Introduction to Dynamic Semantics

Oslo, 15 September, 2006

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Motivating Dynamic Semantics

A sentence is not an island.

Sentences are embedded in larger *discourses*.
They are anaphorically related to other sentences in the same discourse.

Motivating Dynamic Semantics

For example:

(1) John owns a donkey. He feeds it at night.

Notice the anaphoric connection between the indefinite NP ‘a donkey’ and the subsequent pronoun ‘it’.

Motivating Dynamic Semantics

(2) is a good (enough) paraphrase of (1):

(2) John owns a donkey. John feeds it at night.

Motivating Dynamic Semantics

But neither (3) nor (4) is as good:

(3) John owns a donkey. John feeds a donkey at night.

(4) John owns Benjamin (the donkey). John feeds Benjamin at night.

Motivating Dynamic Semantics

Can't seem to eliminate the pronoun 'it' (bound by the indefinite 'a donkey') from (1).

Motivating Dynamic Semantics

This becomes a problem once we decide to regiment (1) in the notation of First-Order Logic (FOL):

(5) $\exists x(\underline{\text{donkey}(x) \wedge \text{owns}(\text{John}, x)})$ $\quad$ $\text{feeds}(\text{John}, x)$

(6) $\exists x(\underline{\text{donkey}(x) \wedge \text{owns}(\text{John}, x)}) \wedge \text{feeds}(\text{John}, x)$

Motivating Dynamic Semantics

What we want:

(7) $\exists x(\underline{\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \wedge \text{feeds}(\text{John}, x)})$

Motivating Dynamic Semantics

The problem is that, to get this meaning, we must first compose a part of the first sentence with the second sentence, and then combine what we have with the remaining part of the first sentence:

(7): [a donkey] [John owns][He feeds it]

Motivating Dynamic Semantics

If we restrict ourselves to completing sentences before we compose them with other sentences, then the best we can do is (6).

(6): [John owns][a donkey] [He feeds it]

Motivating Dynamic Semantics

Who needs it? Discourse semantics is too hard. I'm going to stick with the semantics of sentences.

Motivating Dynamic Semantics

But the donkey is known for its stubbornness…

(8) If John owns a donkey, he feeds it.
(9) Every farmer who owns a donkey feeds it.

Motivating Dynamic Semantics

Incorrect first-orderizations:

(10) $\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x)) \rightarrow \text{feeds}(\text{John}, x)$

(11) $\forall y(\exists x(\text{farmer}(y) \wedge \text{donkey}(x) \wedge \text{owns}(y, x)) \rightarrow \text{feeds}(\text{John}, x))$

In both, the final 'x' is not in the scope of '$\exists x$'.

Motivating Dynamic Semantics

Correct first-orderizations:

(12) $\forall x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \rightarrow \text{feeds}(\text{John}, x))$

(13) $\forall y \forall x(\text{farmer}(y) \wedge \text{donkey}(x) \wedge \text{owns}(y, x) \rightarrow \text{feeds}(\text{John}, x))$

Motivating Dynamic Semantics

Moral: the limitations of FOL (on the standard semantics) can be seen even within sentences.

Nor are ‘donkey’ sentences rare animals. They are as common as the beast of burden itself.

Motivating Dynamic Semantics

A solution:

'Dynamic semantics'

[due (independently) to Kamp (1981) and Heim (1982)]

What is dynamic semantics?

Dynamic Semantics

Consider the phenomenon of *context-sensitivity*.

The same sentence can be true or false, depending on the context.

Dynamic Semantics

'I am standing.'

True as uttered by Sam.

False as uttered by Herman.

Dynamic Semantics

The *meaning* of a sentence can be thought of as a function (cf. Kaplan (1989)),

that takes in a *context*…
…and gives back a *truth-value* (T or F).

Dynamic Semantics

A parallel phenomenon.

Right now, the sentence below is false:

‘Herman said that snow is black.’

Dynamic Semantics

But now Herman says, ‘Snow is black.’

In the context arising immediately *after* his utterance, the earlier sentence is true:

‘Herman said that snow is black.’

Dynamic Semantics

Call the context immediately before Herman's utterance of 'Snow is black', c_1.

And call the context immediately after Herman's utterance, c_2.

Dynamic Semantics

Clearly, the sentence ‘Herman said snow is black’ is *context-sensitive*, since it is true in c_2 but not in c_1.

Dynamic Semantics

Equally clearly, Herman's utterance of 'Snow is black' *changed the context* from c_1 to c_2.

(c_1 must differ from c_2 since it delivers a different truth-value to the sentence above).

Dynamic Semantics

Dynamics takes the semantics of context-sensitivity one step further, to a semantics of *context change*.

Dynamic Semantics

According to dynamic semantics, the meaning of a sentence is an ‘update’,

that takes in a *context*, and gives back a …

CONTEXT.

Dynamic Semantics

But hang on, what does this new view of meaning have to do with the problems with which we began?

(1) John owns a donkey. He feeds it at night.
(8) If John owns a donkey, he feeds it.
(9) Every farmer who owns a donkey feeds it.

Dynamic Semantics

Take the first case:

(1) John owns a donkey. He feeds it at night.

We want it to translate into the FOL:

(7) $\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \wedge \text{feeds}(\text{John}, x))$

Dynamic Semantics

But the best we can do (compositionally) is:

(6) $\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x)) \wedge \text{feeds}(\text{John}, x)$

Dynamic Semantics

What if I told you that, on a dynamic semantics for FOL, the following equivalence holds:

$$\exists x(\phi) \wedge \psi \quad \Leftrightarrow_{DS} \quad \exists x(\phi \wedge \psi)$$

Dynamic Semantics

Since (6) and (7) fit the schema on the left and right hand sides, respectively, they are equivalent on dynamic semantics:

$$\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x)) \wedge \text{feeds}(\text{John}, x)$$

$$\Leftrightarrow_{\text{DS}}$$

$$\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \wedge \text{feeds}(\text{John}, x))$$

Dynamic Semantics

The equivalence means that indefinites can bind indefinitely rightwards across $\wedge$'s:

$$\exists x(\phi) \wedge \psi \wedge \xi \wedge \chi$$
$$\Leftrightarrow_{DS} \exists x(\phi \wedge \psi) \wedge \xi \wedge \chi$$
$$\Leftrightarrow_{DS} \exists x(\phi \wedge \psi \wedge \xi) \wedge \chi$$
$$\Leftrightarrow_{DS} \exists x(\phi \wedge \psi \wedge \xi \wedge \chi)$$

Dynamic Semantics

And what about the other cases?

(8) If John owns a donkey, he feeds it.
(9) Every farmer who owns a donkey feeds it.

Dynamic Semantics

For these the equivalence below will suffice:

$$\exists x(\phi) \to \psi \;\; \Leftrightarrow_{\mathrm{DS}} \forall x(\phi \to \psi)$$

(Only sans the usual restriction to cases where 'ψ' doesn't contain 'x' free.)

Dynamic Semantics

The 2nd equivalence allows us to turn existentials in the antecedent of a conditional into universals taking scope over the whole conditional (but no further).

$$\exists x(\mathrm{donkey}(x) \wedge \mathrm{owns}(\mathrm{John}, x)) \rightarrow \mathrm{feeds}(\mathrm{John}, x)$$

$$\Leftrightarrow_{\mathrm{DS}}$$

$$\forall x(\mathrm{donkey}(x) \wedge \mathrm{owns}(\mathrm{John}, x) \rightarrow \mathrm{feeds}(\mathrm{John}, x))$$

Dynamic Semantics

$$\forall y(\exists x(\text{farmer}(y) \wedge \text{donkey}(x) \wedge \text{owns}(y, x)) \\ \rightarrow \text{feeds}(y, x))$$

$$\Leftrightarrow_{\text{DS}}$$

$$\forall y \forall x(\text{farmer}(y) \wedge \text{donkey}(x) \wedge \text{owns}(y, x) \\ \rightarrow \text{feeds}(y, x))$$

Dynamic Semantics

We will now proceed to show you how to construct a dynamic semantics for FOL on which these hold:

$$\exists x(\phi) \wedge \psi \Leftrightarrow_{\mathrm{DS}} \exists x(\phi \wedge \psi)$$

$$\exists x(\phi) \rightarrow \psi \Leftrightarrow_{\mathrm{DS}} \forall x(\phi \rightarrow \psi)$$

Dynamic Predicate Logic (DPL)

The particular version of dynamic semantics we will look at is Dynamic Predicate Logic (DPL – Groenendijk & Stokhof 1991).

DPL: The Plan.

- semantic values in DPL vs. FOL

- definition of DPL semantics
- relations between DPL connectives
- formula equivalence in DPL:

  $$\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$$

  $$\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$$

- Discourse Representation Structures (DRS's) in DPL

Dynamic Predicate Logic (DPL)

DPL semantics is minimally different from the standard Tarskian semantics for first-order logic.

- instead of interpreting a formula as a set of variable assignments (i.e. the set of variable assignments that satisfy the formula in the given model), we interpret it as a **binary relation between assignments.**

DPL: Semantics.

Why binary relations between **assignments**?

For our narrow purposes (i.e. cross-sentential and ‘donkey’ anaphora), a variable assignment is an effective model of a *context*.

All we ask from a context here is that it keep track of anaphoric relations – hence assignments.

DPL: Semantics.

Why a binary **relation** between assignments?

Dynamic semantics associates a sentence with the manner in which it updates any context (i.e. its context change potential).

The update is modeled as a relation (not a function) because it is non-deterministic:

updating from a context c_1 has different possible outcomes.

DPL: Semantics.

$c_1 \rightarrow c_2$ [Benjamin]

$c_1 \rightarrow c_3$ [Lucius]

$c_1 \rightarrow c_4$ [Patience]

'John owns a donkey',

where John actually owns three donkeys:
Benjamin, Lucius and Patience.

DPL: The Plan.

√ semantic values in DPL vs. FOL

➔ definition of DPL semantics

- relations between DPL connectives
- formula equivalence in DPL:

  $$\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$$

  $$\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$$

- Discourse Representation Structures (DRS's) in DPL

DPL: Semantics.

The definition of the DPL interpretation function $\|\phi\|_{DPL}{}^{M}$ relative to a standard first-order model $\boldsymbol{M}$=<$\boldsymbol{D}^M$, $\boldsymbol{I}^M$>, where:

$\boldsymbol{D}$ is the domain of entities

$\boldsymbol{I}$ is the interpretation function which assigns to each n-place relation R a subset of $\boldsymbol{D}^n$:

DPL: Semantics.

1. For any pair of ***M***-variable assignments $<g, h>$:

a. **Atomic formulas ('lexical' relations and identity):**

$$\| R(x_1, \ldots, x_n) \|^{<g,\, h>} = \mathrm{T}$$
$$\text{iff } g=h \text{ and } <g(x_1), \ldots, g(x_n)> \in \boldsymbol{I}(R)$$

$$\| x_1 = x_2 \|^{<g,\, h>} = \mathrm{T} \text{ iff } g=h \text{ and } g(x_1)=g(x_2)$$

DPL: Semantics.

b. **Connectives:**

Dynamic Conjunction

$\|\phi \wedge \psi\|^{<g,\, h>} = \mathrm{T}$ iff

there is a k s.t. $\|\phi\|^{<g,\, k>} = \mathrm{T}$ and $\|\psi\|^{<k,\, h>} = \mathrm{T}$

DPL: Semantics.

Dynamic Negation

$\|\sim\phi\|\langle g, h\rangle = \mathrm{T}$ iff
$g{=}h$ and there is no k s.t. $\|\phi\|\langle g, k\rangle = \mathrm{T}$

i.e. $\|\sim\phi\|\langle g, h\rangle = \mathrm{T}$ iff $g{=}h$ and $g \notin \mathbf{Dom}(\|\phi\|)$,

where:
$\mathbf{Dom}(\|\phi\|) := \{g\text{: there is an } h \text{ s.t. } \|\phi\|\langle g, h\rangle = \mathrm{T}\}$

DPL: Semantics.

c. **Existential Quantifier:**

$\|\exists x(\phi)\| \langle g, h\rangle = \text{T}$ iff

there is a k s.t. $g[x]k$ and $\|\phi\| \langle k, h\rangle = \text{T}$

where $g[x]k$ means that k differs from g at most with respect to the value it assigns to x,

i.e. for any variable v, if $v \neq x$ then $g(v)=k(v)$.

DPL: Semantics.

d. **Truth:**

A formula ϕ is true with respect to an input assignment g iff

there is an output assignment h s.t. $\|\phi\|^{<g,\, h>} = \mathrm{T}$

i.e. ϕ is true with respect to g iff $g \in \mathbf{Dom}(\|\phi\|)$.

NB: Dynamic meanings are more *fine-grained* than truth-conditions.

DPL: Semantics.

Dynamic Conjunction:

- **not commutative:**

$$\| \sim \mathbf{F}x \wedge \exists x(\mathbf{F}x) \| \neq \| \exists x(\mathbf{F}x) \wedge \sim \mathbf{F}x \|$$

Exercise: Prove this.

DPL: Semantics.

Dynamic Conjunction:

- **not idempotent:**

$$\|\sim\mathbf{F}x \wedge \exists x(\mathbf{F}x)\| \neq \|\sim\mathbf{F}x \wedge \exists x(\mathbf{F}x) \wedge \sim\mathbf{F}x \wedge \exists x(\mathbf{F}x)\|$$

Exercise: Prove this.

DPL: The Plan.

√ semantic values in DPL vs. FOL

√ definition of DPL semantics

→ relations between DPL connectives

- formula equivalence in DPL:

  $$\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$$

  $$\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$$

- Discourse Representation Structures (DRS's) in DPL

DPL: Abbreviations.

2.a. **Abbreviations – connectives:**

Anaphoric closure: $!\phi := {\sim}{\sim}\phi$

i.e. $\|!\phi\| = \{\langle g, h\rangle : g = h$ and $g \in \mathbf{Dom}(\|\phi\|)\}$

Exercise: Prove this.

DPL: Abbreviations.

2.a. **Abbreviations – connectives:**

Disjunction: $\phi \vee \psi := {\sim}({\sim}\phi \wedge {\sim}\psi)$

i.e. $\|\phi \vee \psi\| = \{<g, h>: g{=}h$ and $g \in \mathbf{Dom}(\|\phi\|) \cup \mathbf{Dom}(\|\psi\|)\}$

Exercise: Prove this.

DPL: Abbreviations.

Implication: $\phi \rightarrow \psi := \sim(\phi \wedge \sim\psi)$

i.e. $\| \phi \rightarrow \psi \| = \{<g, h>: g = h$ and
for any k s.t. $\| \phi \| <g, k> = T$,
there is an l s.t. $\| \psi \| <k, l> = T\}$

Exercise: Prove this.

DPL: Abbreviations.

Implication as inclusion:

$\| \phi \rightarrow \psi \| = \{ <g, h>: g = h \text{ and } (\phi)^g \subseteq \mathbf{Dom}(\| \psi \|) \}$

where

$(\phi)^g := \{ h: \| \phi \| <g, h> = \mathrm{T} \}$

Exercise: Prove this.

DPL: Abbreviations.

b. **Abbreviation – universal quantifier:**

$\forall x(\phi) := \sim\exists x(\sim\phi)$

i.e. $\|\forall x(\phi)\| = \{\langle g, h\rangle : g = h$ and
for any k s.t. $g[x]k$,
there is an l s.t. $\|\phi\|\langle k, l\rangle = T\}$

Exercise: Prove this.

DPL: Abbreviations.

Exercise:

Show that $\|\forall x(\phi)\| = \|[x] \rightarrow \phi\|$, where:

$\|[x]\| = \{<g, h>$: for any variable v,
if $v \neq x$ then $g(v)=h(v)\}$

DPL: The Plan.

- √ semantic values in DPL vs. FOL
- √ definition of DPL semantics
- √ relations between DPL connectives

➔ formula equivalence in DPL:

$$\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$$

$$\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$$

- Discourse Representation Structures (DRS's) in DPL

DPL: Equivalence.

Let's return to the general equivalences we wanted to prove.

Equivalence:

Two formulas are DPL-equivalent, symbolized as '$\Leftrightarrow_{DPL}$', iff they denote the same set of pairs of variable assignments,

i.e. iff they denote the same binary relation over assignments.

DPL: Equivalence.

That is:

$$\phi \Leftrightarrow_{\mathrm{DPL}} \psi \quad \text{iff} \quad \|\phi\|_{DPL} = \|\psi\|_{DPL}$$

More explicitly:

$$\phi \Leftrightarrow \psi \quad \text{iff} \quad \text{for any pair of assignments } \langle g, h\rangle\text{:}$$
$$\|\phi\| \langle g, h\rangle = \|\psi\| \langle g, h\rangle$$

i.e. both $\|\phi\| \langle g, h\rangle$ and $\|\psi\| \langle g, h\rangle$ are T
or both are F

DPL: Equivalence.

Since DPL denotations determine truth-conditions, two DPL-equivalent formulas will have the same truth-conditions.

Recall that:

ϕ is *true* with respect to g iff $g \in \mathbf{Dom}(\|\phi\|)$.

Thus:

Suppose $\phi \Leftrightarrow \psi$. Then $\|\phi\| = \|\psi\|$.

Then $\mathbf{Dom}(\|\phi\|) = \mathbf{Dom}(\|\psi\|)$.

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

$\exists x(\phi) \wedge \psi \Leftrightarrow_{DPL} \exists x(\phi \wedge \psi)$

l.h.s. denotes:

$\{<g, h> :$ there is an l s.t. $\|\exists x(\phi)\|<g, k> = \mathrm{T}$ and $\|\psi\|<l, h> = \mathrm{T}\}$

$\{<g, h> :$ there is a k and an l s.t. $g[x]k$ and $\|\phi\|<k, l> = \mathrm{T}$ and $\|\psi\|<l, h> = \mathrm{T}\}$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

$$g \xrightarrow[{[x]}]{} k \xrightarrow[\|\ \phi\ \|]{} l \xrightarrow[\|\psi\|]{} h$$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

r.h.s. denotes:

$\{<g, h> :$ there is a k s.t. $g[x]k$ and $\|\phi \wedge \psi\|<k, h> =$ T$\}$

$\{<g, h> :$ there is a k and an l s.t. $g[x]k$ and $\|\phi\|<k, l> =$ T and $\|\psi\|<l, h> =$ T$\}$

l.h.s. = r.h.s.

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

$$g \xrightarrow[{[x]}]{} k \xrightarrow[\|\phi\|]{} l \xrightarrow[\|\psi\|]{} h$$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

Now let's ensure that DPL gives the intuitively correct truth-conditions to '$\exists x(\phi \wedge \psi)$'.

We will instantiate the schema with our favorite example:

(7) $\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \wedge \text{feeds}(\text{John}, x))$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

(7): $\{\langle g, h\rangle$: there is a k and an l s.t. $g[x]k$
and $\|\text{donkey}(x) \wedge \text{owns}(\text{John}, x)\| \langle k, l\rangle = \text{T}$
and $\|\text{feeds}(\text{John}, x)\| \langle l, h\rangle = \text{T}\}$

$\{\langle g, h\rangle$: there are k, l and m s.t. $g[x]k$
and $\|\text{donkey}(x)\| \langle k, m\rangle$
and $\|\text{owns}(\text{John}, x)\| \langle m, l\rangle = \text{T}$
and $\|\text{feeds}(\text{John}, x)\| \langle l, h\rangle = \text{T}\}$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

(7) $\exists x(\text{donkey}(x) \wedge \text{owns}(\text{John}, x) \wedge \text{feeds}(\text{John}, x))$

Now we apply the definition of truth (1d).

(7) is true with respect to an input assignment g iff there is an output assignment h and intermediate assignments k, l and m s.t.

$$g[x]k \text{ and } \|\text{donkey}(x)\|<k, m>$$
$$\text{and } \|\text{owns}(\text{John}, x)\|<m, l> = \text{T}$$
$$\text{and } \|\text{feeds}(\text{John}, x)\|<l, h> = \text{T}$$

DPL: $\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$.

iff there is an h s.t.

$g[x]h$ and $h(x) \in I(\text{donkey})$

and $\langle \text{John}, h(x) \rangle \in I(\text{owns})$

and $\langle \text{John}, h(x) \rangle \in I(\text{feeds})$

iff there is an individual a s.t.

$a \in I(\text{donkey})$ and $\langle \text{John}, a \rangle \in I(\text{owns})$

and $\langle \text{John}, a \rangle \in I(\text{feeds})$

DPL: $\exists x(\phi) \to \psi \Leftrightarrow \forall x(\phi \to \psi)$.

And now for the second equivalence:

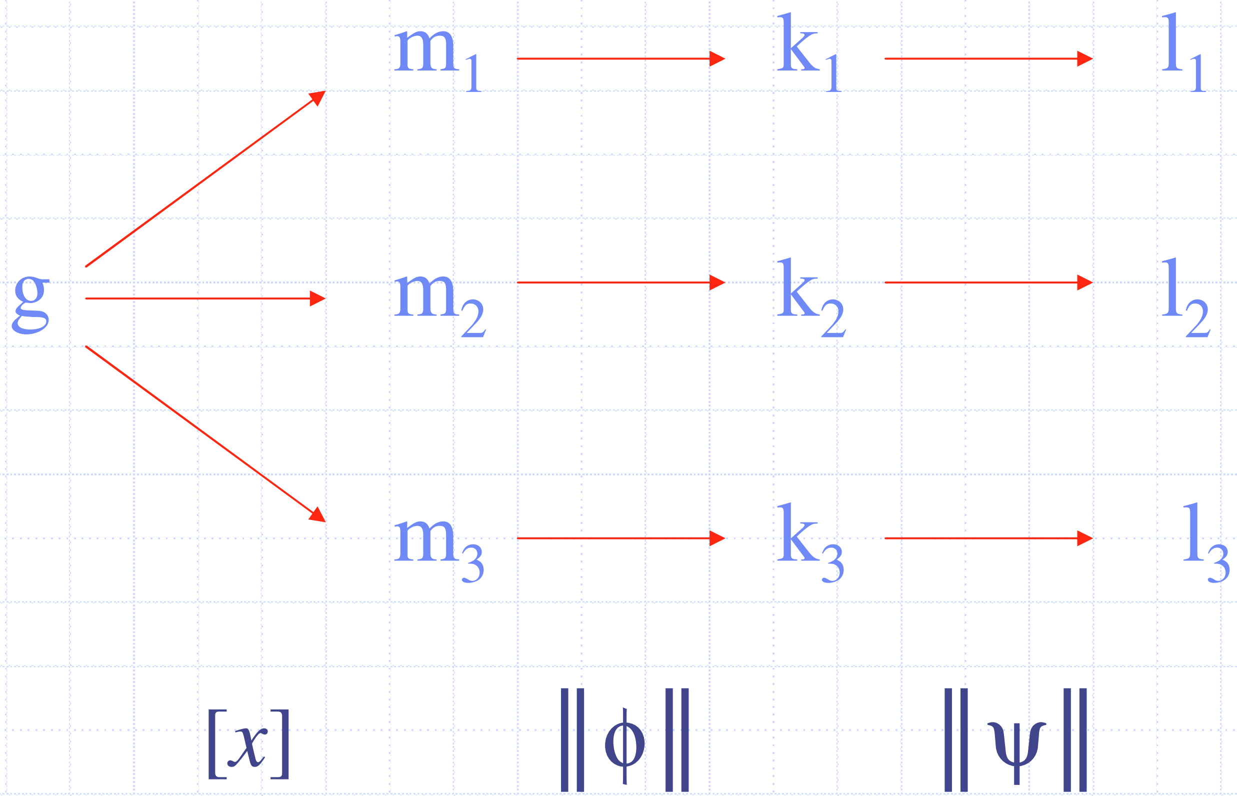
$\exists x(\phi) \to \psi \quad \Leftrightarrow \forall x(\phi \to \psi)$

l.h.s. denotes:

$\{<g, h>: g{=}h$ and for any k s.t. $\|\exists x(\phi)\|<g, k> = \mathrm{T}$,
there is an l s.t. $\|\psi\|<k, l> = \mathrm{T}\}$

$\{<g, h>: g{=}h$ and for any k, m s.t.
$g[x]m$ and $\|\phi\|<m, k> = \mathrm{T}$,
there is an l s.t. $\|\psi\|<k, l> = \mathrm{T}\}$

DPL: $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$.



DPL: $\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$.

r.h.s denotes:

$\{<g, h>: g=h$ and for any m s.t. $g[x]m$,
there is an n s.t. $\|\phi \rightarrow \psi\| <m, n> = T\}$

$\{<g, h>: g=h$ and for any k, m s.t.
$g[x]m$ and $\|\phi\| <m, k> = T$,
there is an l s.t. $\|\psi\| <k, l> = T\}$

l.h.s. = r.h.s.

DPL: $\exists x(\phi) \to \psi \Leftrightarrow \forall x(\phi \to \psi)$.

$$
\begin{tikzcd}
 & m_1 \arrow[r] & k_1 \arrow[r] & l_1 \\
g \arrow[ur] \arrow[r] \arrow[dr] & m_2 \arrow[r] & k_2 \arrow[r] & l_2 \\
 & m_3 \arrow[r] & k_3 \arrow[r] & l_3
\end{tikzcd}
$$

$[x]$ $\qquad \| \phi \|$ $\qquad \| \psi \|$

DPL: The Plan.

- √ semantic values in DPL vs. FOL
- √ definition of DPL semantics
- √ relations between DPL connectives
- √ formula equivalence in DPL:

$$\exists x(\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$$

$$\exists x(\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$$

➔ Discourse Representation Structures (DRS's) in DPL

DPL: Representing DRS's.

To represent Discourse Representation Structures (DRS's), i.e. 'boxes', in DPL, we first need to define:

- the **semantic** notion of *test*

- the **syntactic** notion of *condition*.

DPL: Representing DRS's.

Tests: A wff ϕ is a *test* iff $\|\phi\| \subseteq \{<g, g>: g \in \boldsymbol{G}\}$, where $\boldsymbol{G}$ is the set of all $\boldsymbol{M}$-variable assignments,

Conditions: The set of *conditions* is the smallest set of wff's:

- containing atomic formulas and negative formulas (i.e. negation '~' is the main connective)
- and closed under dynamic conjunction.

DPL: Representing DRS's.

Negative formulas include:

- $\sim\phi$
- anaphoric closure, since $!\phi := \sim\sim\phi$
- disjunctions, since $\phi \vee \psi := \sim(\sim\phi \wedge \sim\psi)$
- implications, since $\phi \rightarrow \psi := \sim(\phi \wedge \sim\psi)$
- universal quantifications, since $\forall x(\phi) := \sim\exists x(\sim\phi)$

DPL: Representing DRS's.

The relation between **tests** (semantic notion) and **conditions** (syntactic notion):

Among non-contradictory formulas,

ϕ is a **condition** iff ϕ is a **test.**

where: ϕ is *contradictory* iff $\|\phi\| = \emptyset$

DPL: Representing DRS's.

Tests / Conditions are externally static – they do not pass on bindings to conjuncts yet to come:

(14) Every donkey is in the corral. #It is happy.

(15) It is not true that John owns a donkey.
#He feeds it at night.

DPL: Representing DRS's.

Conjunctions and existential quantifiers are externally dynamic – they pass on bindings to conjuncts yet to come:

(16) A farmer owns a donkey. He feeds it at night.

DPL: Representing DRS's.

But **test / conditions** can be internally dynamic, i.e. they can pass bindings between sub-formulas:

(17) Every farmer who owns a donkey feeds it at night.

DPL: Representing DRS's.

We indicate that a formula is a **condition** by placing square brackets around it,

e.g. $[\phi]$ is a wff iff ϕ is a *condition* and $\|[\phi]\| = \|\phi\|$

That is, square brackets are just a graphical way of showing that a formula is a condition.

DPL: Representing DRS's.

Abbreviation: $[\phi_1, \ldots, \phi_m] := [\phi_1] \wedge \ldots \wedge [\phi_m]$

Exercise: Prove that conjunction is commutative over conditions,

i.e. $\|[\phi_1] \wedge [\phi_2]\| = \|[\phi_2] \wedge [\phi_1]\|$.

Exercise: Prove that conjunction is idempotent over conditions,

i.e. $\|[\phi]\| = \|[\phi] \wedge [\phi]\|$.

DPL: Representing DRS's.

Abbreviation: $[x_1, \ldots, x_n] := [x_1] \wedge \ldots \wedge [x_n]$,

where:

$\|[x]\| = \{<g, h>$: for any variable v, if $v \neq x$ then $g(v)=h(v)\}$

$[x]$ is called a **random assignment** of value to x.

Exercise: Prove that conjunction is commutative and idempotent over random assignments, i.e.:

$\|[x_1] \wedge [x_2]\| = \|[x_2] \wedge [x_1]\|$ and $\|[x]\| = \|[x] \wedge [x]\|$.

DPL: Representing DRS's.

DRS's, a.k.a. boxes:

$$[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] := [x_1, \ldots, x_n] \wedge [\phi_1, \ldots, \phi_m]$$

$$\| [x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] \| :=$$

$$\{\langle g, h\rangle: g[x_1, \ldots, x_n]h \text{ and } \|\phi_1\|\langle h, h\rangle = T \text{ and } \ldots \|\phi_m\|\langle h, h\rangle = T\}$$

Exercise: Prove that

$$[x_1, \ldots, x_n \mid \phi_1, \ldots, \phi_m] \Leftrightarrow \exists x_1 \ldots \exists x_n([\phi_1, \ldots, \phi_m])$$

DPL: The Duality of $\exists$ and $\forall$.

The existential and universal quantifiers are partly duals:

$$\sim\exists x(\phi) \Leftrightarrow \forall x(\sim\phi)$$

(**Exercise**: Prove this.)

Clearly, $\exists x(\sim\phi) \Leftrightarrow \sim\forall x(\phi)$ doesn't hold:

$\sim\forall x(\phi)$ is a test, while $\exists x(\sim\phi)$ isn't.