Sentence-internal *Different* as Quantifier-internal Anaphora

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Rutgers University • March 4, 2011

1 Introduction: the broader project

- capturing the particular ways in which natural language interpretation proceeds is usually taken to involve rich abstract representations and fairly complex operations over such representations
- under this view, two general goals of semantics are:
  - identify patterns of interpretation that seem to involve such abstract (non-overt / latent) representations and operations
  - design logical systems in which the 'right' range of representations and operators can be defined and in which these representations and operators interact in the 'right' way
- the broader project behind the talk today:
  - describe the patterns of interpretation and outline the emerging typology involved in relating:
  - (*i*) various kinds of distributive quantifiers and
  - (*ii*) various kinds of distributivity-dependent items
  - formally capture these patterns and typology in a logical system that involves:
  - (*i*) fine-grained, structured contexts of evaluation that distributivity-dependent items are sensitive to
  - (*ii*) a family of distributivity operators over such contexts that are the basic components of different kinds of quantifiers

*I want to thank S. AnderBois, P. Alrenga, D. Altshuler, P. Anand, R. Bhatt, S. Chung, U. Cohen Priva, C. Condoravdi, S. Cumming, M. Ellsworth, D. Farkas, J. Ito, A. Kothari, S. Lauer, M.-C. de Marneffe, J. McCloskey, A. Mester, M. Moodie, R. Nouwen, R. Pancheva, A. Pereltsvaig, J. Rett, I. Sag, O. Săvescu, L. Vicente, M. Wagers, the audiences at WCCFL 27 and the UC Berkeley S-Circle and the participants in the fall 2008 *Decomposing Quantification* seminar and the winter 2009 Semantics B course at UCSC for comments and / or discussion and / or judgments and / or examples. I am particularly indebted to C. Barker and J. Dotlačil for their detailed comments, to R. Henderson for bringing the close connection between sentence-internal singular *different* and *every / each* vs *one by one* to my attention, to E. Zimmermann for his advice and support throughout this project and to J.T. Brasoveanu for his support during its final stages. The usual disclaimers apply.

2 Sentence-external and sentence-internal readings of *different*

Goals for today:

- argue that deictic / sentence-external and sentence-internal readings of morphologically singular *different* should receive a unified account
- provide such a unified—and compositional—account

The two readings of singular *different* are exemplified in (1), (2) and (3) below:

(1) a. Mary recited 'The Raven'.
b. Then, Linus recited a different poem.
(deictic / sentence-external: different from 'The Raven')

(2) a. Mary recited 'The Raven'.
b. Then, every boy recited a different poem.
(deictic / sentence-external: different from 'The Raven')

(3) Every boy recited a different poem.
(sentence-internal: for any two boys a and b, a's poem is different from b's poem)

Different in (1b)/(2b) is sentence-external:

- it is anaphoric to the discourse referent (dref) introduced by the proper name 'The Raven' in the previous sentence (1a)/(2a)
- it relates two drefs and requires their values, i.e., the actual entities, to be distinct

Different in (3) is sentence-internal:

- it relates values of only one dref, namely the dref introduced by the narrow-scope indefinite *a poem*.
- these values, i.e., the recited poems, covary with the values of the dref introduced by the universal quantifier *every boy*

- *different* requires the poems to be distinct relative to distinct boys

These two kinds of readings have been known to exist at least since Carlson (1987), but no unified account has been proposed to date (see Alrenga 2007, Barker 2007, Matushansky 2007 and Dotlačil 2010 for recent discussions).

Main proposal:

- distributive quantification temporarily makes available two drefs within its nuclear scope, the values of which are required by sentence-internal uses of *different* to be distinct ...
- ... in much the same way that sentence-external uses require the values of two drefs to be distinct

The account of these readings will be formulated in a dynamic system that:

- provides the semantic values of natural language expressions in terms of *sets of sequences* of individuals and not single sequences (as classical Tarskian semantics would have it)
- models these sequences of individuals as *stacks* and not as total or partial variable assignments

Using sets of sequences instead of single sequences enables us to:

- store the entire set of boys that sentence (3) quantifies over (each boy is stored in a particular sequence / assignment)
- simultaneously constrain multiple members of this set

Modeling these sequences as stacks enables us to:

- consider multiple $\langle boy, poem \rangle$ pairs simultaneously because we can concatenate them in one bigger stack
- the concatenation operation $*$ is easily definable over stacks, but not over total / partial variable assignments

Sentence (3) above is analyzed as follows:

(4) $\emptyset \xRightarrow{\text{Every boy}} \boxed{\begin{array}{|c|}\hline boy_1 \\ \hline boy_2 \\ \hline boy_3 \\ \hline \end{array}} \xRightarrow{\textbf{dist}(\text{recited a different poem})}$

$$\left\{ \begin{array}{l} \boxed{boy_1}\boxed{poem_1} * \boxed{boy_2}\boxed{poem_2} \;\&\; poem_1 \neq poem_2 \\ \boxed{boy_1}\boxed{poem_1} * \boxed{boy_3}\boxed{poem_3} \;\&\; poem_1 \neq poem_3 \\ \boxed{boy_2}\boxed{poem_2} * \boxed{boy_1}\boxed{poem_1} \;\&\; poem_2 \neq poem_1 \\ \boxed{boy_2}\boxed{poem_2} * \boxed{boy_3}\boxed{poem_3} \;\&\; poem_2 \neq poem_3 \\ \qquad\qquad \text{etc.} \end{array} \right\} \xRightarrow[\text{sequences}]{\text{collect}}$$

$$\begin{array}{|c|c|}\hline boy_1 & poem_1 \\ \hline boy_2 & poem_2 \\ \hline boy_3 & poem_3 \\ \hline \end{array} \quad \text{where} \quad \begin{array}{l} boy_1 \text{ recited } poem_1 \\ boy_2 \text{ recited } poem_2 \\ boy_3 \text{ recited } poem_3 \end{array} \quad \text{and} \quad \begin{array}{l} poem_1 \neq poem_2 \\ poem_1 \neq poem_3 \\ poem_2 \neq poem_3 \end{array}$$

The account of singular *different* generalizes to plural *different* and *same*:

- opening the larger project of formally investigating the typology of quantificational distributors and distributivity-dependent items and the richer contexts of evaluation needed to support this typology
- we will talk about the empirical generalizations concerning plural *different* and *same* today, but not about the actual account of those generalizations (the relevant formulas are provided in appendix A; see Brasoveanu 2011 for the account)

Roadmap

> section **3** introduces the empirical generalizations

> section **4** outlines the account of sentence-external and sentence-internal readings for singular *different*

> section **5** examines some predictions of the analysis more closely

> section **6** concludes

3 Varieties of Items with Internal and External Readings and Varieties of Distributivity

The plan for this section:

- start with cross-linguistic generalizations relating distributive interpretations and internal and external readings of distributivity-dependent items
- then focus on English, in particular on the differences between the distribution and interpretation of singular *different*, plural *different* and singular / plural *same*

3.1 The relation between distributivity and internal and external readings crosslinguistically

The morphological realization of sentence-internal and sentence-external readings of singular *different* crosslinguistically:[1]

(*i*) if a language has a lexical item that can have sentence-internal readings under quantifiers like *every / each boy* (morphologically singular and semantically distributive), then that item can also have sentence-external readings

- e.g., the English singular *different* or the German *anders*
- some languages, e.g., Russian, do not have such lexical items, so they express sentence-internal readings by means of an item like *own*

(*ii*) a language can have a lexical item that has only sentence-external readings

- e.g., the English *other / another*, the French *autre* or the Russian *drugoe*

Main point:

- sentence-internal readings under morphologically singular and semantically distributive quantifiers pattern together with sentence-external readings

Implicational universal:

- if a language has a lexical item that can have sentence-internal readings under singular and distributive quantifiers, then that item can also have sentence-external readings (the converse does not hold)

Consequences for the semantics of singular *different*:

- we should derive both sentence-external and sentence-internal readings of singular *different* from the same meaning
- this meaning should be closely related to the meaning of anaphoric, sentence-external only items like *other*
- it should contain some additional meaning component ensuring that only *different*, but not *other*, can have a sentence-internal reading

We turn to a specific proposal to this effect after a discussion of the distribution and interpretation of singular and plural *different* and *same* in English.

[1] Based on a small survey of Bulgarian, French, German, Greek, Hebrew, Hindi, Hungarian, Romanian, Russian and Spanish.

3.2 Varieties of distributivity and *different* vs *same*

3.2.1 Singular *different*

Sentence-internal readings of singular *different* **are licensed** by:

(*i*) distributive quantifiers (Carlson 1987)—e.g., *every boy* in (3) above, *each boy* in (5) below and *every day* in (6)

(5) Each boy recited a different poem. (sentence-internal ✓)

(6) Linus recited a different poem every / each day. (sentence-internal ✓)

(*ii*) distributively interpreted plurals with an overt distributor like *each* (Carlson 1987)—e.g., (7) below

(7) The boys each recited a different poem. (or: The boys recited a different poem each.) (sentence-internal ✓)

(*iii*) the construction *N after N* (*week after week* etc.)—e.g., (8) and (9) below; (8) is from the Corpus of Contemporary American English (COCA, `www.americancorpus.org`)

(8) [Two companies, Xerox and E Ink, which is owned in part by the Hearst Corporation and Motorola, are manufacturing early models of a paperlike plastic sheet whose tiny black capsules can be formed and reformed into letters and symbols. An electric impulse or radio wave alters the configuration.] Eventually, you'll be able to read a different book on the same sheet of paper week after week.

(9) Year after year / Time after time, Linus submitted a different grant proposal, but they were never accepted.

(*iv*) *whenever*—e.g., the COCA examples below[2]

(10) Whenever those TV cameras come into the ice rink, you see a different young man.

(11) [The father told the deputy that the son drove off in his car. The deputy advised the couple to kick their son out of their home. The deputy has crossed paths with the son before. On those occasions, the son had told the deputy that he doesn't get along with his family.] He seems to have a different job whenever the deputy has spoken to him.

Sentence-internal readings of singular *different* **are not licensed** by:

(*i*) singular DPs (Carlson 1987)—e.g., (12) below (this is also true for plural *different* and singular / plural *same*)

[2] I am indebted to Jorge Hankamer for this observation.

(12) Mary recited a different poem. (sentence-external only)

(*ii*) collectively interpreted plurals (Carlson 1987)—e.g., (13) below (this is also true for plural *different* and singular / plural *same*)

(13) The boys gathered around a different fire. (sentence-external only)

(*iii*) covert distributivity operators of the kind usually assumed to derive the distributive interpretation of the second VP-conjunct *had an espresso* in examples like (14) below—e.g., (15) below (Moltmann 1992)

(14) The girls met and had an espresso.

(15) The boys / Two boys / The two boys recited a different poem. (sentence-external only)

(*iv*) morphologically plural distributors like *all (of) the* or *both*—e.g., (16) below

(16) All (of) the / Both boys recited a different poem. (sentence-external only)

(*v*) conjunctions (Moltmann 1992)—e.g., (17), (18) and (19) below

(17) Linus and Mary recited a different poem. (sentence-external only)

(18) Linus chose and recited a different poem. (sentence-external only)

(19) A different boy went to the store and bought ice cream. (sentence-external only)

(*vi*) distributors like *one by one*, *one at a time*, *one after another*, *one after the other*, *separately* or *individually*—e.g., (20) and (21)[3]

(20) One by one / One at a time / One after another / One after the other, the boys recited a different poem. (sentence-external only)

(21) Linus and Mary separately / individually chose a different poem. (sentence-external only)

(*vii*) comparatives—e.g., (22) (based on an example from Carlson 1987)

(22) Bob and Mike are more impressive than a different painter. (sentence-external only)

- adding an overt *each* to some of these sentences is felicitous and, as expected, sentence-internal readings of singular *different* are licensed in such cases, e.g.,
  - *The boys / Two boys / The two boys recited a different poem each.*
  - *Linus and Mary each recited a different poem.*

[3] I am indebted to Robert Henderson for bringing the contrast between *every / each* and *one by one* to my attention.

	sing. *different*
every, each	✓
day after day, week after week, time after time	✓
whenever	✓
pl. (in)definites	#
on those n occasions	#
NP conjunction	#
VP conjunction	#
comparatives	#
both, *all*(?)	#
one by one, one at a time, one after another, one after the other	#
separately, individually	#

3.2.2 Plural *different*

Sentence-internal readings of plural *different*—or singular / plural *same*—are not licensed exclusively by *each* or *every* distributors. They **are licensed** by:

(*i*) distributors in the class of *each* / *every*

(23) Every boy recited (three) different poems. (sentence-internal ✓)

(24) Every boy recited the same poem / the same (three) poems. (sentence-internal ✓)

(*ii*) distributively-interpreted plurals with covert distributivity operators (Carlson 1987)

(25) The boys / Two boys / The two boys recited different poems. (sentence-internal ✓)

(26) The boys / Two boys / The two boys recited the same poem(s). (sentence-internal ✓)

(*iii*) conjunctions (Carlson 1987)

(27) Linus and Mary recited different poems / the same poem(s). (sentence-internal ✓)

(28) Linus chose and recited different poems / the same poem(s). (sentence-internal ✓)

(29) Different boys / The same boy(s) went to the store and bought ice cream. (sentence-internal ✓)

(*iv*) comparatives (Carlson 1987)

(30) Bob and Mike are more impressive than different painters / the same painter(s). (Carlson 1987) (sentence-internal ✓)

Sentence-internal readings of plural *different* **are not licensed** by:

(*i*) morphologically plural distributors like *both / all*

(31) Both boys / All (of) the boys recited different poems. (sentence-external only)

(*ii*) aspectual modifiers like *(continuously) for six hours*,[4] *twice*, *repeatedly* and *over and over (again)* (these aspectual modifiers also fail to license singular *different*)

(32) Linus recited different poems (continuously) for six hours / a different poem (continuously) for six hours. (sentence-external only)

(33) Different people / A different person entered my house twice. (Carlson 1987[5]) (sentence-external only)

(34) Linus repeatedly recited different poems / a different poem. (sentence-external only)

NB: we ignore the 'various' / 'a diversity' reading of plural *different* throughout the talk.

[4] I am indebted to Judith Fiedler for bringing this kind of examples to my attention.

[5] As Carlson (1987) observes, the adverb *twice* contrasts with *on those two occasions*, which licenses sentence-internal plural *different* (*On those two occasions, different people searched my house*), and with *on each of those two occasions*, which licenses sentence-internal singular *different* (*On each of those two occasions, a different person searched my house.*)

	sing. *different*	pl. *different*
every, each	✓	✓
day after day, week after week, time after time	✓	✓
whenever	✓	✓
pl. (in)definites	#	✓
on those n occasions	#	✓
NP conjunction	#	✓
VP conjunction	#	✓
comparatives	#	✓
both, all(?)	#	#
one by one, one at a time, one after another, one after the other	#	N/A
separately, individually	#	N/A
(continuously) for n hours	#	#
twice	#	#
repeatedly, over and over (again)	#	#

3.2.3 Singular / plural *same*

In contrast, sentence-internal readings of singular / plural *same* **are licensed** by:

(*i*) morphologically plural distributors like *both / all*

(35) Both boys / All (of) the boys recited the same poem(s). (sentence-internal ✓)

(36) [The cost issue is addressed to some degree in the TV commercial, which compares 100 potato chips and 100 Pringles crisps.]
Both cost the same. (COCA, sentence-internal ✓)

(37) "Your eyes are as bright as the twin moons, but both the same size," he said. She giggled. (COCA, sentence-internal ✓)

(38) Tradition requires that the carver give both memorials the same facial features. (COCA, sentence-internal ✓)

(39) I couldn't scream and I couldn't breathe and I was trying to do both at the same time. (COCA, sentence-internal ✓)

(40) [Glasses are often an important identifier in a portrait.]
The challenge is to get both lenses the same shape and make sure they add to, rather than dominate, the face. (COCA, sentence-internal ✓)

(*ii*) aspectual modifiers like *(continuously) for six hours*, *twice*, *repeatedly* and *over and over (again)*

(41) Linus recited the same poem / the same (two) poems (continuously) for six hours. (sentence-internal ✓)

(42) The same person / The same people entered my house twice. (sentence-internal ✓)

(43) Linus repeatedly recited the same poem / the same (two) poems. (sentence-internal ✓)

3.2.4 Summary of the English generalizations

We extracted a three-level generalization about the licensors of sentence-internal readings for singular *different* vs plural *different* vs *same*:

- sentence-internal *same* is the most permissive with respect to distributive licensors
- sentence-internal singular *different* is the most restrictive
- sentence-internal plural *different* is somewhere in between

A summary of these empirical findings is provided in the table below, which also includes *similar* and comparatives as two other items that can have sentence-internal readings.

- *similar* behaves like sentence-internal plural *different* (not like *same*!)
- comparatives behave like sentence-internal singular *different*
- note that we look at comparatives both as quantificational licensors of sentence-internal readings (the row labeled as such) and as items that can have sentence-internal readings (e.g., *Every day I get better*)

	sing. *different*	pl. *different*	*same*	*similar*	comparatives
every, each	✓	✓	✓	✓	✓
day after day, week after week, time after time	✓	✓	✓	✓	✓
whenever	✓	✓	✓	✓	✓
pl. (in)definites	#	✓	✓	✓	#
on those n occasions	#	✓	✓	✓	#
NP conjunction	#	✓	✓	✓	#
VP conjunction	#	✓	✓	✓	#
comparatives	#	✓	✓	✓	#
both, all(?)	#	#	✓	#	#
one by one, one at a time, one after another, one after the other	#	N/A	N/A	N/A	#
separately, individually	#	N/A	N/A	N/A	#
(continuously) for n hours	#	#	✓	#	#
twice	#	#	✓	#	#
repeatedly, over and over (again)	#	#	✓	#	#

4 Sentence-internal *different* as quantifier-internal anaphora

This section provides an account of:

- the first level of the table above, i.e., the generalization that sentence-internal singular *different* requires overt quantificational distributivity of the *every* / *each* kind to be licensed
- the cross-linguistic implicational universal above indicating that sentence-internal and sentence-external readings of singular *different* should receive a unified account

The main proposal:

- distributive quantification temporarily makes available two drefs within its nuclear scope
- the values of these drefs are required by sentence-internal singular *different* to be distinct ...
- ... just as sentence-external singular *different* requires the values of two drefs to be distinct (see appendix A for more details)

4.1 Sentence-external readings as cross-sentential anaphora

Deictic / sentence-external readings are just an instance of cross-sentential anaphora, of the same kind as the typical discourse in (44) below.

(44) a. A^{u_0} man came in.
b. He$_{u_0}$ sat down.

(45) $[u_0 \mid \text{MAN}\{u_0\}, \text{COME-IN}\{u_0\}]$;
$[\text{SIT-DOWN}\{u_0\}]$

- the DRT (Kamp 1981, Kamp & Reyle 1993) / FCS (Heim 1982) / DPL (Groenendijk & Stokhof 1991) style analysis of this discourse is provided in (45)
- the indefinite in sentence (44a) introduces a dref u_0, symbolized by the superscript on the indefinite article
- this dref is retrieved by the pronoun in (44b), symbolized by the subscript on the anaphoric pronoun
- the discourse as a whole is represented by two conjoined Discourse Representation Structures (DRSs)
- DRSs are pairs of the form:

(46) [**new drefs** | **conditions**]

- the first member consists of the newly introduced drefs, the second member consists of the conditions that the previously introduced drefs have to satisfy
- dynamic conjunction ";" ensures that the anaphoric information contributed by the first DRS in (45)—i.e., the fact that u_0 stores a man that came in—is available to the second DRS

The analysis of deictic / sentence-external readings follows the same format:

- the proper name 'The Raven' in (47a) below introduces a new dref u_1 storing the poem THE-RAVEN
- this dref is retrieved by the adjective *different* in (47b)

(47) a. Mary^{u_0} recited 'The Raven'^{u_1}.
b. Then, every^{u_2} boy recited a^{u_3} different$_{u_1}$ poem.

The adjective *different* constrains the value of the anaphorically retrieved dref u_1 in two ways:

(*i*) it requires u_1 to satisfy the conditions contributed by the nominal phrase following *different*—i.e., it requires u_1 to be a poem

- to see this, replace the indefinite *a poem* in (47b) with the indefinite *a different passage of Scripture*—this yields the infelicitous sentence in (48b) below

(48) a. Mary^{u_0} recited 'The Raven'^{u_1}.
b. Then, every^{u_2} boy recited a^{u_3} different$_{u_1}$ passage of Scripture.
(sentence-external reading not available)

- this requirement is a presupposition, as shown by the standard S-tests for presupposition projection,[6] e.g., the question in (49b) is also infelicitous in the context of sentence (47a) (on the external reading of singular *different*)

(49) a. Mary^{u_0} recited 'The Raven'^{u_1}.
b. Did every^{u_2} boy recite a^{u_3} different$_{u_1}$ passage of Scripture?
(sentence-external reading not available)

(*ii*) *different* requires the value of the anaphorically retrieved dref u_1 to be distinct from the value of the dref contributed by the indefinite article that precedes *different*—in this case, that dref is u_3

- this requirement is part of the asserted / at-issue content, as the S-tests also show
- e.g., consider *different* under negation in sentence (50b) below

(50) a. Mary^{u_0} recited 'The Raven'^{u_1}, as she$_{u_0}$ promised ...
b. ... but Linus^{u_2} didn't recite a^{u_3} different$_{u_1}$ poem, despite what he$_{u_2}$ promised.

[6] To the extent that the S-tests actually test for presuppositional status as opposed to other kinds of not-at-issue content.

– this sentence says that the poem recited by Linus is not distinct from 'The Raven'—i.e., the distinctness requirement contributed by *different* is in the scope of negation

The representation that is compositionally assigned to discourse (47) above is provided in (51) below.

- the $\mathbf{max}^{u_2}$ operator introduces the dref u_2 and requires it to store the (maximal) set of boys, i.e., the restrictor set of the quantifier *every*^{u_2} *boy*
- the **dist** operator is discussed in the next subsection

(51) $[u_0, u_1 \mid u_0 = \text{MARY}, u_1 = \text{THE-RAVEN}, \text{RECITE}\{u_0, u_1\}]$;
$\mathbf{max}^{u_2}([\mathbf{atoms\text{-}only}\{u_2\}, \text{BOY}\{u_2\}])$;
$\mathbf{dist}_{u_2}([u_3 \mid \mathbf{atoms\text{-}only}\{u_3\}, \mathbf{singleton}\{u_3\}, \mathbf{disjoint}\{u_1, u_3\}, \text{POEM}\{u_3\}, \text{RECITE}\{u_2, u_3\}])$

4.2 Sentence-internal readings as quantifier-internal anaphora

Main idea: sentence-internal readings of singular *different* are parallel to the sentence-external ones.

- they also involve anaphora and relate two drefs, requiring their values to be distinct
- singular distributive quantifiers like *every*^{u_0} *boy* introduce a distributive operator $\mathbf{dist}_{u_0}$ relative to which the nuclear scope of the quantifier is evaluated, as shown in (52) below
- this distributivity operator is the one that temporarily makes available two distinct drefs for poems

(52) Every^{u_0} boy $\mathbf{dist}_{u_0}$(recited a^{u_1} different$^2_{u_1}$ poem).

(53) $\emptyset \xRightarrow{\text{Every}^{u_0}\text{boy}}$ $\begin{array}{|c|}\hline u_0 \\ \hline boy_1 \\ \hline boy_2 \\ \hline boy_3 \\ \hline \end{array}$ $\xRightarrow{\mathbf{dist}_{u_0}(\text{recited a}^{u_1}\text{different}^2_{u_1}\text{poem})}$

$$\left\{ \begin{array}{l} \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_1 & poem_1 \\ \hline \end{array} * \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_2 & poem_2 \\ \hline \end{array} \;\&\; poem_1 \neq poem_2 \\ \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_1 & poem_1 \\ \hline \end{array} * \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_3 & poem_3 \\ \hline \end{array} \;\&\; poem_1 \neq poem_3 \\ \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_2 & poem_2 \\ \hline \end{array} * \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_1 & poem_1 \\ \hline \end{array} \;\&\; poem_2 \neq poem_1 \\ \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_2 & poem_2 \\ \hline \end{array} * \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_3 & poem_3 \\ \hline \end{array} \;\&\; poem_2 \neq poem_3 \\ \text{etc.} \end{array} \right\}$$

$$\xRightarrow{\text{sum all updates}} \begin{array}{|c|c|}\hline u_0 & u_1 \\ \hline boy_1 & poem_1 \\ \hline boy_2 & poem_2 \\ \hline boy_3 & poem_3 \\ \hline \end{array}$$

where $\begin{array}{l} boy_1 \text{ recited } poem_1 \\ boy_2 \text{ recited } poem_2 \\ boy_3 \text{ recited } poem_3 \end{array}$ and $\begin{array}{l} poem_1 \neq poem_2 \\ poem_1 \neq poem_3 \\ poem_2 \neq poem_3 \end{array}$

- we start with no discourse information, represented by the empty discourse-initial information state $\emptyset$
- the quantifier *every*^{u_0} *boy* introduces a new dref u_0 that stores the restrictor set of the quantifier, i.e., the set of boys
- the $\mathbf{dist}_{u_0}$ operator checks in a *distributive, pointwise* manner whether the restrictor set of the quantifier satisfies the nuclear scope DRS
- that is, we temporarily introduce two new drefs, each storing one and only one boy from the restrictor set u_0
- then, we predicate the nuclear scope DRS of each temporary dref and simultaneously make all the necessary updates
- in particular, we associate each of the two boys under consideration with their corresponding u_1-poems
- the adjective *different*$^2_{u_1}$ is anaphoric to the dref u_1 introduced by the immediately preceding indefinite article and is interpreted *in situ*, i.e., within the indefinite *a*^{u_1} ... *poem*
- *different*$^2_{u_1}$ tests that, for the two u_0-boys that we are currently considering, their corresponding u_1-poems are distinct

- the superscript 2 on *different* is the one that tells us where to look for the poems: they are stored by the drefs u_1 and u_{1+2}, i.e., u_3
- this is because the concatenation operator $*$ in (53) above concatenates boy-poem sequences, thereby displacing the second poem under consideration two positions to the right
- the result of one instance of sequence concatenation is provided in (54) below; we see that $poem_2$ is the value of dref u_3 after the two boy-poem sequences are concatenated

(54)

u_0	u_1
boy_1	$poem_1$

$*$

u_0	u_1
boy_2	$poem_2$

$=$

u_0	u_1	u_2	u_3
boy_1	$poem_1$	boy_2	$poem_2$

The superscript on sentence-internal *different* is not arbitrary:

- it reflects how many drefs have been introduced prior to the occurrence of sentence-internal *different*
- in this case, it is 2 because we have previously introduced the two drefs u_0 and u_1

The final two steps of the update in (53):

- repeat the above procedure for any two distinct individuals stored in the restrictor set u_0
- when done checking all pairs of u_0-individuals, sum together all the updates thus obtained

The procedural flavor of this informal description of (53) is just an expository device.

- the actual definition of the **dist** operator (provided in appendix A) directly encodes the non-procedural, guiding intuition that ...
- this particular quantificational variety of distributivity does not merely involve selecting one individual at a time from the restrictor set and checking that the nuclear scope holds of this individual

Instead:

- it involves selecting *pairs* of distinct individuals and *simultaneously* evaluating the nuclear scope relative to each individual

Thus, sentence-internal *different* provides a window into the internal structure of distributive quantification.

We now have an explanation for the fact that sentence-internal singular *different* is licensed only in the nuclear scope of overt distributive quantifiers like *every* and *each*:

- the very process of distributively evaluating their nuclear scope temporarily constructs the same kind of contexts that license anaphoric, sentence-external readings

In a nutshell, the analysis is just this: sentence-internal readings are quantifier-internal / distributivity-internal anaphora.

Since both sentence-external and sentence-internal readings involve the same meaning for singular *different*, we also capture the (hypothesized) implicational universal that:

- if a language has a lexical item that can have sentence-internal readings under morphologically singular and semantically distributive quantifiers, then this item can also have sentence-external readings

More on sentence-external readings in the next section.

The compositionally obtained representation of sentence (52) above is provided in (55) below.

(55) $\mathbf{max}^{u_0}([\mathbf{atoms\text{-}only}\{u_0\}, \textsc{boy}\{u_0\}]);$
$\mathbf{dist}_{u_0}([u_1 \mid \mathbf{atoms\text{-}only}\{u_1\}, \mathbf{singleton}\{u_1\}, \mathbf{disjoint}\{u_{1+2}, u_1\},$
$\textsc{poem}\{u_1\}, \textsc{recite}\{u_0, u_1\}])$

(56) $different^2_{u_1} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \underline{P(u_{1+2})};\ [\mathbf{disjoint}\{u_{1+2}, u_1\}]$

The translation of singular *different* is provided in (56) above:

- *different* is analyzed as an adjective, i.e., a nominal modifier, reflected in the (**et**)(**et**) type of its translation
- the presupposition contributed by *different* is underlined[7]
- this presupposition (omitted in (55) above) is automatically satisfied in sentence-internal cases, i.e., in the scope of **dist** operators, as long as *different* has the correct superscript
- in fact, this presupposition constrains the possible values for the superscript on *different* and it plays an important role in ruling out many incorrect resolutions for this superscript

[7] I assume a presupposition resolution procedure of the kind proposed in van der Sandt (1992).

5 Consequences and predictions of the analysis

This section examines some predictions of the analysis more closely:

- the contrast between *different*, on one hand, and *other* and pronouns on the other hand
- the connection between the scope of distributive quantifiers and the availability of sentence-internal readings for singular *different*

5.1 *Different* vs *other* and pronouns

Items like $other_{u_n}$ can only have sentence-external readings.

- sentence (57b) below can only be anaphorically interpreted and cannot have the sentence-internal reading that is possible for *Every boy recited a different poem*

(57) a. Mary^{u_0} recited 'The Raven'^{u_1}.
b. Then, every^{u_2} boy recited an^{u_3}other$_{u_1}$ poem.

(58) $other_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \underline{P(u_n)};\ [\mathbf{disjoint}\{u_n, v\}]$

- under the present view, this is due to the fact that *other* does not have the additional meaning component that we encoded as a superscript on $different^m_{u_n}$

The additional, 'superscripted' meaning component that *different* has and *other* lacks allows for both sentence-internal and sentence-external readings:

- sentence-internal readings: m is a positive integer and the analysis proceeds as shown in the previous subsection
- sentence-external readings: m is a suitable negative integer such that $0 \leq n + m$
  - this ensures that the index on the dref u_{n+m} is 0 or a positive integer
  - in this case, the dref u_{n+m} is in fact one of the drefs introduced in the previous discourse
  - the dref u_{n+m} functions very much like the dref u_n that sentence-external $other_{u_n}$ is indexed with

In (59) below (repeated from (47) above), *different* has a sentence external reading because it is anaphoric to the dref $u_{3+(-2)}$—i.e., the dref u_1.

(59) a. Mary^{u_0} recited 'The Raven'^{u_1}.
b. Then, every^{u_2} boy recited a^{u_3} different$^{-2}_{u_3}$ poem.

That is, we obtain:

- the sentence-external reading in (47)/(59) above if the superscript is -2
- the sentence-internal reading in (52) above if the superscript is 2

The difference between items that can have only sentence-external readings and items like *different*:

- items like *different* have a special ability to look either 'downstream' in the current sequence of evaluation (result: sentence-internal readings) or 'upstream' (result: sentence-external readings)
- we formalized this special ability by indexing them with an extra superscript m that is used in a specific way
  - this superscript can be a positive or a negative integer
  - it is always added to the index of the dref u_n introduced by the immediately preceding indefinite article
  - that is, *different* is always anaphoric to the drefs u_n and u_{n+m}
- the superscript is the device that enables *different* to take advantage of the particular environment temporarily created by distributive quantifiers, i.e., to be 'bound' in this way and have sentence-internal readings

In contrast, *other* and all ordinary anaphoric items, e.g., pronouns, definites etc., are not lexically specified as having this ability.

- they can only access the 'upstream' sequence of evaluation constructed up to the point where they are interpreted
- formally, there is no superscript on them, so they can only have sentence-external readings

Bound-pronoun readings count as sentence-external since they arise by dref coindexation.

- e.g., *other* and regular pronouns can be bound by a universal quantifier and these bound readings are represented as shown below

(60) Every^{u_0} boy was playing with an^{u_1}other$_{u_0}$ boy.

(61) Every^{u_0} boy was playing with his$_{u_0}$ friend.

5.2 Weak Crossover (WCO) effects and sentence-internal *different*

Analyzing singular *different* and pronominal items in distinct ways correctly predicts that they pattern differently with respect to WCO effects:

- pronouns exhibit WCO effects
  - e.g., *his* in (62) below cannot have a bound reading—(62) cannot be interpreted as: every boy is such that his mother loves him

(62) His mother loves every boy.

  - the bound reading of the pronoun is unavailable despite the fact that the quantifier *every boy* can take scope over the subject—e.g., (63) below can in interpreted as: every boy is such that someone loves him

(63) Someone loves every boy.

- sentence-internal singular *different* does not exhibit WCO effects (this fact has been known at least since Dowty 1985)—as shown by the COCA examples below

(64) A different production team staged each of the four operas independently, with four different casts.
[compare with: Its composer staged each opera.]

(65) A different team of scientists works on each ecoregional plan, resulting in a proliferation of methods.

(66) Use a different knife to serve each cheese.

(67) Heat distribution from a boiler is clean, quiet and easily zoned - a different thermostat can be placed in every room.

We predict the presence of WCO effects with pronouns (or sentence-external-only items like *other*):

- their anaphoric potential is analyzed in terms of dref coindexation
- so we can state the usual WCO constraint, e.g., a pronoun can be bound by a quantifier it is coindexed with only if the quantifier c-commands the pronoun from an A-position

We predict the absence of WCO effects with sentence-internal singular *different*:

- no dref coindexation is established between *different* and the distributive quantifier licensing it—as opposed to pronouns, where such coindexation is a necessary condition for bound readings
- singular *different* is always coindexed with the immediately preceding indefinite article, so the WCO constraint does not apply

5.3 The scope of distributive quantifiers and sentence-internal *different*

The close connection between the scopal properties of *every* quantifiers and sentence-internal singular *different* provides additional support for the proposed analysis.

Main generalization: sentence-internal singular *different* requires its licensor to be able to take scope in / over the clause containing *different*. (Moltmann 1992, building on Dowty 1985 and Carlson 1987)

- e.g., sentence-internal readings are unavailable for the examples in (68) and (69) below (from Moltmann 1992)

(68) A different witness believed every defendant to be guilty / that every defendant was guilty. (sentence-external only)

(69) A different professor wrote a book about every artist / a book that was about every artist. (sentence-external only)

This is parallel to the generalization about the scopal properties of *every* in Farkas (1981):

- the scope of *every* is clause-bounded, i.e., even more local than movement
- e.g., in (70) and (71) below (from Farkas 1981), the universal quantifier cannot take scope over the indefinite despite the fact that extraction is possible from the position of the universal quantifier

(70) John told a reporter that Peter lives in every French town.
[compare with: Where did John tell a reporter that Peter lives?]

(71) A professor wants every student to get a job.
[compare with: Who does a professor want to get a job?]

The connection between the licensing of sentence-internal singular *different* and the scope of *every* extends to non-surface scope:

- universal quantifiers can take scope over indefinites in the same clause even if they do not c-command them (the typical example: *A woman loves every man*)
- similarly, universal quantifiers do not have to c-command *different* to license its sentence-internal reading—see the COCA examples in the WCO discussion above

But if we fix the scope of the universal quantifier and rule out inverse scope, we alter the licensing of sentence-internal singular *different*.

- consider the double object constructions in (72) and (73) below (from Beck 2000)
- only surface scope is possible in double-object constructions (Larson 1990, following D. Lebeaux)

(72) I offered a different girl every marble. (sentence-external only)

(73) I offered every girl a different marble. (sentence-internal ✓)

- in contrast, oblique dative constructions allow for non-surface scope
- as expected, sentence-internal readings are possible in this case—see (74) below

(74) I offered a different marble to every girl. (sentence-internal ✓)

The proposed account captures this parallel between the availability of sentence-internal singular *different* and the scope of *every*:

- the sequence concatenation needed for sentence-internal readings of *different* is available only in the scope of distributive quantifiers

6 Conclusion

- the proposed account of singular *different* is the first unified compositional account of sentence-external and sentence-internal readings
- it captures the (hypothesized) implicational universal that, if a language has an item with a sentence-internal reading under morphologically singular and semantically distributive quantifiers like *every* / *each*, then that item can also have a sentence-external reading
- other properties of *different* are also captured: the connection between sentence-internal readings and the scope of distributors and the differences between *different* and anaphoric items like *other* or pronouns
- the account generalizes to plural *different* and *same*, opening up a larger project of formally investigating the typology of quantificational distributors and distributivity-dependent items in natural languages
- sentence-internal readings provide a new window into the internal structure of distributive quantification

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A Plural Compositional DRT (PCDRT)

A.1 Stack-based Dynamic Ty2

We work with a Dynamic Ty2 logic, i.e., basically, with the Logic of Change in Muskens (1996), which reformulates dynamic semantics (Kamp 1981, Heim 1982) in Gallin's Ty2 (Gallin 1975). We have three basic Types:

(*i*) e (individuals, including the set of natural numbers $\mathbb{N}$)—variables: $x, y, \ldots$; constants: LINUS, MARY, $\ldots$; variables over natural numbers: $m, n, \ldots$,
(*ii*) t (truth values)—$\mathbb{T}, \mathbb{F}$ and
(*iii*) s (stacks)—variables: $i, j, \ldots$.

Four axioms ensure that the entities of type s behave as stacks.

Ax1 (stack identity in terms of projection functions):
$\forall i_s \forall i'_s(\forall n((i)_n = (i')_n) \rightarrow i = i')$

Ax2 (stacks have finite length):
$\forall i_s(\exists n(\mathbf{lng}(i) = n))$[8]

Ax3 (the empty stack exists):
$\exists i_s(\mathbf{lng}(i) = 0)$

Ax4 (enough stacks):
$\forall i_s \forall n \forall x_e(x \neq \# \rightarrow \exists j_s(i[u_n]j \wedge (j)_n = x))$

I take the domain of type e to be the power set of a given non-empty set IN of entities together with the universal falsifier #, i.e., $D_e = \wp^+(\mathsf{IN}) \cup \{\#\}$, where $\wp^+(\mathsf{IN}) := \wp(\mathsf{IN}) \setminus \emptyset$.[9]

The sum of two individuals $x_e \oplus y_e$ is the union of the sets x and y. For a set of atomic and / or non-atomic individuals X_{et}, the sum of the individuals in X (i.e., their union) is $\oplus X$.

The part-of relation over individuals $x \leq y$ (x is a part of y) is the partial order induced by inclusion $\subseteq$ over the set $\wp^+(\mathsf{IN})$. Note that the universal falsifier # is not a part of any individual.

The atomic individuals are the singleton subsets of IN, identified by the predicate $\mathbf{atom}(x) := \forall y_e \leq x(y = x)$. Note that the predicate **atom** does not apply to the universal falsifier #.

A.2 Basic PCDRT

Discourse referents (drefs) u_0, u_1 etc. of type se are just projection functions over stacks.

[8] This is equivalent to $\forall i_s(\mathbf{lng}(i) \neq \#)$.
[9] See Schwarzschild (1996), for example, for more discussion of domain-level plurality.

The 'empty' info state $\emptyset$ that stores no anaphoric information—which is the default discourse-initial info state—is the singleton set containing only the empty stack, i.e., $\{i_{\#}\}$.

Conditions are sets of info states, i.e., sets of sets of stacks (terms of type $(st)t$). DRSs are binary relations between info states / sets of stacks (i.e., terms of type $(st)((st)t)$).

(1) $u_n := \lambda i_s.\ (i)_n$, e.g., $u_0 := \lambda i_s.\ (i)_0$, $u_1 := \lambda i_s.\ (i)_1$ etc.

(2) The empty stack: $i_{\#} := \iota i.\ \mathbf{lng}(i) = 0$

(3) $i[u_n]j := \forall m < n((j)_m = (i)_m) \wedge \forall m > n((j)_m = (i)_{m-1})$[10]

(4) $I[u_n]J := \forall i_s \in I(\exists j_s \in J(i[u_n]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u_n]j))$

(5) $[u_n] := \lambda I_{st}.\lambda J_{st}.\ I[u_n]J$

(6) $I_{u_{m_1}\neq\#,\ldots,u_{m_n}\neq\#} := \{i_s \in I : u_{m_1}i \neq \# \wedge \ \ldots\ \wedge\ u_{m_n}i \neq \#\}$

(7) Lexical relations: for any n-ary relation R of type e^nt, where $e^0t := t$ and $e^{n+1}t := e(e^nt)$,
$R\{u_{m_1}, \ldots, u_{m_n}\} := \lambda I_{st}.\ I_{u_{m_1}\neq\#,\ldots,u_{m_n}\neq\#} \neq \emptyset \wedge$
$\forall i_s \in I_{u_{m_1}\neq\#,\ldots,u_{m_n}\neq\#}(R(u_{m_1}i, \ldots, u_{m_n}i))$

(8) $I_{u_n=x} := \{i_s \in I : u_ni = x\}$

(9) $I_{u_n=X} := \{i_s \in I : u_ni \in X\}$

(10) $I_{u_n\neq x} := \{i_s \in I : u_ni \neq x\}$

(11) $u_nI := \{u_ni : i_s \in I_{u_n\neq\#}\}$

(12) Identity between drefs and individuals (needed for proper names):
$u_n = x := \lambda I_{st}.\ u_nI = \{x\}$

(13) Identity between drefs: $u_n = u_m := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u_ni = u_mi)$

(14) Atomic DRSs: $[C] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge CJ$

(15) Tests: $[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge C_1J \wedge\ \ldots\ \wedge C_mJ$

(16) Dynamic conjunction: $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$

(17) Multiple dref introduction: $[u_{m_1}, \ldots, u_{m_n}] := [u_{m_1}];\ \ldots\ ; [u_{m_n}]$

(18) DRSs: $[u_{m_1}, \ldots, u_{m_n} \mid C_1, \ldots, C_m] := [u_{m_1}, \ldots, u_{m_n}]; [C_1, \ldots, C_m]$

(19) Truth: A DRS D of type $\mathbf{t}$ is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

[10] Or we can use the stronger version: $i[u_n]j := \forall m < n((j)_m = (i)_m \wedge (i)_m \neq \#) \wedge \forall m > n((j)_m = (i)_{m-1})$.

A.3 Maximization and Distributivity

(20) $\mathbf{max}^{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ ([u_n]; D)IJ \wedge$
$\forall K_{st}(([u_n]; D)IK \rightarrow u_nK \subseteq u_nJ)$

(21) $\mathbf{dist}_{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ u_nI = u_nJ \wedge I_{u_n=\#} = J_{u_n=\#} \wedge$
$(|u_nI| = 1 \rightarrow DI_{u_n\neq\#}J_{u_n\neq\#}) \wedge$
$\forall x_e \in u_nI \forall x'_e \in u_nI(x \neq x' \rightarrow D(I_{u_n=x} * J_{u_n=x'})(J_{u_n=x} * J_{u_n=x'}))$

(22) $\mathbf{dist\text{-}COMP}_{u_n}(D) := \lambda I_{st}.\lambda J_{st}.\ u_nI = u_nJ \wedge I_{u_n=\#} = J_{u_n=\#} \wedge$
$(|u_nI| = 1 \rightarrow DI_{u_n\neq\#}J_{u_n\neq\#}) \wedge$
$(|u_nI| \geq 2 \rightarrow \forall x_e \in u_nI(D(I_{u_n=x} * J_{u_n=u_nI\setminus\{x\}})(J_{u_n=x} * J_{u_n=u_nI\setminus\{x\}})))$[11]

(23) $\mathbf{dist\text{-}WHOLE} := \lambda I_{st}.\lambda J_{st}.\ u_nI = u_nJ \wedge I_{u_n=\#} = J_{u_n=\#} \wedge$
$(|u_nI| = 1 \rightarrow DI_{u_n\neq\#}J_{u_n\neq\#}) \wedge$
$(|u_nI| \geq 2 \rightarrow \forall x_e \in u_nI(D(I_{u_n=x} * J_{u_n\neq\#})(J_{u_n=x} * J_{u_n\neq\#})))$

A.4 Compositionality

The compositional aspect of interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e., names and sentences. Abbreviate them as $\mathbf{e}$ and $\mathbf{t}$.

An extensional static logic identifies $\mathbf{e}$ with e and $\mathbf{t}$ with t. The translation of the English noun *poem* is of type $\mathbf{et}$, i.e., et: *poem* $\rightsquigarrow \lambda x_e.\ \textsc{poem}_{et}(x)$. The generalized determiner *every* is of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, i.e., $(et)((et)t)$: *every* $\rightsquigarrow \lambda S_{et}.\lambda S'_{et}.\ \forall x_e(S(x) \rightarrow S'(x))$.

PCDRT assigns the following dynamic types to the 'meta-types' $\mathbf{e}$ and $\mathbf{t}$: $\mathbf{t}$ abbreviates $(st)((st)t)$, i.e., a sentence is interpreted as a DRS, and $\mathbf{e}$ abbreviates se, i.e., a name is interpreted as a dref. The denotation of the noun *poem* is still of type $\mathbf{et}$, the determiner *every* is still of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$ etc.

A.5 Singular *Different*—Translations

(24) *poem* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [\textsc{poem}_{et}\{v\}]$,
i.e., *poem* $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I = J \wedge \textsc{poem}_{et}\{v\}J$

(25) *recite* $\rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [\textsc{recite}\{v, v'\}])$

(26) $\mathbf{atoms\text{-}only}\{u_n\} := \lambda I_{st}.\ I_{u_n\neq\#} \neq \emptyset \wedge \forall i_s \in I_{u_n\neq\#}(\mathbf{atom}(u_ni))$

(27) *every*^{u_n} $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_n}([\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n));\ \mathbf{dist}_{u_n}(P'(u_n))$

[11] Equivalently: $\mathbf{dist\text{-}COMP}_{u_n}(D) :=$
$\lambda I_{st}.\lambda J_{st}.\ u_nI = u_nJ \wedge I_{u_n=\#} = J_{u_n=\#} \wedge (|u_nI| = 1 \rightarrow DI_{u_n\neq\#}J_{u_n\neq\#}) \wedge$
$(|u_nI| \geq 2 \rightarrow \forall x_e \in u_nI(D(I_{u_n=x} * (J_{u_n\neq\#} \setminus J_{u_n=x}))(J_{u_n=x} * (J_{u_n\neq\#} \setminus J_{u_n=x}))))$.

(28) every_{u_n} (anaphoric determiner) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \underline{[\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n)};\ \mathbf{dist}_{u_n}(P'(u_n))$
(presuppositions are underlined)

(29) each^{u_n} (determiner) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_n}([\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n));\ \mathbf{dist}_{u_n}(P'(u_n))$

(30) each_{u_n} (anaphoric determiner) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \underline{[\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n)};\ \mathbf{dist}_{u_n}(P'(u_n))$

(31) each (floating quantifier) $\rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ [\mathbf{atoms\text{-}only}\{v\}];\ \mathbf{dist}_v(P(v))$

(32) $\oplus u_{n'} = \oplus u_n := \lambda I_{st}.\ I_{u_n\neq\#} \neq \emptyset \wedge I_{u_n=\#} = I_{u_{n'}=\#} \wedge \oplus u_{n'} I = \oplus u_n I$

(33) each^{u_n} (as a floating quantifier) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ [u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \oplus u_n = \oplus v];\ \mathbf{dist}_{u_n}(P(u_n))$

(34) $\mathbf{singleton}\{u_n\} := \lambda I_{st}.\ |u_n I| = 1$

(35) $a^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}];\ P(u_n);\ P'(u_n)$

(36) $\mathit{Linus}^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \mid u_n = \textsc{linus}];\ P(u_n)$

(37) $\mathit{it}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}];\ P(u_n)$

(38) $u_n \neq \emptyset := \lambda I_{st}.\ u_n I \neq \emptyset$

(39) $\mathit{they}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \neq \emptyset];\ P(u_n)$

(40) $u_{n'} = \oplus u_n := \lambda I_{st}.\ I_{u_n\neq\#} \neq \emptyset \wedge I_{u_n=\#} = I_{u_{n'}=\#} \wedge$
$\forall i_s \in I_{u_n\neq\#}(u_{n'} i = \oplus u_n I)$

(41) $\mathit{they}^{u_{n'}}_{\oplus u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \neq \emptyset];\ [u_{n'} \mid u_{n'} = \oplus u_n];\ P(u_{n'})$

(42) $u_{n''} = u_n \oplus u_{n'} := \lambda I_{st}.\ I_{u_n\neq\#} \neq \emptyset \wedge I_{u_{n'}\neq\#} \neq \emptyset \wedge$
$I_{u_n=\#} = I_{u_{n'}=\#} = I_{u_{n''}=\#} \wedge$
$\forall i_s \in I_{u_n\neq\#}(u_{n''} i = u_n i \oplus u_{n'} i)$

(43) $\mathit{they}^{u_{n''}}_{u_n\oplus u_{n'}} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_n \neq \emptyset, u_{n'} \neq \emptyset];\ [u_{n''} \mid u_{n''} = u_n \oplus u_{n'}];\ P(u_{n''})$

(44) $\mathbf{disjoint}\{u_n, u_{n'}\} := \lambda I_{st}.\ I \neq \emptyset \wedge$
$\{x \leq \oplus u_n I : \mathbf{atom}(x)\} \cap \{x' \leq \oplus u_{n'} I : \mathbf{atom}(x')\} = \emptyset$

(45) $\mathit{other}_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \underline{P(u_n)};\ [\mathbf{disjoint}\{u_n, v\}]$

(46) $\mathit{different}^m_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \underline{P(u_{n+m})};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}]$,
where u_n has to be the dref introduced by the indefinite article immediately preceding *different*

A.6 Singular *Different*—Sample Derivations

(47) $\mathit{other}_{u_{n'}}\ \mathit{poem} \rightsquigarrow \lambda v_{\mathbf{e}}.\ [\textsc{poem}\{v\}];\ \underline{[\textsc{poem}\{u_{n'}\}]};\ [\mathbf{disjoint}\{u_{n'}, v\}]$

(48) $\mathit{different}^m_{u_n}\ \mathit{poem} \rightsquigarrow \lambda v_{\mathbf{e}}.\ [\textsc{poem}\{v\}];\ \underline{[\textsc{poem}\{u_{n+m}\}]};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}]$

(49) $\mathit{an}^{u_n}\ \mathit{other}_{u_{n'}}\ \mathit{poem}$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ [u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}, \textsc{poem}\{u_n\}];\ P'(u_n);$
$\underline{[\textsc{poem}\{u_{n'}\}]};\ [\mathbf{disjoint}\{u_{n'}, u_n\}]$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \underline{[\textsc{poem}\{u_{n'}\}]};$
$[u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}, \mathbf{disjoint}\{u_{n'}, u_n\}, \textsc{poem}\{u_n\}];$
$P'(u_n)$

(50) $a^{u_n}\ \mathit{different}^m_{u_n}\ \mathit{poem}$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ [u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}, \textsc{poem}\{u_n\}];\ P'(u_n);$
$\underline{[\textsc{poem}\{u_{n+m}\}]};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}]$
$\rightsquigarrow \lambda P'_{\mathbf{et}}.\ \underline{[\textsc{poem}\{u_{n+m}\}]};$
$[u_n \mid \mathbf{atoms\text{-}only}\{u_n\}, \mathbf{singleton}\{u_n\}, \mathbf{disjoint}\{u_{n+m}, u_n\}, \textsc{poem}\{u_n\}];$
$P'(u_n)$

A.7 Plural *Different* and *Same*—Translations

(51) $u_3 = u_1 \cup u_2 := \lambda I_{st}.\ I \neq \emptyset \wedge u_3 I = u_1 I \cup u_2 I$

(52) $\mathit{different}^m_{u_n}|_{u_{n'}} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \mathbf{dist}_{u_{n'}}(\underline{P(u_{n+m})};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}])$,
where the dref $u_{n'}$ that we distribute over is anaphorically retrieved.

(53) $\mathit{null\text{-}plural\text{-}indef}^{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u_n \mid u_n \neq \emptyset];\ P(u_n);\ P'(u_n)$

(54) $\mathit{different}^{u_n,m}|_{u_{n'}} \rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u_n \mid u_n \neq \emptyset];\ P(u_n);$
$\mathbf{dist}_{u_{n'}}(\underline{P(u_{n+m})};\ [\mathbf{disjoint}\{u_{n+m}, u_n\}]);\ P'(u_n)$

(55) $\mathbf{5\text{-}atoms}\{u_1\} := \lambda I_{st}.\ |\{x \leq \oplus u_1 I : \mathbf{atom}(x)\}| = 5$

(56) $\mathit{five}^{u_1} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u_1 \mid \mathbf{5\text{-}atoms}\{u_1\}];\ P(u_1);\ P'(u_1)$

(57) $\mathbf{identical}\{u_n, u_{n'}\} := \lambda I_{st}.\ I \neq \emptyset \wedge$
$\{x \leq \oplus u_n I : \mathbf{atom}(x)\} = \{x' \leq \oplus u_{n'} I : \mathbf{atom}(x')\}$

(58) $\mathit{same}^m_{u_n} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \underline{P(u_{n+m})};\ [\mathbf{identical}\{u_{n+m}, u_n\}]$,
where u_n has to be the dref introduced (or anaphorically retrieved) by the definite article immediately preceding *same*.

(59) $\mathit{same}^m_{u_n}|_{u_{n'}} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ P(v);\ \mathbf{dist}_{u_{n'}}(\underline{P(u_{n+m})};\ [\mathbf{identical}\{u_{n+m}, u_n\}])$,
where the dref $u_{n'}$ that we distribute over is anaphorically retrieved.

(60) both^{u_n} (determiner) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_n}([\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n));$
$[\mathbf{2\text{-}atoms}\{u_n\}];\ \mathbf{dist\text{-}WHOLE}_{u_n}(P'(u_n))$

(61) both (floating quantifier) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ [\mathbf{atoms\text{-}only}\{v\}, \mathbf{2\text{-}atoms}\{v\}];\ \mathbf{dist\text{-}WHOLE}_v(P(v))$

(62) the^{u_n} (Russell) $\rightsquigarrow$
$\lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_n}([\mathbf{atoms\text{-}only}\{u_n\}];\ P(u_n));\ [\mathbf{singleton}\{u_n\}];\ P'(u_n)$