. This is shown in (13). The translation of sentence (1), which derives the intuitively-correct cumulative truth-conditions, is provided in (14). In fact, (14) is truth-conditionally equivalent to the formula without post-suppositions in (15), where the cardinality constraints contributed by the modified numerals are explicitly shown to be outside the scope of the maximization operators $\mathbf{M}x$ and $\mathbf{M}y$.

13. $\exists^{x=n}[\phi]\ (\psi) := \mathbf{M}x(\phi \wedge \psi) \wedge {}^{x=n}$

14. $\exists^{x=3}[\textsc{boy}(x)](\exists^{y=5}[\textsc{movie}(y)](\textsc{see}(x,y))) := \mathbf{M}x(\textsc{boy}(x) \wedge \mathbf{M}y(\textsc{movie}(y) \wedge \textsc{see}(x,y)) \wedge {}^{y=5}) \wedge {}^{x=3}$

15. $\mathbf{M}x(\textsc{boy}(x) \wedge \mathbf{M}y(\textsc{movie}(y) \wedge \textsc{see}(x,y))) \wedge x = 3 \wedge y = 5$

The paper shows how the account generalizes to other modified numerals and to cumulative readings of universal quantifiers, e.g., the cumulative reading of *Exactly three^x students read every^y article*.

References: [**1**] van den Berg, M. (1996). *Some Aspects ...*. PhD diss. [**2**] Büring, D. (2008). The Least *at least* Can Do. WCCFL 26. [**3**] Ferreira, M. (2007). Scope Splitting and Cumulativity. ESSLLI 2007. [**4**] Geurts, B. & R. Nouwen (2007). *At Least* et al. *Language* 83. [**5**] Krifka, M. (1999). At Least Some Determiners Arent Determiners. *The Semantics/Pragmatics Interface ...*. [**6**] Landman, F. (2000). *Events and Plurality*, Kluwer.