Fusional Reduction

& the Logic of Ranking Arguments

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The Setting

- **OT**: a grammar *decides* between alternatives.
- **Constraints**: criteria of decision.
- **Penalties**: a constraint detects only flaws.
- **Better Than** on a single constraint: *fewer* flaws on it.

When Constraints Collide

- **Disagreements**: btw constraints, about what's *worse*:
  - $(CVC)_\sigma$ *vs.* C→Ø codas *vs.* deletions
  - C→Ø *vs.* Ø→V delete *vs.* insert
- **Better Than**: over the *entire conflicting mass of criteria* ?
  - Need to resolve all discord
- **Rank them all.**
  - Impose a linear order on the constraint set

Optimality

- **Better Than** over a ranked constraint set
  - **A** is *better than* **B** on a ranking
    iff **A** is better than **B** on the highest-ranked constraint
    that distinguishes them.
- **Optimal**. **A** is optimal over a ranked constraint set
  iff **A** is *better than every* distinct alternative **B**
  over that ranking.
- The optimal alternative is the grammar's choice.

Two Analytical Challenges

- A **Grammar** is a linear order on a constraint set
  - Two analytical problems then arise:
- **The Selection Problem**
  - Given the ranking order,
    which candidate is optimal?
- **The Ranking Problem**
  - Given a (desired) optimum
    which rankings will produce it?

Solving the Selection Problem

- A simple sequential filtration finds the optima.
  - Take the best, ignore the rest.
    - *Slogan due to Gigerenzer & Goldstein* 1996
  - Start with 1st constraint & continue down the hierarchy
    - Taking the best among the previous best, and so on.
- Easily represented in a violation tableau (VT)
  - Annotated at the point of suboptimum demise

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
berg → berg	0	2	0
berg → perk	1	0	2

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
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Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
berg → berg	0	2	0
berg → perk	1 **W!**	0	2

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
berg → berg	0	2	0
berg → perk	1 **W!**	0	2

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
berg → berg	0	2	0
berg → perk	1 **W!**	0	2

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
berg → berk	0	1	1
berg → berg	0	2 **W!**	0
berg → perk	1 **W!**	0	2

Selecting the Optimum

- A violation tableau (VT). Assume ranked.

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
✓ berg → berk	0	1	1
berg → berg	0	2 W!	0
berg → perk	1 W!	0	2

The Ranking Problem

- We have a winner (in mind): the 'desired optimum'
  - From observation & linguistic analysis
- And we have:
  - The constraints
  - The alternative candidates
- Which rankings choose the desired winner?

Solving the Ranking Problem

- What rankings will make **A** better than **B** ?
  - Assume no ranking known:

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
want: **A**: berg → berk	0	1	1
B: berg → perk	1	0	2

Finding the Friends of A

- If **A** is *better than* **B** on a ranking
  Some constraint preferring **A** to **B**
  ranks above *all* constraints preferring **B** to **A**.
- In any such ranking,
  - the highest-ranked constraint distinguishing A and B
    decides in favor of A and against B.

.

- What do we need to know about the constraints
  to sort this out?

The Constraint's Eye View

- A constraint views the A/B competition in one of 3 ways:
- **W**: **A** is better than **B**
  - The desired optimum wins. *B is worse in violations.*
- **L**: **B** is better than **A**.
  - The desired optimum *loses*! *B has less violation*!
- **e**: No decision.
  - A and B are violationwise identical.

Solving the Ranking Problem

- Calculate how each constraint views A vs. B
  - Assume no ranking known:

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
want: **A**: berg → berk	0	1	1
B: berg → perk	1 **W**	0 **L**	2 **W**

The Comparative Tableau

- Eliminate violation data
  - Its work is done. We only care about *more* vs. *less*

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
A ~ B	W	L	W

The Elementary Ranking Condition

- ERC: Some W must dominate all L's.
  - holds of each tableau row

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
A ~ B	W	L	W

ERC: **Ident/O** >>***ObVoi** -OR- **Ident**>>***ObVoi**

The Candidate Set, Revisited

- Annotating the VT

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
want:A: berg → berk	0	1	1
C: berg → berg	0	2 W	0 L
B: berg → perk	1 W	0 L	2 W

The Candidate Set, Revisited

- The full CT

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
A ~ B	W	L	W
A ~ C		W	L

The Candidate Set, Revisited

- Conclusion: necessarily **Ident/O:Voi >> *ObVoi >> *Ident:Voi**

/berg/	Ident/O:Voi	*ObVoi	Ident:Voi
A ~ B	W	L	W
A ~ C		W	L

Issue: Out of many, One

T4	C_1	C_2	C_3
r_4	W	L	L
r_2	e	W	L

T2	C_1	C_2	C_3
r_1	W	L	W
r_2	e	W	L

T3	C_1	C_2	C_3
r_3	W	L	e
r_2	e	W	L

These all designate the same ranking:

$C_1 >> C_2 >> C_3$

Issue: Out of many, Yet More

	C_1	C_2	C_3
r_1	W	L	W
r_3	W	L	e
r_4	W	L	L
r_2	e	W	L

This also designates the same ranking:

$C_1 >> C_2 >> C_3$

- CT's like this often arise ecologically, in the course of analysis

Of All the CTs in the World …

- We want the most concise, most informative representation of the ranking conditions inherent in the data.
- The learner is happy with a ranking that works.
  - Sufficient conditions for success will suffice.
- The analyst must know more: both nec. & suff.
  - The structure of grammars lies in the exact conditions
  - E.g. when the *necessary* conditions for one optimum entails the *sufficient* conditions for another.
    the presence of the 1st entails the presence of the 2nd.

The Basis

- For any set Σ of ERCs
  a Basis B for Σ is a *minimum cardinality* set of ERCs such that B defines the same rankings as Σ.
- Any Basis for $C_1 >> C_2 >> C_3$ has *two* ERCS
- Several such exist.

All the Bases

T1	C_1	C_2	C_3
r_4	W	L	L
r_2	e	W	L

T2	C_1	C_2	C_3
r_1	W	L	W
r_2	e	W	L

T3	C_1	C_2	C_3
r_3	W	L	e
r_2	e	W	L

Bases for $C_1 >> C_2 >> C_3$

Not a Basis

	C_1	C_2	C_3
r_1	W	L	W
r_3	W	L	e
r_4	W	L	L
r_2	e	W	L

Flabbily designates the same ranking:

$C_1 >> C_2 >> C_3$

- Heavily redundant: only *one* of top 3 rows r_1, r_3, r_4 is needed

All Bases are not created equal

T1	C_1	C_2	C_3
r_4	W	L	L
r_2	e	W	L

Basis for $C_1 >> C_2 >> C_3$

Most Informative Basis (MIB)

- Gives *total* domination info for each row's W-set
- Most L's, minimal # of W's

All Bases are not created equal

T2	C_1	C_2	C_3
r_4	W	L	W
r_2	e	W	L

Basis for $C_1 \gg C_2 \gg C_3$

Least Informative Basis

- As many spurious local disjunctions (**W**'s) as can be tolerated
- The most W's
- Who needs it?

All Bases are not created equal

T2	C_1	C_2	C_3
r_4	W	L	e
r_2	e	W	L

Basis for $C_1 \gg C_2 \gg C_3$

Skeletal Basis

- Eliminates all info derived from transitivity of ranking order
- The most *e*'s, minimal W's
- Nice: but best approached through the MIB

Admirable Qualities of the MIB

- The MIB is minimal in size, lacks all redundancy.
- Each MIB row contains a unique W-set
- The total ranking info for that W-set is displayed
- Every disjunctive W in a row represents a ranking option that is realized in some licit linearization
- The MIB ties *ranking conditions* to the motivating *data*.

Obtaining the MIB

- How does this desirable object emerge from data?
- Related work from different perspectives includes
  - Hayes, B. 2003, Four Rules of Inference
    - On this, see Prince 2006, ROA-882
  - Riggle, J. 2004. Generation, Recognition, and Learning in Finite State Optimality Theory

FRed Obtains the MIB

Fusion

Fusion (Prince 2002a, b):

Fusion	**W**	**e**	**L**
W	W	W	L
e	W	e	L
L	L	L	L

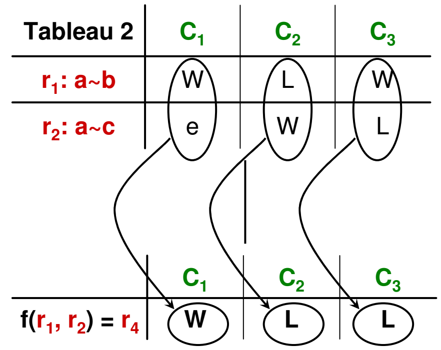
Fusion: select the minimal element relative to the order **L**<**W**<**e.**

Fusion

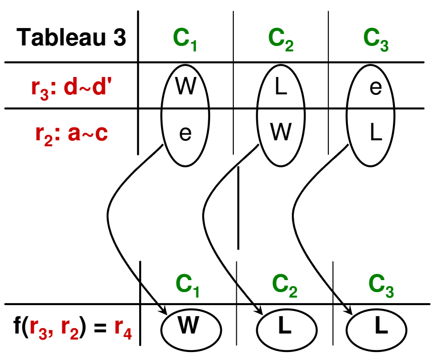
Fusion: select the minimal element relative to the order **L**<**W**<**e**, that is …

- **L** is dominant: for any X in {**W**, **e**, **L**}, f(**L**, X)=**L**
- **e** is identity: for any X in {**W**, **e**, **L**}, f(**e**, X)=X
- f(**W**, **W**)= **W**
- fusion of two rows r_1 and r_2 is obtained by **constraintwise** fusion and is abbreviated as: f(r_1, r_2), for example …

Fusion



Fusion



Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
f(r_1, r_2)=r_4	**W**	**L**	**L**

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
f(r_3, r_2)=r_4	**W**	**L**	**L**

The fusion f(r_1, r_2)=f(r_3, r_2) retains all the ranking information in r_1 / r_3 and strengthens it,

i.e. it locally **maximizes information** in rows r_1 and r_3 based on the rest of the tableau:

we require not only that C_1>>C_2 (as r_3 does), but also that C_1>>C_3.

Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
f(r_1, r_2)=r_4	**W**	**L**	**L**

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
f(r_3, r_2)=r_4	**W**	**L**	**L**

Thus, we will use **fusion** to obtain the **Most Informative Basis** (**MIB**) of Tableau 2 / Tableau 3.

But:

fusion is not enough!

Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
f(r_1, r_2)=r_4	**W**	**L**	**L**

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
f(r_3, r_2)=r_4	**W**	**L**	**L**

Fusion is not enough because we want the MIB to be **equivalent** to the initial tableau, but …

… the ranking information provided by r_2 (i.e. C_2>>C_3) is lost in the fusion f(r_1, r_2)=f(r_3, r_2), which requires only that C_1>>C_2 and C_1>>C_3.

Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
f(r_1, r_2)=r_4	**W**	**L**	**L**

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
f(r_3, r_2)=r_4	**W**	**L**	**L**

So:

we keep row r_2 together with the fusion r_4=f(r_1, r_2)=f(r_3, r_2),

which yields Tableau 4!

Fusion

Tableau 4	C_1	C_2	C_3
f(r_1, r_2)=r_4	W	L	L
r_2: a~c	e	W	L

Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	C_1	C_2	C_3
f(r_1, r_2)=r_4	W	L	L

	C_1	C_2	C_3
r_2: a~c	e	W	L

We keep row r_2 together with the fusion r_4=f(r_1, r_2) …

Fusion

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L

	C_1	C_2	C_3
f(r_1, r_2)=r_4	W	L	L

	C_1	C_2	C_3
r_2: a~c	e	W	L

Tableau 4	C_1	C_2	C_3
f(r_1, r_2)=r_4	W	L	L
r_2: a~c	e	W	L

… which yields Tableau 4.

The Fusional Reduction Algorithm

The **Fusional Reduction** (**FRed**) algorithm generalizes this two-step strategy, namely:

- **use fusion** to obtain maximally informative rows
- **retain** all the rows that contain **information lost in the fusion**

The Fusional Reduction Algorithm

The question is:

How do we identify the rows that lose information in the fusion?

Answer:

By identifying **info loss configurations** …

The Fusional Reduction Algorithm

And an **info loss configuration** is …

a constraint (i.e. a column in a tableau) that fuses to **W** and contains some **e**'s.

That is,

an info loss configuration is any column in a tableau that contains **only e**'s and **W**'s – and **at least** one **e** and one **W**.

The Fusional Reduction Algorithm

Info loss configuration: a constraint that fuses to **W** and contains some **e**'s.

For example:

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
$f(r_1, r_2)=r_4$	W	L	L

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
$f(r_3, r_2)=r_4$	W	L	L

The Fusional Reduction Algorithm

And the information that is lost in the fusion is contributed by the rows with an **e**.

The **e** rows in an info loss configuration form the **info residue** of that info loss configuration.

For example:

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L
$f(r_1, r_2)=r_4$	W	L	L

Tableau 3	C_1	C_2	C_3
r_3: d~d'	W	L	e
r_2: a~c	e	W	L
$f(r_3, r_2)=r_4$	W	L	L

The Fusional Reduction Algorithm

The basic Fusional Reduction (FRed) algorithm.

To obtain the **Most Informative Basis** (**MIB**):

- **whole fusion:** fuse all tableau rows and construct a branch for the fusion
- **info loss:** identify all the info loss configurations; for each info loss configuration, construct a branch with the info residue

The Fusional Reduction Algorithm

And this is exactly what we did before with Tableau 2.

whole fusion

info loss config

Tableau 2	C_1	C_2	C_3
r_1: a~b	W	L	W
r_2: a~c	e	W	L

info residue

	C_1	C_2	C_3
$f(r_1, r_2)=r_4$	W	L	L

	C_1	C_2	C_3
r_2: a~c	e	W	L

The Fusional Reduction Algorithm

There are two further issues:

- cases in which the whole fusion is useless
- cases in which the info residue consists of more than one row

Let's examine them in turn …

The Fusional Reduction Algorithm

First issue: the whole fusion is useless.

For example,

when the info residues repeat the entire initial tableau.

In this case, **we discard the whole fusion**.

The Fusional Reduction Algorithm

Tableau 5	C_1	C_2	C_3
r_5	W	e	L
r_2	e	W	L

	C_1	C_2	C_3
f(r_5, r_2)	W	W	L

	C_1	C_2	C_3
r_2	e	W	L

	C_1	C_2	C_3
r_5	W	e	L

The Fusional Reduction Algorithm

Second issue: the info residue consists of more than one row.

For example …

The Fusional Reduction Algorithm

Tableau 6	C_1	C_2	C_3	C_4
r_6	W	L	e	e
r_7	e	W	L	W
r_8	e	e	W	L

	C_1	C_2	C_3	C_4
f(r_6, r_7, r_8)	W	L	L	L

	C_1	C_2	C_3	C_4
r_7	e	W	L	W
r_8	e	e	W	L

In this case, **we recurse on the info residue**.

The Fusional Reduction Algorithm

Tableau 6	C_1	C_2	C_3	C_4
r_6	W	L	e	e
r_7	e	W	L	W
r_8	e	e	W	L

	C_1	C_2	C_3	C_4
f(r_6, r_7, r_8)	W	L	L	L

	C_1	C_2	C_3	C_4
r_7	e	W	L	W
r_8	e	e	W	L

	C_1	C_2	C_3	C_4
f(r_7, r_8)	e	W	L	L

	C_1	C_2	C_3	C_4
r_8	e	e	W	L

The Fusional Reduction Algorithm

The Fusional Reduction (FRed) algorithm.

To obtain the **Most Informative Basis** (**MIB**):

- **whole fusion:** fuse all tableau rows and construct a branch for the fusion
- **info loss:** identify all the **info loss configurations**; for each info loss configuration, construct a branch with the **info residue**
- **check** if the **whole fusion** has more **L**'s than **the fusion of all the info residues**: keep it if it does, discard it if it doesn't
- **recurse** on each of the info residues, i.e., for each of them, go through the above steps (whole fusion, info loss, check)

FRed Reduces

- Till now, FRed has only *fused* the world
- But FRed can also drastically reduce it

Demo time!

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