Plural Discourse Reference

Adrian Brasoveanu,* UC Santa Cruz, abrsvn@ucsc.edu

1 Plural Reference and Plural Discourse Reference

The goal of this additional section is to further explore the *semantic* notion of plurality and argue that, in addition to plural reference to collections, natural language interpretation involves a notion of plural discourse reference that is essential for the interpretation of quantificationally dependent pronouns (and anaphoric expressions in general, e.g., definites, partitives, reciprocals etc.), irrespective of whether these pronouns are *morphologically* singular or plural.

Plural discourse reference is reference to a quantificational dependency between sets of objects (e.g., atomic individuals or collections, but also times, eventualities, possible worlds etc.) that is established and subsequently elaborated upon in discourse. Consider, for example, the sentence in (1) below – where antecedents are superscripted with the discourse referent (dref) they introduce, while anaphors are subscripted with the dref they retrieve (following the convention in Barwise 1987).

(1) Linus bought a^{u_1} gift for every^{u_2} girl in his class and asked their$_{u_2}$ deskmates to wrap them$_{u_1}$.

The first conjunct in (1) introduces a quantificational dependency between the set u_2 of girls in Linus's class and the set u_1 of gifts bought by Linus: each u_2-girl is correlated with the u_1-gift(s) that Linus bought for her. This correlation / dependency is elaborated upon in the second conjunct: for each u_2-girl, Linus asked her deskmate to wrap her u_1-gift(s).

The idea of plural discourse reference surfaces in various guises in the dynamic semantics literature (e.g., Rooth 1987, Kamp & Reyle 1993, Dekker 1994, Krifka 1996, van Benthem 1996, Nouwen 2003, Asher & Wang 2003), but is for the first time brought to the fore as such in the Dynamic Plural Logic (DPlL) of van den Berg (1996).

The basic proposal in DPlL is to model plural discourse reference as sets of variable assignments. That is, unlike classical dynamic semantics, i.e., Discourse Representation Theory (DRT; Kamp 1981, Kamp & Reyle 1993), File Change Semantics (FCS; Heim 1982) and Dynamic Predicate Logic (DPL; Groenendijk & Stokhof 1991), DPlL takes natural language expressions to be evaluated relative to sets of assignments and not relative to single assignments and models their context-change potentials as updates of sets of assignments (taken collectively) and not as updates of single assignments (or, equivalently, as updates of sets of assignments taken distributively). These sets $I, J, \ldots$ of assignments $i_1, i_2, \ldots, j_1, j_2, \ldots$, i.e., these *plural* information states, can be represented as matrices with assignments / sequences as rows:

Plural Info State I	$\ldots$	u_1	u_2	u_3	$\ldots$
i_1	$\ldots$	α_1	β_1	γ_1	$\ldots$
i_2	$\ldots$	α_2	β_2	γ_2	$\ldots$
i_3	$\ldots$	α_3	β_3	γ_3	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

Plural info states / matrices are two-dimensional and encode two kinds of discourse information: values and structure. The values are the sets of objects that are stored in the columns of the matrix, e.g., relative to the plural info state I above, the dref u_1 stores the set of individuals $\{\alpha_1, \alpha_2, \ldots\}$ since u_1 is assigned an individual by each assignment / row, the dref u_2 stores the set of individuals $\{\beta_1,$

*I want to thank Johan van Benthem and Alice ter Meulen for their detailed comments and Donka Farkas and Bill Ladusaw for discussion.

$\beta_2, \ldots\}$ etc. Each individual can be atomic or non-atomic / a collection, i.e., singular or plural at the domain level. The structure / quantificational dependency is encoded in the rows of the matrix, which induce n-ary relations between objects: for example, for each row in I, the individual assigned to the dref u_1 by that row is correlated with the individual assigned to the dref u_2 by the same row – so, the plural info state I induces the binary relation $\{\langle\alpha_1, \beta_1\rangle, \langle\alpha_2, \beta_2\rangle, \ldots\}$. Similarly, I induces the ternary relation $\{\langle\alpha_1, \beta_1, \gamma_1\rangle, \langle\alpha_2, \beta_2, \gamma_2\rangle, \ldots\}$ between the drefs u_1, u_2 and u_3 etc.

2 Multiple Interdependent Anaphora

Most of the empirical arguments in the literature for an independent notion of plural discourse reference rely on morphologically plural anaphora of the kind instantiated in (1) above. Plural anaphora, however, does not provide a clear-cut argument for distinguishing plural reference (to collections) and plural discourse reference, since either of them could be involved in the interpretation of (1). Nor does it provide a forceful argument for a semantic – as opposed to a pragmatic – encoding of discourse-level reference to quantificational dependencies: it might be that the second conjunct in (1) is cumulatively interpreted (in the sense of Scha 1981) and that the correlation between girls and gifts, brought to salience by the first conjunct, is only pragmatically supplied when we interpret the second conjunct.

This is why sentences with *multiple* instances of *singular* anaphora provide more compelling evidence for a semantics based on plural info states: the fact that the anaphors are morphologically singular enables us to factor out plural reference to collections and the fact that we have multiple anaphoric connections that are simultaneous and interdependent (while being embedded under a quantifier) motivates a *semantics* that relies on plural info states, i.e., that crucially involves plural discourse reference.

2.1 Multiple Singular Donkey Anaphora

For example, sentences with multiple instances of singular donkey anaphora like (2) and (3) below support the idea of a semantics based on plural info states.

(2) Every^{u_1} person who buys a^{u_2} book on `amazon.com` and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$.

(3) Every^{u_1} boy who bought a^{u_2} gift for a^{u_3} girl in his class asked her$_{u_3}$ deskmate to wrap it$_{u_2}$.

Donkey anaphora provided one of the main incentives for developing a dynamic semantics for natural language (for more details, see the chapter *Representing Discourse in Context*). To see why, consider the example of donkey anaphora in (4) below: the indefinite *a*^{u_2} *'Harry Potter' book* seems to be able to semantically bind the pronoun *it*$_{u_2}$ despite the fact that the pronoun is not in its syntactic scope (i.e., the indefinite does not c-command the pronoun). The fact that the pronoun is not in the scope of the indefinite is shown by the minimally different sentence in (5), the infelicity of which (symbolized by #) is due to the failure of the quantifier *every*^{u_2} *'Harry Potter' book* to bind the pronoun *it*$_{u_2}$.

(4) Every^{u_1} boy who read a^{u_2} 'Harry Potter' book recommended it$_{u_2}$ to his friends.

(5) #Every^{u_1} boy who read every^{u_2} 'Harry Potter' book recommended it$_{u_2}$ to his friends.

Going dynamic enables us to account for donkey anaphora because, unlike in static semantics, we can take the variable assignment modified by the indefinite *a*^{u_2} *'Harry Potter' book* in the restrictor of the quantificational determiner *every*^{u_1} and pass it on to the nuclear scope of the quantificational determiner. In this way, we are able to interpret the pronoun *it*$_{u_2}$ as if it was in the scope of the indefinite.

This (classical) dynamic account of donkey anaphora faces a couple of problems, one of which is the availability of both weak and strong readings for donkey sentences. The weak vs strong contrast is exemplified by sentence (6) below, which has a weak reading, and the classical donkey sentence in (7), which has a strong reading. Sentence (6) has a weak reading in the sense that its most salient interpretation is that every person who has a dime will put *some* dime s/he has in the meter – and not all her / his dimes. Sentence (7) has a strong reading in the sense that its most salient interpretation is: every farmer beats *every* donkey s/he owns. This contrast is problematic for the classical dynamic account because this account can derive only strong donkey readings.

(6) Every^{u_1} person who has a^{u_2} dime will put it$_{u_2}$ in the meter. (Pelletier & Schubert 1989)

(7) Every^{u_1} farmer who owns a^{u_2} donkey beats it$_{u_2}$. (based on Geach 1962)

There are a variety of proposals in the literature to revise the notion of quantification in classical dynamic semantics in such a way that both weak and strong donkey readings are allowed (see, for example, Heim 1990, van Eijck & de Vries 1992, Kamp & Reyle 1993, Kanazawa 1994, Chierchia 1995). All the revised systems can handle simple weak or strong donkey sentences – however, just like classical dynamic semantics, they rely on singular info states, i.e., they update single assignments and not sets of assignments, and cannot compositionally account for *mixed* weak & strong relative-clause donkey sentences like the one in (2) above.

Consider sentence (2) more closely: its most salient interpretation is that, for *every* book (strong reading) that any credit-card owner buys on `amazon.com`, there is *some* credit card (weak reading) that s/he uses to pay for the book. Note, in particular, that the credit card can vary from book to book, e.g., I can use my MasterCard to buy set theory books and my Visa to buy detective novels, which means that even weak indefinites like *a*^{u_3} credit card can introduce non-singleton sets.

For each buyer, the two sets of objects, i.e., all the books purchased on `amazon.com` and some of the credit cards that the buyer has, are correlated and the dependency between these sets – left implicit in the restrictor of the quantification – is specified in the nuclear scope: each book is correlated with the credit card that was used to pay for it. This paraphrase of the meaning of sentence (2) is formalized in classical (static) first-order logic as shown in (8) below.

(8) $\forall x(pers(x) \wedge \exists y(bk(y) \wedge buy(x,y)) \wedge \exists z(card(z) \wedge hv(x,z))$
$\rightarrow \forall y'(bk(y') \wedge buy(x,y') \rightarrow \exists z'(card(z') \wedge hv(x,z') \wedge use\_to\_pay(x,z',y'))))$

Given that (2) is intuitively interpreted as shown in (8) above, a plausible hypothesis is that singular donkey anaphora involves plural reference, i.e., reference to collections / non-atomic individuals – as proposed in Lappin & Francez (1994), for example. That is, (multiple) singular donkey anaphora is analyzed in much the same way as the (multiple) plural anaphora in sentence (9) below, where the two plural pronouns *them*$_{u_2}$ and *them*$_{u_1}$ are anaphoric to the plural individuals obtained by summing the domains (i.e., restrictors) of the quantifier *every*^{u_2} *girl in his class* and of the narrow scope indefinite *a*^{u_1} *gift*, respectively.

(9) Linus bought a^{u_1} gift for every^{u_2} girl in his class and asked them$_{u_2}$ / the$_{u_2}$ girls to wrap them$_{u_1}$ / the$_{u_1}$ gifts.

This kind of approach analyzes the mixed-reading donkey sentence in (2) as follows. The strong donkey anaphora to u_2-books involves the maximal sum individual (i.e., the maximal collection) y containing all and only the books bought by a given u_1-person. At the same time, the weak donkey anaphora to u_3-credit cards involves a non-maximal individual z (possibly non-atomic) containing some of the credit cards that said u_1-person has. Finally, the nuclear scope of (2) is cumulatively interpreted, i.e., given the maximal sum y of books and the sum z of some credit cards, we have: (*i*) for any atomic individual y' such that $y' \leq y$ (i.e., y' is a part of the collection y), there is an atom z' such that $z' \leq z$ and z' was used to pay for y' and, also, (*ii*) for any atom $z' \leq z$, there is an atom $y' \leq y$ such that z' was used to pay for y'.

As noticed in Kanazawa (2001), such a plural reference approach to weak / strong donkey anaphora incorrectly predicts that the infelicitous sentence in (10) below (based on Kanazawa 2001:396, (56)) should be acceptable – at least in a situation in which all donkey-owning farmers have more than one donkey. This is because singular donkey anaphora is guaranteed in such a situation to involve reference to non-atomic individuals, hence to be compatible with collective predicates like *gather*.

(10) #Every^{u_1} farmer who owns a^{u_2} donkey gathers it$_{u_2}$ around the fire at night.

One way to maintain the plural reference approach and derive the infelicity of (10) is to assume (following a suggestion in Neale 1990) that singular donkey pronouns always distribute over the non-atomic individual they are anaphoric to. For example, the singular pronoun it_{u_2} in (10) contributes a distributivity operator and requires each donkey atom in the maximal sum of u_2-donkeys to be gathered around the fire at night. The infelicity of (10) follows from the fact that collective predicates apply only to collections / non-atomic individuals.

But this domain-level (as opposed to discourse-level) distributivity strategy will not help us with respect to (3) above. Sentence (3) contains two instances of strong donkey anaphora: we are considering every gift and every girl. Moreover, the restrictor of the quantification in (3) introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. Finally, the nuclear scope retrieves not only the two sets of objects, but also the dependency between (i.e., the structure associated with) them: each gift was wrapped by the deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to structure / dependencies in addition to donkey anaphora to values / objects.

Importantly, the structure associated with the two sets of atoms, i.e., the dependency between gifts and girls that is introduced in the restrictor and elaborated upon in the nuclear scope of the quantification, is semantically encoded and not pragmatically inferred (we would – incorrectly – expect this kind of pragmatic, cumulativity-based approach to work for (3) in view of sentences like (9) above). That is, the nuclear scope of the quantification in (3) is not interpreted cumulatively and the correlation between the sets of gifts and girls is not left vague / underspecified and subsequently made precise only at the pragmatic level, based on various extra-linguistic factors.

To see that the structure / dependency in (3) is semantically – and not pragmatically – encoded, consider the following situation: suppose that Linus buys two gifts, one for Megan and the other for Gabby; moreover, the two girls are deskmates. Intuitively, sentence (3) is true if Linus asked Megan to wrap Gabby's gift and Gabby to wrap Megan's gift and it is false if Linus asked each girl to wrap her own gift. But if the 'wrapping' relation between gifts and girls were semantically vague / underspecified and only pragmatically supplied (as it is in sentence (9) above), we would predict sentence (3) to be intuitively true even in the second kind of situation.

In sum, we need plural discourse reference (in addition to plural reference) to (*i*) account for singular weak / strong donkey anaphora to structured sets of individuals (see (2) and (3) above) and (*ii*) derive the incompatibility between singular donkey anaphora and collective predicates (see (10) above).

Plural info states enable us to capture the semantic non-singularity intuitions associated with morphologically singular donkey anaphora and to give a compositional account of mixed weak & strong donkey sentences by locating the weak / strong donkey ambiguity at the level of the indefinite articles. A weak indefinite stores in a plural info state some of the individuals that satisfy its restrictor and nuclear scope, i.e., a non-maximal witness set that satisfies the nuclear scope, while a strong indefinite article stores in a plural info state all the individuals that satisfy its restrictor and nuclear scope.

We account for the incompatibility between singular donkey anaphora and collective predicates by taking singular donkey anaphora to be (*i*) distributive at the discourse level, i.e., predicates need to be satisfied relative to each individual assignment i in a plural info state I, and (*ii*) singular / atomic at the domain level, that is, for each $i \in I$, the individual i assigns to the dref u, symbolized as ui, is atomic. Since collective predicates apply only to collections / non-atomic individuals, they are felicitous if either (*i*) the individuals stored by each variable assignment are non-atomic, i.e., we have domain-level plurality, or (*ii*) they are interpreted collectively at the discourse level, e.g., we sum all the individuals stored in a plural info state and require the resulting sum individual $ui_1 \oplus ui_2 \oplus \ldots$ to be gathered around the fire. Neither case obtains with singular donkey anaphora.

2.2 Multiple Plural Donkey Anaphora

Having established the need for plural discourse reference, the question arises whether we can do away with plural reference by deriving collections (sets of individuals) from plural info states (sets of assignments). This is the last question raised in the original chapter (section **6**).

Dynamic approaches that countenance plural discourse reference usually treat the two notions of plurality asymmetrically. They fall into roughly two classes. The first kind of approaches (e.g., van den Berg 1996, Nouwen 2003, Asher & Wang 2003) make plural reference dependent on plural discourse reference, i.e., they allow variable assignments to store only atomic individuals. Collections / non-atomic individuals can be accessed in discourse only by summing over plural info states. The second kind of approaches (e.g., Krifka 1996), make plural discourse reference dependent on plural reference: the central notion of parametrized sum individuals (due to Rooth 1987 and developed in Krifka 1996) associates each atom that is part of a collection, i.e. a non-atomic / sum individual, with a variable assignment that 'parametrizes' / is dependent on that atom. For example, the universal quantifier in Geach's original donkey sentence introduces a collection containing all and only the farmer-atoms that are donkey owners – and each farmer-atom is associated / parametrized with one or more variable assignments that each store (relative to a new dref) a donkey-atom that the farmer owns.

Both kinds of approaches have difficulties with plural donkey anaphora stemming from their asymmetric treatment of domain-level and discourse-level plurality. The second kind of approaches find it difficult to account for the incompatibility between singular donkey anaphora and collective predicates exemplified in (10) because the discourse-level plurality associated with strong donkey anaphora requires domain-level plurality, which in turn predicts that the collective predicate *gather* should be felicitous. These approaches also have difficulties with examples of donkey anaphora to structure like (3), in which the order / 'relative scope' of the anaphors does not reproduce the order / 'relative scope' of the antecedents – because the nested structure of the dependencies stored in parametrized sum individuals predicts that we can anaphorically retrieve the entities stored in the parametrizing assignments only if we first retrieve the collection / sum individual that those assignments actually parametrize.

The first kind of approaches have a different set of problems: they do not generalize to morphologically plural donkey anaphora. In particular, they have difficulties (*i*) accounting for plural sage plant examples and (*ii*) capturing the intuitive parallels between singular and plural donkey anaphora. Consider first the singular and plural sage plant examples in (11) (see Heim 1982:89, (12)) and (12) (based on example (49) in Kanazawa 2001:393, adapted from Lappin & Francez 1994) below.

(11) Everybody^{u_1} who bought a^{u_2} sage plant here bought eight^{u_3} others along with it$_{u_2}$.

(12) Everybody^{u_1} who bought two^{u_2} sage plants here bought seven^{u_3} others along with them$_{u_2}$.

Dynamic approaches that countenance only plural discourse reference (but not plural reference, i.e., collections) can account for singular sage plant examples. In the case of the plural example in (12), however, they need to 'distribute' over the purchased sage plants in such a way that they look at all the *pairs* of sage plant atoms and predicate the nuclear scope *bought seven*^{u_3} *others...* of such pairs. This, in turn, requires an operator that distributes over all the pairs of assignments in a plural info state (not over individual assignments) – and it is not clear how to define such an operator or what particular lexical item in (12) contributes it.

Similarly, these approaches cannot capture the intuitive parallel between the multiple plural donkey sentence in (13) below and the multiple singular sentence in (3) above. Note that the collective predicate *fight (each other)* in (13) is felicitous because, in contrast to example (10), we have domain-level non-atomicity introduced by the plural cardinal indefinite *two*^{u_3} *boys*.

(13) Every^{u_1} parent who gives a^{u_2} balloon / three^{u_2} balloons to two^{u_3} boys expects them$_{u_3}$ to end up fighting (each other) for it$_{u_2}$ / them$_{u_2}$.

Thus, (multiple) plural donkey anaphora provides evidence that natural language interpretation requires both plural discourse reference and plural reference and that these two semantic notions of plurality should be formalized as two independent (yet interacting) meaning components. This enables us to derive the correct interpretations for plural sage plant examples and instances of multiple plural donkey anaphora while capturing the intuitive parallels between them and their singular counterparts.

Finally, allowing for both notions of plurality opens the way to an account of weak / strong plural donkey readings that is parallel to the account of weak / strong singular donkey readings. For example, cardinal indefinites like *two* can be either strong, e.g., *two*^{u_3} *boys* in (13) above, or weak, e.g., *two*^{u_2} *dimes* in (14) below – which is a minimal variation on the classical weak donkey sentence in (6) above.

(14) Every^{u_1} driver who had two^{u_2} dimes put them$_{u_2}$ in the meter.

2.3 Multiple Anaphora Generalized

Introducing and incrementally elaborating on dependencies between multiple interrelated objects is a common occurrence in natural language discourse and it is not restricted to dependencies between individuals. For example, the discourse in (15) below (from Karttunen 1976) introduces and elaborates on a dependency between individuals (the courted women) and events (the conventions during which the women are courted). The naturally-occurring discourse in (16)[1] instantiates a similar dependency between children and the various events they participate in, e.g., being congratulated *again*, being given *another* pizza etc. Finally, the discourse in (17) (based on examples in Roberts 1989), introduces and elaborates on a dependency between possibilities – possible scenarios in which a wolf comes in – and individuals – the wolves featuring in each of the possible scenarios.

(15) **a.** Harvey courts a^{u_1} woman at every$^{\epsilon_2}$ convention. **b.** She$_{u_1}$ always$_{\epsilon_2}$ comes to the banquet with him. **c.** The$_{u_1}$ woman is usually$_{\epsilon_2}$ also very pretty.

(16) **a.** [In the BOOK IT! program] You set monthly reading goals for each child in the class. **b.** As soon as a monthly reading goal has been met, you present the child with a pizza award certificate. **c.** The child takes the certificate to a *Pizza Hut* restaurant, where he or she is congratulated and given a free, one-topping pizza. **d.** On the first visit, the child also receives recognition of their accomplishment and a surprise gift. **e.** On each subsequent visit, the child is again congratulated and given another pizza and a sticker to recognize reading achievement.

(17) **a.** A^{u_1} wolf might^{p_1} come in. **b.** It$_{u_1}$ would$_{p_1}$ attack Harvey first.

3 Ontology and Logic

We can think of drefs, i.e., variables, and variable assignments, i.e., assignments of values to drefs, in two ways. Classical dynamic semantics (DRT / FCS / DPL) takes drefs to be atomic, basic entities and variable assignments to be composite objects, namely functions from drefs to appropriate values. Taking drefs to be the basic building blocks is pre-theoretically appealing: as Karttunen (1976) and Webber (1978) first argued, natural language interpretation involves an irreducible notion of discourse-level reference and the referents / entities that are introduced, constrained and related to each other in discourse are distinct from the actual referents / entities (in the static, Fregean / Tarskian sense).

The ontology and logic of the classical dynamic formalization do not, however, perspicuously reflect the main point of the present section, namely the existence of a notion of plural discourse reference that is independent from and parallel to domain-level plural reference. We will, therefore, follow Landman (1986) and Muskens (1996), and take assignments to be atomic, basic entities, while drefs will be modeled as composite objects, namely as functions from assignments to appropriate static entities.

The ontological commitment to the existence of singular and plural info states implicit in this formalization of discourse-level plurality is parallel to the ontological commitment to individuals and collections in Link-style theories of domain-level plurality (see Link 1983).

We will see that, although they make different ontological commitments, the two ways of formalizing drefs and variable assignments are very similar from a technical point of view.

3.1 Dynamic Ty2

Following Muskens (1996), we will use classical (many-sorted) type logic as the underlying logic for the entire dynamic system. This logic, together with the definition of dref types, the set of axioms that ensure the proper behavior of variable assignments etc. is labeled Dynamic Ty2 because, just as the Logic of Change of Muskens (1996), it is a variant of Gallin's Ty2 (Gallin 1975).

[1] The example is a lightly edited version of the text available at `http://www.pizzahutmhat.com/bookit/about.html`.

There are three basic types: type t (truth-values), type e (atomic and non-atomic individuals) and type s, modeling variable assignments as they are used in DPL (Groenendijk & Stokhof 1991). A suitable set of axioms, provided in (20) below, ensures that the entities of type s behave as assignments.

The recursive definition in (18) below isolates a proper subset of types as dref types: these are functions from assignments (type s) to static objects of arbitrary types. We restrict our drefs to such functions because, if we allow for arbitrary dref types, e.g., $s(st)$, we might run into counterparts of Russell's paradox (see Muskens 1995:179-180, fn. 10).

(18) **Dynamic Ty2** – the set of dref types **DRefTyp** and the set of types **Typ**.

a. **BasSTyp** (basic static types): $\{t, e\}$ (truth-values and individuals).
b. **STyp** (static types): the smallest set including **BasSTyp** and s.t., if $\sigma, \tau \in \mathbf{STyp}$, then $(\sigma\tau) \in \mathbf{STyp}$.
c. **BasTyp** (basic types): $\mathbf{BasSTyp} \cup \{s\}$ (variable assignments).
d. **DRefTyp** (dref types): the smallest set s.t., if $\tau \in \mathbf{STyp}$, then $(s\tau) \in \mathbf{DRefTyp}$.
e. **Typ** (types): the smallest set including **BasTyp** and s.t., if $\sigma, \tau \in \mathbf{Typ}$, then $(\sigma\tau) \in \mathbf{Typ}$.

(19) **Dynamic Ty2** – terms (subscripts on terms indicate their type).

a. Basic expressions: for any type $\tau \in \mathbf{Typ}$, there is a denumerable set of τ-constants $\mathbf{Con}_\tau$ and a denumerably infinite set of τ-variables $\mathbf{Var}_\tau = \{v_{\tau,0}, v_{\tau,1}, \ldots\}$, e.g.:
   i. $\mathbf{Con}_e = \{linus, mary, dobby, \ldots\}$, $\mathbf{Con}_{et} = \{donkey, farmer, \ldots, walk, arrive, \ldots\}$, $\mathbf{Con}_{e(et)} = \{own, beat, \ldots\}$
   ii. $\mathbf{Con}_{se} = \{u_1, u_2, \ldots, u, u', \ldots\}$ (drefs for individuals are constants)
   iii. $\mathbf{Var}_e = \{x, y, \ldots\}$, $\mathbf{Var}_s = \{i, j, \ldots\}$; in general, $\mathbf{Var}_\tau = \{v_1, v_2, \ldots, v, v', \ldots\}$ for any $\tau \in \mathbf{Typ}$
b. For any type $\tau \in \mathbf{Typ}$, the set of τ-terms $\mathbf{Term}_\tau$ is the smallest set s.t.:
   i. $\mathbf{Con}_\tau \cup \mathbf{Var}_\tau \subseteq \mathbf{Term}_\tau$
   ii. $\alpha(\beta) \in \mathbf{Term}_\tau$ if $\alpha \in \mathbf{Term}_{\sigma\tau}$ and $\beta \in \mathbf{Term}_\sigma$, for any $\sigma \in \mathbf{Typ}$
   iii. $(\lambda v.\ \alpha) \in \mathbf{Term}_\tau$ if $\tau = (\sigma\rho)$, $v \in \mathbf{Var}_\sigma$ and $\alpha \in \mathbf{Term}_\rho$, for any $\sigma, \rho \in \mathbf{Typ}$
   iv. $(\alpha = \beta) \in \mathbf{Term}_\tau$ if $\tau = t$ and $\alpha, \beta \in \mathbf{Term}_\sigma$, for any $\sigma \in \mathbf{Typ}$
   v. $(i[\delta]j) \in \mathbf{Term}_\tau$ if $\tau = t$ and $i, j \in \mathbf{Var}_s$ and $\delta \in \mathbf{Term}_\sigma$, for any $\sigma \in \mathbf{DRefTyp}$
c. Abbreviation: $Dobby_{se} := \lambda i_s.\ dobby_e$, $Mary_{se} := \lambda i_s.\ mary_e$ etc.

We take a first-order approach to domain-level plurality and lump together atomic individuals and collections / non-atomic individuals into the domain of type e, formalized as the power set of a given non-empty set IN of entities. In this way, we can simplify the types we assign to natural language expressions, which in turn enables us to focus on discourse-level plurality. However, everything we will say is compatible with a higher-order approach to domain-level plurality (see the original chapter for a detailed discussion of higher-order vs first-order approaches to domain-level plurality).

In more detail, the domain of type e is $\wp^+(\mathsf{IN}) := \wp(\mathsf{IN}) \backslash \{\emptyset\}$. The sum of two individuals $x_e \oplus y_e$ is the union of the sets x and y, e.g., $\{\mathsf{megan}\} \oplus \{\mathsf{gabby}\} = \{\mathsf{megan}, \mathsf{gabby}\}$. For a set of atomic / non-atomic individuals X_{et}, the sum of the individuals in X (i.e., their union) is $\oplus X$, e.g., $\oplus \{\{\mathsf{megan}, \mathsf{gabby}\}, \{\mathsf{gabby}\}, \{\mathsf{linus}\}\} = \{\mathsf{megan}, \mathsf{gabby}, \mathsf{linus}\}$. The part-of relation over individuals $x \leq y$ (x is a part of y) is the partial order induced by inclusion $\subseteq$ over the set $\wp^+(\mathsf{IN})$. Atomic individuals are the singleton subsets of IN, identified by means of the predicate $\mathbf{atom}(x) := \forall y \leq x(y = x)$.

A dref for individuals u is a function of type se from assignments i_s to individuals x_e. Intuitively, the individual $u_{se}i_s$ is the individual that the assignment i assigns to the dref u. Dynamic info states I, J etc. are plural: they are sets of variable assignments, i.e., terms of type st. An individual dref u stores a set of atomic and / or non-atomic individuals with respect to a plural info state I, abbreviated as $uI := \{u_{se}i_s : i_s \in I_{st}\}$, i.e., uI is the image of the set of assignments I under the function u.

(20) **Dynamic Ty2** – frames, models, assignments, interpretation and truth.

a. A standard frame $\boldsymbol{F}$ for Dynamic Ty2 is a set $\{D_\tau : \tau \in \mathbf{Typ}\}$ s.t. D_t, D_e and D_s are pairwise disjoint sets ($D_t = \{\mathrm{T}, \mathrm{F}\}$) and, for any $\sigma, \tau \in \mathbf{Typ}$,
   $D_{\sigma\tau} = \{f : f \text{ is a total function from } D_\sigma \text{ to } D_\tau\}$

b. A model $\mathfrak{M}$ for Dynamic Ty2 is a pair $\left\langle \boldsymbol{F}^{\mathfrak{M}}, [\![\cdot]\!]^{\mathfrak{M}} \right\rangle$ s.t.:

   i. $\boldsymbol{F}^{\mathfrak{M}}$ is a standard frame for Dynamic Ty2

   ii. $[\![\cdot]\!]^{\mathfrak{M}}$ assigns an object $[\![\alpha]\!]^{\mathfrak{M}} \in D_{\tau}^{\mathfrak{M}}$ to each $\alpha \in \mathbf{Con}_{\tau}$, for any $\tau \in \mathbf{Typ}$, i.e., $[\![\cdot]\!]^{\mathfrak{M}}$ respects typing

   iii. $\mathfrak{M}$ satisfies the following axioms / axiom schemata:

   Ax1. $\mathbf{udref}(\delta)$, for any unspecific dref name δ of any type $\tau \in \mathbf{DRefTyp}$[2] (e.g., $\mathbf{udref}(u_1)$, $\mathbf{udref}(u_2)$ etc., but $\neg\mathbf{udref}(Dobby)$, $\neg\mathbf{udref}(Mary)$ etc.)

   Ax2. $\mathbf{udref}(\delta) \wedge \mathbf{udref}(\delta') \rightarrow \delta \neq \delta'$, for any two distinct dref names δ and δ' of type τ, for any type $\tau \in \mathbf{DRefTyp}$ (drefs have unique dref names, i.e., we ensure that we do not accidentally update δ' when we update δ)[3]

   Ax3. $\forall i_s \forall j_s (i[\ ]j \rightarrow i = j)$ (identity of 'assignments': two 'assignments' i and j are identical if they they don't differ with respect to the value of any dref)

   Ax4. $\forall i_s \forall v_{s\tau} \forall f_{\tau} (\mathbf{udref}(v) \rightarrow \exists j_s (i[v]j \wedge vj = f))$ (enough 'assignments')

c. An $\mathfrak{M}$-assignment θ is a function that assigns to each variable $v \in \mathbf{Var}_{\tau}$ an element $\theta(v) \in D_{\tau}^{\mathfrak{M}}$, for any $\tau \in \mathbf{Typ}$. Given an $\mathfrak{M}$-assignment θ, if $v \in \mathbf{Var}_{\tau}$ and $d \in D_{\tau}^{\mathfrak{M}}$, then $\theta^{v/d}$ is the $\mathfrak{M}$-assignment identical to θ except that it assigns d to v.

d. The interpretation function $[\![\cdot]\!]^{\mathfrak{M}\theta}$ is defined as follows:

   i. $[\![\alpha]\!]^{\mathfrak{M}\theta} = [\![\alpha]\!]^{\mathfrak{M}}$ if $\alpha \in \mathbf{Con}_{\tau}$, for any $\tau \in \mathbf{Typ}$

   ii. $[\![\alpha]\!]^{\mathfrak{M}\theta} = \theta(\alpha)$ if $\alpha \in \mathbf{Var}_{\tau}$, for any $\tau \in \mathbf{Typ}$

   iii. $[\![\alpha(\beta)]\!]^{\mathfrak{M}\theta} = [\![\alpha]\!]^{\mathfrak{M}\theta}([\![\beta]\!]^{\mathfrak{M}\theta})$

   iv. $[\![\lambda v.\ \alpha]\!]^{\mathfrak{M}\theta} = \left\langle [\![\alpha]\!]^{\mathfrak{M},\theta^{v/d}} : d \in D_{\sigma}^{\mathfrak{M}} \right\rangle$, if $v \in \mathbf{Var}_{\sigma}$

   v. $[\![\alpha = \beta]\!]^{\mathfrak{M}\theta} = \mathrm{T}$ if $[\![\alpha]\!]^{\mathfrak{M}\theta} = [\![\beta]\!]^{\mathfrak{M}\theta}$; F otherwise

   vi. $[\![i[\delta]j]\!]^{\mathfrak{M}\theta} = \mathrm{T}$ if $\delta \in \mathbf{Term}_{\sigma}$, $\sigma \in \mathbf{DRefTyp}$, $[\![\forall v_{\sigma}(\mathbf{udref}(v) \wedge v \neq \delta \rightarrow vi = vj)]\!]^{\mathfrak{M}\theta} = \mathrm{T}$ and $[\![\forall v_{\tau}(\mathbf{udref}(v) \rightarrow vi = vj)]\!]^{\mathfrak{M}\theta} = \mathrm{T}$, for all $\tau \neq \sigma, \tau \in \mathbf{DRefTyp}$; F otherwise

e. **Truth.** A formula $\phi \in \mathbf{Term}_t$ is true in $\mathfrak{M}$ relative to θ iff $[\![\phi]\!]^{\mathfrak{M}\theta} = \mathrm{T}$. A formula $\phi \in \mathbf{Term}_t$ is true in $\mathfrak{M}$ iff it is true in $\mathfrak{M}$ relative to any θ.

Drefs are modeled like individual concepts in Montague semantics: just as the sense of the definite description *the chair of the UC Santa Cruz linguistics department* (where, following Frege, sense is a way of giving the reference) is modeled as an individual concept, i.e., as a function from indices of evaluation to individuals, the meaning of a pronoun is basically a dref, i.e., a discourse-relative individual concept, which is modeled as a function from discourse salience states to individuals (in Dynamic Ty2, a discourse salience state is just a Tarskian, total variable assignment).

Modeling drefs as functions that take assignments as arguments (i.e., entities of type s) and return static objects as values, e.g., individuals (type e), is not as different from the DRT / FCS / DPL way of modeling drefs and variable assignments as it might seem. Classically, drefs are modeled as variables and a variable x is basically an instruction to look at the current index of evaluation (i.e., the current variable assignment) g, and retrieve whatever individual g associates with x, i.e., $g(x)$. So, instead of working directly with variables, we can work with their 'type-lifted' versions, i.e., instead of x, we can take a dref to be a function of the form $\lambda g.\ g(x)$, which is the (set-theoretic) x^{th} projection function that projects the sequence g onto the coordinate x.

This is what happens in Dynamic Ty2: we model variable assignments as atomic entities (of type s) and drefs as functions taking info states as arguments and returning appropriate static entities as values. This way of modeling drefs and assignments is preferable because it makes formally explicit the parallel between domain-level singularity / plurality, encoded by $\wp^{+}(\mathsf{IN})$, and discourse-level singularity / plurality, encoded by $\wp^{+}(D_s)$, where D_s is the domain of assignments.

[2]**udref** is a non-logical constant intuitively identifying the 'variable' drefs, i.e., the non-constant functions of type $s\sigma$ (for any $\sigma \in \mathbf{STyp}$) intended to model DPL-like variables. In fact, **udref** stands for an infinite family of non-logical constants of type τt, for any $\tau \in \mathbf{DRefTyp}$. Alternatively, we can assume a polymorphic type logic with infinite sum types, in which **udref** is a polymorphic function.

[3]Importantly, recall that unspecific drefs δ, δ' etc. (which model DPL-style variables) are non-logical *constants* in Dynamic Ty2, not variables. **Ax2** requires any two such distinct constants to denote distinct functions.

3.2 Dynamic Conditions

A sentence is interpreted as a Discourse Representation Structure (DRS), i.e., as a relation of type $(st)((st)t)$ between an input info state I_{st} and an output info state J_{st}. As shown in (21) below, a DRS is represented as a [**new drefs** | **conditions**] pair, which abbreviates a term of type $(st)((st)t)$ that places two kinds of constraints on the output state J: (i) J differs from the input state I at most with respect to the new drefs and (ii) J satisfies all the conditions. An example is provided in (22).

(21) $[\textbf{new drefs} \mid \textbf{conditions}] := \lambda I_{st}.\lambda J_{st}.\ I[\textbf{new drefs}]J \wedge \textbf{conditions}J$

(22) $[u_1, u_2 \mid person\{u_1\}, book\{u_2\}, buy\{u_1, u_2\}] := \lambda I_{st}.\lambda J_{st}.\ I[u_1, u_2]J \wedge person\{u_1\}J \wedge book\{u_2\}J \wedge buy\{u_1, u_2\}J$

DRSs of the form [**conditions**] that do not introduce new drefs are tests and they abbreviate terms of the form $\lambda I_{st}.\lambda J_{st}.\ I = J \wedge \textbf{conditions}J$, e.g., $[book\{u_2\}] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge book\{u_2\}J$.

Conditions, e.g., lexical relations like $buy\{u_1, u_2\}$, are sets of plural info states, i.e., they are terms of type $(st)t$. Lexical relations are unselectively distributive with respect to the plural info states they accept, where "unselective" is used in the sense of Lewis (1975). That is, lexical relations universally quantify over variable assignments – or cases, to use the terminology of Lewis (1975): a lexical relation accepts a plural info state I iff it accepts, in a pointwise manner, every single assignment $i \in I$, as shown in (23) below. The first conjunct in (23), i.e., $I \neq \emptyset$, rules out the possibility that the universal quantification in the second conjunct $\forall i_s \in I(\dots)$ is vacuously satisfied.

The curly braces used in the representation of conditions indicate that the static constant R of type $e^n t$ does not directly apply to the drefs $u_1, \dots, u_n$ of type se that are its arguments.[4] Thus, this brace convention is closely related to the Montagovian brace convention.

(23) $R\{u_1, \dots, u_n\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(R(u_1 i, \dots, u_n i))$, for any constant R of type $e^n t$.

(24) $\mathfrak{I}$ is a complete ideal without a bottom element (abbreviated as c-ideal) with respect to the partial order induced by set inclusion $\subseteq$ on the set of sets $\wp^+(D_s)$ iff (i) $\mathfrak{I} \subseteq \wp^+(D_s)$ and (ii) $\mathfrak{I}$ is closed under non-empty subsets and under arbitrary unions.

(25) For any c-ideal $\mathfrak{I}$, we have that: $\mathfrak{I} = \wp^+(\bigcup \mathfrak{I})$. That is, c-ideals are complete (atomic) Boolean algebras without a bottom element.

(26) **Lexical relations as c-ideals**. For any constant R of type $e^n t$ and sequence of drefs $\langle u_1, \dots, u_n \rangle$, let $\mathbb{I}(R, \langle u_1, \dots, u_n \rangle) := \lambda i_s.\ R(u_1 i, \dots, u_n i)$, abbreviated $\mathbb{I}^R$ whenever the sequence $\langle u_1, \dots, u_n \rangle$ can be recovered from context. Then, $R\{u_1, \dots, u_n\} = \wp^+(\mathbb{I}^R)$.[5]

Given unselective distributivity, the denotation of lexical relations has a lattice-theoretic ideal structure. The definition of lexical relations in (23) above ensures that they always denote c-ideals in the atomic lattice $\wp(D_s)$. We can in fact characterize them in terms of the supremum of their denotation, as shown in (26) above (which freely switches between function talk and set talk). The fact that lexical relations denote c-ideals endows the notion of natural language dynamic meaning with a range of desirable formal properties. For example, as we will see in the next subsection, DRSs, which are binary relations between sets of assignments of type $(st)((st)t)$, can be defined in terms of simpler binary relations between assignments of type $s(st)$.

3.3 New Drefs and DRSs

The other component of the definition of DRSs in (21) above is new dref introduction. We already have a Dynamic Ty2 notion of dref introduction, i.e., random assignment of value to a dref u. This notion, symbolized as $i[u]j$, relates two assignments i_s and j_s and can be informally paraphrased as: assignments i and j differ at most with respect to the value they assign to the dref u (see (20) above for the exact definition).

The problem posed by new dref introduction in dynamic systems based on plural info states is how to generalize the Dynamic Ty2 notion of new dref introduction, which is a relation between

[4]Types of the form $e^n t$ are defined as the smallest set of types such that $e^0 t := t$ and $e^{m+1} t := e(e^m t)$ (see Muskens 1996).

[5]Convention: $\wp^+(\emptyset_{st}) := \emptyset_{(st)t}$.

assignments, to a relation between sets of assignments (i.e., between plural info states). Various options have been explored in the literature (see van den Berg 1996, Krifka 1996, Nouwen 2003, Brasoveanu 2007) and they generally are stronger versions of the minimal definition in (27) below. The definition in (27) is minimal in the sense that it is just the pointwise, cumulative-quantification style generalization of the Dynamic Ty2 notion.

(27) $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

Informally, $I[u]J$ means that each input assignment i has a $[u]$-successor output assignment j and, vice-versa, each output assignment j has a $[u]$-predecessor input assignment i. This ensures that we preserve the values and structure associated with the previously introduced drefs u', u'' etc. The definition in (27) treats the structure and value components of a plural info state in parallel, since we non-deterministically introduce both of them, namely: (*i*) some new (random) values for u and, also, (*ii*) some new (random) structure associating the u-values and the values of any other (previously introduced) drefs u', u'' etc.

The definition in (27) is motivated on both empirical and theoretical grounds. Empirically, it enables us to account for mixed-reading donkey sentences like (2) above. Recall that, intuitively, we want to allow the credit cards to vary from book to book. That is, we want the restrictor of the *every*-quantification in (2) to non-deterministically introduce some set of u_3-cards and non-deterministically associate them with the u_2-books and let the nuclear scope filter the non-deterministically assigned values and structure by requiring each u_3-card to be used to pay for the corresponding u_2-book.

Theoretically, the definition in (27) is the natural generalization of the Dynamic Ty2 definition insofar as it preserves its formal properties: just as $i[u]j$ is an equivalence relation of type $s(st)$ between assignments, $I[u]J$ is an equivalence relation of type $(st)((st)t)$ between sets of assignments. Moreover, the fact that $[u]$ is an equivalence relation enables us to simplify the definition of DRSs as shown in (30) below.

The dynamic definition of truth – which has the expected form, namely existential quantification over output info states (aka existential closure) – is provided in (31).

(28) **Dynamic conjunction**. $D;D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$

(29) $[u_1,\ldots,u_n] := \lambda I_{st}.\lambda J_{st}.\ ([u_1];\ldots;[u_n])IJ$

(30) **DRSs in terms of c-ideals over relations of type** $s(st)$.
For any DRS $D := [u_1,\ldots,u_n \,|\, C_1,\ldots,C_m]$, where the conditions $C_1,\ldots,C_m$ are c-ideals, let $\mathbb{R}^D := \lambda i_s.\lambda j_s.\ i[u_1,\ldots,u_n]j \wedge j \in (\bigcup C_1 \cap \ldots \cap \bigcup C_m)$.[6] Then:
$D = \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R}_{s(st)} \neq \emptyset(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^D)$
$= \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R}_{s(st)} \in \wp^+(\mathbb{R}^D)(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}))$.[7]

(31) **Truth**. A DRS D is true with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

4 Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics become available. More precisely, the compositional aspect of interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e., names and sentences. Let us abbreviate them as **e** and **t**. An extensional static logic with domain-level plurality identifies **e** with e (atomic and non-atomic individuals) and **t** with t (truth-values). The denotation of the noun ***book*** is of type **et**, i.e., et: ***book*** $\rightsquigarrow \lambda x_e.\ book_{et}(x)$. The generalized determiner ***every*** is of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, i.e., $(et)((et)t)$: ***every*** $\rightsquigarrow \lambda X_{et}.\lambda X'_{et}.\ \forall x_e(X(x) \rightarrow X'(x))$.

We go dynamic with respect to both value and structure by making the 'meta-types' **e** and **t** more complex, i.e., by assigning finer-grained meanings to names and sentences. The 'meta-type' talk should not be taken literally – formally, **e** and **t** are just abbreviations, i.e., syntactic sugar meant to show

[6]Where $i[u_1,\ldots,u_n]j := i([u_1];\ldots;[u_n])j$. In this case, dynamic conjunction ; is defined as relation composition over terms of type $s(st)$, i.e., $[u];[u'] := \lambda i_s.\lambda j_s.\ \exists h_s(i[u]h \wedge h[u']j)$, where $[u]$ and $[u']$ are Dynamic Ty2 terms of type $s(st)$.

[7]Where $\mathbf{Dom}(\mathbb{R}) := \{i_s : \exists j_s(\mathbb{R}ij)\}$ and $\mathbf{Ran}(\mathbb{R}) := \{j_s : \exists i_s(\mathbb{R}ij)\}$.

in a perspicuous way how the very general Montagovian solution to the compositionality problem is formalized in this case. In particular, we assign the following dynamic types to the 'meta-types' $\mathbf{e}$ and $\mathbf{t}$: $\mathbf{t} := (st)((st)t)$, i.e., a sentence is interpreted as a DRS, and $\mathbf{e} := se$, i.e., a name is interpreted as a dref for individuals. The denotation of the noun ***book*** is still of type $\mathbf{et}$, as shown in (32) below. The denotations of pronouns, indefinite articles and generalized determiners are provided in the following three subsections.

(32) ***book*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [book\{v\}]$, i.e., ***book*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I = J \wedge book\{v\}\, J$

4.1 Pronouns

A pronoun anaphoric to a dref u is interpreted as the Montagovian quantifier-lift of the dref u (of type $\mathbf{e}$), i.e., its type is $(\mathbf{et})\mathbf{t}$. Singular number morphology on pronouns contributes domain-level atomicity, as shown in (33) below. For simplicity, the $\mathbf{atom}\{u\}$ condition is formalized as part of the assertion and not as a presupposition. Plural number morphology on pronouns makes a fairly weak contribution: it just indicates the absence of a domain-level atomicity requirement. The stronger requirement of domain-level non-atomicity that is associated with many uses of plural pronouns can be derived in various ways, e.g., following Sauerland (2003), we can assume that a Maximize Presupposition principle of the kind proposed in Heim (1991) requires us to use singular pronouns whenever we can.

(33) $\mathbf{atom}\{u\} := \lambda I_{st}.\ \mathbf{atom}(\oplus uI)$

(34) $\boldsymbol{he}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{atom}\{u\}]; P(u)$

(35) $\boldsymbol{they}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ P(u)$[8]

The fact that singular pronouns contribute an **atom** condition enables us to derive the incompatibility between collective predicates and singular pronouns exemplified in (10) above, while allowing for collective predicates with plural pronouns, as in (13). Also, the **atom** condition on singular pronouns captures the intuition that deictic (i.e., discourse-initial) uses of singular pronouns refer to atomic individuals. In particular, it is crucial that the **atom** condition is collectively interpreted relative to a plural info state I, i.e., that it is collective at the discourse level. This ensures two things: (*i*) any two assignments $i, i' \in I$ assign the same individual x to u, i.e., $\forall i_s, i'_s \in I(ui = ui')$, and (*ii*) the individual x assigned to u throughout the info state I is an atomic individual, i.e., $\forall i_s \in I(\mathbf{atom}(ui))$.

4.2 Indefinites

The translation of indefinite articles has the expected type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, as shown in (36) below. An indefinite article takes two dynamic properties P (the restrictor) and P' (the nuclear scope) as arguments and returns a DRS (i.e., a term of type $\mathbf{t}$) as value. This DRS consists of two sub-DRSs that are dynamically conjoined: the first one, namely $[u]$, introduces a new dref u (the dref with which the indefinite article is indexed); the second sub-DRS, i.e., $\mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u))$, constrains the value of this newly introduced dref. Just as in the case of pronouns, singular number morphology on indefinites contributes domain-level atomicity, i.e., a condition $\mathbf{atom}\{u\}$. This condition, however, is within the scope of a discourse-level distributivity operator **dist**, defined in (37) below.

(36) $\boldsymbol{a}^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u))$

(37) $\mathbf{dist}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists \mathcal{R}_{s((st)t)} \neq \emptyset(I = \mathbf{Dom}(\mathcal{R}) \wedge J = \bigcup \mathbf{Ran}(\mathcal{R}) \wedge \forall k_s \forall L_{st}(\mathcal{R}kL \rightarrow D\{k\}\, L))$

- D is a DRS (type $(st)((st)t)$)
- $\mathcal{R}$ is a relation between assignments and sets of assignments (type $s((st)t)$) such that $\mathbf{Dom}(\mathcal{R}) := \{k_s : \exists L_{st}(\mathcal{R}kL)\}$ (type st) and $\mathbf{Ran}(\mathcal{R}) := \{L_{st} : \exists k_s(\mathcal{R}kL)\}$ (type $(st)t$)
- $\mathcal{R}$ encodes a partial function from assignments $k \in I$ to sets of assignments L, i.e., $\mathcal{R}$ is such that $\forall k_s \in \mathbf{Dom}(\mathcal{R}) \forall L_{st} \forall L'_{st}(RkL \wedge RkL' \rightarrow L = L')$
- $\{k\}$ is the singleton set of assignments (type st) containing only k

[8] Anaphoric definite articles receive similar translations: $\boldsymbol{the}_{\mathbf{sg}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{atom}\{u\}]; P(u); P'(u)$ and $\boldsymbol{the}_{\mathbf{pl}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ P(u); P'(u)$.

Distributively updating an input info state I with a DRS D means that we update each assignment $i \in I$ with the DRS D and then take the union of the resulting output info states. Thus, the operator **dist** is unselectively distributive at the discourse level: distributive at the discourse level in the sense that it distributes over plural info states and unselective in the sense of Lewis (1975) – we update one case, i.e., one assignment i, at a time.

We need the **dist** operator in the translation of indefinites because singular (weak and strong) donkey anaphora is neutral with respect to semantic number – recall that, in (2) above, we are not quantifying only over people that buy exactly one book and have exactly one credit card, but over people that buy one or more books and use one or more of their credit cards to buy them. The fact that the **dist** operator takes scope over the **atom** $\{u\}$ condition contributed by singular number morphology effectively neutralizes the atomicity requirement, which has to be satisfied only relative to each assignment $i \in I$ and not relative to the entire plural info state I, thereby capturing the semantic number neutrality of donkey anaphora.

The translation in (36) above provides the meaning for weak indefinite articles, i.e., the meaning needed for weak donkey readings. The translation for strong indefinite articles is provided in (38) below. The only difference between weak and strong indefinites is the absence vs presence of a maximization operator **max**, defined in (39) below, that takes scope over both the restrictor and the nuclear scope of the indefinites. Attributing the weak / strong ambiguity to the indefinites enables us to give a compositional account of the mixed-reading sentence in (2) above because we locally decide for each indefinite whether it receives a weak or a strong reading.

(38) $a^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}([\mathbf{atom}\,\{u\}];\ P(u);\ P'(u)))$

(39) $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];D)IJ \wedge \forall K_{st}(([u];D)IK \rightarrow uK \subseteq uJ)$

The first conjunct in (39) introduces u as a new dref and makes sure that each individual in uJ satisfies D, i.e., uJ stores only individuals that satisfy D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e., any other set of individuals that satisfies D) is included in uJ. So, uJ stores all the individuals that satisfy D.

The DRS $\mathbf{max}^u(D)$ can be thought of as dynamic λ-abstraction over individuals: the abstracted variable is the dref u, the scope is the DRS D and the result of the abstraction is a set of individuals uJ containing all and only the individuals that satisfy D.

The **max** operator ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref u the maximal set of individuals satisfying both the restrictor property P and the nuclear scope property P'. In contrast, a weak indefinite will non-deterministically store some set of individuals satisfying its restrictor and nuclear scope. Since the only difference between weak and strong indefinites is the absence vs presence of the **max** operator, we can think of indefinites as underspecified with respect to maximization: the decision to introduce **max** or not is made online depending on the discourse and utterance context – much like aspectual coercion or the selection of a particular type for the denotation of an expression are context-driven online processes.

The weak and strong meanings for cardinal indefinites differ from the ones for indefinite articles only with respect to the domain-level requirement. As (40) and (41) below show, each cardinal indefinite comes with its corresponding domain-level condition requiring the newly introduced individuals to have a particular number of atoms. For example, in the case of *two*, the condition $\mathbf{2\_atoms}\,\{u\}$ requires each individual to contain exactly two atomic parts.

(40) $two^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{2\_atoms}\,\{u\}];\ P(u);\ P'(u))$

(41) $two^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}([\mathbf{2\_atoms}\,\{u\}];\ P(u);\ P'(u)))$

(42) $\mathbf{2\_atoms}\,\{u\} := \lambda I_{st}.\ \mathbf{2\_atoms}(\oplus uI)$, where $\mathbf{2\_atoms}(x_e) := |\,\{y_e : y \leq x \wedge \mathbf{atom}(y)\}\,| = 2$

4.3 Generalized Quantification

The notions of dynamic generalized quantification defined in the dynamic semantics literature fall into broad classes. The first class of notions is defined in frameworks based on singular info states, e.g., DRT / FCS / DPL, and takes generalized quantification to be internally dynamic (this is needed for donkey anaphora) and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property, while the nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope dynamic property, so that the anaphoric connections between them are captured (for more details, see the chapter *Representing Discourse in Context*).

The second class of notions is defined in frameworks based on plural info states and takes generalized quantification to be both internally and externally dynamic (see van den Berg 1996, Krifka 1996, Nouwen 2003). The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property and the nuclear scope set of individuals is the maximal *structured subset* of the restrictor set of individuals that satisfies the nuclear scope dynamic property.

Given that the notion of a dref being a structured subset of another dref required for the second kind of definitions involves non-trivial complexities that are orthogonal to the issues at hand, we will define selective generalized quantification following the format of the DRT / FCS / DPL-style definition. However, since we are working in a system based on plural info states, the definition of dynamic quantification provided by the translation in (43) and the condition in (44) below is intermediate between the two kinds of definitions explored in the literature and, thus, it is useful in formally exhibiting the commonalities and differences between them.

(43) $\mathtt{det}^u \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{det}_u(\mathbf{dist}(P(u)), \mathbf{dist}(P'(u)))]$

(44) $\mathbf{det}_u(D, D') := \lambda I_{st}.\ I \neq \emptyset \wedge \mathbf{DET}(u[DI], u[(D; D')I])$

- **DET** is the corresponding static determiner
- $u[DI] := \{\oplus uJ : ([u \mid \mathbf{atom}\{u\}]; D)IJ\}$

The condition $\mathbf{det}_u(D, D')$ defined in (44) above has four components: the dref u that we quantify over, the restrictor DRS D, the nuclear scope DRS D' and the static generalized determiner **DET** that relates two sets of individuals.[9]

This condition tests that the static determiner **DET** relates the restrictor set of atomic individuals $u[DI]$ and the nuclear scope set of atomic individuals $u[(D; D')I]$. The restrictor set $u[DI]$ is the set of atomic individuals assigned to the dref u that satisfy the restrictor DRS D. The nuclear scope set $u[(D; D')I]$ is the set of atomic individuals assigned to the dref u that satisfy the dynamically conjoined restrictor and nuclear scope DRSs D and D'. Dynamically conjoining the restrictor and nuclear scope DRSs ensures that the donkey pronouns in the nuclear scope can be successfully linked to their antecedents in the restrictor.

Since the determiners defined in (43)-(44) above relate sets of individuals, they contribute a selective kind of generalized quantification ("selective" in the sense of Lewis 1975, i.e., quantification over individuals, not over cases / assignments) and, therefore, avoid the proportion problem of classical DRT / FCS / DPL. Also, the determiners are neutral with respect to weak vs strong donkey readings (they are compatible with either one) and the selection of a donkey reading is exclusively determined by indefinite articles.

The dynamic determiners defined above have two important characteristics. First, they are domain-level atomic and discourse-level distributive relative to the dref u they quantify over – this is ensured by the condition $\mathbf{atom}\{u\}$ in the definition of $u[DI]$ in (44). Secondly, they are discourse-level distributive relative to all the drefs introduced and / or retrieved in their restrictor and nuclear scope. In particular, they are discourse-level distributive relative to donkey anaphora – this is ensured by the **dist** operators in (43) taking scope over the restrictor and nuclear scope DRSs $P(u)$ and $P'(u)$.

4.4 The Analysis of Multiple Interdependent Anaphora

The compositionally obtained representation for the most salient reading of the mixed-reading donkey sentence in (2) above is given in (45) below (the representation is simplified based on various type-logical equivalences). Under this (pragmatically most plausible) reading, the indefinite $\mathtt{a}^{\mathbf{str}:u_2}$ $\mathtt{book}$ is strong and the indefinite $\mathtt{a}^{\mathbf{wk}:u_3}$ $\mathtt{credit\ card}$ is weak. Based on the representation in (45), we derive the intuitively correct truth conditions, provided in (46).

[9] For example, the determiner **EVERY** requires the first set of individuals to be included in the second set, **NO** requires their intersection to be empty etc. See the chapter *Generalized Quantifiers* for more details.

(45) $[\mathbf{every}_{u_1}(\mathbf{dist}([person\{u_1\}];\mathbf{max}^{u_2}(\mathbf{dist}([\mathbf{atom}\{u_2\}, book\{u_2\}, buy\{u_1,u_2\}]));$
$[u_3];\mathbf{dist}([\mathbf{atom}\{u_3\}, card\{u_3\}, have\{u_1,u_3\}])),$
$\mathbf{dist}([\mathbf{atom}\{u_2\}, \mathbf{atom}\{u_3\}, use\_to\_pay\{u_1,u_2,u_3\}]))]$

(46) $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e \forall y_e(\mathbf{atom}(x) \wedge person(x) \wedge \mathbf{atom}(y) \wedge book(y) \wedge buy(x,y) \wedge \exists z_e(\mathbf{atom}(z) \wedge card(z) \wedge have(x,z)) \rightarrow \exists z'_e(\mathbf{atom}(z') \wedge card(z') \wedge have(x,z') \wedge use\_to\_pay(x,y,z')))$

The update in (45) proceeds as follows. After the input info state is updated with the restrictor of the quantification in (2), we obtain a plural info state that stores, for each u_1-person that is a book buyer and a card owner: (*i*) the maximal set of purchased book-atoms, stored relative to the dref u_2 (since the indefinite $a^{\mathbf{str}:u_2}$ *book* is strong), (*ii*) some non-deterministically introduced set of credit-card atoms, stored relative to the dref u_3 (since the indefinite $a^{\mathbf{wk}:u_3}$ *credit card* is weak) and, finally, (*iii*) some non-deterministically introduced structure correlating the u_2 and u_3 atoms.

The nuclear scope of the quantification in (2) is anaphoric to both values (in this case, atomic individuals) and structure / dependencies: we test that the non-deterministically introduced values for u_3 and the non-deterministically introduced structure associating u_3 and u_2 satisfy the nuclear scope update (the structure is tested by means of the **dist** operator). That is, we test that, for each assignment in the info state, the u_3-card stored in that assignment is used to pay for the u_2-book stored in the same assignment. Thus, the nuclear scope update elaborates on the dependency between u_3 and u_2 that was non-deterministically introduced in the restrictor.

The pseudo-scopal relation between $a^{\mathbf{str}:u_2}$ *book* and $a^{\mathbf{wk}:u_3}$ *credit card* emerges as a consequence of the fact that we use plural information states, which store and pass on information about both objects and dependencies between them. The relation between the two indefinites is "pseudo-scopal" in the sense that the weak indefinite semantically co-varies with the strong indefinite (people can use different cards to buy different books) – but syntactically, the strong indefinite cannot take scope over the weak indefinite because this would violate the Coordinate Structure Constraint.[10]

The representation for sentence (3) is parallel to the one for sentence (2) except for the fact that both indefinites ($a^{\mathbf{str}:u_2}$ *gift* and $a^{\mathbf{str}:u_3}$ *girl*) are strong. The analysis of the plural donkey example in (13) above is completely parallel to the analysis of (3). Similarly, the singular and plural weak donkey sentences in (6) and (14) above receive parallel analyses (see Brasoveanu 2008 for more details and for the account of these examples and the sage plant sentences in (11) and (12)). The incompatibility between singular (but not plural!) donkey anaphora and collective predicates exemplified in (10) above follows from the fact that the singular number morphology on donkey pronouns contributes an **atom** condition that contradicts the collective, i.e., non-atomic nature, of the verb *gather*. Finally, we can account for examples involving anaphora to events, times or possibilities, e.g., (15), (16) and (17) above, by simply adding new basic static types (for events, times, possible worlds etc.), which automatically makes available the drefs necessary for their analysis.

5 Conclusion

Natural language interpretation requires two independent yet parallel notions of plurality, plural reference and plural discourse reference. The interpretation of multiple simultaneous anaphoric connections in the scope of quantifiers motivates a semantics that relies on plural info states, i.e., that crucially involves discourse-level plurality, and the fact that these anaphoric connections involve reference to both individuals and collections motivates a semantics that also involves domain-level plurality. Future research will hopefully investigate the interplay between these two notions from a cross-linguistic perspective (especially given the recent work on domain-level plurality in Zweig 2009, Champollion 2010 and Farkas & de Swart 2010, building on Schein 1993, Schwarzschild 1996, Kratzer 2000 and Landman 2000 among others) – and, also, their connections with recent compositionality-related debates in game-theoretical semantics and independence-friendly and dependence logic.

[10] The Coordinate Structure Constraint ensures that, although the declarative sentence *You ate the eggs and the bacon* is acceptable, the question **What did you eat the eggs and _?* is unacceptable because we cannot asymmetrically displace (material from) only one conjunct in a conjunction.

References

Asher, N. & L. Wang (2003). Ambiguity and Anaphora with Plurals in Discourse. In the *Proceedings of SALT XIII*, R. Young & Y. Zhou, Ithaca: CLC Publications, 19-36.

Barwise, J. (1987). Noun phrases, Generalized Quantifiers and Anaphora, in *Generalized Quantifiers*, P. Gärdenfors (ed.), Dordrecht: Kluwer, 1-29.

van Benthem, J. (1996). *Exploring Logical Dynamics.* CSLI.

van den Berg, M. (1996). *Some Aspects of the Internal Structure of Discourse.* PhD dissertation, University of Amsterdam.

Brasoveanu, A. (2007). *Structured Nominal and Modal Reference.* PhD dissertation, Rutgers University.

Brasoveanu, A. (2008). Donkey Pluralities. In *Linguistics and Philosophy* 31, 129-209.

Champollion, L. (2010). Cumulative Readings of *Every* Do Not Provide Evidence for Events and Thematic roles. To appear in *Proceedings of the 17th Amsterdam Colloquium.*

Chierchia, G. (1995). *The Dynamics of Meaning.* University of Chicago Press.

Dekker, P. (1994). Predicate Logic with Anaphora. In the *Proceedings of SALT IV*, L. Santelmann & M. Harvey (eds.), DMLL, Cornell University, 79-95.

van Eijck, J. & F.-J. de Vries (1992). Dynamic Interpretation and Hoare Deduction. In *Journal of Logic, Language and Information* 1, 1-44.

Farkas, D.F. & H. de Swart (2010). The Semantics and Pragmatics of Plurals. In *Semantics and Pragmatics*, to appear.

Gallin, D. (1975). *Intensional and Higher-Order Modal Logic.* Amsterdam: North-Holland Mathematics Studies.

Geach, P. (1962). *Reference and Generality.* Ithaca: Cornell University Press.

Groenendijk, J. & M. Stokhof (1991). Dynamic Predicate Logic. In *Linguistics and Philosophy* 14, 39-100.

Heim, I. (1982).*The Semantics of Definite and Indefinite Noun Phrases.* PhD dissertation, UMass Amherst.

Heim, I. (1990). E-Type Pronouns and Donkey Anaphora. In *Linguistics and Philosophy* 13, 137-177.

Heim, I. (1991). Artikel und Definitheit. In *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, A. von Stechow & D. Wunderlich (eds.), Berlin: Walter de Gruyter.

Kamp, H. (1981). A theory of truth and semantic representation. In *Formal Methods in the Study of Language. Part 1*, Groenendijk, J., T. Janssen & M. Stokhof (eds.), Amsterdam: Mathematical Center, 277-322.

Kamp, H. & U. Reyle (1993). *From Discourse to Logic.* Dordrecht: Kluwer.

Kanazawa, M. (1994). Weak vs Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting. In *Linguistics and Philosophy* 17, 109-158.

Kanazawa, M. (2001). Singular Donkey Pronouns are Semantically Singular. In *Linguistics and Philosophy* 24, 383-403.

Karttunen, L. (1976). Discourse Referents. In *Syntax and Semantics Vol. 7*, J.D. McCawley (ed.), New York: Academic Press, 363-385.

Kratzer, A. (2000). *The Event Argument and the Semantics of Verbs.* UMass ms., available at `semanticsarchive.net`.

Krifka, M. (1996). Parametric Sum Individuals for Plural Anaphora. In *Linguistics and Philosophy* 19, 555-598.

Landman, F. (1986). *Towards a Theory of Information. The Statues of Partial Objects in Semantics*, GRASS 6, Dordrecht: Foris.

Landman, F. (2000). *Events and Plurality*, Kluwer.

Lappin, S. & Francez, N. (1994). E-type Pronouns, I-sums and Donkey Anaphora. In *Linguistics and Philosophy* 17, 391-428.

Lewis, D. (1975). Adverbs of Quantification. In *Formal Semantics of Natural Language*, E. Keenan (ed.), Cambridge: Cambridge University Press, 3-15.

Link, G. (1983). The logical analysis of plurals and mass terms: A lattice-theoretical approach. In *Meaning, Use and Interpretation of Language*, R. Baüerle, C. Schwarze & A. von Stechow (eds.), Berlin: Walter de Gruyter, 302-323.

Muskens, R. (1995). Tense and the Logic of Change. In *Lexical Knowledge in the Organization of Language*, U. Egli, P.E. Pause, C. Schwarze, A. von Stechow & G. Wienold (eds.), Amsterdam: Benjamins, 147-183.

Muskens, R. (1996). Combining Montague Semantics and Discourse Representation. In *Linguistics and Philosophy* 19, 143-186.

Neale, S. (1990). *Descriptions*, Cambridge: MIT Press.

Nouwen, R. (2003). *Plural Pronominal Anaphora in Context.* PhD Dissertation, University of Utrecht.

Roberts, C. (1989). Modal Subordination and Pronominal Anaphora in Discourse. In *Linguistics and Philosophy* 12, 683-721.

Rooth, M. (1987). Noun Phrase Interpretation in Montague Grammar, File Change Semantics and Situation Semantics. In *Generalized Quantifiers*, P. Grdenfors (ed.), Dordrecht: Kluwer, 237-268.

Pelletier, F.J. & L.K. Schubert (1989). Generically Speaking. In *Properties, Types and Meanings. Vol. 2*, G. Chierchia, B.H. Partee & R. Turner (eds.), Dordrecht: Kluwer, 193-268.

Sauerland, U. (2003). A New Semantics for Number. In the *Proceedings of SALT XIII*, R. Young & Y. Zhou (eds.), Ithaca: CLC Publications, 258-275.

Scha, R. (1981). Distributive, Collective and Cumulative Quantification. In *Formal Methods in the Study of Language. Part 2*, J. Groenendijk, T. Janssen & M. Stokhof (eds.), University of Amsterdam, 483-512.

Schein, B. (1993). *Plurals and Events.* MIT Press.

Schwarzschild, R. (1996). *Pluralities.* Dordrecht/Boston/London: Kluwer Academic Publishers.

Webber, B. (1978). *A Formal Approach to Discourse Anaphora*, PhD dissertation, Harvard University (published in the series *Outstanding Dissertations in Linguistics*, NY: Garland Publishing).

Zweig, E. (2009). Number-Neutral Bare plurals and the Multiplicity Implicature. In *Linguistics and Philosophy*, DOI: 10.1007/s10988-009-9064-3.